UNIVERSITY OF OXFORD
Mathematical Institute

HONOUR SCHOOL OF MATHEMATICS

SUPPLEMENT TO THE UNDERGRADUATE HANDBOOK – 2010 Matriculation

SYNOPSES OF LECTURE COURSES

Part C 2013-14
for examination in 2014

These synopses can be found at:
http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/

Issued October 2013

Handbook for the Undergraduate Mathematics Courses
Supplement to the Handbook
Honour School of Mathematics
Syllabus and Synopses for Part C 2013–14
for examination in 2014

Contents

1 Foreword **4**

Honour School of Mathematics 4

"Units" 4

Registration 4

2 Mathematics Department units **6**

C1.1a: Model Theory — Prof Zilber — 16MT 6

C1.1b: Gödel's Incompleteness Theorems — Dr Paseau — 16HT 7

C1.2a: Analytic Topology — Dr Suabedissen — 16MT 8

C1.2b: Axiomatic Set Theory — Dr Suabedissen — 16HT 9

C2.1a: Lie Algebras — Dr Szendroi — 16MT 10

C2.1b: Representation Theory of Symmetric Groups — Prof. Henke —16HT 11

C2.2a: Commutative Algebra — Prof Segal — 16MT 13

C2.2b Homological Algebra — Dr Dyckerhoff — 16HT 13

C2.3b: Infinite Groups — Dr Nikolov — 16HT 14

C3.1a: Algebraic Topology — Prof. Tillmann — 16MT 15

C3.2b: Geometric Group Theory — Dr Papazoglou — 16HT 16

C3.3b: Differentiable Manifolds — Prof. Hitchin — 16HT 17

C3.4a: Algebraic Geometry — Dr Berczi — 16MT 18

C3.4b: Lie Groups — Dr Ritter — 16HT 20

C4.1a: Functional Analysis — Prof. Kristensen — 16MT 21

C4.1b Linear Operators — Prof. Batty — 16HT 22

C5.1a Methods of Functional Analysis for PDEs — Prof. Seregin — 16MT 23
C5.1b Fixed Point Methods for Nonlinear PDEs — Dr Nguyen — 16HT . 25
C5.2b: Calculus of Variations — Prof. Shkoller — 16HT 26
C5.3b: Hyperbolic Equations — Dr Wang — 16HT 27
C6.1a: Solid Mechanics — Prof Goriely — 16 MT 29
C6.1b: Elasticity and Plasticity — Dr Howell — 16HT 30
C6.2a: Statistical Mechanics — Dr Fowler — 16MT 31
C6.2b: Networks — Dr Porter — 16HT 32
C6.3a: Perturbation Methods — Dr Oliver — 16MT 34
C6.3b: Applied Complex Variables — Dr Oliver — 16HT 35
C6.4a: Topics in Fluid Mechanics — Dr Vella — 16MT 36
C6.4b: Stochastic Modelling of Biological Processes – Dr Erban – 16HT . . 37
C6.5b: Mathematical Mechanical Biology — Prof Goriely — 16HT 39
C7.1b: Quantum Theory and Quantum Computers — Prof. Ekert and Prof Mason — 16HT 40
C7.2a: General Relativity I — Dr Lipstein — 16MT 42
C7.2b: General Relativity II — Dr de la Ossa — 16HT 43
C7.4: Theoretical Physics 44
C7.4a: Theoretical Physics I — Prof. Essler and Dr Uli Haisch — 24MT 44
C7.4b: Theoretical Physics II — Prof. Essler and Dr Uli Haisch — 16HT 45
C8.1a: Mathematical Geoscience — Dr Fowler and Prof. Sander — 16MT . 46
C8.1b: Mathematical Physiology — Dr Gaffney — 16HT 47
C9.1a: Modular Forms — Dr Lauder — 16MT 48
C9.1b Elliptic Curves — Prof. Kim — 16HT 49
C9.2a: Analytic Number Theory —Prof. Heath-Brown—16MT 51
C10.1a: Stochastic Differential Equations —Prof. Hambly—16MT 52
C10.1b: Brownian Motion and Conformal Invariance — Dr Belyaev— 16HT 54
C11.1a: Combinatorics — Prof. Scott — 16MT 55
C11.1b: Probabilistic Combinatorics — Prof. McDiarmid — 16HT 56
C12.1a Numerical Linear Algebra — Prof. Tanner — 16MT 57
C12.1b Continuous Optimization — Dr Cartis — 16HT 58
C12.2a Approximation of Functions — Prof Trefethen — 16MT 59

C12.2b Finite Element Methods for Partial Differential Equations — Prof Suli — 16HT . . . 60

CD : Dissertations on a Mathematical Topic . . . 62

3 Other Units **63**

MS: Statistics Units . . . 63

Computer Science: Units . . . 63

Philosophy: Double Units . . . 64

OD : Dissertations on a Mathematically related Topic . . . 64

4 Language Classes: French and Spanish **65**

1 Foreword

The synopses for Part C will be available on the website at:

http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/

before the start of Michaelmas Term 2013.

See the current edition of the Examination Regulations for the full regulations governing these examinations.

Examination Conventions can be found at: http://www.maths.ox.ac.uk/notices/undergrad

In the unlikely event that any course receives a very low registration we may offer this course as a reading course (this would include some lectures but fewer classes).

Honour School of Mathematics

"Units"

Students staying on to take Part C will take the equivalent of eight units. One unit is the equivalent of a 16 hour lecture course. The equivalent of six units must be taken from the schedule of "Mathematics Department units" and may include a dissertation on a mathematical topic. Up to two units may be taken from the schedule of "Other Units".

Most Mathematics Department lecture courses are independently available as units, the exceptions being:

1. C7.4 Theoretical Physics - this is available as a double-unit only.

All the units described in this booklet are "M–Level".

Language Classes

Mathematics students may apply to take classes in a foreign language. In 2013-14 classes will be offered in French and German. Students' performances in these classes will not contribute to the degree classification awarded. However, successful completion of the course may be recorded on students' transcripts. See section 5 for more details.

Registration

Classes

Students will have to register in advance for the courses they wish to take. Students will have to register by Friday of Week 10 of Trinity Term 2013 using the online system which can be accessed at https://www.maths.ox.ac.uk/courses/registration/. Students will then be asked to sign up for classes at the start of Michaelmas Term 2013. Further information about this will be sent via email before the start of term.

Note on Intercollegiate Classes

Where undergraduate registrations for lecture courses fall below 5, classes will not run as part of the intercollegiate scheme but will be arranged informally by the lecturer.

Lectures

Every effort will be made when timetabling lectures to ensure that lectures do not clash. However, because of the large number of options in Part C this may sometimes be unavoidable. In the event of clashes being necessary, then students will be notified of the clashes by email and in any case options will only be allowed to clash when the take-up of both options is unlikely or inadvisable.

2 Mathematics Department units

C1.1a: Model Theory — Prof Zilber — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2B60).

Recommended Prerequisites

This course presupposes basic knowledge of First Order Predicate Calculus up to and including the Soundness and Completeness Theorems. A familiarity with (at least the statement of) the Compactness Theorem would also be desirable.

Overview

The course deepens a student's understanding of the notion of a mathematical structure and of the logical formalism that underlies every mathematical theory, taking B1 Logic as a starting point. Various examples emphasise the connection between logical notions and practical mathematics.

The concepts of completeness and categoricity will be studied and some more advanced technical notions, up to elements of modern stability theory, will be introduced.

Learning Outcomes

Students will have developed an in depth knowledge of the notion of an algebraic mathematical structure and of its logical theory, taking B1 Logic as a starting point. They will have an understanding of the concepts of completeness and categoricity and more advanced technical notions.

Synopsis

Structures. The first-order language for structures. The Compactness Theorem for first-order logic. Elementary embeddings. Löwenheim–Skolem theorems. Preservation theorems for substructures. Model Completeness. Quantifier elimination.

Categoricity for first-order theories. Types and saturation. Omitting types. The Ryll Nardzewski theorem characterizing aleph-zero categorical theories. Theories with few types. Ultraproducts.

Reading

1. D. Marker, *Model Theory: An Introduction* (Springer, 2002).

2. W. Hodges, *Shorter Model Theory* (Cambridge University Press, 1997).

3. J. Bridge, *Beginning Model Theory* (Oxford University Press, 1977). (Out of print but can be found in libraries.)

Further reading

1. All topics discussed (and much more) can also be found in W. Hodges, *Model Theory* (Cambridge University Press, 1993).

C1.1b: Gödel's Incompleteness Theorems — Dr Paseau — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2A60)

Recommended Prerequisites

This course presupposes knowledge of first-order predicate logic up to and including soundness and completeness theorems for a formal system of first-order predicate logic (B1 Logic).

Overview

The starting point is Gödel's mathematical sharpening of Hilbert's insight that manipulating symbols and expressions of a formal language has the same formal character as arithmetical operations on natural numbers. This allows the construction for any consistent formal system containing basic arithmetic of a 'diagonal' sentence in the language of that system which is true but not provable in the system. By further study we are able to establish the intrinsic meaning of such a sentence. These techniques lead to a mathematical theory of formal provability which generalizes the earlier results. We end with results that further sharpen understanding of formal provability.

Learning Outcomes

Understanding of arithmetization of formal syntax and its use to establish incompleteness of formal systems; the meaning of undecidable diagonal sentences; a mathematical theory of formal provability; precise limits to formal provability and ways of knowing that an unprovable sentence is true.

Synopsis

Gödel numbering of a formal language; the diagonal lemma. Expressibility in a formal language. The arithmetical undefinability of truth in arithmetic. Formal systems of arithmetic;

arithmetical proof predicates. Σ_0-completeness and Σ_1-completeness. The arithmetical hierarchy. ω-consistency and 1-consistency; the first Gödel incompleteness theorem. Separability; the Rosser incompleteness theorem. Adequacy conditions for a provability predicate. The second Gödel incompleteness theorem; Löb's theorem. Provable Σ_1-completeness. Provability logic; the fixed point theorem. The ω-rule.

Reading

1. Lecture notes for the course.

Further Reading

1. Raymond M. Smullyan, *Gödel's Incompleteness Theorems* (Oxford University Press, 1992).
2. George S. Boolos and Richard C. Jeffrey, *Computability and Logic* (3rd edition, Cambridge University Press, 1989), Chs 15, 16, 27 (pp 170–190, 268-284).
3. George Boolos, *The Logic of Provability* (Cambridge University Press, 1993).

C1.2a: Analytic Topology — Dr Suabedissen — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2A61)

Recommended Prerequisites

Part A Topology; a basic knowledge of Set Theory, including cardinal arithmetic, ordinals and the Axiom of Choice, will also be useful.

Overview

The aim of the course is to present a range of major theory and theorems, both important and elegant in themselves and with important applications within topology and to mathematics as a whole. Central to the course is the general theory of compactness and Tychonoff's theorem, one of the most important in all mathematics (with applications across mathematics and in mathematical logic) and computer science.

Synopsis

Bases and initial topologies (including pointwise convergence and the Tychonoff product topology). Separation axioms, continuous functions, Urysohn's lemma. Separable, Lindelöf and second countable spaces. Urysohn's metrization theorem. Filters and ultrafilters.

Tychonoff's theorem. Compactifications, in particular the Alexandroff One-Point Compactification and the Stone–Čech Compactification. Connectedness and local connectedness. Components and quasi-components. Totally disconnected compact spaces, Boolean algebras and Stone spaces. Paracompactness (brief treatment).

Reading

1. S. Willard, *General Topology* (Addison–Wesley, 1970), Chs. 1–8.

2. N. Bourbaki, *General Topology* (Springer-Verlag, 1989), Ch. 1.

C1.2b: Axiomatic Set Theory — Dr Suabedissen — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2B61).

Recommended Prerequisites

This course presupposes basic knowledge of First Order Predicate Calculus up to and including the Soundness and Completeness Theorems, together with a course on basic set theory, including cardinals and ordinals, the Axiom of Choice and the Well Ordering Principle.

Overview

Inner models and consistency proofs lie at the heart of modern Set Theory, historically as well as in terms of importance. In this course we shall introduce the first and most important of inner models, Gödel's constructible universe, and use it to derive some fundamental consistency results.

Synopsis

A review of the axioms of ZF set theory. The recursion theorem for the set of natural numbers and for the class of ordinals. The Cumulative Hierarchy of sets and the consistency of the Axiom of Foundation as an example of the method of inner models. Levy's Reflection Principle. Gödel's inner model of constructible sets and the consistency of the Axiom of Constructibility ($V = L$). The fact that $V = L$ implies the Axiom of Choice. Some advanced cardinal arithmetic. The fact that $V = L$ implies the Generalized Continuum Hypothesis.

Reading

For the review of ZF set theory:

1. D. Goldrei, *Classic Set Theory* (Chapman and Hall, 1996).

For course topics (and much more):

1. K. Kunen, *Set Theory: An Introduction to Independence Proofs* (North Holland, 1983) (now in paperback). Review: Chapter 1. Course topics: Chapters 3, 4, 5, 6 (excluding section 5).

Further Reading

1. K. Hrbacek and T. Jech, *Introduction to Set Theory* (3rd edition, M Dekker, 1999).

C2.1a: Lie Algebras — Dr Szendroi — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2A62)

Recommended Prerequisites

Part B course B2a. A thorough knowledge of linear algebra and the second year algebra courses; in particular familiarity with group actions, quotient rings and vector spaces, isomorphism theorems and inner product spaces will be assumed. Some familiarity with the Jordan–Hölder theorem and the general ideas of representation theory will be an advantage.

Overview

Lie Algebras are mathematical objects which, besides being of interest in their own right, elucidate problems in several areas in mathematics. The classification of the finite-dimensional complex Lie algebras is a beautiful piece of applied linear algebra. The aims of this course are to introduce Lie algebras, develop some of the techniques for studying them, and describe parts of the classification mentioned above, especially the parts concerning root systems and Dynkin diagrams.

Learning Outcomes

Students will learn how to utilise various techniques for working with Lie algebras, and they will gain an understanding of parts of a major classification result.

Synopsis

Definition of Lie algebras, small-dimensional examples, some classical groups and their Lie algebras (treated informally). Ideals, subalgebras, homomorphisms, modules.

Nilpotent algebras, Engel's theorem; soluble algebras, Lie's theorem. Semisimple algebras and Killing form, Cartan's criteria for solubility and semisimplicity, Weyl's theorem on complete reducibility of representations of semisimple Lie algebras.

The root space decomposition of a Lie algebra; root systems, Cartan matrices and Dynkin diagrams. Discussion of classification of irreducible root systems and semisimple Lie algebras.

Reading

1. J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory,* Graduate Texts in Mathematics 9 (Springer-Verlag, 1972, reprinted 1997). Chapters 1–3 are relevant and part of the course will follow Chapter 3 closely.
2. B. Hall, *Lie Groups, Lie Algebras, and Representations. An Elementary Introduction,* Graduate Texts in Mathematics 222 (Springer-Verlag, 2003).
3. K. Erdmann, M. J. Wildon, *Introduction to Lie Algebras* (Springer-Verlag, 2006), ISBN: 1846280400.

Additional Reading

1. J.-P. Serre, *Complex Semisimple Lie Algebras* (Springer, 1987). Rather condensed, assumes the basic results. Very elegant proofs.
2. N. Bourbaki, *Lie Algebras and Lie Groups* (Masson, 1982). Chapters 1 and 4–6 are relevant; this text fills in some of the gaps in Serre's text.
3. William Fulton, Joe Harris, *Representation theory: a first course*, GTM, Springer.

C2.1b: Representation Theory of Symmetric Groups — Prof. Henke —16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2B62).

Recommended Prerequisites

A thorough knowledge of linear algebra and the second year algebra courses; in particular familiarity with the symmetric groups, (symmetric) group actions, quotient vector spaces, isomorphism theorems and inner product spaces will be assumed. Some familiarity with basic representation theory from B2 (group algebras, simple modules, reducibility, Maschke's theorem, Wedderburn's theorem, characters) will be an advantage.

Overview

The representation theory of symmetric groups is a special case of the representation theory of finite groups. Whilst the theory over characteristic zero is well understood, this is not so over fields of prime characteristic. The course will be algebraic and combinatorial in flavour, and it will follow the approach taken by G. James. One main aim is to construct and parametrise the simple modules of the symmetric groups over an arbitrary field.

Synopsis

Counting standard tableaux of fixed shape: Young diagrams and tableaux, standard-tableaux, Young–Frobenius formula, hook formula. Robinson–Schensted-Knuth algorithm and correspondence.

Construction of fundamental modules for symmetric groups: Action of symmetric groups on tableaux, tabloids and polytabloids; permutation modules on cosets of Young subgroups. Specht modules, and their standard bases. Examples and applications.

Simplicity of Specht modules in characteristic zero and classification of simple S_n-module over characteristic zero. Characters of symmetric groups, Murnaghan–Nakayama rule.

Submodule Theorem, construction of simple S_n-modules over a field of prime characteristic. Decomposition matrices. Examples and applications.

Reading

1. W. Fulton, *Young Tableaux,* London Mathematical Society Student Texts 35 (Cambridge University Press, 1997). From Part I and II.

2. D. Knuth, *The Art of Computer Programming, Volume 3* (Addison–Wesley, 1998). From Chapter 5.

3. B. E. Sagan, *The Symmetric Group: Representations, Combinatorial Algorithms, and Symmetric Functions,* Graduate Texts in Mathematics 203 (Springer–Verlag, 2000). Chapters 1 – 2.

Additional Reading

1. W. Fulton, J. Harris, *Representation Theory: A first course,* Graduate Texts in Mathematics, Readings in Mathematics 129 (Springer–Verlag, 1991). From Part I.

2. G. James, *The Representation Theory of the Symmetric Groups,* Lecture Notes in Mathematics 682 (Springer–Verlag, 1978).

3. G. James, A. Kerber, *The Representation Theory of the Symmetric Groups*, Encyclopaedia of Mathematics and its Applications 16, (Addison–Wesley, 1981). From Chapter 7.

4. R. Stanley, *Enumerative Combinatorics. Volume 2*, Cambridge Studies in Advanced Mathematics 62 (Cambridge University Press, 1999).

C2.2a: Commutative Algebra — Prof Segal — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code tbc).

Recommended Prerequisites

A thorough knowledge of the second-year algebra courses, in particular rings, ideals and fields.

Overview

Amongst the most familiar objects in mathematics are the ring of integers and the polynomial rings over fields. These play a fundamental role in number theory and in algebraic geometry, respectively. The course explores the basic properties of such rings, and introduces the key concept of a module, which generalizes both abelian groups and the idea of a linear transformation on a vector space.

Synopsis

Introduction to modules. The structure of modules over a principal ideal ring. Prime ideals, maximal ideals, nilradical and Jacobson radical. Noetherian rings; Hilbert basis theorem. Minimal primes. Artin-Rees Lemma; Krull intersection theorem. Integral extensions. Prime ideals in integral extensions. Noether Normalization Lemma. Hilbert Nullstellensatz, maximal ideals. Krull dimension; Principal ideal theorem.; dimension of an affine algebra.

Reading

1. Atiyah, Macdonald, *Introduction to Commutative Algebra*, (Addison-Wesley, 1969).

2. Eisenbud *Commutative Algebra: with a View Toward Algebraic Geometry*, Grad. Texts Math. 150, (Springer-Verlag, 1995) Chapters 4, 5 and 13.

C2.2b Homological Algebra — Dr Dyckerhoff — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code tbc).

Synopsis

Chain complexes: complexes of R-modules, operations on chain complexes, long exact sequences, chain homotopies, mapping cones and cylinders (4 hours) Derived functors: delta functors, projective and injective resolutions, left and right derived functors (5 hours) Tor and Ext: Tor and flatness, Ext and extensions, universal coefficients theorems, Koszul resolutions (4 hours) Group homology and cohomology: definition, interpretation of H^1 and H^2, universal central extensions, the Bar resolution (3 hours).

Reading

Weibel, Charles *An introduction to Homological algebra* (see Google Books)

C2.3b: Infinite Groups — Dr Nikolov — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code tbc).

Recommended Prerequisites

A thorough knowledge of the second-year algebra courses; in particular, familiarity with group actions, quotient rings and quotient groups, and isomorphism theorems will be assumed. Familiarity with the Commutative Algebra course will be helpful but not essential.

Overview

The concept of a group is so general that anything which is true of all groups tends to be rather trivial. In contrast, groups that arise in some specific context often have a rich and beautiful theory. The course introduces some natural families of groups, various questions that one can ask about them, and various methods used to answer these questions; these involve among other things rings and trees.

Synopsis

Free groups and their subgroups; finitely generated groups: counting finite-index subgroups; finite presentations and decision problems; Linear groups: residual finiteness; structure of soluble linear groups; Nilpotency and solubility: lower central series and derived series; structural and residual properties of finitely generated nilpotent groups and polycyclic groups; characterization of polycyclic groups as soluble Z-linear groups; Finitely generated groups acting on rooted trees: Gupta-Sidki groups and the General Burnside Problem.

Reading

1. D. J. S. Robinson, *A course in the theory of groups*, 2nd ed., Graduate texts in Mathematics, (Springer-Verlag, 1995). Chapters 2, 5, 6, 15.
2. D. Segal, *Polycyclic groups*, (CUP, 2005) Chapters 1 and 2.

C3.1a: Algebraic Topology — Prof. Tillmann — 16MT

Level: M-level. **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code tbc).

Recommended Prerequisites

Helpful but not essential: Part A Topology, B3.1a Topology and Groups.

Overview

Homology theory is a subject that pervades much of modern mathematics. Its basic ideas are used in nearly every branch, pure and applied. In this course, the homology groups of topological spaces are studied. These powerful invariants have many attractive applications. For example we will prove that the dimension of a vector space is a topological invariant and the fact that 'a hairy ball cannot be combed'.

Learning Outcomes

At the end of the course, students are expected to understand the basic algebraic and geometric ideas that underpin homology and cohomology theory. These include the cup product and Poincaré Duality for manifolds. They should be able to choose between the different homology theories and to use calculational tools such as the Mayer-Vietoris sequence to compute the homology and cohomology of simple examples, including projective spaces, surfaces, certain simplicial spaces and cell complexes. At the end of the course, students should also have developed a sense of how the ideas of homology and cohomology may be applied to problems from other branches of mathematics.

Synopsis

Chain complexes of free Abelian groups and their homology. Short exact sequences. Delta (and simplicial) complexes and their homology. Euler characteristic.

Singular homology of topological spaces. Relative homology and the Five Lemma. Homotopy invariance and excision (details of proofs not examinable). Mayer-Vietoris Sequence. Equivalence of simplicial and singular homology.

Degree of a self-map of a sphere. Cell complexes and cellular homology. Application: the hairy ball theorem.

Cohomology of spaces and the Universal Coefficient Theorem (proof not examinable). Cup products. Künneth Theorem (without proof). Topological manifolds and orientability. The fundamental class of an orientable, closed manifold and the degree of a map between manifolds of the same dimension. Poincaré Duality (without proof).

Reading

1. A. Hatcher, *Algebraic Topology* (Cambridge University Press, 2001). Chapters 3 and 4.

2. G. Bredon, *Topology and Geometry* (Springer, 1997). Chapters 4 and 5.

3. J. Vick, *Homology Theory,* Graduate Texts in Mathematics 145 (Springer, 1973).

C3.2b: Geometric Group Theory — Dr Papazoglou — 16HT

Level: M-level. **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code tbc).

Recommended Prerequisites.

The Topology & Groups course is a helpful, though not essential prerequisite.

Overview.

The aim of this course is to introduce the fundamental methods and problems of geometric group theory and discuss their relationship to topology and geometry.

The first part of the course begins with an introduction to presentations and the list of problems of M. Dehn. It continues with the theory of group actions on trees and the structural study of fundamental groups of graphs of groups.

The second part of the course focuses on modern geometric techniques and it provides an introduction to the theory of Gromov hyperbolic groups.

Synopsis.

Free groups. Group presentations. Dehn's problems. Residually finite groups.

Group actions on trees. Amalgams, HNN-extensions, graphs of groups, subgroup theorems for groups acting on trees.

Quasi-isometries. Hyperbolic groups. Solution of the word and conjugacy problem for hyperbolic groups.

If time allows: Small Cancellation Groups, Stallings Theorem, Boundaries.

Reading.

1. J.P. Serre, *Trees* (Springer Verlag 1978).

2. M. Bridson, A. Haefliger, *Metric Spaces of Non-positive Curvature, Part III* (Springer, 1999), Chapters I.8, III.H.1, III. *Gamma* 5.

3. H. Short *et al.*, 'Notes on word hyperbolic groups', *Group Theory from a Geometrical Viewpoint, Proc. ICTP Trieste* (eds E. Ghys, A. Haefliger, A. Verjovsky, World Scientific 1990)

   available online at: http://www.cmi.univ-mrs.fr/ hamish/

4. C.F. Miller, *Combinatorial Group Theory*, notes: http://www.ms.unimelb.edu.au/ cfm/notes/cgt-notes.pdf.

Additional Reading.

1. G. Baumslag, *Topics in Combinatorial Group Theory* (Birkhauser, 1993).

2. O. Bogopolski, *Introduction to Group Theory* (EMS Textbooks in Mathematics, 2008).

3. R. Lyndon, P. Schupp, *Combinatorial Group Theory* (Springer, 2001).

4. W. Magnus, A. Karass, D. Solitar,*Combinatorial Group Theory: Presentations of Groups in Terms of Generators and Relations* (Dover Publications, 2004).

5. P. de la Harpe, *Topics in Geometric Group Theory*, (University of Chicago Press, 2000).

C3.3b: Differentiable Manifolds — Prof. Hitchin — 16HT

Level: M-level. **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code tbc).

Recommended Prerequisites

2nd year core algebra, topology, multivariate calculus. Useful but not essential: groups in action, geometry of surfaces.

Overview

A manifold is a space such that small pieces of it look like small pieces of Euclidean space. Thus a smooth surface, the topic of the B3 course, is an example of a (2-dimensional) manifold.

Manifolds are the natural setting for parts of classical applied mathematics such as mechanics, as well as general relativity. They are also central to areas of pure mathematics such as topology and certain aspects of analysis.

In this course we introduce the tools needed to do analysis on manifolds. We prove a very general form of Stokes' Theorem which includes as special cases the classical theorems of Gauss, Green and Stokes. We also introduce the theory of de Rham cohomology, which is central to many arguments in topology.

Learning Outcomes

The candidate will be able to manipulate with ease the basic operations on tangent vectors, differential forms and tensors both in a local coordinate description and a global coordinate-free one; have a knowledge of the basic theorems of de Rham cohomology and some simple examples of their use; know what a Riemannian manifold is and what geodesics are.

Synopsis

Smooth manifolds and smooth maps. Tangent vectors, the tangent bundle, induced maps. Vector fields and flows, the Lie bracket and Lie derivative.

Exterior algebra, differential forms, exterior derivative, Cartan formula in terms of Lie derivative. Orientability. Partitions of unity, integration on oriented manifolds.

Stokes' theorem. De Rham chomology. Applications of de Rham theory including degree.

Riemannian metrics. Isometries. Geodesics.

Reading

1. M. Spivak, *Calculus on Manifolds*, (W. A. Benjamin, 1965).
2. M. Spivak, *A Comprehensive Introduction to Differential Geometry*, Vol. 1, (1970).
3. W. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, 2nd edition, (Academic Press, 1986).
4. M. Berger and B. Gostiaux, *Differential Geometry: Manifolds, Curves and Surfaces.* Translated from the French by S. Levy, (Springer Graduate Texts in Mathematics, 115, Springer–Verlag (1988)) Chapters 0–3, 5–7.
5. F. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, (Springer Graduate Texts in Mathematics, 1994).
6. D. Barden and C. Thomas, *An Introduction to Differential Manifolds.* (Imperial College Press, London, 2003.)

C3.4a: Algebraic Geometry — Dr Berczi — 16MT

Level: M-level. **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code tbc)

Recommended Prerequisites

Part A Group Theory and Introduction to Fields (B3 Algebraic Curves useful but not essential).

Overview

Algebraic geometry is the study of algebraic varieties: an algebraic variety is roughly speaking, a locus defined by polynomial equations. One of the advantages of algebraic geometry is that it is purely algebraically defined and applied to any field, including fields of finite characteristic. It is geometry based on algebra rather than calculus, but over the real or complex numbers it provides a rich source of examples and inspiration to other areas of geometry.

Synopsis

Affine algebraic varieties, the Zariski topology, morphisms of affine varieties. Irreducible varieties.

Projective space and general position points. Projective varieties, affine cones over projective varieties. The Zariski topology on projective varieties. The projective closure of affine variety. Morphisms of projective varieties. Projective equivalence.

Veronese morphism: definition, examples. Veronese morphisms are isomorphisms onto their image; statement, and proof in simple cases. Subvarieties of Veronese varieties. Segre maps and products of varieties, Categorical products: the image of Segre map gives the categorical product.

Coordinate rings. Hilbert's Nullstellensatz. Correspondence between affine varieties (and morphisms between them) and finitely generate reduced k-algebras (and morphisms between them). Graded rings and homogeneous ideals. Homogeneous coordinate rings.

Categorical quotients of affine varieties by certain group actions. The maximal spectrum.

Discrete invariants projective varieties: degree dimension, Hilbert function. Statement of theorem defining Hilbert polynomial.

Quasi-projective varieties, and morphisms of them. The Zariski topology has a basis of affine open subsets. Rings of regular functions on open subsets and points of quasi-projective varieties. The ring of regular functions on an affine variety in the coordinate ring. Localisation and relationship with rings of regular functions.

Tangent space and smooth points. The singular locus is a closed subvariety. Algebraic re-formulation of the tangent space. Differentiable maps between tangent spaces.

Function fields of irreducible quasi-projective varieties. Rational maps between irreducible varieties, and composition of rational maps. Birational equivalence. Correspondence between dominant rational maps and homomorphisms of function fields. Blow-ups: of affine space at appoint, of subvarieties of affine space, and general quasi-projective varieties along general subvarieties. Statement of Hironaka's Desingularisation Theorem. Every irreducible variety is birational to hypersurface. Re-formulation of dimension. Smooth points are a dense open subset.

Reading

KE Smith et al, *An Invitation to Algebraic Geometry*, (Springer 2000), Chapters 1–8.

Further Reading

1. M Reid, *Undergraduate Algebraic Geometry*, LMS Student Texts 12, (Cambridge 1988).
2. K Hulek, *Elementary Algebraic Geometry*, Student Mathematical Library 20. (American Mathematical Society, 2003).

C3.4b: Lie Groups — Dr Ritter — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code tbc).

Recommended Prerequisites

Part A Group Theory, Topology and Multivariable Calculus.

Overview

The theory of Lie Groups is one of the most beautiful developments of pure mathematics in the twentieth century, with many applications to geometry, theoretical physics and mechanics. The subject is an interplay between geometry, analysis and algebra. Lie groups are groups which are simultaneously manifolds, that is geometric objects where the notion of differentiability makes sense, and the group multiplication and inversion are differentiable maps. The majority of examples of Lie groups are the familiar groups of matrices. The course does not require knowledge of differential geometry: the basic tools needed will be covered within the course.

Learning Outcomes

Students will have learnt the fundamental relationship between a Lie group and its Lie algebra, and the basics of representation theory for compact Lie groups. This will include a firm understanding of maximal tori and the Weyl group, and their role for representations.

Synopsis

Brief introduction to manifolds. Classical Lie groups. Left-invariant vector fields, Lie algebra of a Lie group. One-parameter subgroups, exponential map. Homomorphisms of Lie groups and Lie algebras. Ad and ad. Compact connected abelian Lie groups are tori. The Campbell-Baker-Hausdorff series (statement only).

Lie subgroups. Definition of embedded submanifolds. A subgroup is an embedded Lie subgroup if and only if it is closed. Continuous homomorphisms of Lie groups are smooth. Correspondence between Lie subalgebras and Lie subgroups (proved assuming the Frobenius theorem). Correspondence between Lie group homomorphisms and Lie algebra homomorphisms. Ado's theorem (statement only), Lie's third theorem.

Basics of representation theory: sums and tensor products of representations, irreducibility, Schur's lemma. Compact Lie groups: left-invariant integration, complete reducibility. Representations of the circle and of tori. Characters, orthogonality relations. Peter-Weyl theorem (statement only).

Maximal tori. Roots. Conjugates of a maximal torus cover a compact connected Lie group (proved assuming the Lefschetz fixed point theorem). Weyl group. Reflections. Weyl group of $U(n)$. Representations of a compact connected Lie group are the Weyl-invariant representations of a maximal torus (proof of inclusion only). Representation ring of T^n and $U(n)$.

Killing form. Remarks about the classification of compact Lie groups.

Reading

1. J. F. Adams, *Lectures on Lie Groups* (University of Chicago Press, 1982).
2. T. Bröcker and T. tom Dieck, *Representations of Compact Lie Groups* (Graduate Texts in Mathematics, Springer, 1985).

Further Reading

1. R. Carter, G. Segal and I. MacDonald, *Lectures on Lie Groups and Lie Algebras* (LMS Student Texts, Cambridge, 1995).
2. W. Fulton, J. Harris, *Representation Theory: A First Course* (Graduate Texts in Mathematics, Springer, 1991).
3. F. W. Warner, *Foundations of Differentiable Manifolds and Lie Groups* (Graduate Texts in Mathematics, 1983).

C4.1a: Functional Analysis — Prof. Kristensen — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2A64)

Recommended Prerequisites

Part A Topology, B4 Analysis

Overview

This course builds on B4, by extending the theory of Banach spaces and operators. As well as developing general methods that are useful in Operator Theory, we shall look in more detail at the structure and special properties of "classical" sequence-spaces and function-spaces.

Synopsis

Normed spaces and Banach spaces; dual spaces, subspaces, direct sums and completions; quotient spaces and quotient operators.

Baire's Category Theorem and its consequences (review).

Classical Banach spaces and their duals; smoothness and uniform convexity of norms.

Compact sets and compact operators. Ascoli's theorem.

Hahn–Banach extension and separation theorems; the bidual space and reflexivity.

Weak and weak* topologies. The Banach–Alaoglu theorem and Goldstine's theorem. Weak compactness.

Schauder bases; examples in classical spaces. Gliding-hump arguments.

Fredholm operators.

Reading

1. M. Fabian et al., *Functional Analysis and Infinite-Dimensional Geometry* (Canadian Math. Soc, Springer 2001), Chapters 1,2,3,6,7.

Alternative Reading

1. N. L. Carothers, *A Short Course on Banach Space Theory*, (LMS Student Text, Cambridge University Press 2004).

C4.1b Linear Operators — Prof. Batty — 16HT

Level: M-level **Method of assessment:** Written examination.

Weight: Unit (OSS paper code to follow)

Recommended Prerequisites

Essential: B4a, B4b. Useful: C4.1a

Overview

Many of the linear operators that arise in mathematical physics and models from other sciences are not bounded operators. Typically they are defined on a dense subspace of a Banach or Hilbert space. They may be closed operators, but sometimes it is necessary to find the appropriate closed extension of the operator and the domain of the extension may be unclear. This course describes some of the theory of unbounded operators, particularly spectral properties of closed operators and ways to convert them into bounded operators.

Learning Outcomes

Synopsis

Review of bounded operators and spectrum. Holomorphic functional calculus.

Unbounded operators; closed and closable operators; adjoints, spectrum.

Operators on Hilbert space; symmetric, self-adjoint, essentially self-adjoint. Spectral theorem and functional calculus. Quadratic forms, simple differential and Schrödinger operators.

Sectorial operators, semigroup generators

Perturbation theory: bounded, relatively bounded perturbations.

Reading

E.B. Davies, Linear operators and their spectra, CUP, 2007

Further Reading

M. Schechter, Principles of functional analysis, 2nd ed, AMS, 2002

M. Reed & B. Simon, Methods of modern mathematical physics I,II, Academic Press, 1972, 1975

T. Kato, [A short introduction to] Perturbation theory for linear operators, Springer [1982], 2nd ed, 1995.

E.B. Davies, Differential operators and spectral theory, CUP, 1995

C5.1a Methods of Functional Analysis for PDEs — Prof. Seregin — 16MT

Only available to students who have not offered C5.1a Methods of Functional Analysis for PDEs at Part B.

Level: M-level **Method of assessment:** Written examination.

Weight: Unit (OSS paper code 2A65)

Recommended Prerequisites

Part A Integration. There will be a 'Users' Guide to Integration' on the subject website and anyone who has not done Part A Integration can read it up over the summer vacation. In addition some knowledge of functional analysis, in particular Banach spaces (as in B4) and compactness (as in Part A Topology), is useful. We will however recall the relevant definitions as we go along so these prerequisites are not strictly needed.

Overview

The course will introduce some of the modern techniques in partial differential equations that are central to the theoretical and numerical treatments of linear and nonlinear partial differential equations arising in science, geometry and other fields.

It provides valuable background for the Part C courses on Calculus of Variations, Fixed Point Methods for Nonlinear PDEs, and Finite Element Methods.

Learning Outcomes

Students will learn techniques and results about Lebesgue and Sobolev Spaces, distributions and weak derivatives, embedding theorems, traces, weak solution to elliptic PDE's, existence, uniqueness, and smoothness of weak solutions.

Synopsis

Why functional analysis methods are important for PDE's?

Revision of relevant definitions and statements from functional analysis: completeness, seperability, compactness, and duality.

Revision of relevant definitions and statements from Lebesgue integration theory: sequences of measurable functions, Lebesgue and Riesz theorems.

Lebesgue spaces: completeness, dense sets, linear functionals and weak convergence.

Distributions and distributional derivatives.

Sobolev spaces: mollifications and weak derivatives, completeness, Friedrichs inequality, star-shaped domains and dense sets, extension of functions with weak derivatives.

Embedding of Sobolev spaces into Lebesgue spaces: Poincare inequality, Reillich-Kondrachov-Sobolev theorems on compactness.

Traces of functions with weak derivatives.

Dirichlet boundary value problems for elliptic PDE's, Fredholm Alternative (uniqueness implies existence).

Smoothness of weak solutions: embedding from Sobolev spaces into spaces of continuous functions, interior regularity of distributional solutions to elliptic equations with smooth coefficients.

Reading

Lawrence C. Evans, *Partial differential equations*, (Graduate Studies in Mathematics 2004), American Mathematical Society

Elliott H. Lieb and Michael Loss, *Analysis*, 2nd Edition, (Graduate Studies in Mathematics 2001), American Mathematical Society

Additional Reading

E. Kreyszig, *Introductory Functional Analysis with Applications*, Wiley (revised edition, 1989)

P.D. Lax *Functional analysis* (Wiley-Interscience, New York, 2002).

J. Rauch, *Partial differential equations*, (Springer–Verlag, New York, 1992).

C5.1b Fixed Point Methods for Nonlinear PDEs — Dr Nguyen — 16HT

Level: M-level **Method of assessment:** Written examination.

Weight: Unit (OSS paper code 2B75)

Recommended Prerequisites

C5.1a: Methods of Functional Analysis for PDEs. Some knowledge of functional analysis, in particular Banach spaces (as in B4) and compactness (as in Part A Topology), is useful.

Overview

This course gives an introduction to the techniques of nonlinear functional analysis with emphasis on the major fixed point theorems and their applications to nonlinear differential equations and variational inequalities, which abound in applications such as fluid and solid mechanics, population dynamics and geometry.

Learning Outcomes

Besides becoming acquainted with the fixed point theorems of Banach, Brouwer and Schauder, students will see the abstract principles in a concrete context. Hereby they also reinforce techniques from elementary topology, functional analysis, Banach spaces, compactness methods, calculus of variations and Sobolev spaces.

Synopsis

Examples of nonlinear differential equations and variational inequalities. Contraction Mapping Theorem and applications. Brouwer's fixed point theorem, proof via Calculus of Variations and Null-Lagrangians. Compact operators and Schauder's fixed point theorem. Applications of Schauder's fixed point theorem to nonlinear elliptic equations. Variational inequalities and monotone operators. Applications of monotone operator theory to nonlinear elliptic equations (p-Laplacian, stationary Navier-Stokes)

Reading

1. Lawrence C. Evans, *Partial Differential Equations,* Graduate Studies in Mathematics (American Mathematical Society, 2004).
2. E. Zeidler, *Nonlinear Functional Analysis I & II* (Springer-Verlag, 1986/89).
3. M. S. Berger, *Nonlinearity and Functional Analysis* (Academic Press, 1977).
4. K. Deimling, *Nonlinear Functional Analysis* (Springer-Verlag, 1985).
5. L. Nirenberg, *Topics in Nonlinear Functional Analysis,* Courant Institute Lecture Notes (American Mathematical Society, 2001).
6. R.E. Showalter, *Monotone Operators in Banach Spaces and Nonlinear Partial Differential Equations,* Mathematical Surveys and Monographs, vol.49 (American Mathematical Society, 1997).

C5.2b: Calculus of Variations — Prof. Shkoller — 16HT

Weight: Unit, OSS paper code 2B65

Recommended Prerequisites

C5.1a: Methods of Functional Analysis for PDEs. Some familiarity with the Lebesgue integral is essential, and some knowledge of elementary functional analysis (e.g. Banach spaces and their duals, weak convergence) an advantage.

B5.1a: Dynamical Systems and Energy Minimization: Some knowledge of local minimizers in the 1D calculus of variations is helpful. This material will be reviewed in the course.

Overview

The aim of the course is to give a modern treatment of the calculus of variations from a rigorous perspective, blending classical and modern approaches and applications.

Learning Outcomes

Students will learn rigorous results in the classical and modern one-dimensional calculus of variations and see possible behaviour and application of these results in examples. They will see some examples of multi-dimensional problems.

Synopsis

Classical and modern examples of variational problems (e.g. brachistochrone, models of phase transformations).

One-dimensional problems, function spaces and definitions of weak and strong relative minimizers. Necessary conditions; the Euler-Lagrange and Du Bois-Reymond equations, theory of the second variation, the Weierstrass condition. Sufficient conditions; field theory and sufficiency theorems for weak and strong relative minimizers. The direct method of the calculus of variations and Tonelli's existence theorem. Regularity of minimizers. Examples of singular minimizers and the Lavrentiev phenomenon. Problems whose infimum Is not attained. Relaxation and generalized solutions. Isoperimetric problems and Lagrange multipliers. Invariant variational problems, Noether's theorem, conservation laws.

Multi-dimensional problems, done via some examples.

Reading

1. G. Buttazzo, M. Giaquinta, S. Hildebrandt, *One-dimensional Variational Problems,* Oxford Lecture Series in Mathematics, Vol. 15 (Oxford University Press, 1998). Ch 1, Sections 1.1, 1.2 (treated differently in course), 1.3, Ch 2 (background), Ch 3, Sections 3.1, 3.2, Ch 4, Sections 4.1, 4.3.

Further Reading

1. U. Brechtken-Manderscheid, *Introduction to the Calculus of Variations* (Chapman & Hall, 1991).

2. H. Sagan, *Introduction to the Calculus of Variations* (Dover, 1992).

3. J. Troutman, *Variational Calculus and Optimal Control* (Springer-Verlag, 1995).

4. L. C. Evans, *Partial Differential Equations* (American Mathematical Society, 2010).

C5.3b: Hyperbolic Equations — Dr Wang — 16HT

Weight: Unit, OSS paper code to follow

Recommended Prerequisites

Part A Integration, Part A Topology, it would also be useful if the students had attended B4 (Banach and Hilbert Spaces) B5b (Applied PDEs) and C5.1a (Functional Analytic Methods for PDEs). We expand on the themes briefly discussed in B5b (Applied PDES), and provide a rigorous treatment in the frame work of Sobolev spaces.

Overview

We introduce geometric and analysis approaches to hyperbolic equations, by discussing model problems from wave equations and conservation laws. These approaches have been applied and extended extensively in recent research, and lie in the heart of theory of hyperbolic PDEs.

Learning Outcomes

Synopsis

1. Sobolev space and Sobolev inequalities
2. Nonlinear first order equations: Eikonal equations and method of characteristics
3. Introduction to conservation laws in one space dimension (shocks, simple waves, rarefaction waves, Riemann problem)
4. Theory of linear wave equation : The solution of Cauchy problem, energy estimates, finite speed of propagation, domain of determination, ligntcone and null frames, hyperbolic rotation and Lorentz vector fields, Klainerman inequality.
5. Weak solution of wave equation, and local well-posedness
6. Littlewood-Paley theory and harmonic analysis technique for wave equation (off syllabus - not required for exam)

Reading

We follow the main structure of [1], and refer to [3] and [2, Chapter 3,5,7] for detailed exposition. We use notes by Tao, T. to present the last topic.

1. Alinhac, S, *Hyperbolic partial differential equations, an elementary introduction*, 2008, http://www.math.u-psud.fr/ alinhac/tot1.pdf
2. Evans, L, *Partial differential equations.* Second edition. Graduate Studies in Mathematics, 19. American Mathematical Society, 2010.
3. John, F, *Partial differential equations.* Fourth edition. Applied Mathematical Sciences, 1. Springer-Verlag, New York, 1982

C6.1a: Solid Mechanics — Prof Goriely — 16 MT

[This course will run if teaching resources allow]

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit. OSS paper code 2466.

Prerequisites

There are no formal prerequisites. In particular it is not necessary to have taken any courses in fluid mechanics, though having done so provides some background in the use of similar concepts. Use is made of (i) elementary linear algebra in (e.g., eigenvalues, eigenvectors and diagonalization of symmetric matrices, and revision of this material, for example from the Mods Linear Algebra course, is useful preparation); and (ii) some 3D calculus (mainly differentiation of vector-valued functions of several variables). All necessary material is summarized in the course.

Overview

Solid mechanics is a vital ingredient of materials science and engineering, and is playing an increasing role in biology. It has a rich mathematical structure. The aim of the course is to derive the basic equations of elasticity theory, the central model of solid mechanics, and give some interesting applications to the behaviour of materials.The course is useful preparation for C6.1b Elasticity and Plasticity. Taken together the two courses will provide a broad overview of modern solid mechanics, with a variety of approaches.

Learning Outcomes

Students will learn basic techniques of modern continuum mechanics, such as kinematics of deformation, stress, constitutive equations and the relation between nonlinear and linearized models. The emphasis on the course is on the structure of the models, but some applications are also discussed.

Synopsis

Kinematics: Lagrangian and Eulerian descriptions of motion, deformation gradient, invertibility

Analysis of strain: polar decomposition, stretch tensors, Cauchy–Green tensors Stress Principle: forces in continuum mechanics, balance of forces, Cauchy stress tensor, the Piola–Kirchhoff stress

Constitutive Models: stress-strain relations, hyperelasticity and stored energy function, boundary value problems, the variational problem, frame indifference, material symmetry, isotropic materials

Further topics: incompressible elasticity, linearized elasticity and the shape-memory effect in crystalline solids.

Reading

1. O. Gonzales and A. Stuart, *A first course in continuum mechanics*, (Cambridge University Press, 2008).
2. M. E. Gurtin, *A introduction to continuum mechanics*, (Academic Press, 1981).

Further Reading

1. P. G. Ciarlet, *Mathematical Elasticity.* Vol. I Three-dimensional Elasticity, (North-Holland, 1988)
2. S. S. Antman, Nonlinear *Problems of Elasticity*, (Springer, 1995)
3. J. E. Marsden and T.J.R. Hughes, *Mathematical Foundations of Elasticity*, Prentice–Hall, 1983

C6.1b: Elasticity and Plasticity — Dr Howell — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit. OSS paper code 2467.

Recommended Prerequisites

Familiarity will be assumed with Part A Complex Analysis, Differential Equations and Calculus of Variations, as well as B568 Introduction to Applied Mathematics. A basic understanding of stress tensors from either B6a Viscous Flow or C6.2a Solid Mechanics will also be required. The following courses are also helpful: B5a Techniques of Applied Mathematics, B5b Applied Partial Differential Equations, C6.3a Perturbation Methods, C6.3b Applied Complex Variables.

Overview

The course starts with a rapid overview of mathematical models for basic solid mechanics. Benchmark solutions are derived for static problems and wave propagation in linear elastic materials. It is then shown how these results can be used as a basis for practically useful problems involving thin beams and plates. Simple geometrically nonlinear models are then introduced to explain buckling, fracture and contact. Models for yield and plasticity are then discussed, both microscopically and macroscopically.

Synopsis

Review of tensors, conservation laws, Navier equations. Antiplane strain, torsion, plane strain. Elastic wave propagation, Rayleigh waves. Ad hoc approximations for thin materials; simple bifurcation theory and buckling. Simple mixed boundary value problems, brittle fracture and smooth contact. Perfect plasticity theories for granular materials and metals.

Reading

1. P. D. Howell, G. Kozyreff and J. R. Ockendon, *Applied Solid Mechanics* (Cambridge University Press, 2008).
2. S. P. Timoshenko and J. N. Goodier, *Theory of Elasticity* (McGraw-Hill, 1970).
3. L.D. Landau and E.M. Lifshitz, *Theory of Elasticity* (Pergamon Press, 1986).

C6.2a: Statistical Mechanics — Dr Fowler — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit. OSS paper code tbc.

Recommended Prerequisites

A familiarity with classical mechanics, probability and fluid mechanics will be helpful.

Overview

Statistical mechanics is a subject which has fundamental and powerful connections with probability, mechanics, stochastic processes, fluid mechanics, thermodynamics, quantum mechanics (though we avoid this), and even philosophy. It is also notoriously inaccessible to applied mathematicians. This course will endeavour to trace a rational path towards classical statistical mechanics, beginning with classical mechanics, and then developing the concepts of thermodynamics through study of the Boltzmann equation. In passing, we derive the Navier-Stokes equations, before developing a mechanically-based formulation of thermodynamics and its famous second law concerning entropy. The latter parts of the course develop a variety of applications of current interest.

Learning Outcomes

Students will have developed a sound knowledge and appreciation of some of the tools, concepts, and computations used in the study of statistical mechanics. They will also get some exposure to some modern research topics in the field.

Synopsis

Classical mechanics: Newton's second law, D'Alembert's principle, Lagrange's equations, Hamilton's equations. Probability: probability density functions, moment generating function, central limit theorem. Fluid mechanics: material derivative, Euler and Navier-Stokes equations, energy equation. Random walks, Brownian motion, diffusion equation. Loschmidt's paradox.

Liouville equation, BBGKY hierarchy, Boltzmann equation. The collision integral for a hard sphere gas. Boltzmann H theorem. Maxwellian distribution. Definition of entropy and temperature. Gibbs and Helmholtz free energies. Thermodynamic relations.

Classical statistical mechanics. Ergodic theorem, equiprobability. Microcanonical ensemble for the hard sphere gas, entropy. Canonical ensemble.

Selected applications and extensions: for example, chemical potential, phase change, binary alloys, surface energy, radiative transfer, polymer solution theory, Arrhenius kinetics, nucleation theory, percolation theory, renormalisation.

Reading

1. David Chandler, *Introduction to Modern Statistical Mechanics* (Oxford University Press 1987)

2. M. Kardar, *Statistical Physics of Particles* (Cambridge University Press 2007)

3. M. Kardar, *Statistical Physics of Fields* (Cambridge University Press 2007)

4. F. Schwabl, *Statistical Mechanics* 2nd ed. (Springer-Verlag 2006)

5. J.P. Sethna, *Statistical Mechanics: Entropy, Order Parameters, and Complexity* (Oxford University Press 2006) [available online at http://pages.physics.cornell.edu/sethna/StatMech]

C6.2b: Networks — Dr Porter — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit. OSS paper code tbc.

Recommended Prerequisites

None [in particular, C6.2a (Statistical Mechanics) is not required], though some intuition from modules like C6.2a, the Part B graph theory course, and probability courses (at the level that everybody has to take anyway) can be useful. However, everything is self-contained, and none of these courses are required. Some computational experience is also helpful, and ideas from linear algebra will certainly be helpful.

Overview

This course aims to provide an introduction to network science, which can be used to study complex systems of interacting agents. Networks are interesting both mathematically and computationally, and they are pervasive in physics, biology, sociology, information science, and myriad other fields. The study of networks is one of the “rising stars” of scientific endeavors, and networks have become among the most important subjects for applied mathematicians to study. Most of the topics to be considered are active modern research areas.

Learning Outcomes

Students will have developed a sound knowledge and appreciation of some of the tools, concepts, and computations used in the study of networks. The study of networks is predominantly a modern subject, so the students will also be expected to develop the ability to read and understand current (2013) research papers in the field.

Synopsis

1. Introduction and Basic Concepts (1-2 lectures): nodes, edges, adjacencies, weighted networks, unweighted networks, degree and strength, degree distribution, other types of networks

2. Small Worlds (2 lectures): clustering coefficients, paths and geodesic paths, Watts-Strogatz networks [focus is on modelling and heuristic calculations]

3. Toy Models of Network Formation (2 lectures): preferential attachment, generalizations of preferential attachment, network optimization

4. Additional Summary Statistics and Other Useful Concepts (2 lectures): modularity and assortativity, degree-degree correlations, centrality measures, communicability, reciprocity and structural balance

5. Random Graphs (2 lectures): Erdős-Rényi graphs, configuration model, random graphs with clustering, other models of random graphs or hypergraphs; application of generating-function methods [focus is on modelling and heuristic calculations; material in this section forms an important basis for sections 6 and 7]

6. Community Structure and Mesoscopic Structure (2 lectures): linkage clustering, optimization of modularity and other quality functions, overlapping communities, other methods and generalizations

7. Dynamics on (and of) Networks (3-4 lectures): general ideas, models of biological and social contagions, percolation, voter and opinion models, temporal networks, other topics

8. Additional Topics (0-2 lectures): games on networks, exponential random graphs, network inference, other topics of special interest to students [depending on how much room there is and interest of current students]

Reading

(most important are [2] and [3]):

1. A. Barrat et al, *Dynamical Processes on Complex Networks*, Cambridge University Press, 2008
2. M. E. J. Newman, *Networks: An Introduction*, Oxford University Press, 2010 [also, Newman's 2003 review article in SIAM Review for "older" topics]
3. M. A. Porter, *A Terse Introduction to Networks*, Springer, in preparation
4. Various papers and review articles (e.g. Bocaletti et al, *Physics Reports*, 2006 as well as reviews on more specific topics, such as Porter et al, *Notices* AMS, 2009 for community structure and Holme & Saramaki, arXiv paper, 2011 for temporal networks).
5. Other networks books are also useful. (I will point interested students to them if they ask, but I have listed enough things here. They can also look in the references in the textbook I am writing.)

C6.3a: Perturbation Methods — Dr Oliver — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2A68)

Recommended Prerequisites

Part A Differential Equations and Core Analysis (Complex Analysis). B5, B6 and B8 are helpful but not officially required.

Overview

Perturbation methods underlie numerous applications of physical applied mathematics: including boundary layers in viscous flow, celestial mechanics, optics, shock waves, reaction-diffusion equations, and nonlinear oscillations. The aims of the course are to give a clear and systematic account of modern perturbation theory and to show how it can be applied to differential equations.

Synopsis

Asymptotic expansions. Asymptotic evaluation of integrals (including Laplace's method, method of stationary phase, method of steepest descent). Regular and singular perturbation theory. Multiple-scale perturbation theory. WKB theory and semiclassics. Boundary layers and related topics. Applications to nonlinear oscillators. Applications to partial differential equations and nonlinear waves.

Reading

1. E.J. Hinch, *Perturbation Methods* (Cambridge University Press, 1991), Chs. 1–3, 5–7.
2. C.M. Bender and S.A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers* (Springer, 1999), Chs. 6, 7, 9–11.
3. J. Kevorkian and J.D. Cole, *Perturbation Methods in Applied Mathematics* (Springer-Verlag, 1981), Chs. 1, 2.1–2.5, 3.1, 3.2, 3.6, 4.1, 5.2.

C6.3b: Applied Complex Variables — Dr Oliver — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2B68).

Recommended Prerequisites

The course requires second year core analysis (complex analysis). It continues the study of complex variables in the directions suggested by contour integration and conformal mapping. Part A Fluid Dynamics and Waves and Part C Perturbation Methods are desirable

Overview

The course begins where core second-year complex analysis leaves off, and is devoted to extensions and applications of that material. It is assumed that students will be familiar with inviscid two-dimensional hydrodynamics (Part A Fluid Dynamics and Waves) to the extent of the existence of a harmonic stream function and velocity potential in irrotational icompressible flow, and Bernoulli's equation.

Synopsis

Review of core complex analysis, especially continuation, multifunctions, contour integration, conformal mapping and Fourier transforms.

Riemann mapping theorem (in statement only). Schwarz-Christoffel formula. Solution of Laplace's equation by conformal mapping onto a canonical domain. Applications to inviscid hydrodynamics: flow past an aerofoil and other obstacles by conformal mapping; free streamline flows of hodograph plane. Unsteady flow with free boundaries in porous media.

Application of Cauchy integrals and Plemelj formulae. Solution of mixed boundary value problems motivated by thin aerofoil theory and the theory of cracks in elastic solids. Reimann-Hilbert problems. Cauchy singular integral equations. Transform methods, complex Fourier transform. Contour integral solutions of ODE's. Wiener-Hopf method.

Reading

1. G.F. Carrier, M. Krook and C.E. Pearson, *Functions of a Complex Variable*(Society for Industrial and Applied Mathematics, 2005.) ISBN 0898715954.
2. M. J. Ablowitz and A. S. Fokas, *Complex Variables: Introduction and Applications* (2nd edition, Cambridge University Press., Cambridge, 2003). ISBN 0521534291.
3. J. Ockendon, Howison, Lacey and Movichan, *Applied Partial Differential Equations* (Oxford, 1999) Pages 195–212.

C6.4a: Topics in Fluid Mechanics — Dr Vella — 16MT

Level: M-level **Method of Assessment:** Written examination,

Weight: Unit. OSS paper code 2A74.

Prerequisites

B6 fluid mechanics.

Overview

The course will expand and illuminate the 'classical' fluid mechanics taught in the third year course B6, and illustrate its modern application in a number of different areas in industry and geoscience.

Synopsis

Thin film flows: coatings and foams. Lubrication theory: gravity flows, Marangoni effects. Droplet dynamics, contact lines, menisci. Drying and wetting.

Flow in porous media: Darcy's law; thermal and solutal convection; gravity-driven flow and carbon sequestration.

Rotating flows: atmosphere and oceans. Waves, geostrophy, quasi-geostrophy, baroclinic instability.

Reading

1. L.G Leal, *Advanced Transport Phenomena*,(Cambridge University Press, Cambridge, 2007).

2. O.M. Phillips, *Geological Fluid Dynamics*,(Cambridge University Press, Cambridge, 2009).

3. J.S. Turner, *Buoyancy Effects in Fluids*, (Cambridge University Press, Cambridge, 1973).

4. D.G Andrews, *An Introduction to Atmospheric Physics*, (Cambridge University Press, Cambridge, 2010).

5. G.R. Vallis, *Atmospheric and Oceanic Fluid Dynamics*, (Cambridge University Press, Cambridge, 2006).

Further Reading

1. G. K. Batchelor, H. K. Moffatt and M. G. Worster (eds.), *Perspectives in Fluid Dynamics* (Cambridge University Press, Cambridge, 2000).

C6.4b: Stochastic Modelling of Biological Processes – Dr Erban – 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit, (OSS paper code tbc)

Recommended Prerequisites

A basic understanding of probability is sufficient. The course is designed in such a way that a Part C student should be able to understand it without taking special stochastic or biological classes. Computer simulations play an important role in stochastic modelling in biology. Previous experience with programming is not needed, but enthusiasm and willingness to implement simple models on the computer will significantly enhance understanding of the underlying mathematical concepts.

Overview

This course provides an overview of stochastic methods which are used for modelling biological systems. The course starts with stochastic modelling of chemical reactions, introducing stochastic simulation algorithms and mathematical methods which can be used for analysis of stochastic models (chemical master equation). Systems with increasing level of complexity are used to illustrate the theory. Then stochastic differential equations are introduced (from the computational point of view), explaining their connections with modelling chemical systems and the Fokker-Planck equation. Different models of molecular diffusion (on-lattice and off-lattice models, velocity jump processes) and their properties are studied, before moving to stochastic reaction-diffusion models. Compartment-based and molecular-based approaches to stochastic reaction-diffusion modelling (Brownian dynamics) are discussed together with properties of stochastic spatially-distributed models (pattern formation). The final lectures include discussion of bacterial chemotaxis, Metropolis-Hastings algorithm and multiscale modelling.

Learning Outcomes

The student will learn: (i) about biological systems which are often described in terms of stochastic models; (ii) mathematical techniques which are used for the analysis of stochastic models; (iii) how the models can be efficiently simulated using a computer; (iv) connections and differences between different stochastic methods, and between stochastic and deterministic modelling.

Synopses

Stochastic simulation of chemical reactions: well-stirred systems, Gillespie algorithm, chemical master equation, analysis of simple systems, deterministic vs. stochastic modelling, systems with multiple favourable states, stochastic resonance, stochastic focusing.

Stochastic differential equations: numerical methods, Fokker-Planck equation, first exit time, backward Kolmogorov equation, chemical Fokker-Planck equation.

Diffusion: Brownian motion, on-lattice and off-lattice models, compartment-based approach, velocity jump processes, Einstein-Smoluchowski relation, diffusion to adsorbing surfaces, reactive boundary conditions.

Stochastic reaction-diffusion models: compartment-based reaction-diffusion algorithm, reaction-diffusion master equation, pattern formation, morphogen gradients, Turing patterns, molecular-based approaches to reaction-diffusion modelling, Brownian dynamics, reaction radius.

Bacterial chemotaxis: reaction-diffusion-advection processes, velocity jump processes with internal dynamics, agent-based modelling.

Metropolis-Hastings algorithm: Markov chain Monte Carlo methods.

Multiscale modelling: efficient stochastic modelling of chemical reactions, multiscale SSA with partial equilibrium assumption, hybrid modelling approaches.

Reading

1. R. Erban, J. Chapman and P. Maini: "A practical guide to stochastic simulations of reaction-diffusion processes, available as http://arxiv.org/abs/0704.1908, 2007 (course lecture notes extend this material)

Further Reading

1. H. Berg: "Random Walks in Biology", new, expanded edition, Princeton University Press, 1993,

2. D. Gillespie: "Markov Processes, an Introduction for Physical Scientists", Academic Press, Inc.,1992

3. A.Chorin and O.Hald: "Stochastic Tools for Mathematics and Science", 2nd Edition, Springer, 2009

C6.5b: Mathematical Mechanical Biology — Prof Goriely — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit. OSS paper code tbc.

Recommended Prerequisites

Fluid Mechanics: Part A Fluid Dynamics and Wave and at least one of Part B fluids is recommended. Solid mechanics: One of the Part C course (Solid mechanics or Elasticity/Plasticity is recommended) Mathematical biology or Physiology is desirable but not necessary as the material for a particular biological system will be part of the course.

Overview

The course will be motivated by outstanding problems in physiology and biology but the emphasis is on the mathematical tools needed to answer some biologically relevant problems.

Learning Outcomes

Synopsis

1. Bio-Fluids (2 weeks) (a) Low Reynolds Number: Motility, Scallop theorem. (b) Complex biofluids: active and non-Newtonian fluids (c) Circulation: Blood flow, microcirculation, networks

2. Bio-Solids (2 weeks) (a) Large deformation biomechanics (b) Active stresses and morphoelasticity (c) Bio-filaments, bio-membranes (d) Application to morphogenesis, bone, arteries, heart, lung, brain mechanics

3. Multiphase/Multiphysics methods (2 weeks) (a) Coupling fluids and solids: poroelastic tissue (b) Coupling fluid, solids and chemistry: tissue swelling (c) A general thermodynamics approach (d) Application to tissue engineering, wound healing.

4. Topics (2 weeks) The last 2 weeks will be dedicated to a single biological system and will use the methods seen in the first 6 weeks. The topic will change from year to year but will include in the first years: Tumour Growth, Modelling the Cell, Tissue Engineering, The cardiovascular system (heart and arteries), Brain mechanics, Cell Mechanics. It will be based on the current research by Oxford faculty.

Reading

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C7.1b: Quantum Theory and Quantum Computers — Prof. Ekert and Prof Mason — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit, OSS paper code 2A78.

Prerequisites

B7.1a Quantum Mechanics.

Overview

This course builds directly on the first course in quantum mechanics and covers a series of important topics, particularly features of systems containing several particles. The behaviour of identical particles in quantum theory is more subtle than in classical mechanics, and an understanding of these features allows one to understand the periodic table of elements and the rigidity of matter. It also introduces a new property of entanglement linking particles which can be quite widely dispersed.

There are rarely neat solutions to problems involving several particles, so usually one needs some approximation methods. In very complicated systems, such as the molecules of gas in a container, quantum mechanical uncertainty is compounded by ignorance about other details of the system and requires tools of quantum statistical mechanics.

Two state quantum systems enable one to encode binary information in a new way which permits superpositions. This leads to a quantum theory of information processing, and by exploiting entanglement to other ideas such as quantum teleportation.

Learning Outcomes

Students will be able to demonstrate knowledge and understanding of quantum mechanics of many particle systems, statistics, entanglement, and applications to quantum computing.

Synopsis

Identical particles, symmetric and anti-symmetric states, Fermi-Dirac and Bose-Einstein statistics and atomic structure.

Heisenberg representation, interaction representation, time dependent perturbation theory and Feynman–Dyson expansion. Approximation methods, Rayleigh-Schrö)dinger time-independent perturbation theory and variation principles. The virial theorem. Helium.

Mixed states, density operators. The example of spin systems. Purification. Gibbs states and the KMS condition.

Entanglement. The EPR paradox, Bell's inequalities, Aspect's experiment.

Quantum information processing, qubits and quantum computing. The no-cloning theorem, quantum teleportation. Quantum logic gates. Quantum operations. The quantum Fourier transform.

Reading

K. Hannabuss, *Introduction to Quantum Mechanics* (oup, 1997). Chapters 10–12 and 14, 16, supplemented by lecture notes on quantum computers on the web.

Further Reading

A popular non-technical account of the subject:

A. Hey and P. Walters, *The New Quantum Universe* (Cambridge, 2003).

Also designed for an Oxford course, though only covering some material:

I. .P Grant, *Classical and Quantum Mechanics*, Mathematical Institute Notes (1991).

A concise account of quantum information theory:

S. Stenholm and K.-A. Suominen, *Quantum Approach to Informatics* (Wiley, 2005).

An encyclopaedic account of quantum computing:

M. A. Nielsen and I. L. Chuang, *Quantum Computation* (Cambridge University Press, 2000).

Even more paradoxes can be found in:

Y. Aharanov and D. Rohrlich, *Quantum Paradoxes* (Wiley–VCH, 2005).

Those who read German can find further material on entanglement in:

J. Audretsch, *Verschränkte Systeme* (Wiley–VCH, 2005).

Other accounts of the first part of the course:

L. I. Schiff, *Quantum Mechanics* (3rd edition, Mc Graw Hill, 1968).

B. J. Bransden and C. J. Joachain, *Introduction to Quantum Mechanics* (Longman, 1995).

A. I. M. Rae, *Quantum Mechanics* (4th edition, Institute of Physics, 1993).

John Preskill's on-line lecture notes (http://www.theory.caltech.edu/ preskill/ph219/index.html).

C7.2a: General Relativity I — Dr Lipstein — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit. OSS paper code 2B78.

Recommended Prerequisites

B7.2a Relativity and Electromagnetism.

Overview

The course is intended as an elementary introduction to general relativity, the basic physical concepts of its observational implications, and the new insights that it provides into the nature of space time, and the structure of the universe. Familiarity with special relativity and electromagnetism as covered in the B7 course will be assumed. The lectures will review Newtonian gravitation, tensor calculus and continuum physics in special relativity, physics in curved space time and the Einstein field equations. This will suffice for an account of simple applications to planetary motion, the bending of light and the existence of black holes.

Learning Outcomes

This course starts by asking how the theory of gravitation can be made consistent with the special-relativistic framework. Physical considerations (the principle of equivalence, general covariance) are used to motivate and illustrate the mathematical machinery of tensor calculus. The technical development is kept as elementary as possible, emphasising the use of local inertial frames. A similar elementary motivation is given for Einstein's equations and the Schwarzschild solution. Orbits in the Schwarzschild solution are given a unified treatment which allows a simple account of the three classical tests of Einstein's theory. Finally, the analysis of extensions of the Schwarzschild solution show how the theory of black holes emerges and exposes the radical consequences of Einstein's theory for space-time structure. Cosmological solutions are not discussed.

The learning outcomes are an understanding and appreciation of the ideas and concepts described above.

Synopsis

Review of Newtonian gravitation theory and problems of constructing a relativistic generalisation. Review of Special Relativity. The equivalence principle. Tensor formulation of special relativity (including general particle motion, tensor form of Maxwell's equations and the energy momentum-tensor of dust). Curved space time. Local inertial coordinates. General coordinate transformations, elements of Riemannian geometry (including connections, curvature and geodesic deviation). Mathematical formulation of General Relativity, Einstein's equations (properties of the energy-momentum tensor will be needed in the case of dust only). The Schwarzschild solution; planetary motion, the bending of light, and black holes.

Reading

1. L.P. Hughston and K.P. Tod, *An Introduction to General Relativity,* LMS Student Text 5 (London Mathematical Society, Cambridge University Press, 1990), Chs 1–18.
2. N.M.J. Woodhouse, *Notes on Special Relativity*, Mathematical Institute Notes. Revised edition; published in a revised form as *Special Relativity, Lecture notes in Physics m6* (Springer-Verlag, 1992), Chs 1–7

Further Reading

1. B. Schutz, *A First Course in General Relativity* (Cambridge University Press, 1990).
2. R.M. Wald, *General Relativity* (Chicago, 1984).
3. W. Rindler, *Essential Relativity* (Springer-Verlag, 2nd edition, 1990).

C7.2b: General Relativity II — Dr de la Ossa — 16HT

Prerequisites

B7.1a, C7.2a General Relativity I

Aims & Objectives

In this, the second course in General Relativity, we have two principal aims. We first aim to increase our mathematical understanding of the theory of relativity and our technical ability to solve problems in it. This leads to a greater understanding of the Schwarzschild solution and an introduction to its rotating counterpart, the Kerr solution. Then we apply the theory to a wider class of physical situations, notably to the problem of constructing a cosmological model to represent the universe itself in the large.

Synopsis

The Lie derivative and isometries. Linearised General Relativity and the metric of an isolated body. The Schwarzschild solution and its extensions; Eddington-Finkelstein coordinates and the Kruskal extension. Stationary, axisymmetric metrics and orthogonal transitivity; the Kerr solution and its properties; interpretation as rotating black hole. The Einstein field equations with matter; the energy-momentum tensor for a perfect fluid; equations of motion form the conservation law. Cosmological principles, homogeneity and isotropy; cosmological models; the Friedman–Robertson–Walker solutions; observational consequences.

Method of Examination

4 examination questions.

Reading

1. L. P. Hughston and K. P. Tod, *An Introduction to General Relativity*, LMS Student Text 5, CUP (1990), Chs.19, 20, 22-26.
2. R. M. Wald, *General Relativity*, Univ of Chicago Press (1984).

C7.4: Theoretical Physics

Note: This double unit is offered by the Physics Department.

Level: M-level **Method of Assessment:** Written examination.

Weight: Double unit only. OSS paper code 2756.

Recommended Prerequisites

Part A Quantum Theory, Part A Classical Mechanics, B7.1a Quantum Mechanics, C7.1b Quantum Theory and Quantum Computers, B7.2a Special Relativity and Electromagnetism.

Overview

This course is intended to give an introduction to some aspects of many-particle systems, field theory and related ideas. These form the basis of our current theoretical understanding of particle physics, condensed matter and statistical physics. An aim is to present some core ideas and important applications in a unified way. These applications include the classical mechanics of continuum systems, the quantum mechanics and statistical mechanics of many-particle systems, and some basic aspects of relativistic quantum field theory.

C7.4a: Theoretical Physics I — Prof. Essler and Dr Uli Haisch — 24MT

Synopsis

1. Path Integrals in Quantum Mechanics (6 lectures; FE)

   - Mathematical tools for describing systems with an infinite number of degrees of freedom: functionals, functional differentiation; Multi-dimensional Gaussian integrals.

   - Quantum mechanical propagator as a path integral. Semiclassical limit. Free particle.

   - Quantum statistical mechanics in terms of path integrals. Harmonic oscillator.

- Perturbation theory for non-Gaussian functional integrals. Anharmonic oscillator. Feynman diagrams.

2. Quantum Many-Particle Systems (7 lectures; FE)

   - Second Quantization: bosons and fermions, Fock space, single-particle and two-particle operators.

   - Applications to the Fermi gas, weakly interacting Bose condensates, magnons in (anti)ferromagnets, and to superconductivity.

   - quantum field theory as a low-energy description of quantum many-particle systems.

3. Classical Field Theory (5 lectures; UH)

   - Group theory and Lie algebra primer: basic concepts, SU(N), Lorentz group.

   - Elements of classical field theory: fields, Lagrangians, Hamiltonians, principle of least action, equations of motion, Noether's theorem, space-time symmetries.

   - Applications: scalar fields, spontaneous symmetry breaking, U(1) symmetry, Goldstone?s theorem, SU(2) U(1) symmetry, vector fields, Maxwell's theory, scalar electrodynamics.

4. Canonical Quantisation (6 lectures; UH)

   - Free real and complex scalar fields: Klein-Gordon field as harmonic oscillators, Heisenberg picture.

   - Propagators and Wick's theorem: correlators, causality, Green's functions.

   - Free vector fields: gauge fixing, Feynman propagator.

C7.4b: Theoretical Physics II — Prof. Essler and Dr Uli Haisch — 16HT

Synopsis

1. Interacting Quantum Fields (6 lectures; UH)

   - Perturbation theory: classification of interactions, interaction picture, Feynman diagrams. - Applications: tree-level decay and scattering processes of scalar and U(1) gauge fields. - Path integrals: effective action, Feynman diagrams from path integrals.

2. Statistical Physics, Phase Transitions and Stochastic Processes (10 lectures; FE)

   - Transfer matrices: one-dimensional systems in classical statistical mechanics. Transfer matrices in D=2 and their relation to path integrals.

   - Phase transition in the 2D Ising model: Peierls argument.

   - Landau Theory of phase transitions: phase diagrams, first-order and continuous phase transitions. Landau-Ginzburg-Wilson free energy functionals. Examples including liquid crystals. Critical phenomena and scaling theory.

   - Stochastic processes: the Langevin and Fokker-Planck equation. Brownian motion of single particle.

[Total: 40 lectures]

Reading

The lecturers are aware of no book that presents all parts of this course in a unified way and at an appropriate level. For this reason, detailed lecture notes will be made available.

C8.1a: Mathematical Geoscience — Dr Fowler and Prof. Sander — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2A81)

Recommended Prerequisites

B6 highly recommended.

Overview

The aim of the course is to illustrate the techniques of mathematical modelling in their particular application to environmental problems. The mathematical techniques used are drawn from the theory of ordinary differential equations and partial differential equations. However, the course does require the willingness to become familiar with a range of different scientific disciplines. In particular, familiarity with the concepts of fluid mechanics will be useful.

Synopsis

Applications of mathematics to environmental or geophysical problems involving the use of models with ordinary and partial differential equations. Examples to be considered are: Climate dynamics. River flow and sediment transport. Glacier dynamics.

Reading

1. A. C. Fowler, *Mathematical Geoscience* (Springer, 2011).
2. K. Richards, *Rivers* (Methuen, 1982).
3. G. B. Whitham, *Linear and Nonlinear Waves* (Wiley, New York, 1974).
4. K. M. Cuffey and W. S. B. Paterson, *The Physics of Glaciers* (4th edition, Butterworth-Heinemann, 2011).
5. J. T. Houghton, *The Physics of Atmospheres* (3rd ed., Cambridge University Press., Cambridge, 2002).

C8.1b: Mathematical Physiology — Dr Gaffney — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2B81).

Recommended Prerequisites

B8a highly recommended.

Overview

The course aims to provide an introduction which can bring students within reach of current research topics in physiology, by synthesising a coherent description of the physiological background with realistic mathematical models and their analysis. The concepts and treatment of oscillations, waves and stability are central to the course, which develops ideas introduced in the more elementary B8a course. In addition, the lecture sequence aims to build understanding of the workings of the human body by treating in sequence problems at the intracellular, intercellular, whole organ and systemic levels.

Synopsis

Review of enzyme reactions and Michaelis–Menten theory. Trans-membrane ion transport: Hodgkin–Huxley and Fitzhugh–Nagumo models.

Excitable media; wave propagation in neurons

Calcium dynamics; calcium-induced calcium release. Intracellular oscillations and wave propagation.

The electrochemical action of the heart. Spiral waves, tachycardia and fibrillation

Discrete delays in physiological systems. The Glass–Mackey model of respiration. Regulation of stem cell and blood cell production. Dynamical diseases.

Reading

The principal text is:

1. J. Keener and J. Sneyd, *Mathematical Physiology* (Springer-Verlag, 1998). Chs. 1, 4, 5, 9–12, 14–17. [Or: Second edition Vol I: Chs. 1, 2, 4, 5, 6, 7. Vol II: Chs. 11, 13, 14. (Springer-Verlag, 2009)]

Subsidiary mathematical texts are:

1. J. D. Murray, *Mathematical Biology* (Springer-Verlag, 2nd ed., 1993). [Third edition, Vols I and II, (Springer-Verlag, 2003).]

2. L. A. Segel, *Modeling Dynamic Phenomena in Molecular and Cellular Biology* (Cambridge University Press, 1984).

3. L. Glass and M. C. Mackey, *From Clocks to Chaos* (Princeton University Press, 1988).

4. P. Grindrod, *Patterns and Waves* (oup, 1991).

General physiology texts are:

1. R. M. Berne and M. N. Levy, *Principles of Physiology* (2nd ed., Mosby, St. Louis, 1996).

2. J. R. Levick, *An Introduction to Cardiovascular Physiology* (3rd ed. Butterworth–Heinemann, Oxford, 2000).

3. A. C. Guyton and J. E. Hall, *Textbook of Medical Physiology* (10th ed., W. B. Saunders Co., Philadelphia, 2000).

C9.1a: Modular Forms — Dr Lauder — 16MT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code tbc)

Prerequisites

Part A Analysis and Algebra (core material) and Part A Group Theory. Part A Number Theory is useful but not essential. B3b Algebraic Curves is recommended, or alternatively some background reading on Riemann surfaces.

Overview

The course aims to introduce students to the beautiful theory of modular forms, one of the cornerstones of modern number theory. This theory is a rich and challenging blend of methods from complex analysis and linear algebra, and an explicit application of group actions and the theory of Riemann surfaces.

Learning Outcomes

The student will learn about modular curves and spaces of modular forms, and understand in special cases how to compute their genus and dimension, respectively. They will see how modular curves parameterise families of elliptic curves, and that modular forms can be described explicitly via their q-expansions, and they will be familiar with explicit examples of modular forms. They will learn about the rich algebraic structure on spaces of modular forms, given by Hecke operators and the Petersson inner product.

Synopsis

Geometry of modular curves: linear fractional transformations; the modular group; the modular curves; principal congruence subgroups G; Riemann surfaces X_G and their genus; examples.

Modular forms: definitions; function theory on Riemann surfaces; dimension formulae for spaces of modular forms.

Examples of modular forms: Poincaré series; Eisenstein series; coefficients of the Eisenstein series for the modular group; Ramanujan's delta function; some arithmetic applications.

An inner product, lattices and elliptic curves: Petersson inner product; functions on lattices; Eisenstein series revisited; Elliptic curves; modular points for congruence subgroups (no proofs).

Hecke operators (level 1): Hecke operators on lattices; Hecke operators on modular forms; Hecke operators are Hermitian.

Reading

1. F. Diamond and J. Shurman, *A First Course in Modular Forms,* Graduate Texts in Mathematics 228, Springer-Verlag, 2005. (The course covers much of the material in Chapters 1-5, but takes a more intuitive approach to the geometry.)

2. R.C. Gunning, *Lectures on Modular Forms*, Annals of mathematical studies 48, Princeton University Press, 1962. (The first part of the course is based upon Chapters I-IV.)

3. J.-P. Serre, *A Course in Arithmetic,* Graduate Texts in Mathematics 7, Springer-Verlag, 1973. (Chapter VII is a classic treatment of level 1 modular forms.)

C9.1b Elliptic Curves — Prof. Kim — 16HT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2B82).

Recommended Prerequisites

It is helpful, but not essential, if students have already taken a standard introduction to algebraic curves and algebraic number theory. For those students who may have gaps in their background, I have placed the file "Preliminary Reading" permanently on the Elliptic Curves webpage, which gives in detail (about 30 pages) the main prerequisite knowledge for the course. Go first to: http://www.maths.ox.ac.uk/courses/material then click on "C9.1b Elliptic Curves" and then click on the pdf file "Preliminary Reading".

Overview

Elliptic curves give the simplest examples of many of the most interesting phenomena which can occur in algebraic curves; they have an incredibly rich structure and have been the testing ground for many developments in algebraic geometry whilst the theory is still full of deep unsolved conjectures, some of which are amongst the oldest unsolved problems in mathematics. The course will concentrate on arithmetic aspects of elliptic curves defined over the rationals, with the study of the group of rational points, and explicit determination of the rank, being the primary focus. Using elliptic curves over the rationals as an example, we will be able to introduce many of the basic tools for studying arithmetic properties of algebraic varieties.

Learning Outcomes

On completing the course, students should be able to understand and use properties of elliptic curves, such as the group law, the torsion group of rational points, and 2-isogenies between elliptic curves. They should be able to understand and apply the theory of fields with valuations, emphasising the p-adic numbers, and be able to prove and apply Hensel's Lemma in problem solving. They should be able to understand the proof of the Mordell–Weil Theorem for the case when an elliptic curve has a rational point of order 2, and compute ranks in such cases, for examples where all homogeneous spaces for descent-via-2-isogeny satisfy the Hasse principle. They should also be able to apply the elliptic curve method for the factorisation of integers.

Synopsis

Non-singular cubics and the group law; Weierstrass equations.
Elliptic curves over finite fields; Hasse estimate (stated without proof).
p-adic fields (basic definitions and properties).
1-dimensional formal groups (basic definitions and properties).
Curves over p-adic fields and reduction mod p.
Computation of torsion groups over $\mathbb{Q}$; the Nagell–Lutz theorem.
2-isogenies on elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.
Weak Mordell–Weil Theorem for elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.
Height functions on Abelian groups and basic properties.
Heights of points on elliptic curves defined over $\mathbb{Q}$; statement (without proof) that this gives a height function on the Mordell–Weil group.
Mordell–Weil Theorem for elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.
Explicit computation of rank using descent via 2-isogeny.
Public keys in cryptography; Pollard's $(p-1)$ method and the elliptic curve method of factorisation.

Reading

1. J.W.S. Cassels, *Lectures on Elliptic Curves,* LMS Student Texts 24 (Cambridge University Press, 1991).
2. N. Koblitz, *A Course in Number Theory and Cryptography,* Graduate Texts in Mathematics 114 (Springer, 1987).
3. J.H. Silverman and J. Tate, *Rational Points on Elliptic Curves,* Undergraduate Texts in Mathematics (Springer, 1992).
4. J.H. Silverman, *The Arithmetic of Elliptic Curves,* Graduate Texts in Mathematics 106 (Springer, 1986).

Further Reading

1. A. Knapp, *Elliptic Curves, Mathematical Notes 40* (Princeton University Press, 1992).
2. G, Cornell, J.H. Silverman and G. Stevans (editors), *Modular Forms and Fermat's Last Theorem* (Springer, 1997).
3. J.H. Silverman, *Advanced Topics in the Arithmetic of Elliptic Curves,* Graduate Texts in Mathematics 151 (Springer, 1994).

C9.2a: Analytic Number Theory —Prof. Heath-Brown—16MT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2A82)

Recommended Prerequisites

Complex analysis (holomorphic and meromorphic functions, Cauchy's Residue Theorem, Evaluation of integrals by contour integration, Uniformly convergent sums of holomorphic functions). Elementary number theory (Unique Factorization Theorem).

Overview

The course aims to introduce students to the theory of prime numbers, showing how the irregularities in this elusive sequence can be tamed by the power of complex analysis. The course builds up to the Prime Number Theorem which is the corner-stone of prime number theory, and culminates in a description of the Riemann Hypothesis, which is arguably the most important unsolved problem in modern mathematics.

Learning Outcomes

Students will learn to handle multiplicative functions, to deal with Dirichlet series as functions of a complex variable, and to prove the Prime Number Theorem and simple variants.

Synopsis

Introductory material on primes.

Arithmetic functions — Möbius function, Euler function, Divisor function, Sigma function — multiplicativity.

Dirichlet series — Euler products — von Mangoldt function.

Riemann Zeta-function — analytic continuation to $\mathrm{Re}(s) > 0$.

Non-vanishing of $\zeta(s)$ on $\mathrm{Re}(s) = 1$.

Proof of the prime number theorem.

The Riemann hypothesis and its significance.

The Gamma function, the functional equation for $\zeta(s)$, the value of $\zeta(s)$ at negative integers.

Reading

1. T.M. Apostol, *Introduction to Analytic Number Theory,* Undergraduate Texts in Mathematics (Springer-Verlag, 1976). Chapters 2,3,11,12 and 13.

2. M. Ram Murty, *Problems in Analytic Number Theory* (Springer, 2001). Chapters 1 – 5.

3. G.H. Hardy and E.M. Wright, *An Introduction to the Theory of Numbers* (Fifth edition, Oxford University Press, 1979). Chapters 16 ,17 and 18.

4. G.J.O. Jameson, *The Prime Number Theorem,* LMS Student Texts 53 (Cambridge University Press, 2003).

C10.1a: Stochastic Differential Equations —Prof. Hambly—16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2A83)

Prerequisites

Part A integration and B10a Martingales Through Measure Theory, is expected.

Overview

Stochastic differential equations have been used extensively in many areas of application, including finance and social science as well as chemistry. This course develops the basic theory of Itô's calculus and stochastic differential equations.

Learning Outcomes

The student will have developed an appreciation of stochastic calculus as a tool that can be used for defining and understanding diffusive systems.

Synopsis

Brownian motion: basic properties, reflection principle, quadratic variation. Itô's calculus: stochastic integrals with respect to martingales, Itô's lemma, Levy's characterisation of Brownian motion, exponential martingales, exponential inequality, Girsanov's Theorem, the Martingale Representation Theorem. Stochastic differential equations: strong and weak solutions, questions of existence and uniqueness, diffusion processes.

Reading — Main Texts

1. Dr Qian's online notes:

   www.maths.ox.ac.uk/courses/course/15721

2. B. Oksendal, *Stochastic Differential Equations: An introduction with applications* (Universitext, Springer, 6th edition). Chapters II, III, IV, V, part of VI, Chapter VIII (F).

3. F. C. Klebaner, *Introduction to Stochastic Calculus with Applications* (Imperial College Press, 1998, second edition 2005). Sections 3.1 – 3.5, 3.9, 3.12. Chapters 4, 5, 11.

Alternative Reading

1. H. P. McKean, *Stochastic Integrals* (Academic Press, New York and London, 1969).

Further Reading

1. N. Ikeda & S. Watanabe, *Stochastic Differential Equations and Diffusion Processes* (North–Holland Publishing Company, 1989).

2. I. Karatzas and S. E. Shreve, *Brownian Motion and Stochastic Calculus,* Graduate Texts in Mathematics 113 (Springer-Verlag, 1988).

3. L. C. G. Rogers & D. Williams, *Diffusions, Markov Processes and Martingales Vol 1 (Foundations) and Vol 2 (Ito Calculus)* (Cambridge University Press, 1987 and 1994).

4. D Revus and M. Yor, *Continuous Martingales and Brownian Motion*, (Springer, Third Edition, 1999)

C10.1b: Brownian Motion and Conformal Invariance — Dr Belyaev— 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2B83).

Prerequisites

Essential Prerequisites: Part A Analysis and knowledge of Brownian Motion.

Recommended Prerequisites: Part A Differential Equations, B10a Martingales through Measure and C10.1a Stochastic Differential Equations.

Overview

This course is devoted to connections between two-dimensional Brownian Motion, Complex Analysis and Lattice Models. An important example is the standard random walk on a square lattice. It is known, that after proper rescaling, the random walk converges to Brownian Motion (this is a corollary of the Central Limit Theorem). It was observed by Lévy that Brownian Motion is conformally invariant i.e. the image of the Brownian trajectory under an analytic map looks like a Brownian trajectory. In particular, this implies that the limiting object has more symmetry than the original random walk on the lattice. There is a big class of other lattice models which are conjectured to have scaling limits and these limits are conjectured to be conformally invariant.

This course will consist of two parts: in the first part we will discuss the conformal invariance of Brownian Motion. In the second part we will learn about the very recent and exciting theory of Schramm-Loewner Evolution (SLE). This theory will provide the necessary tools to study conformally invariant limits of various lattice models.

Learning Outcomes

The students will develop an understanding of the role the Brownian Motion plays in different areas of mathematics and physics. They will be familiar with basic ideas and techniques of Schramm-Loewner Evolution.

Synopsis

Brief introduction to Brownian Motion, continuous martingales, and Ito formula. Conformal invariance of Brownian Motion. Brief introduction to conformal maps and Loewner Evolution. Lattice models: percolation, Ising, Loop-erased Random Walk etc. Schramm's principle and the introduction of SLE (Schramm-Loewner Evolution or Stochastic Loewner Evolution). Properties of SLE. [Convergence of percolation to SLE(6)]

Reading

This is a very young and actively developing area of research. Unfortunately this means that there are very few books, in fact, there is only one proper book and a couple of lecture notes.

1. G. Lawler, *Conformally Invariant Processes in the Plane*, Mathematical Surveys and Monographs, Vol 114 (2005)

2. W. Werner, *Random planar curves and Schramm-Loewner evolutions* http://arxiv.org/abs/math/0303354

C11.1a: Combinatorics — Prof. Scott — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code tbc).

Recommended Prerequisites

Part B Graph Theory is helpful, but not required.

Overview

An important branch of discrete mathematics concerns properties of collections of subsets of a finite set. There are many beautiful and fundamental results, and there are still many basic open questions. The aim of the course is to introduce this very active area of mathematics, with many connections to other fields.

Learning Outcomes

The student will have developed an appreciation of the combinatorics of finite sets.

Synopsis

Chains and antichains. Sperner's Lemma. LYM inequality. Dilworth's Theorem.

Shadows. Kruskal-Katona Theorem.

Intersecting families. Erdos-Ko-Rado Theorem. Cross-intersecting families.

VC-dimension. Sauer-Shelah Theorem.

t-intersecting families. Fisher's Inequality. Frankl-Wilson Theorem.

Application to Borsuk's Conjecture.

Projections and the Bollobás-Thomason Box Theorem.

Reading

1. Bela Bollobás, *Combinatorics*, CUP, 1986.
2. Stasys Jukna, *Extremal Combinatorics*, Springer, 2007

C11.1b: Probabilistic Combinatorics — Prof. McDiarmid — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS code tbc)

Recommended Prerequisites

Part B Graph Theory and Part A Probability. C11.1a Combinatorics is not as essential prerequisite for this course, though it is a natural companion for it.

Overview

Probabilistic combinatorics is a very active field of mathematics, with connections to other areas such as computer science and statistical physics. Probabilistic methods are essential for the study of random discrete structures and for the analysis of algorithms, but they can also provide a powerful and beautiful approach for answering deterministic questions. The aim of this course is to introduce some fundamental probabilistic tools and present a few applications.

Learning Outcomes

The student will have developed an appreciation of probabilistic methods in discrete mathematics.

Synopsis

First-moment method, with applications to Ramsey numbers, and to graphs of high girth and high chromatic number.
Second-moment method, threshold functions for random graphs.
Lovász Local Lemma, with applications to two-colourings of hypergraphs, and to Ramsey numbers.
Chernoff bounds, concentration of measure, Janson's inequality.

Branching processes and the phase transition in random graphs.
Clique and chromatic numbers of random graphs.

Reading

1. N. Alon and J.H. Spencer, *The Probabilistic Method* (third edition, Wiley, 2008).

Further Reading

1. B. Bollobás, *Random Graphs* (second edition, Cambridge University Press, 2001).
2. M. Habib, C. McDiarmid, J. Ramirez-Alfonsin, B. Reed, ed., *Probabilistic Methods for Algorithmic Discrete Mathematics* (Springer, 1998).
3. S. Janson, T. Luczak and A. Rucinski, *Random Graphs* (John Wiley and Sons, 2000).
4. M. Mitzenmacher and E. Upfal, *Probability and Computing : Randomized Algorithms and Probabilistic Analysis* (Cambridge University Press, New York (NY), 2005).
5. M. Molloy and B. Reed, *Graph Colouring and the Probabilistic Method* (Springer, 2002).
6. R. Motwani and P. Raghavan, *Randomized Algorithms* (Cambridge University Press, 1995).

C12.1a Numerical Linear Algebra — Prof. Tanner — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2A88)

Recommended Prerequisites

Only elementary linear algebra is assumed in this course. The part A Numerical Analysis course would be helpful, indeed some swift review and extensions of some of the material of that course is included here.

Overview

Linear Algebra is a central and widely applicable part of mathematics. It is estimated that many (if not most) computers in the world are computing with matrix algorithms at any moment in time whether these be embedded in visualization software in a computer game or calculating prices for some financial option. This course builds on elementary linear algebra and in it we derive, describe and analyse a number of widely used constructive methods (algorithms) for various problems involving matrices.

Numerical Methods for solving linear systems of equations, computing eigenvalues and singular values and various related problems involving matrices are the main focus of this course.

Synopsis

Common problems in linear algebra. Matrix structure, singular value decomposition. QR factorization, the QR algorithm for eigenvalues. Direct solution methods for linear systems, Gaussian elimination and its variants. Iterative solution methods for linear systems.

Chebyshev polynomials and Chebyshev semi-iterative methods, conjugate gradients, convergence analysis, preconditioning.

Reading

L. N. Trefethen and D. Bau III, *Numerical Linear Algebra* (SIAM, 1997).

J. W. Demmel, *Applied Numerical Linear Algebra* (SIAM, 1997).

A. Greenbaum, *Iterative Methods for Solving Linear Systems* (SIAM, 1997).

G. H. Golub and C. F. van Loan, *Matrix Computations* (John Hopkins University Press, 3rd edition, 1996).

H. C. Elman, D. J. Silvester and A. J. Wathen, *Finite Elements and Fast Iterative Solvers* (Oxford University Press, 1995), only chapter 2.

C12.1b Continuous Optimization — Dr Cartis — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2B88).

Overview

Optimization deals with the problem of minimising or maximising a mathematical model of an objective function such as cost, fuel consumption etc. under a set of side constraints on the domain of definition of this function. Optimization theory is the study of the mathematical properties of optimization problems and the analysis of algorithms for their solution. The aim of this course is to provide an introduction to nonlinear continuous optimization specifically tailored to the background of mathematics students.

Synopsis

Part 1: Unconstrained Optimization
Optimality conditions, Newton's method for nonlinear systems, Convergence rates, Steepest descent method, General line search methods (alternative search directions, e.g. Newton,

CG, BFG, ...), Trust region methods, Inexact evaluation of linear systems, iterative methods and the role of preconditioners.

Part 2: Constrained Optimization
Optimality/KKT conditions, Lagrange Multipliers, Penalty methods and SQP for equality constrained optimization, Interior penalty / barrier methods for inequality constrained optimization.

Reading List

Lecture notes will be made available for downloading from the course webpage.
To complement the notes, reading assignments will be given from the book of J.Nocedal and S.J.Wright, *Numerical Optimisation*, (Springer, 1999).

C12.2a Approximation of Functions — Prof Trefethen — 16MT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2A89)

Recommended Prerequisites:

None

Overview

How can a function f(x) be approximated over a prescribed domain by a simpler function like a polynomial or a rational function? Such questions were at the heart of analysis in the early 1900s and later grew into a mature subject of approximation theory. Recently they have been invigorated as problems of approximation have become central to computational algorithms for differential equations, linear algebra, optimization and other fields. This course, based on Trefethen's new text in which results are illustrated by Chebfun computations, will focus in a modern but still rigorous way on the fundamental results of interpolation and approximation and their algorithmic application.

Synopsis

Chebyshev interpolants, polynomials, and series. Barycentric interpolation formula. Weierstrass approximation theorem. Convergence rates of polynomial approximations. Hermite integral formula and Runge phenomenon. Lebesgue constants, polynomial rootfinding. Orthogonal polynomials. Clenshaw-Curtis and Gauss quadrature. Rational approximation.

Reading

1. L. N. Trefethen, *Approximation Theory and Approximation Practice*

This course will be based entirely on my new textbook Approximation Theory and Approximation Practice, published by SIAM in 2013. All students in the course are required to have a copy of this book. The lectures and exam will be closely tied to the book, and the problems assigned will be taken from the book.

The first six short chapters are available online: see
http://people.maths.ox.ac.uk/trefethen/books.html.

If you're wondering whether or not to take the course, I suggest you read a few pages to help decide. If you know you're going to take it, please get a copy of the book. Here are some options:

- amazon.co.uk quotes a price of 28.05 pounds.

- Cambridge U. Press quotes a price of 33.00 pounds.

- SIAM quotes a price of 34.30 dollars for members, and all Oxford students are eligible for the member price, since Oxford is an institutional member of SIAM.

A fourth option may be simplest, cheapest, and speediest: the Numerical Analysis Group administrator, Lotti Ekert, has some copies on hand which we are prepared to sell for 20.00 pounds each. Mrs. Ekert is in Room S2.26 of the new Andrew Wiles mathematics building, and you may wish to check availability with her by email before dropping by.

C12.2b Finite Element Methods for Partial Differential Equations — Prof Suli — 16HT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Unit (OSS paper code 2B89).

Recommended Prerequisites

No formal prerequisites are assumed. The course builds on elementary calculus, analysis and linear algebra and, of course, requires some acquaintance with partial differential equations such as the material covered in the Maths Mods Waves and Diffusion course, in particular the Divergence Theorem. Part A Numerical Analysis would be helpful but is certainly not essential. Function Space material will be introduced in the course as needed.

Overview

Computational algorithms are now widely used to predict and describe physical and other systems. Underlying such applications as weather forecasting, civil engineering (design of structures) and medical scanning are numerical methods which approximately solve partial differential equation problems. This course gives a mathematical introduction to one of the more widely used methods: the finite element method.

Synopsis

Finite element methods represent a powerful and general class of techniques for the approximate solution of partial differential equations. The aim of this course is to introduce these methods for boundary value problems for the Poisson and related elliptic partial differential equations.

Attention will be paid to the formulation, the mathematical analysis and the implementation of these methods.

Reading List

S.C. Brenner & L.R. Scott, *The Mathematical Theory of Finite Element Methods.* Springer, 2nd edition, 2002.[Chapters 0,1,2,3; Chapter 4: Secs. 4.1–4.4, Chapter 5: Secs. 5.1–5.7].

H. Elman, D. Silvester & A. Wathen, *Finite Elements and Fast Iterative Solvers.* OUP, 2005. [Mainly Chapters 1 and 5].

C. Johnson, *Numerical Solution of Partial Differential Equations by the Finite Element Method.* CUP, 1990. [Chapters 1–4; Chapter 8:Secs. 8.1–8.4.2; Chapter 9: Secs. 9.1–9.5].

Typed lecture notes covering the entire course (and more):

Endre Süli, *Finite Element Methods for Partial Differential Equations.* Mathematical Institute, University of Oxford, 2011.

are available from `http://web.comlab.ox.ac.uk/people/endre.suli/fem.pdf`

Some of the introductory material is covered in

Endre Süli & David Mayers, *An Introduction to Numerical Analysis*,CUP 2003; Second Printing 2006. [Chapter 11 and in particular Chapter 14].

CD : Dissertations on a Mathematical Topic

Level : M-level

Weight : Unit (5,000 words) or double-unit (10,000).

Students may offer either a unit or a double-unit dissertation on a Mathematical topic for examination at Part C. A unit is equivalent to a 16-hour lecture course and a double-unit is equivalent to a 32-hour lecture course. Students will have approximately 4 hours of supervision for a unit dissertation or 8 hours for a double-unit distributed over Michaelmas and Hilary terms. In addition there are lectures on writing mathematics and using LaTeX in Michaelmas and Hilary terms. See the lecture list for details.

Students considering offering a dissertation should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* available at:

http://www.maths.ox.ac.uk/current-students/undergraduates/projects/.

Application

Students must apply to the Mathematics Projects Committee for approval of their proposed topic in advance of beginning work on their dissertation. Proposals should be addressed to the Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room S0.16, Mathematical Institute and are accepted from the end of Trinity Term. All proposals must be received before 12noon on Friday of Week 0 of Michaelmas Full Term. For CD dissertations candidates should take particular care to remember that the project must have substantial mathematical content. The application form is available at
http://www.maths.ox.ac.uk/current-students/undergraduates/projects/. Once a title has been approved, it may only be changed by approval of the Chairman of the Projects Committee.

Assessment

Each project is blind double marked. The marks are reconciled through discussion between the two assessors, overseen by the examiners. Please see the *Guidance Notes on Extended Essays and Dissertations in Mathematics* for detailed marking criteria and class descriptors.

Submission

THREE copies of your dissertation, identified by your candidate number only, should be sent to the Chairman of Examiners, FHS of Mathematics Part C, Examination Schools, Oxford, to arrive no later than **12noon on Monday of week 10, Hilary Term 2014**. An electronic copy of your dissertation should also be submitted via the Mathematical Institute website. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics.*

3 Other Units

MS: Statistics Units

Students in Part C may take units drawn from Part C of the Honour School of Mathematics and Statistics. For full details of these units see the syllabus and synopses for Part C of the Honour School Mathematics and Statistics, which are available on the web at http://www.stats.ox.ac.uk/current_students/bammath/course_handbooks/

The Statistics units available are as follows:

- MS1b Statistical Data Mining and Machine Learning
- MS2b Stochastic Models in Mathematical Genetics
- MS5a Probability and Statistics for Network Analysis
- MS6a Modern Survival Analysis
- MS6b Advanced Simulation Methods

Computer Science: Units

Students in Part C may take units drawn from Part C of the Honour School of Mathematics and Computing. For full details of these units see the Department of Computer Science's website (http://www.cs.ox.ac.uk/teaching/courses/)

Please note that these three courses will be examined by mini-project (as for MSc students). Mini-projects will be handed out to candidates on the last Monday or Friday of the term in which the subject is being taught, and you will have to hand it in to the Exam Schools by noon on Monday of Week 1 of the following term. The mini-project will be designed to be completed in about four to five days. It will include some questions that are more open-ended than those on a standard sit-down exam. The work you submit should be your own work, and include suitable references.

Please note that the Computer Science courses in Part C are 50% bigger than those in earlier years, i.e. for each Computer Science course in the 3rd year undergraduates are expected to undertake about 10 hours of study per week, but 4th year courses will each require about 15 hours a week of study. Lecturers are providing this extra work in a variety of ways, e.g. some will give 16 lectures with extra reading, classes and/or practicals, whereas others will be giving 24 lectures, and others still will be doing something in between. Students will need to look at each synopsis for details on this.

The Computer Science units available are as follows:

- CCS1a Categories, Proofs and Processes
- CCS3b Quantum Computer Science
- CCS4b Automata, Logics and Games

Philosophy: Double Units

Students in Part C may take options, all double units, drawn from Part C of the Honour School of Mathematics and Philosophy. For full details of these double units see the Faculty of Philosophy's website http://www.philosophy.ox.ac.uk/undergraduate/course_descriptions

The Philosophy units available are as follows:

- Rise of Modern Logic (Double unit)

This course will be examined by a three-hour exam and a submitted essay of up to 5000 words.

OD : Dissertations on a Mathematically related Topic

Level : M-level

Weight : Unit (5,000 words) or double-unit (10,000 words).

Students may offer either a unit or a double-unit dissertation on a Mathematically related topic for examination at Part C. For example, applications of mathematics to another field (eg Maths in Music), historical topics, topics concentrating on the analysis of statistical data, or topics concentrating on the production of computer-generated data are acceptable as topics for an OD dissertation. (Topics in mathematical education are not allowed.)

A unit is equivalent to a 16-hour lecture course and a double-unit is equivalent to a 32-hour lecture course. Students will have approximately 4 hours of supervision for a unit dissertation or 8 hours for a double-unit distributed over Michaelmas and Hilary terms. In addition there are lectures on writing mathematics and using LaTeX in Michaelmas and Hilary terms. See the lecture list for details.

Candidates considering offering a dissertation should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* available at:

http://www.maths.ox.ac.uk/current-students/undergraduates/projects/.

Application

Students must apply to the Mathematics Projects Committee for approval of their proposed topic in advance of beginning work on their dissertation. Proposals should be addressed to the Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room S0.16, Mathematical Institute and are accepted from the end of Trinity Term. All proposals must be received before 12noon on Friday of Week 0 of Michaelmas Full Term. The application form is available at http://www.maths.ox.ac.uk/current-students/undergraduates/projects/.

Once a title has been approved, it may only be changed by approval of the Chairman of the Projects Committee.

Assessment

Each project is blind double marked. The marks are reconciled through discussion between the two assessors, overseen by the examiners. Please see the *Guidance Notes on Extended Essays and Dissertations in Mathematics* for detailed marking criteria and class descriptors.

Submission

THREE copies of your dissertation,identified by your candidate number only, should be sent to the Chairman of Examiners, FHS of Mathematics Part C, Examination Schools, Oxford, to arrive no later than **12noon on Monday of week 10, Hilary Term 2014**. An electronic copy of your dissertation should also be submitted via the Mathematical Institute website. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics.*

4 Language Classes: French and Spanish

Language courses in French and German or Spanish (in alternate years) are offered by the University Language Centre.

Students in the FHS Mathematics may apply to take language classes. In 2013-2014, French and Spanish language classes will be run in MT and HT. We have a limited number of places but if we have spare places we will offer these to joint school students , Mathematics and Computer Science, Mathematics and Philosophy and Mathematics and Statistics.

Two levels of French courses are offered, a lower level for those with a good pass at GCSE, and a higher level course for those with A/S or A level. Acceptance on either course will depend on satisfactory performance in the Preliminary Qualifying Test held in Week 1 of Michaelmas Term (Monday, 17.00-19.00 at the Language Centre). Classes at both levels will take place on Mondays, 17.00-19.00. A single class in German or Spanish at a lower or higher level will be offered on the basis of the performances in the Preliminary Qualifying Test, held at the same time as the French test. Classes will also be held on Mondays, 17-00-19.00.

Performance on the course will not contribute to the class of degree awarded. However, upon successful completion of the course, students will be issued with a certificate of achievement which will be sent to their college.

Places at these classes are limited, so students are advised that they must indicate their interest at this stage. If you are interested please contact Nia Roderick (roderick@maths.ox.ac.uk or tel. 01865 615205), Academic Assistant in the Mathematical Institute, as soon as possible.

Aims and rationale

The general aim of the language courses is to develop the student's ability to communicate (in both speech and writing) in French, German or Spanish to the point where he or she can function in an academic or working environment in a French-speaking, German-speaking or Spanish-speaking country.

The courses have been designed with general linguistic competence in the four skills of reading, writing, speaking and listening in mind. There will be opportunities for participants to develop their own particular interests through presentations and assignments.

Form and Content

Each course will consist of a thirty-two contact hour course, together with associated work. Classes will be held in the Language Centre at two hours per week in Michaelmas and Hilary Terms.

The content of the courses is based on coursebooks together with a substantial amount of supplementary material prepared by the language tutors. Participants should expect to spend an additional two hours per week on preparation and follow-up work.

Each course aims to cover similar ground in terms of grammar, spoken and written language and topics. Areas covered will include:

Grammar:

- all major tenses will be presented and/or revised, including the subjunctive
- passive voice
- pronouns
- formation of adjectives, adverbs, comparatives
- use of prepositions
- time expressions

Speaking

- guided spoken expression for academic, work and leisure contact (e.g. giving presentations, informal interviews, applying for jobs)
- expressing opinions, tastes and preferences
- expressing cause, consequence and purpose

Writing

- Guided letter writing for academic and work contact
- Summaries and short essays

Listening

- Listening practice of recorded materials (e.g. news broadcasts, telephone messages, interviews)
- developing listening comprehension strategies

Topics (with related readings and vocabulary, from among the following)

- life, work and culture
- the media in each country
- social and political systems
- film, theatre and music
- research and innovation
- sports and related topics
- student-selected topics

Teaching staff

The courses are taught by Language Centre tutors or Modern Languages Faculty instructors.

Teaching and learning approaches

Each course uses a communicative methodology which demands active participation from students. In addition, there will be some formal grammar instruction. Students will be expected to prepare work for classes and homework will be set. Students will also be given training in strategies for independent language study, including computer-based language learning, which will enable them to maintain and develop their language skills after the course.

Entry

Two classes in French and one in German or Spanish (probably at Basic and Threshold levels) will be formed according to level of French/German/Spanish at entry. The minimum entry standard is a good GCSE Grade or equivalent. Registration for each option will take place at the start of Michaelmas Term. A preliminary qualifying test is then taken which candidates must pass in order to be allowed to join the course.

Learning Outcomes

The learning outcomes will be based on internationally agreed criteria for specific levels (known as the ALTE levels), which are as follows:

Basic Level (corresponds to ALTE Level 2 "Can-do" statements)

- Can express opinions on professional, cultural or abstract matters in a limited way, offer advice and understand instructions.
- Can give a short presentation on a limited range of topics.
- Can understand routine information and articles, including factual articles in newspapers, routine letters from hotels and letters expressing personal opinions.
- Can write letters or make notes on familiar or predictable matters.

Threshold Level (corresponds to ALTE Level 3 "Can-do" statements)

- Can follow or give a talk on a familiar topic or keep up a conversation on a fairly wide range of topics, such as personal and professional experiences, events currently in the news.
- Can give a clear presentation on a familiar topic, and answer predictable or factual questions.
- Can read texts for relevant information, and understand detailed instructions or advice.
- Can make notes that will be of reasonable use for essay or revision purposes.
- Can make notes while someone is talking or write a letter including non- standard requests.

Assessment

There will a preliminary qualifying test in Michaelmas Term. There are three parts to this test: Reading Comprehension, Listening Comprehension, and Grammar. Candidates who have not studied or had contact with French or German/Spanish for some time are advised to revise thoroughly, making use of the Language Centre's French, German or Spanish resources.

Students' achievement will be assessed through a variety of means, including continuous assessment of oral performance, a written final examination, and a project or assignment chosen by individual students in consultation with their language tutor.

Reading comprehension, writing, listening comprehension and speaking are all examined. The oral component may consist of an interview, a presentation or a candidate's performance in a formal debate or discussion.