Handbook for the Undergraduate Mathematics Courses
Supplement to the Handbook
Honour School of Mathematics
Syllabus and Synopses for Part C 2012–13
for examination in 2013

Contents

1 Foreword

The synopses for Part C will be available on the website at:

http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/

before the start of Michaelmas Term 2012.

See the current edition of the Examination Regulations for the full regulations governing these examinations.

Examination Conventions can be found at: http://www.maths.ox.ac.uk/notices/undergrad

In the unlikely event that any course receives a very low registration we may offer this course as a reading course (this would include some lectures but fewer classes).

Honour School of Mathematics

"Units" and "Half Units"

Students staying on to take Part C will take the equivalent of three units. The equivalent of two units must be taken from the schedule of "Mathematics Department units" and may include a dissertation on a mathematical topic. Up to one unit may be taken from the schedule of "Other Units".

Most Mathematics Department lecture courses are independently available as half units, the exceptions being:

1. C7.4 Theoretical Physics - this is available as a whole-unit only.
2. C11.1b Probabilistic Combinatorics, where C11.1a Graph Theory is an essential pre-requisite.

All the units and half units described in this booklet are "M–Level".

Language Classes

Mathematics students may apply to take classes in a foreign language. In 2012-13 classes will be offered in French and German. Students' performances in these classes will not contribute to the degree classification awarded. However, successful completion of the course may be recorded on students' transcripts. See section 5 for more details.

Registration

Classes

Students will have to register in advance for the classes they wish to take. Students will have to register by Friday of Week 10 of Trinity Term 2012 using the online system which can be accessed at https://www.maths.ox.ac.uk/courses/registration/

Note on Intercollegiate Classes

Where undergraduate registrations for lecture courses fall below 5, classes will not run as part of the intercollegiate scheme but will be arranged informally by the lecturer.

Lectures

Some combinations of subjects are not advised and lectures may clash. Details are given below. We will use the information on your registration forms to aim to keep clashes to a minimum. However, because of the large number of options available in Part C some clashes are inevitable, and we must aim to accommodate the maximum number of student preferences.

Lecture Timetabling in Part C, 2012–13

The Teaching Committee has agreed that the following clashes be allowed.

C1.1a Model Theory & C1.1b Godel's Incompleteness Theorems C1.2a Analytic Topology C1.2b Axiomatic Set Theory C2.1a Lie Algebras C2.1b Representation Theory of Symmetric Groups C2.2a Commutative Algebra C2.2b Homological Algebra C2.3b Infinite Groups C3.1a Algebraic Topology C3.2b Geometric Group Theory C3.4a Algebraic Geometry C9.1a Modular Forms C9.1b Elliptic Curves	may clash with	C6.1a Solid Mechanics C6.1b Elasticity and Plasticity C6.2a Statistical Mechanics C6.3a Perturbation Methods C6.3b Applied Complex Variables C6.4a Special Topics in Fluid Mechanics C8.1a Mathematical Geoscience C8.1b Mathematical Physiology All Statistics options C12.1a Numerical Linear Algebra C12.1b Continuous Optimization C12.2b Finite Element Methods for PDEs C12.3b Approximations of Functions

Mathematics vs. Computer Science		
C3.4a Algebraic Geometry C6.1a Solid Mechanics C6.1b Elasticity and Plasticity C6.3a Perturbation Methods C6.3b Applied Complex Variables	may clash with	CCS1a Categories, Proofs and Processes

Other Units		
All Statistics options	may clash with	All Philosophy papers

2 Mathematics Department units

C1.1a: Model Theory — Prof Zilber — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2B60).

Recommended Prerequisites

This course presupposes basic knowledge of First Order Predicate Calculus up to and including the Soundness and Completeness Theorems. A familiarity with (at least the statement of) the Compactness Theorem would also be desirable.

Overview

The course deepens a student's understanding of the notion of a mathematical structure and of the logical formalism that underlies every mathematical theory, taking B1 Logic as a starting point. Various examples emphasise the connection between logical notions and practical mathematics.

The concepts of completeness and categoricity will be studied and some more advanced technical notions, up to elements of modern stability theory, will be introduced.

Learning Outcomes

Students will have developed an in depth knowledge of the notion of an algebraic mathematical structure and of its logical theory, taking B1 Logic as a starting point. They will have an understanding of the concepts of completeness and categoricity and more advanced technical notions.

Synopsis

Structures. The first-order language for structures. The Compactness Theorem for first-order logic. Elementary embeddings. Löwenheim–Skolem theorems. Preservation theorems for substructures. Model Completeness. Quantifier elimination.

Categoricity for first-order theories. Types and saturation. Omitting types. The Ryll Nardzewski theorem characterizing aleph-zero categorical theories. Theories with few types. Ultraproducts.

Reading

1. D. Marker, *Model Theory: An Introduction* (Springer, 2002).
2. W. Hodges, *Shorter Model Theory* (Cambridge University Press, 1997).

3. J. Bridge, *Beginning Model Theory* (Oxford University Press, 1977). (Out of print but can be found in libraries.)

Further reading

1. All topics discussed (and much more) can also be found in W. Hodges, *Model Theory* (Cambridge University Press, 1993).

C1.1b: Gödel's Incompleteness Theorems — Dr Alex Paseau — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A60)

Recommended Prerequisites

This course presupposes knowledge of first-order predicate logic up to and including soundness and completeness theorems for a formal system of first-order predicate logic (B1 Logic).

Overview

The starting point is Gödel's mathematical sharpening of Hilbert's insight that manipulating symbols and expressions of a formal language has the same formal character as arithmetical operations on natural numbers. This allows the construction for any consistent formal system containing basic arithmetic of a 'diagonal' sentence in the language of that system which is true but not provable in the system. By further study we are able to establish the intrinsic meaning of such a sentence. These techniques lead to a mathematical theory of formal provability which generalizes the earlier results. We end with results that further sharpen understanding of formal provability.

Learning Outcomes

Understanding of arithmetization of formal syntax and its use to establish incompleteness of formal systems; the meaning of undecidable diagonal sentences; a mathematical theory of formal provability; precise limits to formal provability and ways of knowing that an unprovable sentence is true.

Synopsis

Gödel numbering of a formal language; the diagonal lemma. Expressibility in a formal language. The arithmetical undefinability of truth in arithmetic. Formal systems of arithmetic;

arithmetical proof predicates. Σ_0-completeness and Σ_1-completeness. The arithmetical hierarchy. ω-consistency and 1-consistency; the first Gödel incompleteness theorem. Separability; the Rosser incompleteness theorem. Adequacy conditions for a provability predicate. The second Gödel incompleteness theorem; Löb's theorem. Provable Σ_1-completeness. Provability logic; the fixed point theorem. The ω-rule. The Bernays arithmetized completeness theorem; formally undecidable Δ_2^0-sentences of arithmetic.

Reading

1. Lecture notes for the course.

Further Reading

1. Raymond M. Smullyan, *Gödel's Incompleteness Theorems* (oup, 1992).
2. George S. Boolos and Richard C. Jeffrey, *Computability and Logic* (3rd edition, Cambridge University Press, 1989), Chs 15, 16, 27 (pp 170–190, 268-284).
3. George Boolos, *The Logic of Provability* (Cambridge University Press, 1993), pp. 44–49.

C1.2a: Analytic Topology — Dr Suabedissen — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A61)

We find it necessary to run Analytic Topology in MT. Classes for Analytic Topology will be run both in MT and HT, so that students who find themselves overburdened in MT will have the opportunity to attend classes on Analytic Topology in HT.

Recommended Prerequisites

Part A Topology; a basic knowledge of Set Theory, including cardinal arithmetic, ordinals and the Axiom of Choice, will also be useful.

Overview

The aim of the course is to present a range of major theory and theorems, both important and elegant in themselves and with important applications within topology and to mathematics as a whole. Central to the course is the general theory of compactness and Tychonoff's theorem, one of the most important in all mathematics (with applications across mathematics and in mathematical logic) and computer science.

Synopsis

Bases and initial topologies (including pointwise convergence and the Tychonoff product topology). Separation axioms, continuous functions, Urysohn's lemma. Separable, Lindelöf and second countable spaces. Urysohn's metrization theorem. Filters and ultrafilters. Tychonoff's theorem. Compactifications, in particular the Alexandroff One-Point Compactification and the Stone–Čech Compactification. Connectedness and local connectedness. Components and quasi-components. Totally disconnected compact spaces, Boolean algebras and Stone spaces. Paracompactness (brief treatment).

Reading

1. S. Willard, *General Topology* (Addison–Wesley, 1970), Chs. 1–8.
2. N. Bourbaki, *General Topology* (Springer-Verlag, 1989), Ch. 1.

C1.2b: Axiomatic Set Theory — Prof. Zilber — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2B61).

Recommended Prerequisites

This course presupposes basic knowledge of First Order Predicate Calculus up to and including the Soundness and Completeness Theorems, together with a course on basic set theory, including cardinals and ordinals, the Axiom of Choice and the Well Ordering Principle.

Overview

Inner models and consistency proofs lie at the heart of modern Set Theory, historically as well as in terms of importance. In this course we shall introduce the first and most important of inner models, Gödel's constructible universe, and use it to derive some fundamental consistency results.

Synopsis

A review of the axioms of ZF set theory. The recursion theorem for the set of natural numbers and for the class of ordinals. The Cumulative Hierarchy of sets and the consistency of the Axiom of Foundation as an example of the method of inner models. Levy's Reflection Principle. Gödel's inner model of constructible sets and the consistency of the Axiom of Constructibility ($V = L$). The fact that $V = L$ implies the Axiom of Choice. Some advanced cardinal arithmetic. The fact that $V = L$ implies the Generalized Continuum Hypothesis.

Reading

For the review of ZF set theory:

1. D. Goldrei, *Classic Set Theory* (Chapman and Hall, 1996).

For course topics (and much more):

1. K. Kunen, *Set Theory: An Introduction to Independence Proofs* (North Holland, 1983) (now in paperback). Review: Chapter 1. Course topics: Chapters 3, 4, 5, 6 (excluding section 5).

Further Reading

1. K. Hrbacek and T. Jech, *Introduction to Set Theory* (3rd edition, M Dekker, 1999).

C2.1a: Lie Algebras — Dr McGerty — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A62)

Recommended Prerequisites

Part B course B2a. A thorough knowledge of linear algebra and the second year algebra courses; in particular familiarity with group actions, quotient rings and vector spaces, isomorphism theorems and inner product spaces will be assumed. Some familiarity with the Jordan–Hölder theorem and the general ideas of representation theory will be an advantage.

Overview

Lie Algebras are mathematical objects which, besides being of interest in their own right, elucidate problems in several areas in mathematics. The classification of the finite-dimensional complex Lie algebras is a beautiful piece of applied linear algebra. The aims of this course are to introduce Lie algebras, develop some of the techniques for studying them, and describe parts of the classification mentioned above, especially the parts concerning root systems and Dynkin diagrams.

Learning Outcomes

Students will learn how to utilise various techniques for working with Lie algebras, and they will gain an understanding of parts of a major classification result.

Synopsis

Definition of Lie algebras, small-dimensional examples, some classical groups and their Lie algebras (treated informally). Ideals, subalgebras, homomorphisms, modules.

Nilpotent algebras, Engel's theorem; soluble algebras, Lie's theorem. Semisimple algebras and Killing form, Cartan's criteria for solubility and semisimplicity.

The root space decomposition of a Lie algebra; root systems, Cartan matrices and Dynkin diagrams. Classification of irreducible root systems. Description (with few proofs) of the classification of complex simple Lie algebras; examples.

Reading

1. J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory,* Graduate Texts in Mathematics 9 (Springer-Verlag, 1972, reprinted 1997). Chapters 1–3 are relevant and part of the course will follow Chapter 3 closely.

2. B. Hall, *Lie Groups, Lie Algebras, and Representations. An Elementary Introduction,* Graduate Texts in Mathematics 222 (Springer-Verlag, 2003).

3. K. Erdmann, M. J. Wildon, *Introduction to Lie Algebras* (Springer-Verlag, 2006), ISBN: 1846280400.

Additional Reading

1. J.-P. Serre, *Complex Semisimple Lie Algebras* (Springer, 1987). Rather condensed, assumes the basic results. Very elegant proofs.

2. N. Bourbaki, *Lie Algebras and Lie Groups* (Masson, 1982). Chapters 1 and 4–6 are relevant; this text fills in some of the gaps in Serre's text.

C2.1b: Representation Theory of Symmetric Groups — Prof. Henke —16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2B62).

Recommended Prerequisites

A thorough knowledge of linear algebra and the second year algebra courses; in particular familiarity with the symmetric groups, (symmetric) group actions, quotient vector spaces, isomorphism theorems and inner product spaces will be assumed. Some familiarity with basic representation theory from B2 (group algebras, simple modules, reducibility, Maschke's theorem, Wedderburn's theorem, characters) will be an advantage.

Overview

The representation theory of symmetric groups is a special case of the representation theory of finite groups. Whilst the theory over characteristic zero is well understood, this is not so over fields of prime characteristic. The course will be algebraic and combinatorial in flavour, and it will follow the approach taken by G. James. One main aim is to construct and parametrise the simple modules of the symmetric groups over an arbitrary field. Combinatorial highlights include combinatorial algorithms such as the Robinson–Schensted–Knuth correspondence. The final part of the course will discuss some finite-dimensional representations of the general linear group $\mathrm{GL}_n(\mathbb{C})$, and connections with representations of symmetric groups. In particular we introduce tensor products, and symmetric and exterior powers.

Synopsis

Counting standard tableaux of fixed shape: Young diagrams and tableaux, standard-tableaux, Young–Frobenius formula, hook formula. Robinson–Schensted-Knuth algorithm and correspondence.

Construction of fundamental modules for symmetric groups: Action of symmetric groups on tableaux, tabloids and polytabloids; permutation modules on cosets of Young subgroups. Specht modules, and their standard bases. Examples and applications.

Simplicity of Specht modules in characteristic zero and classification of simple S_n-module over characteristic zero. Characters of symmetric groups, Murnaghan–Nakayama rule.

Submodule Theorem, construction of simple S_n-modules over a field of prime characteristic. Decomposition matrices. Examples and applications.

Some finite-dimensional $\mathrm{GL}_n(\mathbb{C})$-modules, in particular the natural module, its tensor powers, and its symmetric and exterior powers. Connections with representations of S_n over $\mathbb{C}$.

Reading

1. W. Fulton, *Young Tableaux,* London Mathematical Society Student Texts 35 (Cambridge University Press, 1997). From Part I and II.
2. D. Knuth, *The Art of Computer Programming, Volume 3* (Addison–Wesley, 1998). From Chapter 5.
3. B. E. Sagan, *The Symmetric Group: Representations, Combinatorial Algorithms, and Symmetric Functions,* Graduate Texts in Mathematics 203 (Springer–Verlag, 2000). Chapters 1 – 2.

Additional Reading

1. W. Fulton, J. Harris, *Representation Theory: A first course,* Graduate Texts in Mathematics, Readings in Mathematics 129 (Springer–Verlag, 1991). From Part I.
2. G. James, *The Representation Theory of the Symmetric Groups,* Lecture Notes in Mathematics 682 (Springer–Verlag, 1978).

3. G. James, A. Kerber, *The Representation Theory of the Symmetric Groups*, Encyclopaedia of Mathematics and its Applications 16, (Addison–Wesley, 1981). From Chapter 7.

4. R. Stanley, *Enumerative Combinatorics. Volume 2*, Cambridge Studies in Advanced Mathematics 62 (Cambridge University Press, 1999).

C2.2a: Commutative Algebra — Prof Segal — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code tbc).

Recommended Prerequisites

A thorough knowledge of the second-year algebra courses, in particular rings, ideals and fields.

Overview

Amongst the most familiar objects in mathematics are the ring of integers and the polynomial rings over fields. These play a fundamental role in number theory and in algebraic geometry, respectively. The course explores the basic properties of such rings, and introduces the key concept of a module, which generalizes both abelian groups and the idea of a linear transformation on a vector space.

Synopsis

Introduction to modules. The structure of modules over a principal ideal ring. Prime ideals, maximal ideals, nilradical and Jacobson radical. Noetherian rings; Hilbert basis theorem. Minimal primes. Artin-Rees Lemma; Krull intersection theorem. Integral extensions. Prime ideals in integral extensions. Noether Normalization Lemma. Hilbert Nullstellensatz, maximal ideals. Krull dimension; Principal ideal theorem.; dimension of an affine algebra.

Reading

1. Atiyah, Macdonald, *Introduction to Commutative Algebra*, (Addison-Wesley, 1969).

2. Eisenbud *Commutative Algebra: with a View Toward Algebraic Geometry*, Grad. Texts Math. 150, (Springer-Verlag, 1995) Chapters 4, 5 and 13.

C2.2b Homological Algebra — Dr Kremnitzer — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code tbc).

Synopsis

Chain complexes: complexes of R-modules, operations on chain complexes, long exact sequences, chain homotopies, mapping cones and cylinders (3 hours) Derived functors: delta functors, projective and injective resolutions, left and right derived functors (4 hours) Tor and Ext: Tor and flatness, Ext and extensions, universal coefficients theorems (3 hours) Group homology and cohomology: definition, interpretation of H^1 and H^2, universal central extensions, the Bar resolution (3 hours) Lie algebra homology and cohomology: Lie algebras and universal enveloping algebras, definition of homology and cohomology and relations to Tor and Ext, H^1 and H^2, universal central extensions (3 hours)

Reading

Weibel, Charles *An introduction to Homological algebra* (see Google Books)

C2.3b: Infinite Groups — Dr Nikolov — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code tbc).

Recommended Prerequisites

A thorough knowledge of the second-year algebra courses; in particular, familiarity with group actions, quotient rings and quotient groups, and isomorphism theorems will be assumed. Familiarity with the Commutative Algebra course will be helpful but not essential.

Overview

The concept of a group is so general that anything which is true of all groups tends to be rather trivial. In contrast, groups that arise in some specific context often have a rich and beautiful theory. The course introduces some natural families of groups, various questions that one can ask about them, and various methods used to answer these questions; these involve among other things rings and trees.

Synopsis

Free groups and their subgroups; finitely generated groups: counting finite-index subgroups; finite presentations and decision problems; Linear groups: residual finiteness; structure of soluble linear groups; Nilpotency and solubility: lower central series and derived series; structural and residual properties of finitely generated nilpotent groups and polycyclic groups; characterization of polycyclic groups as soluble Z-linear groups; Finitely generated groups acting on rooted trees: Gupta-Sidki groups and the General Burnside Problem.

Reading

1. D. J. S. Robinson, *A course in the theory of groups*, 2nd ed., Graduate texts in Mathematics, (Springer-Verlag, 1995). Chapters 2, 5, 6, 15.
2. D. Segal, *Polycyclic groups*, (CUP, 2005) Chapters 1 and 2.

C3.1a: Algebraic Topology — Prof. Tillmann — 16MT

Level: M-level. **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper codes tbc).

Recommended Prerequisites

Helpful but not essential: Part A Topology, B3.1a Topology and Groups.

Overview

Homology theory is a subject that pervades much of modern mathematics. Its basic ideas are used in nearly every branch, pure and applied. In this course, the homology groups of topological spaces are studied. These powerful invariants have many attractive applications. For example we will prove that the dimension of a vector space is a topological invariant and the fact that 'a hairy ball cannot be combed'.

Learning Outcomes

At the end of the course, students are expected to understand the basic algebraic and geometric ideas that underpin homology and cohomology theory. These include the cup product and Poincaré Duality for manifolds. They should be able to choose between the different homology theories and to use calculational tools such as the Mayer-Vietoris sequence to compute the homology and cohomology of simple examples, including projective spaces, surfaces, certain simplicial spaces and cell complexes. At the end of the course, students should also have developed a sense of how the ideas of homology and cohomology may be applied to problems from other branches of mathematics.

Synopsis

Chain complexes of free Abelian groups and their homology. Short exact sequences. Delta (and simplicial) complexes and their homology. Euler characteristic.

Singular homology of topological spaces. Relative homology and the Five Lemma. Homotopy invariance and excision (details of proofs not examinable). Mayer-Vietoris Sequence. Equivalence of simplicial and singular homology.

Degree of a self-map of a sphere. Cell complexes and cellular homology. Application: the hairy ball theorem.

Cohomology of spaces and the Universal Coefficient Theorem (proof not examinable). Cup products. Künneth Theorem (without proof). Topological manifolds and orientability. The fundamental class of an orientable, closed manifold and the degree of a map between manifolds of the same dimension. Poincaré Duality (without proof).

Reading

1. A. Hatcher, *Algebraic Topology* (Cambridge University Press, 2001). Chapters 3 and 4.
2. G. Bredon, *Topology and Geometry* (Springer, 1997). Chapters 4 and 5.
3. J. Vick, *Homology Theory,* Graduate Texts in Mathematics 145 (Springer, 1973).

C3.2b: Geometric Group Theory — Dr Papazoglou — 16HT

Level: M-level. **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper codes tbc).

Recommended Prerequisites.

The Topology & Groups course is a helpful, though not essential prerequisite.

Overview.

The aim of this course is to introduce the fundamental methods and problems of geometric group theory and discuss their relationship to topology and geometry.

The first part of the course begins with an introduction to presentations and the list of problems of M. Dehn. It continues with the theory of group actions on trees and the structural study of fundamental groups of graphs of groups.

The second part of the course focuses on modern geometric techniques and it provides an introduction to the theory of Gromov hyperbolic groups.

Synopsis.

Free groups. Group presentations. Dehn's problems. Residually finite groups.

Group actions on trees. Amalgams, HNN-extensions, graphs of groups, subgroup theorems for groups acting on trees.

Quasi-isometries. Hyperbolic groups. Solution of the word and conjugacy problem for hyperbolic groups.

If time allows: Small Cancellation Groups, Stallings Theorem, Boundaries.

Reading.

1. J.P. Serre, *Trees* (Springer Verlag 1978).
2. M. Bridson, A. Haefliger, *Metric Spaces of Non-positive Curvature, Part III* (Springer, 1999), Chapters I.8, III.H.1, III. *Gamma* 5.
3. H. Short *et al.*, 'Notes on word hyperbolic groups', *Group Theory from a Geometrical Viewpoint, Proc. ICTP Trieste* (eds E. Ghys, A. Haefliger, A. Verjovsky, World Scientific 1990)

   available online at: http://www.cmi.univ-mrs.fr/ hamish/
4. C.F. Miller, *Combinatorial Group Theory*, notes:
   http://www.ms.unimelb.edu.au/ cfm/notes/cgt-notes.pdf.

Additional Reading.

1. G. Baumslag, *Topics in Combinatorial Group Theory* (Birkhauser, 1993).
2. O. Bogopolski, *Introduction to Group Theory* (EMS Textbooks in Mathematics, 2008).
3. R. Lyndon, P. Schupp, *Combinatorial Group Theory* (Springer, 2001).
4. W. Magnus, A. Karass, D. Solitar,*Combinatorial Group Theory: Presentations of Groups in Terms of Generators and Relations* (Dover Publications, 2004).
5. P. de la Harpe, *Topics in Geometric Group Theory*, (University of Chicago Press, 2000).

C3.3b: Differentiable Manifolds — Prof. Hitchin — 16HT

Level: M-level. **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper codes tbc).

Recommended Prerequisites

2nd year core algebra, topology, multivariate calculus. Useful but not essential: groups in action, geometry of surfaces.

Overview

A manifold is a space such that small pieces of it look like small pieces of Euclidean space. Thus a smooth surface, the topic of the B3 course, is an example of a (2-dimensional) manifold.

Manifolds are the natural setting for parts of classical applied mathematics such as mechanics, as well as general relativity. They are also central to areas of pure mathematics such as topology and certain aspects of analysis.

In this course we introduce the tools needed to do analysis on manifolds. We prove a very general form of Stokes' Theorem which includes as special cases the classical theorems of Gauss, Green and Stokes. We also introduce the theory of de Rham cohomology, which is central to many arguments in topology.

Learning Outcomes

The candidate will be able to manipulate with ease the basic operations on tangent vectors, differential forms and tensors both in a local coordinate description and a global coordinate-free one; have a knowledge of the basic theorems of de Rham cohomology and some simple examples of their use; know what a Riemannian manifold is and what geodesics are.

Synopsis

Smooth manifolds and smooth maps. Tangent vectors, the tangent bundle, induced maps. Vector fields and flows, the Lie bracket and Lie derivative.

Exterior algebra, differential forms, exterior derivative, Cartan formula in terms of Lie derivative. Orientability. Partitions of unity, integration on oriented manifolds.

Stokes' theorem. De Rham chomology. Applications of de Rham theory including degree.

Riemannian metrics. Isometries. Geodesics.

Reading

1. M. Spivak, *Calculus on Manifolds*, (W. A. Benjamin, 1965).
2. M. Spivak, *A Comprehensive Introduction to Differential Geometry*, Vol. 1, (1970).
3. W. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, 2nd edition, (Academic Press, 1986).
4. M. Berger and B. Gostiaux, *Differential Geometry: Manifolds, Curves and Surfaces.* Translated from the French by S. Levy, (Springer Graduate Texts in Mathematics, 115, Springer–Verlag (1988)) Chapters 0–3, 5–7.

5. F. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, (Springer Graduate Texts in Mathematics, 1994).

6. D. Barden and C. Thomas, *An Introduction to Differential Manifolds.* (Imperial College Press, London, 2003.)

C3.4a: Algebraic Geometry — Dr Berczi — 16MT

Level: M-level. **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper codes tbc)

Recommended Prerequisites

Part A Group Theory and Introduction to Fields (B3 Algebraic Curves useful but not essential).

Overview

Algebraic geometry is the study of algebraic varieties: an algebraic variety is roughly speaking, a locus defined by polynomial equations. One of the advantages of algebraic geometry is that it is purely algebraically defined and applied to any field, including fields of finite characteristic. It is geometry based on algebra rather than calculus, but over the real or complex numbers it provides a rich source of examples and inspiration to other areas of geometry.

Synopsis

Affine algebraic varieties, the Zariski topology, morphisms of affine varieties. Irreducible varieties.

Projective space and general position points. Projective varieties, affine cones over projective varieties. The Zariski topology on projective varieties. The projective closure of affine variety. Morphisms of projective varieties. Projective equivalence.

Veronese morphism: definition, examples. Veronese morphisms are isomorphisms onto their image; statement, and proof in simple cases. Subvarieties of Veronese varieties. Segre maps and products of varieties, Categorical products: the image of Segre map gives the categorical product.

Coordinate rings. Hilbert's Nullstellensatz. Correspondence between affine varieties (and morphisms between them) and finitely generate reduced k-algebras (and morphisms between them). Graded rings and homogeneous ideals. Homogeneous coordinate rings.

Categorical quotients of affine varieties by certain group actions. The maximal spectrum.

Discrete invariants projective varieties: degree dimension, Hilbert function. Statement of theorem defining Hilbert polynomial.

Quasi-projective varieties, and morphisms of them. The Zariski topology has a basis of affine open subsets. Rings of regular functions on open subsets and points of quasi-projective varieties. The ring of regular functions on an affine variety in the coordinate ring. Localisation and relationship with rings of regular functions.

Tangent space and smooth points. The singular locus is a closed subvariety. Algebraic re-formulation of the tangent space. Differentiable maps between tangent spaces.

Function fields of irreducible quasi-projective varieties. Rational maps between irreducible varieties, and composition of rational maps. Birational equivalence. Correspondence between dominant rational maps and homomorphisms of function fields. Blow-ups: of affine space at appoint, of subvarieties of affine space, and general quasi-projective varieties along general subvarieties. Statement of Hironaka's Desingularisation Theorem. Every irreducible variety is birational to hypersurface. Re-formulation of dimension. Smooth points are a dense open subset.

Reading

KE Smith et al, *An Invitation to Algebraic Geometry*, (Springer 2000), Chapters 1–8.

Further Reading

1. M Reid, *Undergraduate Algebraic Geometry*, LMS Student Texts 12, (Cambridge 1988).
2. K Hulek, *Elementary Algebraic Geometry*, Student Mathematical Library 20. (American Mathematical Society, 2003).

C3.4b: Lie Groups — Prof. Joyce — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code tbc).

Recommended Prerequisites

Part A Group Theory, Topology and Multivariable Calculus.

Overview

The theory of Lie Groups is one of the most beautiful developments of pure mathematics in the twentieth century, with many applications to geometry, theoretical physics and mechanics, and links to both algebra and analysis. Lie groups are groups which are simultaneously manifolds, so that the notion of differentiability makes sense, and the group multiplication and inverse maps are differentiable. However this course introduces the theory in a more concrete way via groups of matrices, in order to minimise the prerequisites.

Learning Outcomes

Students will have learnt the basic theory of topological matrix groups and their representations. This will include a firm understanding of root systems and their role for representations.

Synopsis

The exponential map for matrices, Ad and ad, the Campbell–Baker–Hausdorff series.

Linear Groups, their Lie algebras and the Lie correspondence. Homomorphisms and coverings of linear groups. Examples including $SU(2)$, $SO(3)$ and $SL(2;\mathbb{R}) \cong SU(1,1)$.

The compact and complex classical Lie groups. Cartan subgroups, Weyl groups, weights, roots, reflections.

Informal discussion of Lie groups as manifolds with differentiable group structures; quotients of Lie groups by closed subgroups.

Bi-invariant integration on a compact group (statement of existence and basic properties only). Representations of compact Lie groups. Tensor products of representations. Complete reducibility, Schur's lemma. Characters, orthogonality relations.

Statements of Weyl's character formula, the theorem of the highest weight and the Borel–Weil theorem, with proofs for $SU(2)$ only.

Reading

1. W. Rossmann, *Lie Groups: An Introduction through Linear Groups*, (Oxford, 2002), Chapters 1–3 and 6.

2. A. Baker, *Matrix Groups: An Introduction to Lie Group Theory*, (Springer Undergraduate Mathematics Series).

Further Reading

1. J. F. Adams, *Lectures on Lie Groups* (University of Chicago Press, 1982).

2. R. Carter, G. Segal and I. MacDonald, *Lectures on Lie Groups and Lie Algebras* (LMS Student Texts, Cambridge, 1995).

3. J. F. Price, *Lie Groups and Compact Groups* (LMS Lecture Notes 25, Cambridge, 1977).

C4.1a: Functional Analysis — Prof. Kristensen — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A64)

Recommended Prerequisites

Part A Topology, B4 Analysis

Overview

This course builds on B4, by extending the theory of Banach spaces and operators. As well as developing general methods that are useful in Operator Theory, we shall look in more detail at the structure and special properties of "classical" sequence-spaces and function-spaces.

Synopsis

Normed spaces and Banach spaces; dual spaces, subspaces, direct sums and completions; quotient spaces and quotient operators.

Baire's Category Theorem and its consequences (review).

Classical Banach spaces and their duals; smoothness and uniform convexity of norms.

Compact sets and compact operators. Ascoli's theorem.

Hahn–Banach extension and separation theorems; the bidual space and reflexivity.

Weak and weak* topologies. The Banach–Alaoglu theorem and Goldstine's theorem. Weak compactness.

Schauder bases; examples in classical spaces. Gliding-hump arguments.

Fredholm operators.

Reading

1. M. Fabian et al., *Functional Analysis and Infinite-Dimensional Geometry* (Canadian Math. Soc, Springer 2001), Chapters 1,2,3,6,7.

Alternative Reading

1. N. L. Carothers, *A Short Course on Banach Space Theory*, (LMS Student Text, Cambridge University Press 2004).

C5.1a Methods of Functional Analysis for PDEs — Prof. Seregin — 16MT

Only available to students who have not offered C5.1a Methods of Functional Analysis for PDEs at Part B.

Level: M-level **Method of assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A65)

Recommended Prerequisites

Part A Integration. There will be a 'Users' Guide to Integration' on the subject website and anyone who has not done Part A Integration can read it up over the summer vacation. In addition some knowledge of functional analysis, in particular Banach spaces (as in B4) and compactness (as in Part A Topology), is useful. We will however recall the relevant definitions as we go along so these prerequisites are not strictly needed.

Overview

The course will introduce some of the modern techniques in partial differential equations that are central to the theoretical and numerical treatments of linear and nonlinear partial differential equations arising in science, geometry and other fields.

It provides valuable background for the Part C courses on Calculus of Variations, Fixed Point Methods for Nonlinear PDEs, and Finite Element Methods.

Learning Outcomes

Students will learn techniques and results about Lebesgue and Sobolev Spaces, distributions and weak derivatives, embedding theorems, traces, weak solution to elliptic PDE's, existence, uniqueness, and smoothness of weak solutions.

Synopsis

Why functional analysis methods are important for PDE's?

Revision of relevant definitions and statements from functional analysis: completeness, seperability, compactness, and duality.

Revision of relevant definitions and statements from Lebesgue integration theory: sequences of measurable functions, Lebesgue and Riesz theorems.

Lebesgue spaces:completeness, dense sets, linear functionals and weak convergence.

Distributions and distributional derivatives.

Sobolev spaces: mollifications and weak derivatives, completeness, Friedrichs inequality, star-shaped domains and dense sets, extension of functions with weak derivatives (with no proof).

Embedding of Sobolev spaces into Lebesgue spaces: Poincare inequality, Reillich-Kondrachov-Sobolev theorems on compactness.

Traces of functions with weak derivatives.

Dirichlet boundary value problems for elliptic PDE's, Fredholm Alternative (uniqueness implies existence).

Smoothness of weak solutions: embedding from Sobolev spaces into space of continuous functions, interior regularity of distributional solutions to elliptic equations with constant coefficients.

Reading

Lawrence C. Evans, *Partial differential equations*, (Graduate Studies in Mathematics 2004), American Mathematical Society

Elliott H. Lieb and Michael Loss, *Analysis*, 2nd Edition, (Graduate Studies in Mathematics 2001), American Mathematical Society

Additional Reading

E. Kreyszig, *Introductory Functional Analysis with Applications*, Wiley (revised edition, 1989)

P.D. Lax *Functional analysis* (Wiley-Interscience, New York, 2002).

J. Rauch, *Partial differential equations*, (Springer–Verlag, New York, 1992).

C5.1b Fixed Point Methods for Nonlinear PDEs — Dr Capdeboscq — 16HT

Level: M-level **Method of assessment:** Written examination.

Weight: Half-unit (OSS paper code 2B75)

Recommended Prerequisites

C5.1a: Methods of Functional Analysis for PDEs. Some knowledge of functional analysis, in particular Banach spaces (as in B4) and compactness (as in Part A Topology), is useful.

Overview

This course gives an introduction to the techniques of nonlinear functional analysis with emphasis on the major fixed point theorems and their applications to nonlinear differential equations and variational inequalities, which abound in applications such as fluid and solid mechanics, population dynamics and geometry.

Learning Outcomes

Besides becoming acquainted with the fixed point theorems of Banach, Brouwer and Schauder, students will see the abstract principles in a concrete context. Hereby they also reinforce techniques from elementary topology, functional analysis, Banach spaces, compactness methods, calculus of variations and Sobolev spaces.

Synopsis

Examples of nonlinear differential equations and variational inequalities. Contraction Mapping Theorem and applications. Brouwer's fixed point theorem, proof via Calculus of Variations and Null-Lagrangians. Compact operators and Schauder's fixed point theorem. Applications of Schauder's fixed point theorem to nonlinear elliptic equations. Variational inequalities and monotone operators. Applications of monotone operator theory to nonlinear elliptic equations (p-Laplacian, stationary Navier-Stokes)

Reading

1. Lawrence C. Evans, *Partial Differential Equations,* Graduate Studies in Mathematics (American Mathematical Society, 2004).
2. E. Zeidler, *Nonlinear Functional Analysis I & II* (Springer-Verlag, 1986/89).
3. M. S. Berger, *Nonlinearity and Functional Analysis* (Academic Press, 1977).
4. K. Deimling, *Nonlinear Functional Analysis* (Springer-Verlag, 1985).
5. L. Nirenberg, *Topics in Nonlinear Functional Analysis,* Courant Institute Lecture Notes (American Mathematical Society, 2001).
6. R.E. Showalter, *Monotone Operators in Banach Spaces and Nonlinear Partial Differential Equations,* Mathematical Surveys and Monographs, vol.49 (American Mathematical Society, 1997).

C5.2b: Calculus of Variations — Prof. Chen — 16HT

Weight: Half-unit, OSS paper code 2B65

Recommended Prerequisites

C5.1a: Methods of Functional Analysis for PDEs. Some familiarity with the Lebesgue integral is essential, and some knowledge of elementary functional analysis (e.g. Banach spaces and their duals, weak convergence) an advantage.

B5.1a: Dynamics and Energy Minimization: Some knowledge of local minimizers in the 1D calculus of variations is helpful. This material will be reviewed in the course.

Overview

The aim of the course is to give a modern treatment of the calculus of variations from a rigorous perspective, blending classical and modern approaches and applications.

Learning Outcomes

Students will learn rigorous results in the classical and modern one-dimensional calculus of variations and see possible behaviour and application of these results in examples. They will see some examples of multi-dimensional problems.

Synopsis

Classical and modern examples of variational problems (e.g. brachistochrone, models of phase transformations).

One-dimensional problems, function spaces and definitions of weak and strong relative minimizers. Necessary conditions; the Euler-Lagrange and Du Bois-Reymond equations, theory of the second variation, the Weierstrass condition. Sufficient conditions; field theory and sufficiency theorems for weak and strong relative minimizers. The direct method of the calculus of variations and Tonelli's existence theorem. Regularity of minimizers. Examples of singular minimizers and the Lavrentiev phenomenon. Problems whose infimum Is not attained. Relaxation and generalized solutions. Isoperimetric problems and Lagrange multipliers. Invariant variational problems, Noether's theorem, conservation laws.

Multi-dimensional problems, done via some examples.

Reading

1. G. Buttazzo, M. Giaquinta, S. Hildebrandt, *One-dimensional Variational Problems,* Oxford Lecture Series in Mathematics, Vol. 15 (Oxford University Press, 1998). Ch 1, Sections 1.1, 1.2 (treated differently in course), 1.3, Ch 2 (background), Ch 3, Sections 3.1, 3.2, Ch 4, Sections 4.1, 4.3.

Further Reading

1. U. Brechtken-Manderscheid, *Introduction to the Calculus of Variations* (Chapman & Hall, 1991).

2. H. Sagan, *Introduction to the Calculus of Variations* (Dover, 1992).

3. J. Troutman, *Variational Calculus and Optimal Control* (Springer-Verlag, 1995).

4. L. C. Evans, *Partial Differential Equations* (American Mathematical Society, 2010).

C6.1a: Solid Mechanics — Prof Goriely — 16 MT

[This course will run if teaching resources allow]

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit. OSS paper code 2466.

Recommended Prerequisites
There are no formal prerequisites. In particular it is not necessary to have taken any courses in fluid mechanics, though having done so provides some background in the use of similar concepts. Use is made of (i) elementary linear algebra in (e.g., eigenvalues, eigenvectors and diagonalization of symmetric matrices, and revision of this material, for example from the Mods Linear Algebra course, is useful preparation); and (ii) some 3D calculus (mainly differentiation of vector-valued functions of several variables). All necessary material is summarized in the course.

Overview

Solid mechanics is a vital ingredient of materials science and engineering, and is playing an increasing role in biology. It has a rich mathematical structure. The aim of the course is to derive the basic equations of elasticity theory, the central model of solid mechanics, and give some interesting applications to the behaviour of materials.The course is useful preparation for C6.1b Elasticity and Plasticity. Taken together the two courses will provide a broad overview of modern solid mechanics, with a variety of approaches.

Learning Outcomes

Students will learn basic techniques of modern continuum mechanics, such as kinematics of deformation, stress, constitutive equations and the relation between nonlinear and linearized models.The emphasis on the course is on the structure of the models, but some applications are also discussed.

Synopsis

Kinematics: Lagrangian and Eulerian descriptions of motion, deformation gradient, invertibility

Analysis of strain: polar decomposition, stretch tensors, Cauchy–Green tensors Stress Principle: forces in continuum mechanics, balance of forces, Cauchy stress tensor, the Piola–Kirchhoff stress

Constitutive Models: stress-strain relations, hyperelasticity and stored energy function, boundary value problems, the variational problem, frame indifference, material symmetry, isotropic materials

Further topics: incompressible elasticity, linearized elasticity and the shape-memory effect in crystalline solids.

Reading

1. O. Gonzales and A. Stuart, *A first course in continuum mechanics*, (Cambridge University Press, 2008).
2. M. E. Gurtin, *A introduction to continuum mechanics*, (Academic Press, 1981).

Further Reading

1. P. G. Ciarlet, *Mathematical Elasticity.* Vol. I Three-dimensional Elasticity, (North-Holland, 1988)
2. S. S. Antman, Nonlinear *Problems of Elasticity*, (Springer, 1995)
3. J. E. Marsden and T.J.R. Hughes, *Mathematical Foundations of Elasticity*, Prentice–Hall, 1983

C6.1b: Elasticity and Plasticity — Dr Howell — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit. OSS paper code 2467.

Recommended Prerequisites

Familiarity will be assumed with Part A Complex Analysis, Differential Equations and Calculus of Variations, as well as B568 Introduction to Applied Mathematics. A basic understanding of stress tensors from either B6a Viscous Flow or C6.2a Solid Mechanics will also be required. The following courses are also helpful: B5a Techniques of Applied Mathematics, B5b Applied Partial Differential Equations, C6.3a Perturbation Methods, C6.3b Applied Complex Variables.

Overview

The course starts with a rapid overview of mathematical models for basic solid mechanics. Benchmark solutions are derived for static problems and wave propagation in linear elastic materials. It is then shown how these results can be used as a basis for practically useful problems involving thin beams and plates. Simple geometrically nonlinear models are then introduced to explain buckling, fracture and contact. Models for yield and plasticity are then discussed, both microscopically and macroscopically.

Synopsis

Review of tensors, conservation laws, Navier equations. Antiplane strain, torsion, plane strain. Elastic wave propagation, Rayleigh waves. Ad hoc approximations for thin materials; simple bifurcation theory and buckling. Simple mixed boundary value problems, brittle fracture and smooth contact. Perfect plasticity theories for granular materials and metals.

Reading

1. P. D. Howell, G. Kozyreff and J. R. Ockendon, *Applied Solid Mechanics* (Cambridge University Press, 2008).
2. S. P. Timoshenko and J. N. Goodier, *Theory of Elasticity* (McGraw-Hill, 1970).
3. L.D. Landau and E.M. Lifshitz, *Theory of Elasticity* (Pergamon Press, 1986).

C6.2a: Statistical Mechanics — Dr Porter — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit. OSS paper code tbc.

Prerequisites

Familiarity with classical mechanics and probability. [No lecture prerequisites, so in particular the Classical Mechanics lectures from Part A are not required. Everything will be self-contained.]

Overview

This course aims to provide an introduction to the tools of statistical mechanics, which are used to investigate collective behavior in complex systems of interacting entities. The traditional use of statistical mechanics is to study large numbers of interacting particles when tracking all of them using Newton's laws becomes infeasible. One thus studies ensembles and examines their statistical properties, such as the temperature in a room versus the vibrations of each individual molecule in the room. More recently, ideas of statistical mechanics have given powerful results in areas of study such as social networks and finance.

Learning Outcomes

Students will have developed a sound knowledge and appreciation of some of the tools, concepts, and computations used in the study of statistical mechanics. They will also get some exposure to some modern research topics in the field.

Synopsis

Thermodynamics and Probability (3 lectures): microscopic versus macroscopic viewpoints, the laws of thermodynamics, temperature, entropy, free energy, etc.

Classical Statistical Mechanics (4 lectures): ideal gas, Gibbs paradox, canonical and grand canonical ensembles, Liouville's theorem and ergodicity, Maxwell relations

Nonequilibrium Statistical Mechanics (2-3 lectures): Boltzmann equation, Boltzmann-Grad limit.

Phase Transitions (4 lectures): order parameters, phase transitions, critical phenomena, Ising model, Potts model, renormalization, symmetry breaking

Other Topics and Applications (2-3 lectures): This could vary from year to year, but a good example would be Bose-Einstein condensates or statistical mechanics of random graphs.

Reading

1. David Chandler, *Introduction to Modern Statistical Mechanics* (Oxford University Press 1987)
2. M. Kardar, *Statistical Physics of Particles* (Cambridge University Press 2007)
3. M. Kardar, *Statistical Physics of Fields* (Cambridge University Press 2007)
4. F. Schwabl, *Statistical Mechanics* (Springer-Verlag 2002)
5. J.P. Sethna, Entropy, *Order Parameters, and Complexity* (Oxford University Press 2006) [available online at http://pages.physics.cornell.edu/sethna/StatMech/]

C6.3a: Perturbation Methods — Prof. Maini — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A68)

Recommended Prerequisites

Part A Differential Equations and Core Analysis (Complex Analysis). B5, B6 and B8 are helpful but not officially required.

Overview

Perturbation methods underlie numerous applications of physical applied mathematics: including boundary layers in viscous flow, celestial mechanics, optics, shock waves, reaction-diffusion equations, and nonlinear oscillations. The aims of the course are to give a clear and systematic account of modern perturbation theory and to show how it can be applied to differential equations.

Synopsis

Asymptotic expansions. Asymptotic evaluation of integrals (including Laplace's method, method of stationary phase, method of steepest descent). Regular and singular perturbation theory. Multiple-scale perturbation theory. WKB theory and semiclassics. Boundary layers and related topics. Applications to nonlinear oscillators. Applications to partial differential equations and nonlinear waves.

Reading

1. E.J. Hinch, *Perturbation Methods* (Cambridge University Press, 1991), Chs. 1–3, 5–7.
2. C.M. Bender and S.A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers* (Springer, 1999), Chs. 6, 7, 9–11.
3. J. Kevorkian and J.D. Cole, *Perturbation Methods in Applied Mathematics* (Springer-Verlag, 1981), Chs. 1, 2.1–2.5, 3.1, 3.2, 3.6, 4.1, 5.2.

C6.3b: Applied Complex Variables — Dr Oliver — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit in C6.3b (OSS paper code 2B68).

Recommended Prerequisites

The course requires second year core analysis (complex analysis). It continues the study of complex variables in the directions suggested by contour integration and conformal mapping. Part A Fluid Dynamics and Waves and Part C Perturbation Methods are desirable

Overview

The course begins where core second-year complex analysis leaves off, and is devoted to extensions and applications of that material. It is assumed that students will be familiar with inviscid two-dimensional hydrodynamics (Part A Fluid Dynamics and Waves) to the extent of the existence of a harmonic stream function and velocity potential in irrotational icompressible flow, and Bernoulli's equation.

Synopsis

Review of core complex analysis, especially continuation, multifunctions, contour integration, conformal mapping and Fourier transforms.

Riemann mapping theorem (in statement only). Schwarz-Christoffel formula. Solution of Laplace's equation by conformal mapping onto a canonical domain. Applications to inviscid hydrodynamics: flow past an aerofoil and other obstacles by conformal mapping; free streamline flows of hodograph plane. Unsteady flow with free boundaries in porous media.

Application of Cauchy integrals and Plemelj formulae. Solution of mixed boundary value problems motivated by thin aerofoil theory and the theory of cracks in elastic solids. Reimann-Hilbert problems. Cauchy singular integral equations. Transform methods, complex Fourier transform. Contour integral solutions of ODE's. Wiener-Hopf method.

Reading

1. G.F. Carrier, M. Krook and C.E. Pearson, *Functions of a Complex Variable*(Society for Industrial and Applied Mathematics, 2005.) ISBN 0898715954.
2. M. J. Ablowitz and A. S. Fokas, *Complex Variables: Introduction and Applications* (2nd edition, Cambridge University Press., Cambridge, 2003). ISBN 0521534291.
3. J. Ockendon, Howison, Lacey and Movichan, *Applied Partial Differential Equations* (Oxford, 1999) Pages 195–212.

C6.4a: Special Topics in Fluid Mechanics — Dr Vella — 16MT

Level: M-level **Method of Assessment:** Written examination,

Weight: Half-unit. OSS paper code 2A74.

Recommended Prerequisites

B6 fluid mechanics.

Overview

The course will expand and illuminate the 'classical' fluid mechanics taught in the third year course B6, and illustrate its modern application in a number of different areas in industry and geoscience.

Synopsis

Thin film flows: coatings and foams. Lubrication theory: gravity flows, Marangoni effects. Droplet dynamics, contact lines, menisci. Drying and wetting.

Flow in porous media: Darcy's law; thermal and solutal convection; gravity-driven flow and carbon sequestration.

Rotating flows: atmosphere and oceans. Waves, geostrophy, quasi-geostrophy, baroclinic instability.

Reading

1. L.G Leal, *Advanced Transport Phenomena*,(Cambridge University Press, Cambridge, 2007).
2. O.M. Phillips, *Geological Fluid Dynamics*,(Cambridge University Press, Cambridge, 2009).
3. J.S. Turner, *Buoyancy Effects in Fluids* (Cambridge University Press, Cambridge, 1973).
4. A.E. Gill, *Atmosphere-Ocean Dynamics* (Academic Press, San Diego, 1982).
5. J. Pedlosky, *Geophysical Fluid Dynamics* (Springer-Verlag, Berlin, 1979).

Further Reading

1. G. K. Batchelor, H. K. Moffatt and M. G. Worster (eds.), *Perspectives in Fluid Dynamics* (Cambridge University Press, Cambridge, 2000).

C6.4b: Stochastic Modelling of Biological Processes – Dr Erban – 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit, (OSS paper codes tbc)

Recommended Prerequisites

A basic understanding of probability is sufficient. The course is designed in such a way that a Part C student should be able to understand it without taking special stochastic or biological classes.

Overview

This course provides an overview of stochastic methods which are used for modelling biological systems. The course starts with stochastic modelling of chemical reactions, introducing stochastic simulation algorithms and mathematical methods which can be used for analysis of stochastic models (chemical master equation). Systems with increasing level of complexity are used to illustrate the theory. Then stochastic differential equations are introduced (from the computational point of view), explaining their connections with modelling chemical systems and the Fokker-Planck equation. Different models of molecular diffusion (on-lattice and off-lattice models, velocity jump processes) and their properties are studied, before moving to stochastic reaction-diffusion models. Compartment-based and molecular-based approaches to stochastic reaction-diffusion modelling (Brownian dynamics) are discussed together with properties of stochastic spatially-distributed models (pattern formation). The final lectures include discussion of bacterial chemotaxis, Metropolis-Hastings algorithm and multiscale modelling.

Learning Outcomes

The student will learn: (i) about biological systems which are often described in terms of stochastic models; (ii) mathematical techniques which are used for the analysis of stochastic models; (iii) how the models can be efficiently simulated using a computer; (iv) connections and differences between different stochastic methods, and between stochastic and deterministic modelling.

Synopses

Stochastic simulation of chemical reactions: well-stirred systems, Gillespie algorithm, chemical master equation, analysis of simple systems, deterministic vs. stochastic modelling, systems with multiple favourable states, stochastic resonance, stochastic focusing.

Stochastic differential equations: numerical methods, Fokker-Planck equation, first exit time, backward Kolmogorov equation, chemical Fokker-Planck equation.

Diffusion: Brownian motion, on-lattice and off-lattice models, compartment-based approach, velocity jump processes, Einstein-Smoluchowski relation, diffusion to adsorbing surfaces, reactive boundary conditions.

Stochastic reaction-diffusion models: compartment-based reaction-diffusion algorithm, reaction-diffusion master equation, pattern formation, morphogen gradients, Turing patterns, molecular-based approaches to reaction-diffusion modelling, Brownian dynamics, reaction radius.

Bacterial chemotaxis: reaction-diffusion-advection processes, velocity jump processes with internal dynamics, agent-based modelling.

Metropolis-Hastings algorithm: Markov chain Monte Carlo methods.

Multiscale modelling: efficient stochastic modelling of chemical reactions, multiscale SSA with partial equilibrium assumption, hybrid modelling approaches.

Reading

1. R. Erban, J. Chapman and P. Maini: "A practical guide to stochastic simulations of reaction-diffusion processes, available as http://arxiv.org/abs/0704.1908, 2007 (course lecture notes extend this material)

Further Reading

1. H. Berg: "Random Walks in Biology", new, expanded edition, Princeton University Press, 1993,
2. D. Gillespie: "Markov Processes, an Introduction for Physical Scientists", Academic Press, Inc.,1992
3. A.Chorin and O.Hald: "Stochastic Tools for Mathematics and Science", 2nd Edition, Springer, 2009

C7.1b: Quantum Theory and Quantum Computers — Prof. Ekert — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit, OSS paper code 2A78.

Prerequisites

B7.1a Quantum Mechanics.

Overview

This course builds directly on the first course in quantum mechanics and covers a series of important topics, particularly features of systems containing several particles. The behaviour of identical particles in quantum theory is more subtle than in classical mechanics, and an understanding of these features allows one to understand the periodic table of elements and the rigidity of matter. It also introduces a new property of entanglement linking particles which can be quite widely dispersed.

There are rarely neat solutions to problems involving several particles, so usually one needs some approximation methods. In very complicated systems, such as the molecules of gas in a container, quantum mechanical uncertainty is compounded by ignorance about other details of the system and requires tools of quantum statistical mechanics.

Two state quantum systems enable one to encode binary information in a new way which permits superpositions. This leads to a quantum theory of information processing, and by exploiting entanglement to other ideas such as quantum teleportation.

Learning Outcomes

Students will be able to demonstrate knowledge and understanding of quantum mechanics of many particle systems, statistics, entanglement, and applications to quantum computing.

Synopsis

Identical particles, symmetric and anti-symmetric states, Fermi-Dirac and Bose-Einstein statistics and atomic structure.

Heisenberg representation, interaction representation, time dependent perturbation theory and Feynman–Dyson expansion. Approximation methods, Rayleigh-Schrö)dinger time-independent perturbation theory and variation principles. The virial theorem. Helium.

Mixed states, density operators. The example of spin systems. Purification. Gibbs states and the KMS condition.

Entanglement. The EPR paradox, Bell's inequalities, Aspect's experiment.

Quantum information processing, qubits and quantum computing. The no-cloning theorem, quantum teleportation. Quantum logic gates. Quantum operations. The quantum Fourier transform.

Reading

K. Hannabuss, *Introduction to Quantum Mechanics* (oup, 1997). Chapters 10–12 and 14, 16, supplemented by lecture notes on quantum computers on the web.

Further Reading

A popular non-technical account of the subject:

A. Hey and P. Walters, *The New Quantum Universe* (Cambridge, 2003).

Also designed for an Oxford course, though only covering some material:

I. .P Grant, *Classical and Quantum Mechanics*, Mathematical Institute Notes (1991).

A concise account of quantum information theory:

S. Stenholm and K.-A. Suominen, *Quantum Approach to Informatics* (Wiley, 2005).

An encyclopaedic account of quantum computing:

M. A. Nielsen and I. L. Chuang, *Quantum Computation* (Cambridge University Press, 2000).

Even more paradoxes can be found in:

Y. Aharanov and D. Rohrlich, *Quantum Paradoxes* (Wiley–VCH, 2005).

Those who read German can find further material on entanglement in:

J. Audretsch, *Verschränkte Systeme* (Wiley–VCH, 2005).

Other accounts of the first part of the course:

L. I. Schiff, *Quantum Mechanics* (3rd edition, Mc Graw Hill, 1968).

B. J. Bransden and C. J. Joachain, *Introduction to Quantum Mechanics* (Longman, 1995).

A. I. M. Rae, *Quantum Mechanics* (4th edition, Institute of Physics, 1993).

John Preskill's on-line lecture notes (http://www.theory.caltech.edu/ preskill/ph219/index.html).

C7.2a: General Relativity I — Dr Lipstein — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit. OSS paper code 2B78.

Recommended Prerequisites

B7.2a Relativity and Electromagnetism.

Overview

The course is intended as an elementary introduction to general relativity, the basic physical concepts of its observational implications, and the new insights that it provides into the nature of space time, and the structure of the universe. Familiarity with special relativity and electromagnetism as covered in the B7 course will be assumed. The lectures will review Newtonian gravitation, tensor calculus and continuum physics in special relativity, physics in curved space time and the Einstein field equations. This will suffice for an account of simple applications to planetary motion, the bending of light and the existence of black holes.

Learning Outcomes

This course starts by asking how the theory of gravitation can be made consistent with the special-relativistic framework. Physical considerations (the principle of equivalence, general covariance) are used to motivate and illustrate the mathematical machinery of tensor calculus. The technical development is kept as elementary as possible, emphasising the use of local inertial frames. A similar elementary motivation is given for Einstein's equations and the Schwarzschild solution. Orbits in the Schwarzschild solution are given a unified treatment which allows a simple account of the three classical tests of Einstein's theory. Finally, the analysis of extensions of the Schwarzschild solution show how the theory of black holes emerges and exposes the radical consequences of Einstein's theory for space-time structure. Cosmological solutions are not discussed.

The learning outcomes are an understanding and appreciation of the ideas and concepts described above.

Synopsis

Review of Newtonian gravitation theory and problems of constructing a relativistic generalisation. Review of Special Relativity. The equivalence principle. Tensor formulation of special relativity (including general particle motion, tensor form of Maxwell's equations and the energy momentum-tensor of dust). Curved space time. Local inertial coordinates. General coordinate transformations, elements of Riemannian geometry (including connections, curvature and geodesic deviation). Mathematical formulation of General Relativity, Einstein's equations (properties of the energy-momentum tensor will be needed in the case of dust only). The Schwarzschild solution; planetary motion, the bending of light, and black holes.

Reading

1. L.P. Hughston and K.P. Tod, *An Introduction to General Relativity,* LMS Student Text 5 (London Mathematical Society, Cambridge University Press, 1990), Chs 1–18.

2. N.M.J. Woodhouse, *Notes on Special Relativity*, Mathematical Institute Notes. Revised edition; published in a revised form as *Special Relativity, Lecture notes in Physics m6* (Springer-Verlag, 1992), Chs 1–7

Further Reading

1. B. Schutz, *A First Course in General Relativity* (Cambridge University Press, 1990).

2. R.M. Wald, *General Relativity* (Chicago, 1984).

3. W. Rindler, *Essential Relativity* (Springer-Verlag, 2nd edition, 1990).

C7.2b: General Relativity II — Dr de la Ossa — 16HT

Recommended Prerequisites

B7.1a, C7.2a General Relativity I

Aims & Objectives

In this, the second course in General Relativity, we have two principal aims. We first aim to increase our mathematical understanding of the theory of relativity and our technical ability to solve problems in it. This leads to a greater understanding of the Schwarzschild solution and an introduction to its rotating counterpart, the Kerr solution. Then we apply the theory to a wider class of physical situations, notably to the problem of constructing a cosmological model to represent the universe itself in the large.

Synopsis

The Lie derivative and isometries. Linearised General Relativity and the metric of an isolated body. The Schwarzschild solution and its extensions; Eddington-Finkelstein coordinates and the Kruskal extension. Stationary, axisymmetric metrics and orthogonal transitivity; the Kerr solution and its properties; interpretation as rotating black hole. The Einstein field equations with matter; the energy-momentum tensor for a perfect fluid; equations of motion form the conservation law. Cosmological principles, homogeneity and isotropy; cosmological models; the Friedman–Robertson–Walker solutions; observational consequences.

Method of Examination

4 examination questions.

Reading

1. L. P. Hughston and K. P. Tod, *An Introduction to General Relativity*, LMS Student Text 5, CUP (1990), Chs.19, 20, 22-26.

2. R. M. Wald, *General Relativity*, Univ of Chicago Press (1984).

C7.4: Theoretical Physics

Note: This unit is offered by the Physics Department.

Level: M-level **Method of Assessment:** Written examination.

Weight: Whole-unit only. OSS paper code 2756.

Recommended Prerequisites

Part A Quantum Theory, Part A Classical Mechanics, B7.1a Quantum Mechanics, C7.1b Quantum Theory and Quantum Computers, B7.2a Special Relativity and Electromagnetism.

Overview

This course is intended to give an introduction to some aspects of field theory and related ideas. These are important in particular for treating systems with an infinite number of degrees of freedom. An aim is to present some core ideas and important applications in a unified way. These applications include the classical mechanics of continuum systems, the quantum mechanics and statistical mechanics of many-particle systems, and some basic aspects of relativistic quantum field theory.

C7.4a: Theoretical Physics I — Prof. Lucas — 24MT

Synopsis

1. The mathematical description of systems with an infinite number of degrees of freedom: functionals, functional differentiation, and functional integrals. Multi-dimensional Gaussian integrals. Random fields: properties of a Gaussian field. Perturbation theory for non-Gaussian functional integrals. Path integrals and quantum mechanics. Treatment of free particle and of harmonic oscillator. [5 lectures]

2. Classical field theory: fields, Lagrangians and Hamiltonians. The least action principle and field equations. Space-time and internal symmetries: $U(1)$ example, Noether current. The idea of an irreducible representation of a group. Irreducible representations of $SU(2)$ and application to global internal symmetry. Simple representations of the Lorentz group via $SU(2) \times SU(2)$ without proof. $U(1)$ gauge symmetry, action of scalar QED and derivation of Maxwell's equations in covariant form. [5 lectures]

3. Landau theory and phase transitions: phase diagrams, first-order and continuous phase transitions. Landau–Ginsburg–Wilson free energy functionals. Examples including liquid crystals. Critical phenomena and scaling theory. [5 lectures]

4. The link between quantum mechanics and the statistical mechanics of one-dimensional systems via Wick rotation. Transfer matrices for one-dimensional systems in statistical mechanics. [4 lectures]

5. Stochastic processes and path integrals: the Langevin and Fokker–Planck equation. Brownian motion of single particle. Rouse model of polymer dynamics. [4 lectures]

C7.4b: Theoretical Physics II — Prof. Lucas — 16HT

Synopsis

1. Canonical quantisation and connection to many body theory: quantised elastic waves; quantisation of free scalar field theory; many-particle quantum systems. [4 lectures]
2. Path integrals and quantum field theory: generating functional and free particle propagator for scalar and $U(1)$ gauge fields (in Lorentz gauge). [5 lectures]
3. Perturbation theory at tree level for decay and scattering processes. Examples from pure scalar theories and scalar QED. Goldstone theorem. [4 lectures]
4. Canonical transformations in quantum field theory: Bogoliubov transformations applied to bose condensates, magnons in antiferromagnets, and to BCS theory. [4 lectures]

[Total: 40 lectures]

Reading

The lecturers are aware of no book that presents all parts of this course in a unified way and at an appropriate level. For this reason, detailed lecture notes will be made available.

Some books that cover parts of the course are:

1. D. Bailin and A. Love, *Introduction to Gauge Field Theory* Graduate Student Series in Physics, mainly chapters 1 – 6 for sections 1, 2, 6, 7 and 8.
2. R. P. Feynman, *Statistical Mechanics* A Set of Lectures, mainly chapters 3, 4 and 6 for sections 1, 6 and 9.
3. F. Reif, *Statistical and Thermal Physics*,(Fundamentals of Physics) chapter 15 for section 5.
4. J. M. Yeomans, *Statistical Mechanics of Phase Transitions*, (Oxford Science Publications) chapters 1 – 5 for sections 3 and 4.

C8.1a: Mathematical Geoscience — Dr Fowler — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A81)

Recommended Prerequisites

B6 highly recommended.

Overview

The aim of the course is to illustrate the techniques of mathematical modelling in their particular application to environmental problems. The mathematical techniques used are drawn from the theory of ordinary differential equations and partial differential equations. However, the course does require the willingness to become familiar with a range of different scientific disciplines. In particular, familiarity with the concepts of fluid mechanics will be useful.

Synopsis

Applications of mathematics to environmental or geophysical problems involving the use of models with ordinary and partial differential equations. Examples to be considered are: Climate dynamics. River flow and sediment transport. Glacier dynamics.

Reading

1. A. C. Fowler, *Mathematical Geoscience* (Springer, 2011).
2. K. Richards, *Rivers* (Methuen, 1982).
3. G. B. Whitham, *Linear and Nonlinear Waves* (Wiley, New York, 1974).
4. K. M. Cuffey and W. S. B. Paterson, *The Physics of Glaciers* (4th edition, Butterworth-Heinemann, 2011).
5. J. T. Houghton, *The Physics of Atmospheres* (3rd ed., Cambridge University Press., Cambridge, 2002).

C8.1b: Mathematical Physiology — Dr Gaffney — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2B81).

Recommended Prerequisites

B8a highly recommended.

Overview

The course aims to provide an introduction which can bring students within reach of current research topics in physiology, by synthesising a coherent description of the physiological background with realistic mathematical models and their analysis. The concepts and treatment of oscillations, waves and stability are central to the course, which develops ideas introduced in the more elementary B8a course. In addition, the lecture sequence aims to build understanding of the workings of the human body by treating in sequence problems at the intracellular, intercellular, whole organ and systemic levels.

Synopsis

Review of enzyme reactions and Michaelis–Menten theory. Trans-membrane ion transport: Hodgkin–Huxley and Fitzhugh–Nagumo models.

Excitable media; wave propagation in neurons

Calcium dynamics; calcium-induced calcium release. Intracellular oscillations and wave propagation.

The electrochemical action of the heart. Pacemakers, phase oscillators and the sinoatrial node. Spiral waves, tachycardia and fibrillation

Discrete delays in physiological systems. The Glass–Mackey model of respiration. Regulation of stem cell and blood cell production. Dynamical diseases.

Reading

The principal text is:

1. J. Keener and J. Sneyd, *Mathematical Physiology* (Springer-Verlag, 1998). Chs. 1, 4, 5, 9–12, 14–17. [Or: Second edition Vol I: Chs. 1, 2, 4, 5, 6, 7. Vol II: Chs. 11, 13, 14. (Springer-Verlag, 2009)]

Subsidiary mathematical texts are:

1. J. D. Murray, *Mathematical Biology* (Springer-Verlag, 2nd ed., 1993). [Third edition, Vols I and II, (Springer-Verlag, 2003).]
2. L. A. Segel, *Modeling Dynamic Phenomena in Molecular and Cellular Biology* (Cambridge University Press, 1984).
3. L. Glass and M. C. Mackey, *From Clocks to Chaos* (Princeton University Press, 1988).
4. P. Grindrod, *Patterns and Waves* (oup, 1991).

General physiology texts are:

1. R. M. Berne and M. N. Levy, *Principles of Physiology* (2nd ed., Mosby, St. Louis, 1996).
2. J. R. Levick, *An Introduction to Cardiovascular Physiology* (3rd ed. Butterworth–Heinemann, Oxford, 2000).
3. A. C. Guyton and J. E. Hall, *Textbook of Medical Physiology* (10th ed., W. B. Saunders Co., Philadelphia, 2000).

C9.1a: Modular Forms — Dr White — 16MT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code tbc)

Prerequisites

Part A Analysis and Algebra (core material) and Part A Group Theory. Part A Number Theory is useful but not essential. B3b Algebraic Curves is recommended but some background reading on the notions of a Riemann surface and its genus will suffice.

Overview

The course aims to introduce students to the beautiful theory of modular forms, one of the cornerstones of modern number theory. This theory is a rich and challenging blend of methods from complex analysis and linear algebra, and an explicit application of group actions and the theory of Riemann surfaces.

Learning Outcomes

The student will learn about modular curves and spaces of modular forms, and understand in special cases how to compute their genus and dimension, respectively. They will see how modular curves parameterise families of elliptic curves, and that modular forms can be described explicitly via their q-expansions, and they will be familiar with explicit examples of modular forms. They will learn about the rich algebraic structure on spaces of modular forms, given by Hecke operators and the Petersson inner product, how spaces of modular forms of different level are related, and how modular forms may be used in number theory.

Synopsis

The modular group and the upper half-plane. Lattices and elliptic curves. Modular forms of level 1. Examples of modular forms: Eisenstein series, Ramanujan's function Δ. Congruence subgroups and fundamental domains. Modular forms of higher level. Hecke operators. The Petersson inner product. Old and new forms. Applications in number theory.

Reading

1. F. Diamond and J. Shurman, *A First Course in Modular Forms,* Graduate Texts in Mathematics 228, (Parts of Chapters 1-5), Springer-Verlag, 2005.
2. J.-P. Serre, *A Course in Arithmetic,* (Chapter VII), Graduate Texts in Mathematics 7, Springer-Verlag, 1973.

C9.1b Elliptic Curves — Prof. Kim — 16HT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2B82).

Recommended Prerequisites

It is helpful, but not essential, if students have already taken a standard introduction to algebraic curves and algebraic number theory. For those students who may have gaps in their background, I have placed the file "Preliminary Reading" permanently on the Elliptic Curves webpage, which gives in detail (about 30 pages) the main prerequisite knowledge for the course. Go first to: http://www.maths.ox.ac.uk/courses/material then click on "C9.1b Elliptic Curves" and then click on the pdf file "Preliminary Reading".

Overview

Elliptic curves give the simplest examples of many of the most interesting phenomena which can occur in algebraic curves; they have an incredibly rich structure and have been the testing ground for many developments in algebraic geometry whilst the theory is still full of deep unsolved conjectures, some of which are amongst the oldest unsolved problems in mathematics. The course will concentrate on arithmetic aspects of elliptic curves defined over the rationals, with the study of the group of rational points, and explicit determination of the rank, being the primary focus. Using elliptic curves over the rationals as an example, we will be able to introduce many of the basic tools for studying arithmetic properties of algebraic varieties.

Learning Outcomes

On completing the course, students should be able to understand and use properties of elliptic curves, such as the group law, the torsion group of rational points, and 2-isogenies between elliptic curves. They should be able to understand and apply the theory of fields with valuations, emphasising the p-adic numbers, and be able to prove and apply Hensel's Lemma in problem solving. They should be able to understand the proof of the Mordell–Weil Theorem for the case when an elliptic curve has a rational point of order 2, and compute ranks in such cases, for examples where all homogeneous spaces for descent-via-2-isogeny satisfy the Hasse principle. They should also be able to apply the elliptic curve method for the factorisation of integers.

Synopsis

Non-singular cubics and the group law; Weierstrass equations.
Elliptic curves over finite fields; Hasse estimate (stated without proof).
p-adic fields (basic definitions and properties).
1-dimensional formal groups (basic definitions and properties).
Curves over p-adic fields and reduction mod p.
Computation of torsion groups over $\mathbb{Q}$; the Nagell–Lutz theorem.
2-isogenies on elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.
Weak Mordell–Weil Theorem for elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.
Height functions on Abelian groups and basic properties.
Heights of points on elliptic curves defined over $\mathbb{Q}$; statement (without proof) that this gives a height function on the Mordell–Weil group.
Mordell–Weil Theorem for elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.
Explicit computation of rank using descent via 2-isogeny.
Public keys in cryptography; Pollard's $(p-1)$ method and the elliptic curve method of factorisation.

Reading

1. J.W.S. Cassels, *Lectures on Elliptic Curves,* LMS Student Texts 24 (Cambridge University Press, 1991).

2. N. Koblitz, *A Course in Number Theory and Cryptography,* Graduate Texts in Mathematics 114 (Springer, 1987).

3. J.H. Silverman and J. Tate, *Rational Points on Elliptic Curves,* Undergraduate Texts in Mathematics (Springer, 1992).

4. J.H. Silverman, *The Arithmetic of Elliptic Curves,* Graduate Texts in Mathematics 106 (Springer, 1986).

Further Reading

1. A. Knapp, *Elliptic Curves, Mathematical Notes 40* (Princeton University Press, 1992).

2. G, Cornell, J.H. Silverman and G. Stevans (editors), *Modular Forms and Fermat's Last Theorem* (Springer, 1997).

3. J.H. Silverman, *Advanced Topics in the Arithmetic of Elliptic Curves,* Graduate Texts in Mathematics 151 (Springer, 1994).

C10.1a: Stochastic Differential Equations —Prof. Hambly—16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A83)

Recommended Prerequisites Part A integration and B10a Martingales Through Measure Theory, is expected.

Overview

Stochastic differential equations have been used extensively in many areas of application, including finance and social science as well as chemistry. This course develops the basic theory of Itô's calculus and stochastic differential equations.

Learning Outcomes

The student will have developed an appreciation of stochastic calculus as a tool that can be used for defining and understanding diffusive systems.

Synopsis

Brownian motion: basic properties, reflection principle, quadratic variation. Itô's calculus: stochastic integrals with respect to martingales, Itô's lemma, Levy's theorem on characteristic of Brownian motion, exponential martingales, exponential inequality, Girsanov's theorem, The Martingale Representation Theorem. Stochastic differential equations: strong solutions, questions of existence and uniqueness, diffusion processes, Cameron–Martin formula, weak solution.

Reading — Main Texts

1. Dr Qian's online notes:

   www.maths.ox.ac.uk/courses/course/15721

2. B. Oksendal, *Stochastic Differential Equations: An introduction with applications* (Universitext, Springer, 6th edition). Chapters II, III, IV, V, part of VI, Chapter VIII (F).

3. F. C. Klebaner, *Introduction to Stochastic Calculus with Applications* (Imperial College Press, 1998, second edition 2005). Sections 3.1 – 3.5, 3.9, 3.12. Chapters 4, 5, 11.

Alternative Reading

1. H. P. McKean, *Stochastic Integrals* (Academic Press, New York and London, 1969).

Further Reading

1. N. Ikeda & S. Watanabe, *Stochastic Differential Equations and Diffusion Processes* (North–Holland Publishing Company, 1989).
2. I. Karatzas and S. E. Shreve, *Brownian Motion and Stochastic Calculus,* Graduate Texts in Mathematics 113 (Springer-Verlag, 1988).
3. L. C. G. Rogers & D. Williams, *Diffusions, Markov Processes and Martingales Vol 1 (Foundations) and Vol 2 (Ito Calculus)* (Cambridge University Press, 1987 and 1994).

C10.1b: Brownian Motion and Conformal Invariance — Dr Belyaev— 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2B83).

Prerequisites

Essential Prerequisites: Part A Analysis and knowledge of Brownian Motion.

Recommended Prerequisites: Part A Differential Equations, B10a Martingales through Measure and C10.1a Stochastic Differential Equations.

Overview

This course is devoted to connections between two-dimensional Brownian Motion, Complex Analysis and Lattice Models. An important example is the standard random walk on a square lattice. It is known, that after proper rescaling, the random walk converges to Brownian Motion (this is a corollary of the Central Limit Theorem). It was observed by Lévy that Brownian Motion is conformally invariant i.e. the image of the Brownian trajectory under an analytic map looks like a Brownian trajectory. In particular, this implies that the limiting object has more symmetry than the original random walk on the lattice. There is a big class of other lattice models which are conjectured to have scaling limits and these limits are conjectured to be conformally invariant.

This course will consist of two parts: in the first part we will discuss the conformal invariance of Brownian Motion. In the second part we will learn about the very recent and exciting theory of Schramm-Loewner Evolution (SLE). This theory will provide the necessary tools to study conformally invariant limits of various lattice models.

Learning Outcomes

The students will develop an understanding of the role the Brownian Motion plays in different areas of mathematics and physics. They will be familiar with basic ideas and techniques of Schramm-Loewner Evolution.

Synopsis

Brief introduction to Brownian Motion, continuous martingales, and Ito formula. Conformal invariance of Brownian Motion. Brief introduction to conformal maps and Loewner Evolution. Lattice models: percolation, Ising, Loop-erased Random Walk etc. Schramm's principle and the introduction of SLE (Schramm-Loewner Evolution or Stochastic Loewner Evolution). Properties of SLE. [Convergence of percolation to SLE(6)]

Reading

This is a very young and actively developing area of research. Unfortunately this means that there are very few books, in fact, there is only one proper book and a couple of lecture notes.

1. G. Lawler, *Conformally Invariant Processes in the Plane*, Mathematical Surveys and Monographs, Vol 114 (2005)

2. W. Werner, *Random planar curves and Schramm-Loewner evolutions* http://arxiv.org/abs/math/0303354

C11.1a: Graph Theory — Prof. McDiarmid — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A85).

Recommended Prerequisites

None.

Overview

Graphs are among the simplest mathematical structures, but nevertheless have a very rich and well-developed structural theory. Graph Theory is an important area of mathematics, and also has many applications in other fields such as computer science.

The main aim of the course is to introduce the analysis of discrete structures, and particularly the use of extremal methods.

Learning Outcomes

The student will have developed an appreciation of extremal methods in the analysis and understanding of graphical structures.

Synopsis

Introduction. Trees. Euler circuits. Planar graphs.
Matchings and Hall's Theorem. Connectivity and Menger's Theorem.
Extremal problems. Long paths and cycles. Turán's Theorem. Erdős–Stone Theorem.
Graph colouring. The Theorem of Brooks. The chromatic polynomial.
Ramsey's Theorem.
Szemerédi's Regularity Lemma.

Reading

1. B. Bollobás, *Modern Graph Theory,* Graduate Texts in Mathematics 184 (Springer-Verlag, 1998)

Further Reading

1. J. A. Bondy and U. S. R. Murty, *Graph Theory: An Advanced Course*, Graduate Texts in Mathematics 244 (Springer–Verlag, 2007).
2. R. Diestel, *Graph Theory,* Graduate Texts in Mathematics 173 (third edition, Springer-Verlag, 2005).
3. D. West, *Introduction to Graph Theory* (second edition, Prentice–Hall, 2001).

C11.1b: Probabilistic Combinatorics — Prof. McDiarmid — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half Unit (OSS code tbc)

Recommended Prerequisites

C11.1a Graph Theory and Part A Probability.

Overview

Probabilistic combinatorics is a very active field of mathematics, with connections to other areas such as computer science and statistical physics. Probabilistic methods are essential for the study of random discrete structures and for the analysis of algorithms, but they can also provide a powerful and beautiful approach for answering deterministic questions. The aim of this course is to introduce some fundamental probabilistic tools and present a few applications.

Learning Outcomes

The student will have developed an appreciation of probabilistic methods in discrete mathematics.

Synopsis

First-moment method, with applications to Ramsey numbers, and to graphs of high girth and high chromatic number.
Second-moment method, threshold functions for random graphs.
Lovasz Local Lemma, with applications to two-colourings of hypergraphs (property B), and to Ramsey numbers.
Chernoff bounds, concentration of measure, Janson's inequality.
Branching processes and the phase transition in random graphs.
Clique and chromatic numbers of random graphs.

Reading

1. N. Alon and J.H. Spencer, *The Probabilistic Method* (second edition, Wiley, 2000).

Further Reading

1. B. Bollobás, *Random Graphs* (second edition, Cambridge University Press, 2001).
2. M. Habib, C. McDiarmid, J. Ramirez-Alfonsin, B. Reed, ed., *Probabilistic Methods for Algorithmic Discrete Mathematics* (Springer, 1998).
3. S.Janson, T. Luczak and A.Rucinski, *Random Graphs* (John Wiley and Sons, 2000).
4. M. Mitzenmacher and E. Upfal, *Probability and Computing : Randomized Algorithms and Probabilistic Analysis* (Cambridge University Press, New York (NY), 2005).
5. M. Molloy and B. Reed, *Graph Colouring and the Probabilistic Method* (Springer, 2002).
6. R. Motwani and P. Raghavan, *Randomized Algorithms* (Cambridge University Press, 1995).

C12.1a Numerical Linear Algebra — Dr Wathen — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A88)

Recommended Prerequisites

Only elementary linear algebra is assumed in this course. The part A Numerical Analysis course would be helpful, indeed some swift review and extensions of some of the material of that course is included here.

Overview

Linear Algebra is a central and widely applicable part of mathematics. It is estimated that many (if not most) computers in the world are computing with matrix algorithms at any moment in time whether these be embedded in visualization software in a computer game or calculating prices for some financial option. This course builds on elementary linear algebra and in it we derive, describe and analyse a number of widely used constructive methods (algorithms) for various problems involving matrices.

Numerical Methods for solving linear systems of equations, computing eigenvalues and singular values and various related problems involving matrices are the main focus of this course.

Synopsis

Common problems in linear algebra. Matrix structure, singular value decomposition. QR factorization, the QR algorithm for eigenvalues. Direct solution methods for linear systems, Gaussian elimination and its variants. Iterative solution methods for linear systems.

Chebyshev polynomials and Chebyshev semi-iterative methods, conjugate gradients, convergence analysis, preconditioning.

Reading

L. N. Trefethen and D. Bau III, *Numerical Linear Algebra* (SIAM, 1997).

J. W. Demmel, *Applied Numerical Linear Algebra* (SIAM, 1997).

A. Greenbaum, *Iterative Methods for Solving Linear Systems* (SIAM, 1997).

G. H. Golub and C. F. van Loan, *Matrix Computations* (John Hopkins University Press, 3rd edition, 1996).

H. C. Elman, D. J. Silvester and A. J. Wathen, *Finite Elements and Fast Iterative Solvers* (Oxford University Press, 1995), only chapter 2.

C12.1b Continuous Optimization — Prof. Hauser — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Half-Unit (OSS paper code 2B88).

Overview

Optimization deals with the problem of minimising or maximising a mathematical model of an objective function such as cost, fuel consumption etc. under a set of side constraints on the domain of definition of this function. Optimization theory is the study of the mathematical properties of optimization problems and the analysis of algorithms for their solution. The aim of this course is to provide an introduction to nonlinear continuous optimization specifically tailored to the background of mathematics students.

Synopsis

Part 1: Unconstrained Optimization
Optimality conditions, Newton's method for nonlinear systems, Convergence rates, Steepest descent method, General line search methods (alternative search directions, e.g. Newton, CG, BFG, ...), Trust region methods, Inexact evaluation of linear systems, iterative methods and the role of preconditioners.

Part 2: Constrained Optimization
Optimality/KKT conditions, Lagrange Multipliers, Penalty methods and SQP for equality constrained optimization, Interior penalty / barrier methods for inequality constrained optimization.

Reading List

Lecture notes will be made available for downloading from the course webpage.
To complement the notes, reading assignments will be given from the book of J.Nocedal and S.J.Wright, *Numerical Optimisation*, (Springer, 1999).

C12.2b Finite Element Methods for Partial Differential Equations — Prof Suli — 16HT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2B89).

Recommended Prerequisites

No formal prerequisites are assumed. The course builds on elementary calculus, analysis and linear algebra and, of course, requires some acquaintance with partial differential equations such as the material covered in the Maths Mods Waves and Diffusion course, in particular the Divergence Theorem. Part A Numerical Analysis would be helpful but is certainly not essential. Function Space material will be introduced in the course as needed.

Overview

Computational algorithms are now widely used to predict and describe physical and other systems. Underlying such applications as weather forecasting, civil engineering (design of structures) and medical scanning are numerical methods which approximately solve partial differential equation problems. This course gives a mathematical introduction to one of the more widely used methods: the finite element method.

Synopsis

Finite element methods represent a powerful and general class of techniques for the approximate solution of partial differential equations. The aim of this course is to introduce these methods for boundary value problems for the Poisson and related elliptic partial differential equations.

Attention will be paid to the formulation, the mathematical analysis and the implementation of these methods.

Reading List

S.C. Brenner & L.R. Scott, *The Mathematical Theory of Finite Element Methods.* Springer, 2nd edition, 2002.[Chapters 0,1,2,3; Chapter 4: Secs. 4.1–4.4, Chapter 5: Secs. 5.1–5.7].

H. Elman, D. Silvester & A. Wathen, *Finite Elements and Fast Iterative Solvers.* OUP, 2005. [Mainly Chapters 1 and 5].

C. Johnson, *Numerical Solution of Partial Differential Equations by the Finite Element Method.* CUP, 1990. [Chapters 1–4; Chapter 8:Secs. 8.1–8.4.2; Chapter 9: Secs. 9.1–9.5].

Typed lecture notes covering the entire course (and more):

Endre Süli, *Finite Element Methods for Partial Differential Equations.* Mathematical Institute, University of Oxford, 2011.

are available from `http://web.comlab.ox.ac.uk/people/endre.suli/fem.pdf`

Some of the introductory material is covered in

Endre Süli & David Mayers, *An Introduction to Numerical Analysis*,CUP 2003; Second Printing 2006. [Chapter 11 and in particular Chapter 14].

C12.3a Approximation of Functions — Dr Sobey — 16MT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Half-unit (OSS paper code 2A89)

Recommended Prerequisites:

None

Overview

How can a function f(x) be approximated over a prescribed domain by a simpler function like a polynomial or a rational function? Such questions were at the heart of analysis in the early 1900s and later grew into a mature subject of approximation theory. Recently they have been invigorated as problems of approximation have become central to computational algorithms for differential equations, linear algebra, optimization and other fields. This course, based on Trefethen's new text in which results are illustrated by Chebfun computations, will focus in a modern but still rigorous way on the fundamental results of interpolation and approximation and their algorithmic application.

Synopsis

Chebyshev interpolants, polynomials, and series. Barycentric interpolation formula. Weierstrass approximation theorem. Convergence rates of polynomial approximations. Hermite integral formula and Runge phenomenon. Lebesgue constants, polynomial rootfinding. Orthogonal polynomials. Clenshaw-Curtis and Gauss quadrature. Rational approximation.

Reading

1. L. N. Trefethen, *Approximation Theory and Approximation Practice*

CD : Dissertations on a Mathematical Topic

Level : M-level

Weight : Half-unit (5,000 words) or whole-unit (10,000).

Students may offer either a whole-unit or a half-unit dissertation on a Mathematical topic for examination at Part C. A whole-unit is equivalent to a 32-hour lecture course and a half-unit is equivalent to a 16-hour lecture course. Students will have approximately 8 hours of supervision for a whole-unit dissertation or 4 hours for a half-unit distributed over Michaelmas and Hilary terms. In addtion there are lectures on writing mathematics and using Latex in Michaelmas and Hilary terms. See the lecture list for details.

Students considering offering a dissertation should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* available at:

http://www.maths.ox.ac.uk/current-students/undergraduates/projects/.

Application

Students must apply to the Mathematics Projects Committee for approval of their proposed topic in advance of beginning work on their dissertation. Proposals should be addressed to the Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room DH61, Dartingtion House and are accepted from the end of Trinity Term. All proposals must be received before 12noon on Friday of Week 0 of Michaelmas Full Term. For CD dissertations candidates should take particular care to remember that the project must have substantial mathematical content. The application form is available at
http://www.maths.ox.ac.uk/current-students/undergraduates/projects/. Once a title has been approved, it may only be changed by approval of the Chairman of the Projects Committee.

Assessment

Each project is blind double marked. The marks are reconciled through discussion between the two assessors, overseen by the examiners. Please see the *Guidance Notes on Extended Essays and Dissertations in Mathematics* for detailed marking criteria and class descriptors.

Submission

THREE copies of your dissertation, identified by your candidate number only, should be sent to the Chairman of Examiners, FHS of Mathematics Part C, Examination Schools, Oxford, to arrive no later than **12noon on Friday of week 9, Hilary Term 2013**. An electronic copy of your dissertation should also be submitted via the Mathematical Institute website. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics.*

3 Other Units

MS: Statistics Half-units

Students in Part C may take half units drawn from Part C of the Honour School of Mathematics and Statistics. For full details of these half units see the syllabus and synopses for Part C of the Honour School Mathematics and Statistics, which are available on the web at http://www.stats.ox.ac.uk/current_students/bammath/course_handbooks/

The Statistics half units available are as follows:

- MS1b Statistical Data Mining
- MS2a Bioinformatics and Computational Biology
- MS2b Stochastic Models in Mathematical Genetics
- MS5a Probability and Statistics for Network Analysis
- MS6b Advanced Simulation Methods

Computer Science: Half Units

Students in Part C may take half units drawn from Part C of the Honour School of Mathematics and Computing. For full details of these half units see the Department of Computer Science's website (http://www.cs.ox.ac.uk/teaching/courses/)

Please note that these three courses will be examined by mini-project (as for MSc students). Mini-projects will be handed out to candidates on the last Friday of the term in which the subject is being taught, and you will have to hand it in to the Exam Schools by noon on Monday of Week 1 of the following term. The mini-project will be designed to be completed in about three days. It will include some questions that are more open-ended than those on a standard sit-down exam. The work you submit should be your own work, and include suitable references.

Please note that the Computer Science courses in Part C are 50% bigger than those in earlier years, i.e. for each Computer Science course in the 3rd year undergraduates are expected to undertake about 10 hours of study per week, but 4th year courses will each require about 15 hours a week of study. Lecturers are providing this extra work in a variety of ways, e.g. some will give 16 lectures with extra reading, classes and/or practicals, whereas others will be giving 24 lectures, and others still will be doing something in between. Students will need to look at each synopsis for details on this.

The Computer Science half units available are as follows:

- CCS1a Categories, Proofs and Processes
- CCS3b Quantum Computer Science
- CCS4b Automata, Logics and Games

Philosophy

Students in Part C may take units drawn from Part C of the Honour School of Mathematics and Philosophy. For full details of these units see the Faculty of Philosophy's website http://www.philosophy.ox.ac.uk/undergraduate/course_descriptions

The Philosophy units available are as follows:

- Rise of Modern Logic

This course will be examined by a three-hour exam and a submitted essay of up to 5000 words.

OD : Dissertations on a Mathematically related Topic

Level : M-level

Weight : Half-unit (5,000 words) or whole-unit (10,000 words).

Students may offer either a whole-unit or a half-unit dissertation on a Mathematically related topic for examination at Part C. For example, applications of mathematics to another field (eg Maths in Music), historical topics, topics concentrating on the analysis of statistical data, or topics concentrating on the production of computer-generated data are acceptable as topics for an OD dissertation. (Topics in mathematical education are not allowed.)

A whole-unit is equivalent to a 32-hour lecture course and a half-unit is equivalent to a 16-hour lecture course. Students will have approximately 8 hours of supervision for a whole-unit dissertation or 4 hours for a half-unit distributed over Michaelmas and Hilary terms. In addtion there are lectures on writing mathematics and using Latex in Michaelmas and Hilary terms. See the lecture list for details.

Candidates considering offering a dissertation should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* available at:

http://www.maths.ox.ac.uk/current-students/undergraduates/projects/.

Application

Students must apply to the Mathematics Projects Committee for approval of their proposed topic in advance of beginning work on their dissertation. Proposals should be addressed to the Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room DH61, Dartingtion House and are accepted from the end of Trinity Term. All proposals must be received before 12noon on Friday of Week 0 of Michaelmas Full Term. The application form is available at http://www.maths.ox.ac.uk/current-students/undergraduates/projects/.

Once a title has been approved, it may only be changed by approval of the Chairman of the Projects Committee.

Assessment

Each project is blind double marked. The marks are reconciled through discussion between the two assessors, overseen by the examiners. Please see the *Guidance Notes on Extended Essays and Dissertations in Mathematics* for detailed marking criteria and class descriptors.

Submission

THREE copies of your dissertation,identified by your candidate number only, should be sent to the Chairman of Examiners, FHS of Mathematics Part C, Examination Schools, Oxford, to arrive no later than **12noon on Friday of week 9, Hilary Term 2013**. An electronic copy of your dissertation should also be submitted via the Mathematical Institute website. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics*.

4 Language Classes: French and German

Language courses in French and German offered by the University Language Centre.

Students in the FHS Mathematics may apply to take language classes. In 2012-2013, French and German language classes will be run in MT and HT. We have a limited number of places but if we have spare places we will offer these to joint school students , Mathematics and Computer Science, Mathematics and Philosophy and Mathematics and Statistics.

Two levels of French courses are offered, a lower level for those with a good pass at GCSE, and a higher level course for those with A/S or A level. Acceptance on either course will depend on satisfactory performance in the Preliminary Qualifying Test held in Week 1 of Michaelmas Term (Monday, 17.00-19.00 at the Language Centre). Classes at both levels will take place on Mondays, 17.00-19.00. A single class in German at a lower or higher level will be offered on the basis of the performances in the Preliminary Qualifying Test, held at the same time as the French test. Classes will also be held on Mondays, 17-00-19.00.

Performance on the course will not contribute to the class of degree awarded. However, upon successful completion of the course, students will be issued with a certificate of achievement which will be sent to their college.

Places at these classes are limited, so students are advised that they must indicate their interest at this stage. If you are interested but are unable to attend this presentation for some reason please contact the Academic Administrator in the Mathematical Institute (academic.administrator@maths.ox.ac.uk; (6)15203) as soon as possible.

Aims and rationale

The general aim of the language courses is to develop the student's ability to communicate (in both speech and writing) in French or German to the point where he or she can function in an academic or working environment in a French-speaking or German-speaking country.

The courses have been designed with general linguistic competence in the four skills of reading, writing, speaking and listening in mind. There will be opportunities for participants to develop their own particular interests through presentations and assignments.

Form and Content

Each course will consist of a thirty-two contact hour course, together with associated work. Classes will be held in the Language Centre at two hours per week in Michaelmas and Hilary Terms.

The content of the courses is based on coursebooks together with a substantial amount of supplementary material prepared by the language tutors. Participants should expect to spend an additional two hours per week on preparation and follow-up work.

Each course aims to cover similar ground in terms of grammar, spoken and written language and topics. Areas covered will include:

Grammar:

- all major tenses will be presented and/or revised, including the subjunctive
- passive voice
- pronouns
- formation of adjectives, adverbs, comparatives
- use of prepositions
- time expressions

Speaking

- guided spoken expression for academic, work and leisure contact (e.g. giving presentations, informal interviews, applying for jobs)
- expressing opinions, tastes and preferences
- expressing cause, consequence and purpose

Writing

- Guided letter writing for academic and work contact
- Summaries and short essays

Listening

- Listening practice of recorded materials (e.g. news broadcasts, telephone messages, interviews)
- developing listening comprehension strategies

Topics (with related readings and vocabulary, from among the following)

- life, work and culture
- the media in each country
- social and political systems
- film, theatre and music

- research and innovation
- sports and related topics
- student-selected topics

Teaching staff

The courses are taught by Language Centre tutors or Modern Languages Faculty instructors.

Teaching and learning approaches

Each course uses a communicative methodology which demands active participation from students. In addition, there will be some formal grammar instruction. Students will be expected to prepare work for classes and homework will be set. Students will also be given training in strategies for independent language study, including computer-based language learning, which will enable them to maintain and develop their language skills after the course.

Entry

Two classes in French and one in German at (probably at Basic and Threshold levels) will be formed according to level of French/German at entry. The minimum entry standard is a good GCSE Grade or equivalent. Registration for each option will take place at the start of Michaelmas Term. A preliminary qualifying test is then taken which candidates must pass in order to be allowed to join the course.

Learning Outcomes

The learning outcomes will be based on internationally agreed criteria for specific levels (known as the ALTE levels), which are as follows:

Basic Level (corresponds to ALTE Level 2 "Can-do" statements)

- Can express opinions on professional, cultural or abstract matters in a limited way, offer advice and understand instructions.
- Can give a short presentation on a limited range of topics.
- Can understand routine information and articles, including factual articles in newspapers, routine letters from hotels and letters expressing personal opinions.
- Can write letters or make notes on familiar or predictable matters.

Threshold Level (corresponds to ALTE Level 3 "Can-do" statements)

- Can follow or give a talk on a familiar topic or keep up a conversation on a fairly wide range of topics, such as personal and professional experiences, events currently in the news.
- Can give a clear presentation on a familiar topic, and answer predictable or factual questions.

- Can read texts for relevant information, and understand detailed instructions or advice.
- Can make notes that will be of reasonable use for essay or revision purposes.
- Can make notes while someone is talking or write a letter including non- standard requests.

Assessment

There will a preliminary qualifying test in Michaelmas Term. There are three parts to this test: Reading Comprehension, Listening Comprehension, and Grammar. Candidates who have not studied or had contact with French or German for some time are advised to revise thoroughly, making use of the Language Centre's French or German resources.

Students' achievement will be assessed through a variety of means, including continuous assessment of oral performance, a written final examination, and a project or assignment chosen by individual students in consultation with their language tutor.

Reading comprehension, writing, listening comprehension and speaking are all examined. The oral component may consist of an interview, a presentation or a candidate's performance in a formal debate or discussion.