UNIVERSITY OF OXFORD
Mathematical Institute

HONOUR SCHOOL OF MATHEMATICS

SUPPLEMENT TO THE UNDERGRADUATE HANDBOOK – 2013 Matriculation

SYNOPSES OF LECTURE COURSES

Part C 2016-17
for examination in 2017

These synopses can be found at:
http://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/handbooks-synopses

Issued October 2016

Handbook for the Undergraduate Mathematics Courses
Supplement to the Handbook
Honour School of Mathematics
Syllabus and Synopses for Part C 2016–17
for examination in 2017

Contents

1 Foreword

The synopses for Part C will be available on the website at:

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/handbooks-synopses

before the start of Michaelmas Term 2016.

See the current edition of the Examination Regulations for the full regulations governing these examinations.

Examination Conventions can be found at: https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions

In the unlikely event that any course receives a very low registration we may offer this course as a reading course (this would include some lectures but fewer classes).

1.1 Honour School of Mathematics

1.1.1 "Units"

Students staying on to take Part C will take the equivalent of eight units. One unit is the equivalent of a 16 hour lecture course. The equivalent of four units must be taken from the schedule of "Mathematics Department units" and may include a dissertation on a mathematical topic. Up to four units may be taken from the schedule of "Other Units" with not more than two from each category (Statistics options, Computer Science options, Other options).

Most Mathematics Department lecture courses are independently available as units, the exception being:

1. C7.1 Theoretical Physics - this is available as a double-unit only.

All the units described in this booklet are "M–Level".

1.2 Guidance on Mini-Projects

Dates for release and submission of mini-projects

Mini-projects will be released to students at 12noon on Monday of Week 8 of the term in which the course is taught. You will then have three weeks to work on the mini-project before the submission deadline: 12noon on Monday of Week 11 of the term in which the course is taught.

Format of Mini-Projects

Report-Based Mini-Projects

Some mini-projects will require you to research and write a short report on a topic from the lecture course. You will be given pointers on what your report should cover.

Computation or Task Based Mini-Projects

Some mini-projects will require you to work through a series of questions or may give you a few tasks which you can select from. It is likely that the mini-project will include some questions which are more open-ended than those you would encounter in a written exam.

Working on your Mini-Project

The work you submit for your mini-project should be entirely your own. You may use books, articles or other references but must acknowledge these. Please see http://www.ox.ac.uk/students/academic/guidance/skills/plagiarism for advice on avoiding plagiarism. It is impossible to give precise guidance on length, as this can vary considerably from project to project, depending on how much calculation may be needed (and also possibly on the style file required). It is unlikely, however, that a project can be completed in fewer than five pages in 'ordinary' formatting, and it will more often be in the 10–15 page range. You will have a window of 3 weeks to work on your project, but projects are designed to be completed in around 3 days.

Presentation of Submitted Work

Your submission should be clearly written in sentences with appropriate punctuation, display of formulae, and appropriate use of 'Definition', 'Lemma', 'Theorem', 'Proof', etc. For some mini-projects, you may be asked to typeset your report using LaTeX and to use a specific style file to give a particular format.

You should begin your mini-project with a brief statement of the overall goal of the project, and finish with a conclusion of what you have achieved (or needed to assume) and comment on what other questions your work might lead to.

Your work should be printed single sided, and each page should be numbered. The pages of your project should be held firmly in a stiff cover. For example, it may be ring-bound or placed in a plastic report folder. You must not write your name on your mini project; the only identification should be your candidate number.

You will be asked to submit the paper-copies of your mini-project to Exam Schools and will also need to submit an electronic version via the Mathematical Institute website. You will be sent further instructions on how to do this by the examiners.

Queries about Mini-Projects

If you have any questions about the mini project (e.g., requests for clarification), please email, Helen Lowe at helen.lowe@maths.ox.ac.uk. These will be passed as appropriate to the relevant Assessor and/or the Chairman of Examiners. Any replies will be sent to all students taking that mini project. You must not communicate directly with the Assessor, nor discuss the projects with each other.

1.3 Language Classes

Mathematics students may apply to take classes in a foreign language. In 2016-17 classes will be offered in French and German. Students' performances in these classes will not contribute to the degree classification awarded. However, successful completion of the course may be recorded on students' transcripts. See section 4 for more details.

1.4 Registration

Classes

Students will have to register in advance for the courses they wish to take. Students will have to register by Friday of Week 9 of Trinity Term 2016 using the online system which can be accessed at https://www.maths.ox.ac.uk/courses/registration/. Students will then be asked to sign up for classes at the start of Michaelmas Term 2016. Further information about this will be sent via email before the start of term.

Note on Intercollegiate Classes

Where undergraduate registrations for lecture courses fall below 5, classes will not run as part of the intercollegiate scheme but will be arranged informally by the lecturer.

Lectures

Every effort will be made when timetabling lectures to ensure that lectures do not clash. However, because of the large number of options in Part C this may sometimes be unavoidable. In the event of clashes being necessary, then students will be notified of the clashes by email and in any case options will only be allowed to clash when the take-up of both options is unlikely or inadvisable.

1.5 Course list by term

Table 1: Michaelmas Term Courses

Code	Title	Term
C1.1	Model Theory	MT
C1.3	Analytic Topology	MT
C2.1	Lie Algebras	MT
C2.2	Homological Algebra	MT
C2.7	Category Theory	MT
C3.1	Algebraic Topology	MT
C3.3	Differentiable Manifolds	MT
C3.4	Algebraic Geometry	MT
C3.8	Analytic Number Theory	MT
C4.1	Functional Analysis	MT
C4.3	Functional Analytic Methods for PDEs	MT
C4.8	Complex Analysis: Conformal Maps and Geometry	MT
C5.1	Solid Mechanics	MT
C5.3	Statistical Mechanics	MT
C5.5	Perturbation Methods	MT
C5.7	Topics in Fluid Mechanics	MT
C5.11	Mathematical Geoscience	MT
C5.12	Mathematical Physiology	MT
C6.1	Numerical Linear Algebra	MT
C6.3	Approximation of Functions	MT
C7.1	Theoretical Physics	MT
C7.5	General Relativity I	MT
C8.1	Stochastic Differential Equations	MT
C8.3	Combinatorics	MT

Table 2: Hilary Term Courses

Code	Title	Term
C1.2	Godel's Incompleteness Theorems	HT
C1.4	Axiomatic Set Theory	HT
C2.3	Representation Theory of Semisimple Lie Algebras	HT
C2.4	Infinite Groups	HT
C2.5	Noncommutative Rings	HT
C2.6	Introduction to Schemes	HT
C3.2	Geometric Group Theory	HT
C3.5	Lie Groups	HT
C3.6	Modular Forms	HT
C3.7	Elliptic Curves	HT
C3.9	Computational Algebraic Topology	HT
C4.2	Linear Operators	HT
C4.6	Fixed Point Methods for Nonlinear PDEs	HT
C5.2	Elasticity and Plasticity	HT
C5.4	Networks	HT
C5.6	Applied Complex Variables	HT
C5.9	Mathematical Mechanical Biology	HT
C6.2	Continuous Optimisation	HT
C6.4	Finite Element Methods for PDEs	HT
C7.1	Theoretical Physics	HT
C7.6	General Relativity II	HT
C8.2	Stochastic Analysis and PDEs	HT
C8.4	Probabilistic Combinatorics	HT

2 Mathematics Department units

2.1 C1.1: Model Theory — Prof. Ehud Hrushovski — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

This course presupposes basic knowledge of First Order Predicate Calculus up to and including the Soundness and Completeness Theorems. A familiarity with (at least the statement of) the Compactness Theorem would also be desirable.

Overview

The course deepens a student's understanding of the notion of a mathematical structure and of the logical formalism that underlies every mathematical theory, taking B1 Logic as a starting point. Various examples emphasise the connection between logical notions and practical mathematics.

The concepts of completeness and categoricity will be studied and some more advanced technical notions, up to elements of modern stability theory, will be introduced.

Learning Outcomes

Students will have developed an in depth knowledge of the notion of an algebraic mathematical structure and of its logical theory, taking B1 Logic as a starting point. They will have an understanding of the concepts of completeness and categoricity and more advanced technical notions.

Synopsis

Structures. The first-order language for structures. The Compactness Theorem for first-order logic. Elementary embeddings. Löwenheim–Skolem theorems. Preservation theorems for substructures. Model Completeness. Quantifier elimination.

Categoricity for first-order theories. Types and saturation. Omitting types. The Ryll Nardzewski theorem characterizing aleph-zero categorical theories. Theories with few types. Ultraproducts.

Reading

1. C.C. Chang and H. Jerome Keisler, *Model Theory* (Third Edition (Dover Books on Mathematics)Paperback)
2. Tent and Ziegler, *A Course in Model Theory*, Cambridge University Press, April 2012

2.2 C1.2: Gödel's Incompleteness Theorems — Dr Dan Isaacson — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

This course presupposes knowledge of first-order predicate logic up to and including soundness and completeness theorems for a formal system of first-order predicate logic (B1 Logic).

Overview

The starting point is Gödel's mathematical sharpening of Hilbert's insight that manipulating symbols and expressions of a formal language has the same formal character as arithmetical operations on natural numbers. This allows the construction for any consistent formal system containing basic arithmetic of a 'diagonal' sentence in the language of that system which is true but not provable in the system. By further study we are able to establish the intrinsic meaning of such a sentence. These techniques lead to a mathematical theory of formal provability which generalizes the earlier results. We end with results that further sharpen understanding of formal provability.

Learning Outcomes

Understanding of arithmetization of formal syntax and its use to establish incompleteness of formal systems; the meaning of undecidable diagonal sentences; a mathematical theory of formal provability; precise limits to formal provability and ways of knowing that an unprovable sentence is true.

Synopsis

Gödel numbering of a formal language; the diagonal lemma. Expressibility in a formal language. The arithmetical undefinability of truth in arithmetic. Formal systems of arithmetic; arithmetical proof predicates. Σ_0-completeness and Σ_1-completeness. The arithmetical hierarchy. ω-consistency and 1-consistency; the first Gödel incompleteness theorem. Separability; the Rosser incompleteness theorem. Adequacy conditions for a provability predicate. The second Gödel incompleteness theorem; Löb's theorem. Provable Σ_1-completeness. The ω-rule. The provability logics GL; fixed point theorems for GL. The Bernays arithmetized completeness theorem; undecidable Δ_2-sentences of arithmetic.

Reading

1. Lecture notes for the course.

Further Reading

1. Raymond M. Smullyan, *Gödel's Incompleteness Theorems* (Oxford University Press, 1992).

2. George S. Boolos and Richard C. Jeffrey, *Computability and Logic* (3rd edition, Cambridge University Press, 1989), Chs 15, 16, 27 (pp 170–190, 268-284).

3. George Boolos, *The Logic of Provability* (Cambridge University Press, 1993).

2.3 C1.3: Analytic Topology — Dr Rolf Suabedissen — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Part A Topology; a basic knowledge of Set Theory, including cardinal arithmetic, ordinals and the Axiom of Choice, will also be useful.

Overview

The aim of the course is to present a range of major theory and theorems, both important and elegant in themselves and with important applications within topology and to mathematics as a whole. Central to the course is the general theory of compactness, compactifications and Tychonoff's theorem, one of the most important in all mathematics (with applications across mathematics and in mathematical logic) and computer science.

Synopsis

Bases and initial topologies (including pointwise convergence and the Tychonoff product topology). Separation axioms, continuous functions, Urysohn's lemma. Separable, Lindelöf and second countable spaces. Urysohn's metrization theorem. Filters and ultrafilters. Tychonoff's theorem. Compactifications, in particular the Alexandroff One-Point Compactification and the Stone–Čech Compactification. Completeness, connectedness and local connectedness. Components and quasi-components. Totally disconnected compact spaces. Paracompactness; Bing Metrization Theorem.

Reading

1. S. Willard, *General Topology* (Addison–Wesley, 1970), Chs. 1–8.

2. R. Engelking, *General Topology* (Sigma Series in Pure Mathematics, Vol 6, 1989)

2.4 C1.4: Axiomatic Set Theory — Dr Rolf Suabedissen — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

This course presupposes basic knowledge of First Order Predicate Calculus up to and including the Soundness and Completeness Theorems, together with a course on basic set theory, including cardinals and ordinals, the Axiom of Choice and the Well Ordering Principle.

Overview

Inner models and consistency proofs lie at the heart of modern Set Theory, historically as well as in terms of importance. In this course we shall introduce the first and most important of inner models, Gödel's constructible universe, and use it to derive some fundamental consistency results.

Synopsis

A review of the axioms of ZF set theory. Absoluteness, the recursion theorem. The Cumulative Hierarchy of sets and the consistency of the Axiom of Foundation as an example of the method of inner models. Levy's Reflection Principle. Gödel's inner model of constructible sets and the consistency of the Axiom of Constructibility ($V = L$). $V = L$ is absolute. The fact that $V = L$ implies the Axiom of Choice. Some advanced cardinal arithmetic. The fact that $V = L$ implies the Generalized Continuum Hypothesis.

Reading

For the review of ZF set theory and the prerequisites from Logic:

1. D. Goldrei, *Classic Set Theory* (Chapman and Hall, 1996).
2. K. Kunen, *The Foundations of Mathematics* (College Publications, 2009).

For course topics (and much more):

1. K. Kunen, *Set Theory* (College Publications, 2011) Chapters (I and II).

Further Reading

1. K. Hrbacek and T. Jech, *Introduction to Set Theory* (3rd edition, M Dekker, 1999).

2.5 C2.1: Lie Algebras — Prof. Dan Ciubotaru — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Part B course B2.1 Introduction to Representation Theory. A thorough knowledge of linear algebra and the second year algebra courses; in particular familiarity with group actions, quotient rings and vector spaces, isomorphism theorems and inner product spaces will be assumed. Some familiarity with the Jordan–Hölder theorem and the general ideas of representation theory will be an advantage.

Overview

Lie Algebras are mathematical objects which, besides being of interest in their own right, elucidate problems in several areas in mathematics. The classification of the finite-dimensional complex Lie algebras is a beautiful piece of applied linear algebra. The aims of this course are to introduce Lie algebras, develop some of the techniques for studying them, and describe parts of the classification mentioned above, especially the parts concerning root systems and Dynkin diagrams.

Learning Outcomes

Students will learn how to utilise various techniques for working with Lie algebras, and they will gain an understanding of parts of a major classification result.

Synopsis

Definition of Lie algebras, small-dimensional examples, some classical groups and their Lie algebras (treated informally). Ideals, subalgebras, homomorphisms, modules.

Nilpotent algebras, Engel's theorem; soluble algebras, Lie's theorem. Semisimple algebras and Killing form, Cartan's criteria for solubility and semisimplicity, Weyl's theorem on complete reducibility of representations of semisimple Lie algebras.

The root space decomposition of a Lie algebra; root systems, Cartan matrices and Dynkin diagrams. Discussion of classification of irreducible root systems and semisimple Lie algebras.

Reading

1. J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory,* Graduate Texts in Mathematics 9 (Springer-Verlag, 1972, reprinted 1997). Chapters 1–3 are relevant and part of the course will follow Chapter 3 closely.

2. B. Hall, *Lie Groups, Lie Algebras, and Representations. An Elementary Introduction,* Graduate Texts in Mathematics 222 (Springer-Verlag, 2003).

3. K. Erdmann, M. J. Wildon, *Introduction to Lie Algebras* (Springer-Verlag, 2006), ISBN: 1846280400.

Further Reading

1. J.-P. Serre, *Complex Semisimple Lie Algebras* (Springer, 1987). Rather condensed, assumes the basic results. Very elegant proofs.

2. N. Bourbaki, *Lie Algebras and Lie Groups* (Masson, 1982). Chapters 1 and 4–6 are relevant; this text fills in some of the gaps in Serre's text.

3. William Fulton, Joe Harris, *Representation theory: a first course*, GTM, Springer.

2.6 C2.2 Homological Algebra — Prof. Andre Henriques — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Part A Rings and Modules. Introduction to Representation Theory B2.1 is recommended but not essential.

Overview

Homological algebra is one of the most important tools in mathematics with application ranging from number theory and geometry to quantum physics. This course will introduce the basic concepts and tools of homological algebra with examples in module theory and group theory.

Learning Outcomes

Students will learn about abelian categories and derived functors and will be able to apply these notions in different contexts. They will learn to compute Tor, Ext, and group cohomology and homology.

Synopsis

Chain complexes: complexes of R-modules, operations on chain complexes, long exact sequences, chain homotopies, mapping cones and cylinders (4 hours) Derived functors: delta functors, projective and injective resolutions, left and right derived functors (5 hours) Tor and Ext: Tor and flatness, Ext and extensions, universal coefficients theorems, Koszul resolutions (4 hours) Group homology and cohomology: definition, interpretation of H^1 and H^2, universal central extensions, the Bar resolution (3 hours).

Reading

Weibel, Charles *An introduction to Homological algebra* (see Google Books)

2.7 C2.3: Representation Theory of Semisimple Lie Algebras — Prof. Dan Ciubotaru —16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Past attendance at Lie algebras is recommended, but not required. Past attendance at Introduction to Representation Theory is recommended as well, but not required.

Overview

The representation theory of semisimple Lie algebras plays a central role in modern mathematics with motivation coming from many areas of mathematics and physics, for example, the Langlands program. The methods involved in the theory are diverse and include remarkable interactions with algebraic geometry, as in the proofs of the Kazhdan-Lusztig and Jantzen conjectures.

The course will cover the basics of finite dimensional representations of semisimple Lie algebras (e.g., the Cartan-Weyl highest weight classification) in the framework of the larger Bernstein-Gelfand-Gelfand category $\mathcal{O}$.

Learning Outcomes

The students will have developed a comprehensive understanding of the basic concepts and modern methods in the representation theory of semisimple Lie algebras, including the classification of finite dimensional modules, the classification of objects in category O, character formulas, Lie algebra cohomology and resolutions of finite dimensional modules.

Synopsis

Universal enveloping algebra of a Lie algebra, Poincaré-Birkhoff-Witt theorem, basic definitions and properties of representations of Lie algebras, tensor products.

The example of $sl(2)$: finite dimensional modules, highest weights.

Category $\mathcal{O}$: Verma modules, highest weight modules, infinitesimal characters and Harish-Chandra's isomorphism, formal characters, contravariant (Shapovalov) forms.

Finite dimensional modules of a semisimple Lie algebra: the Cartan-Weyl classification, Weyl character formula, dimension formula, Kostant's multiplicity formula, examples.

Homological algebra: Lie algebra cohomology, Bernstein-Gelfand-Gelfand resolution of finite dimensional modules, Ext groups in category $\mathcal{O}$.

Topics: applications, Bott's dimension formula for Lie algebra cohomology groups, characters of the symmetric group (via Zelevinsky's application of the BGG resolution to Schur-Weyl duality).

Reading

1. Course Lecture Notes.
2. J. Bernstein, "Lectures on Lie algebras", in *Representation Theory, Complex Analysis, and Integral Geometry* (Springer 2012).

Further reading

1. J. Humphreys, *Representations of semisimple Lie algebras in the BGG category $\mathcal{O}$* (AMS, 2008).
2. J. Humphreys, *Introduction to Lie algebras and representation theory* (Springer, 1997).
3. W. Fulton, J. Harris, *Representation Theory* (Springer 1991).

2.8 C2.4: Infinite Groups — Prof. Dan Segal — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

A thorough knowledge of the second-year algebra courses; in particular, familiarity with group actions, quotient rings and quotient groups, and isomorphism theorems will be assumed. Familiarity with the Commutative Algebra course will be helpful but not essential.

Overview

The concept of a group is so general that anything which is true of all groups tends to be rather trivial. In contrast, groups that arise in some specific context often have a rich and beautiful theory. The course introduces some natural families of groups, various questions that one can ask about them, and various methods used to answer these questions; these involve among other things rings and trees.

Synopsis

Free groups and their subgroups; finitely generated groups: counting finite-index subgroups; finite presentations and decision problems; Linear groups: residual finiteness; structure of soluble linear groups; Nilpotency and solubility: lower central series and derived series; structural and residual properties of finitely generated nilpotent groups and polycyclic groups; characterization of polycyclic groups as soluble Z-linear groups; Torsion groups and the General Burnside Problem.

Reading

1. D. J. S. Robinson, *A course in the theory of groups*, 2nd ed., Graduate texts in Mathematics, (Springer-Verlag, 1995). Chapters 2, 5, 6, 15.
2. D. Segal, *Polycyclic groups*, (CUP, 2005) Chapters 1 and 2.

2.9 C2.5: Non-Commutative Rings — Dr Thomas Bitoun — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Prerequisites: Part A Rings and Modules.
Recommended background: Introduction to Representation Theory B2.1, Part B Commutative Algebra (from 2016 onwards).

Overview

This course builds on Algebra 2 from the second year. We will look at several classes of non-commutative rings and try to explain the idea that they should be thought of as functions on "non-commutative spaces". Along the way, we will prove several beautiful structure theorems for Noetherian rings and their modules.

Learning Outcomes

Students will be able to appreciate powerful structure theorems, and be familiar with examples of non-commutative rings arising from various parts of mathematics.

Synopsis

1. Examples of non-commutative Noetherian rings: enveloping algebras, rings of differential operators, group rings of polycyclic groups. Filtered and graded rings. (3 hours)

2. Jacobson radical in general rings. Jacobson's density theorem. Artin-Wedderburn. (3 hours)

3. Ore localisation. Goldie's Theorem on Noetherian domains. (3 hours)

4. Minimal prime ideals and dimension functions. Rees rings and good filtrations. (3 hours)

5. Bernstein's Inequality and Gabber's Theorem on the integrability of the characteristic variety. (4 hours)

Reading

1. K.R. Goodearl and R.B. Warfield, *An Introduction to Noncommutative Noetherian Rings* (CUP, 2004).

Further reading

1. M. Atiyah and I. MacDonald, *Introduction to Commutative Algebra* (Westview Press, 1994).
2. S.C. Coutinho, *A Primer of Algebraic D-modules* (CUP, 1995).
3. J. Björk, *Analytic D-Modules and Applications* (Springer, 1993).

2.10 C2.6: Introduction to Schemes — Prof. Damian Rossler — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Commutative Algebra is essential. Homological Algebra is highly recommended and Category Theory is recommended but the necessary material from both courses can be learnt during the course (see the beginning of the lecture notes for precise references). Algebraic Geometry is recommended but not technically necessary. Algebraic Topology contains many homological techniques also used in C2.6.

Overview

Scheme theory is the foundation of modern algebraic geometry. It unifies algebraic geometry with algebraic number theory. This unification has led to proofs of important conjectures in number theory such as the Weil conjecture by Deligne and the Mordell conjecture by Faltings.

This course will cover the basics of the theory of schemes, with an emphasis on cohomological techniques.

Learning Outcomes

Students will have developed a thorough understanding of the basic concepts and methods of scheme theory. They will be able to work with affine and projective schemes, as well as with coherent sheaves and their cohomology groups.

Synopsis

Sheaves and cohomology of sheaves.

Affine schemes: points, topology, structure sheaf. Schemes: definition, subschemes, morphisms, glueing. Relative schemes: fibred products, Cohomological characterisation of affine schemes.

Projective schemes, morphisms to projective space. Ample line bundles. Cohomological characterisation of ampleness.

Flat morphisms, semicontinuity, Hilbert polynomials. Cohomological characterisation of flatness.

Constructibility and irreducibility. Images of constructible sets.

Separatedness, properness and valuative criteria. Hilbert and Quot schemes.

Reading

1. Robin Hartshorne, *Algebraic Geometry.*
2. Ravi Vakil, *Foundations of Algebraic Geometry*, online notes on the website of Stanford University (open access).

Further reading

1. David Mumford, *The Red Book of Varieties and Schemes.*
2. David Eisenbud and Joe Harris, *The Geometry of Schemes.*

2.11 C2.7: Category Theory — Prof. Kobi Kremnitzer — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

There are no essential prerequisites but past attendance at several Part A pure maths options would be very useful, and the Part B courses Topology and Groups and Introduction to Representation Theory are relevant, as are a number of Part C courses. Category Theory also has links with B1 Logic and Set Theory, but this course will not stress those links.

Overview

Category theory brings together many areas of pure mathematics (and also has close links to logic and to computer science). It is based on the observation that many mathematical topics can be unified and simplified by using descriptions in terms of diagrams of arrows; the arrows represent functions of suitable types. Moreover many constructions in pure mathematics can be described in terms of 'universal properties' of such diagrams.

The aim of this course is to provide an introduction to category theory using a host of familiar examples, to explain how these examples fit into a categorical framework and to use categorical ideas to make new constructions.

Learning Outcomes

Students will have developed a thorough understanding of the basic concepts and methods of category theory. They will be able to work with commutative diagrams, naturality and universality properties, and to apply categorical ideas and methods in a wide range of areas of mathematics.

Synopsis

Introduction: universal properties in linear and multilinear algebra.

Categories, functors, natural transformations. Examples including categories of sets, groups, rings, vector spaces and modules, topological spaces. Groups, monoids and partially ordered sets as categories. Opposite categories and the principle of duality. Covariant, contravariant, faithful and full functors.

Adjoints: definition and examples including free and forgetful functors and abelianisations of groups. Adjunctions via units and counits, adjunctions via initial objects.

Representables: definitions and examples including tensor products. The Yoneda lemma and applications.

Limits and colimits, including products, equalizers, pullbacks and pushouts. Monics and epics. Interaction between functors and limits.

Monads and comonads, Barr-Beck monadicity theorem, faithfully flat descent, pure monomorphisms of rings. The category of affine schemes as the opposite of the category of commutative rings. Examples of non-affine schemes.

Reading

1. T. Leinster, *Basic category theory*, (CUP, 2014) Chapters 1-5

Further reading

1. D. Eisenbud, J. Harris, *The geometry of schemes.*

2. S. Lang, *Linear algebra* 2nd edition, (Addison Wesley, 1971) Chapter XIII, out of print but may be available in college libraries.
3. S. Mac Lane, *Categories for the Working Mathematician*, 2nd ed., (Springer, 1998)
4. S. Awodey, *Category theory*, Oxford Logic Guides (OUP, 2010)
5. D.G. Northcott, *Multilinear algebra* (CUP, reissued 2009)

2.12 C3.1: Algebraic Topology — Prof. Christopher Douglas — 16MT

Level: M-level. **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Helpful but not essential: Part A Topology, B3.5 Topology and Groups.

Overview

Homology theory is a subject that pervades much of modern mathematics. Its basic ideas are used in nearly every branch, pure and applied. In this course, the homology groups of topological spaces are studied. These powerful invariants have many attractive applications. For example we will prove that the dimension of a vector space is a topological invariant and the fact that 'a hairy ball cannot be combed'.

Learning Outcomes

At the end of the course, students are expected to understand the basic algebraic and geometric ideas that underpin homology and cohomology theory. These include the cup product and Poincaré Duality for manifolds. They should be able to choose between the different homology theories and to use calculational tools such as the Mayer-Vietoris sequence to compute the homology and cohomology of simple examples, including projective spaces, surfaces, certain simplicial spaces and cell complexes. At the end of the course, students should also have developed a sense of how the ideas of homology and cohomology may be applied to problems from other branches of mathematics.

Synopsis

Chain complexes of free Abelian groups and their homology. Short exact sequences. Delta complexes and their homology. Euler characteristic.

Singular homology of topological spaces. Relative homology and the Five Lemma. Homotopy invariance and excision (details of proofs not examinable). Mayer-Vietoris Sequence. Equivalence of simplicial and singular homology.

Degree of a self-map of a sphere. Cell complexes and cellular homology. Application: the hairy ball theorem.

Cohomology of spaces and the Universal Coefficient Theorem (proof not examinable). Cup products. Künneth Theorem (without proof). Topological manifolds and orientability. The fundamental class of an orientable, closed manifold and the degree of a map between manifolds of the same dimension. Poincaré Duality (without proof).

Reading

1. A. Hatcher, *Algebraic Topology* (Cambridge University Press, 2001). Chapters 2 and 3.

2. G. Bredon, *Topology and Geometry* (Springer, 1997). Chapters 4 and 5.

3. J. Vick, *Homology Theory,* Graduate Texts in Mathematics 145 (Springer, 1973).

2.13 C3.2: Geometric Group Theory — Prof. Panos Papazoglou — 16HT

Level: M-level. **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites.

The Topology & Groups course is a helpful, though not essential prerequisite.

Overview.

The aim of this course is to introduce the fundamental methods and problems of geometric group theory and discuss their relationship to topology and geometry.

The first part of the course begins with an introduction to presentations and the list of problems of M. Dehn. It continues with the theory of group actions on trees and the structural study of fundamental groups of graphs of groups.

The second part of the course focuses on modern geometric techniques and it provides an introduction to the theory of Gromov hyperbolic groups.

Synopsis.

Free groups. Group presentations. Dehn's problems. Residually finite groups.

Group actions on trees. Amalgams, HNN-extensions, graphs of groups, subgroup theorems for groups acting on trees.

Quasi-isometries. Hyperbolic groups. Solution of the word and conjugacy problem for hyperbolic groups.

If time allows: Small Cancellation Groups, Stallings Theorem, Boundaries.

Reading.

1. J.P. Serre, *Trees* (Springer Verlag 1978).

2. M. Bridson, A. Haefliger, *Metric Spaces of Non-positive Curvature, Part III* (Springer, 1999), Chapters I.8, III.H.1, III. *Gamma* 5.

3. H. Short *et al.*, ‘Notes on word hyperbolic groups’, *Group Theory from a Geometrical Viewpoint, Proc. ICTP Trieste* (eds E. Ghys, A. Haefliger, A. Verjovsky, World Scientific 1990)

   available online at: http://www.cmi.univ-mrs.fr/ hamish/

4. C.F. Miller, *Combinatorial Group Theory*, notes: http://www.ms.unimelb.edu.au/ cfm/notes/cgt-notes.pdf.

Further Reading.

1. G. Baumslag, *Topics in Combinatorial Group Theory* (Birkhauser, 1993).
2. O. Bogopolski, *Introduction to Group Theory* (EMS Textbooks in Mathematics, 2008).
3. R. Lyndon, P. Schupp, *Combinatorial Group Theory* (Springer, 2001).
4. W. Magnus, A. Karass, D. Solitar,*Combinatorial Group Theory: Presentations of Groups in Terms of Generators and Relations* (Dover Publications, 2004).
5. P. de la Harpe, *Topics in Geometric Group Theory*, (University of Chicago Press, 2000).

2.14 C3.3: Differentiable Manifolds — Prof. Dominic Joyce — 16MT

Level: M-level. **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Part A Introduction to Manifolds. Useful but not essential: Part B Geometry of Surfaces.

Overview

A manifold is a space such that small pieces of it look like small pieces of Euclidean space. Thus a smooth surface, the topic of the B3 course, is an example of a (2-dimensional) manifold.

Manifolds are the natural setting for parts of classical applied mathematics such as mechanics, as well as general relativity. They are also central to areas of pure mathematics such as topology and certain aspects of analysis.

In this course we introduce the tools needed to do analysis on manifolds. We prove a very general form of Stokes' Theorem which includes as special cases the classical theorems of Gauss, Green and Stokes. We also introduce the theory of de Rham cohomology, which is central to many arguments in topology.

Learning Outcomes

The candidate will be able to manipulate with ease the basic operations on tangent vectors, differential forms and tensors both in a local coordinate description and a global coordinate-free one; have a knowledge of the basic theorems of de Rham cohomology and some simple examples of their use; know what a Riemannian manifold is and what geodesics are.

Synopsis

Smooth manifolds and smooth maps. Tangent vectors, the tangent bundle, induced maps. Vector fields and flows, the Lie bracket and Lie derivative.

Exterior algebra, differential forms, exterior derivative, Cartan formula in terms of Lie derivative. Orientability. Partitions of unity, integration on oriented manifolds.

Stokes' theorem. De Rham cohomology. Applications of de Rham theory including degree.

Riemannian metrics. Isometries. Geodesics.

Reading

1. M. Spivak, *Calculus on Manifolds*, (W. A. Benjamin, 1965).
2. M. Spivak, *A Comprehensive Introduction to Differential Geometry*, Vol. 1, (1970).
3. W. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, 2nd edition, (Academic Press, 1986).
4. M. Berger and B. Gostiaux, *Differential Geometry: Manifolds, Curves and Surfaces.* Translated from the French by S. Levy, (Springer Graduate Texts in Mathematics, 115, Springer–Verlag (1988)) Chapters 0–3, 5–7.
5. F. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, (Springer Graduate Texts in Mathematics, 1994).
6. D. Barden and C. Thomas, *An Introduction to Differential Manifolds.* (Imperial College Press, London, 2003.)

2.15 C3.4: Algebraic Geometry — Prof. Alexander Ritter — 16MT

Level: M-level. **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Part A Rings and Modules. B3.3 Algebraic Curves useful but not essential.

Overview

Algebraic geometry is the study of algebraic varieties: an algebraic variety is roughly speaking, a locus defined by polynomial equations. One of the advantages of algebraic geometry is that it is purely algebraically defined and applied to any field, including fields of finite characteristic. It is geometry based on algebra rather than calculus, but over the real or complex numbers it provides a rich source of examples and inspiration to other areas of geometry.

Synopsis

Affine algebraic varieties, the Zariski topology, morphisms of affine varieties. Irreducible varieties.

Projective space. Projective varieties, affine cones over projective varieties. The Zariski topology on projective varieties. The projective closure of affine variety. Morphisms of projective varieties. Projective equivalence.

Veronese morphism: definition, examples. Veronese morphisms are isomorphisms onto their image; statement, and proof in simple cases. Subvarieties of Veronese varieties. Segre maps and products of varieties. Categorical products: the image of the Segre map gives the categorical product.

Coordinate rings. Hilbert's Nullstellensatz. Correspondence between affine varieties (and morphisms between them) and finitely generated reduced k-algebras (and morphisms between them). Graded rings and homogeneous ideals. Homogeneous coordinate rings.

Categorical quotients of affine varieties by certain group actions. The maximal spectrum.

Primary decomposition of ideals.

Discrete invariants of projective varieties: degree, dimension, Hilbert function. Statement of theorem defining Hilbert polynomial.

Quasi-projective varieties, and morphisms between them. The Zariski topology has a basis of affine open subsets. Rings of regular functions on open subsets and points of quasi-projective varieties. The ring of regular functions on an affine variety is the coordinate ring. Localisation and relationship with rings of regular functions.

Tangent space and smooth points. The singular locus is a closed subvariety. Algebraic re-formulation of the tangent space. Differentiable maps between tangent spaces.

Function fields of irreducible quasi-projective varieties. Rational maps between irreducible varieties, and composition of rational maps. Birational equivalence. Correspondence between dominant rational maps and homomorphisms of function fields. Blow-ups: of affine space at a point, of subvarieties of affine space, and of general quasi-projective varieties along general subvarieties. Statement of Hironaka's Desingularisation Theorem. Every irreducible variety is birational to a hypersurface. Re-formulation of dimension. Smooth points are a dense open subset.

Reading

KE Smith et al, *An Invitation to Algebraic Geometry*, (Springer 2000), Chapters 1–8.

Further Reading

1. M Reid, *Undergraduate Algebraic Geometry*, LMS Student Texts 12, (Cambridge 1988).
2. K Hulek, *Elementary Algebraic Geometry*, Student Mathematical Library 20. (American Mathematical Society, 2003).
3. A Gathmann, *Algebraic Geometry lecture notes*, online: `www.mathematik.uni-kl.de/en/agag/members/professors/gathmann/notes/alggeom`
4. I Shafarevich, *Basic Algebraic Geometry 1*, (Springer, 1994).
5. D Mumford, *The Red Book of Varieties and Schemes*, (Springer, 2009).

2.16 C3.5: Lie Groups — Prof. Andrew Dancer — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Part A Group Theory, Topology and Introduction to Manifolds are all useful but not essential.

Overview

The theory of Lie Groups is one of the most beautiful developments of pure mathematics in the twentieth century, with many applications to geometry, theoretical physics and mechanics. The subject is an interplay between geometry, analysis and algebra. Lie groups are groups which are simultaneously manifolds, that is geometric objects where the notion of differentiability makes sense, and the group multiplication and inversion are differentiable maps. The majority of examples of Lie groups are the familiar groups of matrices. The course does not require knowledge of differential geometry: the basic tools needed will be covered within the course.

Learning Outcomes

Students will have learnt the fundamental relationship between a Lie group and its Lie algebra, and the basics of representation theory for compact Lie groups. This will include a firm understanding of maximal tori and the Weyl group, and their role for representations.

Synopsis

Brief introduction to manifolds. Classical Lie groups. Left-invariant vector fields, Lie algebra of a Lie group. One-parameter subgroups, exponential map. Homomorphisms of Lie groups and Lie algebras. Ad and ad. Compact connected abelian Lie groups are tori. The Campbell-Baker-Hausdorff series (statement only).

Lie subgroups. Definition of embedded submanifolds. A subgroup is an embedded Lie subgroup if and only if it is closed. Continuous homomorphisms of Lie groups are smooth. Correspondence between Lie subalgebras and Lie subgroups (proved assuming the Frobenius theorem). Correspondence between Lie group homomorphisms and Lie algebra homomorphisms. Ado's theorem (statement only), Lie's third theorem.

Basics of representation theory: sums and tensor products of representations, irreducibility, Schur's lemma. Compact Lie groups: left-invariant integration, complete reducibility. Representations of the circle and of tori. Characters, orthogonality relations. Peter-Weyl theorem (statement only).

Maximal tori. Roots. Conjugates of a maximal torus cover a compact connected Lie group (proved assuming the Lefschetz fixed point theorem). Weyl group. Reflections. Weyl group of $U(n)$. Representations of a compact connected Lie group are the Weyl-invariant representations of a maximal torus (proof of inclusion only). Representation ring of T^n and $U(n)$.

Killing form. Remarks about the classification of compact Lie groups.

Reading

1. J. F. Adams, *Lectures on Lie Groups* (University of Chicago Press, 1982).

2. T. Bröcker and T. tom Dieck, *Representations of Compact Lie Groups* (Graduate Texts in Mathematics, Springer, 1985).

Further Reading

1. R. Carter, G. Segal and I. MacDonald, *Lectures on Lie Groups and Lie Algebras* (LMS Student Texts, Cambridge, 1995).

2. W. Fulton, J. Harris, *Representation Theory: A First Course* (Graduate Texts in Mathematics, Springer, 1991).

3. F. W. Warner, *Foundations of Differentiable Manifolds and Lie Groups* (Graduate Texts in Mathematics, 1983).

2.17 C3.6: Modular Forms — Prof. Alan Lauder — 16HT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Part A Number Theory, Topology and Part B Geometry of Surfaces, Algebraic Curves are useful but not essential.

Overview

The course aims to introduce students to the beautiful theory of modular forms, one of the cornerstones of modern number theory. This theory is a rich and challenging blend of methods from complex analysis and linear algebra, and an explicit application of group actions.

Learning Outcomes

The student will learn about modular curves and spaces of modular forms, and understand in special cases how to compute their genus and dimension, respectively. They will see that modular forms can be described explicitly via their q-expansions, and they will be familiar with explicit examples of modular forms. They will learn about the rich algebraic structure on spaces of modular forms, given by Hecke operators and the Petersson inner product.

Synopsis

1. Overview and examples of modular forms. Definition and basic properties of modular forms.

2. Topology of modular curves: a fundamental domain for the full modular group; fundamental domains for subgroups Γ of finite index in the modular group; the compact surfaces X_Γ; explicit triangulations of X_Γ and the computation of the genus using the Euler characteristic formula; the congruence subgroups $\Gamma(N), \Gamma_1(N)$ and $\Gamma_0(N)$; examples of genus computations.

3. Dimensions of spaces of modular forms: general dimension formula (proof non-examinable); the valence formula (proof non-examinable).

4. Examples of modular forms: Eisenstein series in level 1; Ramanujan's Δ function; some arithmetic applications.

5. The Petersson inner product.

6. Modular forms as functions on lattices: modular forms of level 1 as functions on lattices; Eisenstein series revisited.

7. Hecke operators in level 1: Hecke operators on lattices; Hecke operators on modular forms and their q-expansions; Hecke operators are Hermitian; multiplicity one.

Reading

1. F. Diamond and J. Shurman, *A First Course in Modular Forms,* Graduate Texts in Mathematics 228, Springer-Verlag, 2005.

2. R.C. Gunning, *Lectures on Modular Forms*, Annals of mathematical studies 48, Princeton University Press, 1962.

3. J.S. Milne, *Modular Functions and Modular Forms*: www.jmilne.org/math/CourseNotes/mf.html

4. J.-P. Serre, Chapter VII, *A Course in Arithmetic,* Graduate Texts in Mathematics 7, Springer-Verlag, 1973.

2.18 C3.7 Elliptic Curves — Prof. Victor Flynn — 16HT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

It is helpful, but not essential, if students have already taken a standard introduction to algebraic curves and algebraic number theory. For those students who may have gaps in their background, I have placed the file "Preliminary Reading" permanently on the Elliptic Curves webpage, which gives in detail (about 30 pages) the main prerequisite knowledge for the course. Go first to: http://www.maths.ox.ac.uk/courses/material then click on "C3.7 Elliptic Curves" and then click on the pdf file "Preliminary Reading".

Overview

Elliptic curves give the simplest examples of many of the most interesting phenomena which can occur in algebraic curves; they have an incredibly rich structure and have been the testing ground for many developments in algebraic geometry whilst the theory is still full of deep unsolved conjectures, some of which are amongst the oldest unsolved problems in mathematics. The course will concentrate on arithmetic aspects of elliptic curves defined over the rationals, with the study of the group of rational points, and explicit determination of the rank, being the primary focus. Using elliptic curves over the rationals as an example, we will be able to introduce many of the basic tools for studying arithmetic properties of algebraic varieties.

Learning Outcomes

On completing the course, students should be able to understand and use properties of elliptic curves, such as the group law, the torsion group of rational points, and 2-isogenies between elliptic curves. They should be able to understand and apply the theory of fields with valuations, emphasising the p-adic numbers, and be able to prove and apply Hensel's Lemma in problem solving. They should be able to understand the proof of the Mordell–Weil Theorem for the case when an elliptic curve has a rational point of order 2, and compute ranks in such cases, for examples where all homogeneous spaces for descent-via-2-isogeny satisfy the Hasse principle. They should also be able to apply the elliptic curve method for the factorisation of integers.

Synopsis

Non-singular cubics and the group law; Weierstrass equations.
Elliptic curves over finite fields; Hasse estimate (stated without proof).
p-adic fields (basic definitions and properties).
1-dimensional formal groups (basic definitions and properties).
Curves over p-adic fields and reduction mod p.
Computation of torsion groups over $\mathbb{Q}$; the Nagell–Lutz theorem.
2-isogenies on elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.
Weak Mordell–Weil Theorem for elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.
Height functions on Abelian groups and basic properties.
Heights of points on elliptic curves defined over $\mathbb{Q}$; statement (without proof) that this gives a height function on the Mordell–Weil group.
Mordell–Weil Theorem for elliptic curves defined over $\mathbb{Q}$, with a $\mathbb{Q}$-rational point of order 2.
Explicit computation of rank using descent via 2-isogeny.
Public keys in cryptography; Pollard's $(p-1)$ method and the elliptic curve method of factorisation.

Reading

1. J.W.S. Cassels, *Lectures on Elliptic Curves,* LMS Student Texts 24 (Cambridge University Press, 1991).
2. N. Koblitz, *A Course in Number Theory and Cryptography,* Graduate Texts in Mathematics 114 (Springer, 1987).
3. J.H. Silverman and J. Tate, *Rational Points on Elliptic Curves,* Undergraduate Texts in Mathematics (Springer, 1992).
4. J.H. Silverman, *The Arithmetic of Elliptic Curves,* Graduate Texts in Mathematics 106 (Springer, 1986).

Further Reading

1. A. Knapp, *Elliptic Curves, Mathematical Notes 40* (Princeton University Press, 1992).
2. G, Cornell, J.H. Silverman and G. Stevans (editors), *Modular Forms and Fermat's Last Theorem* (Springer, 1997).
3. J.H. Silverman, *Advanced Topics in the Arithmetic of Elliptic Curves,* Graduate Texts in Mathematics 151 (Springer, 1994).

2.19 C3.8: Analytic Number Theory — Prof. Ben Green — 16MT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Basic ideas of complex analysis. Elementary number theory. Some familiarity with Fourier series will be helpful but not essential.

Overview

The aim of this course is to study the prime numbers using the famous Riemann ζ-function. In particular, we will study the connection between the primes and the zeros of the ζ-function. We will state the Riemann hypothesis, perhaps the most famous unsolved problem in mathematics, and examine its implication for the distribution of primes. We will prove the prime number theorem, which states that the number of primes less than X is asymptotic to $X/\log X$.

Learning Outcomes

In addition to the highlights mentioned above, students will gain experience with different types of Fourier transform and with the use of complex analysis.

Synopsis

Introductory material on primes. Arithmetic functions: Möbius function, Euler's ϕ-function, the divisor function, the σ-function. Multiplicativity. Dirichlet series and Euler products. The von Mangoldt function.

The Riemann ζ-function for $\Re(s) > 1$. Euler's proof of the infinitude of primes. ζ and the von Mongoldt function.

Schwarz functions on $\mathbf{R}$, $\mathbf{Z}$, $\mathbf{R}/\mathbf{Z}$ and their Fourier transforms. *Inversion formulas and uniqueness*. The Poisson summation formula. The θ-function and its functional equation. The Γ-function and the meromorphic continuation and functional equation of the ζ-function. Poles and zeros of ζ and statement of the Riemann hypothesis. Basic estimates for ζ.

Product theorems for entire functions. Hadamard's product formula for ζ and the partial fraction expansion.

The Mellin transform of a smooth compactly supported function. The Mellin inversion formula. Decay of the Mellin transform in vertical strips. Statement and proof of the explicit formula.

The classical zero-free region. Proof of the prime number theorem. Implications of the Riemann hypothesis for the distribution of primes.

Reading

Full printed notes will be provided for the course, including the non-examinable topics (marked with asterisks above). The following books are relevant to the course.

1. G. H. Hardy and E. M. Wright, An introduction to the Theory of Numbers (Sixth edition, OUP 2008). Chapters 16, 17, 18.
2. H. Davenport, Multiplicative number theory (Third Edition, Springer Graduate texts 74), selected parts of the first half.
3. M. du Sautoy, Music of the primes (this is a popular book which could be useful background reading for the course).

2.20 C3.9: Computational Algebraic Topology — Prof Ulrike Tillmann & Prof Samson Abramsky 16HT

Level: M-level **Method of Assessment:** Mini-project (see section 1.2)

Weight: Unit

Prerequisites

Some familiarity with the main concepts from algebraic topology, homological algebra and category theory will be helpful.

Overview

Ideas and tools from algebraic topology have become more and more important in computational and applied areas of mathematics. This course will provide at the masters level an introduction to the main concepts of (co)homology theory, and explore areas of applications in data analysis and in foundations of quantum mechanics and quantum information.

Learning outcomes

Students should gain a working knowledge of homology and cohomology of simplicial sets and sheaves, and improve their geometric intuition. Furthermore, they should gain an awareness of a variety of application in rather different, research active fields of applications with an emphasis on data analysis and contextuality.

Synopsis

The course has two parts. The first part will introduce students to the basic concepts and results of (co)homology, including sheaf cohomology. In the second part applied topics are introduced and explored.

Core: Homology and cohomology of chain complexes. Algorithmic computation of boundary maps (with a view of the classification theorm for finitely generated modules over a PID). Chain homotopy. Snake Lemma. Simplicial complexes. Other complexes (Delaunay, Cech). Mayer-Vietoris sequence. Poincare duality. Alexander duality. Acyclic carriers. Discrete Morse theory. (6 lectures)

Topic A: Persistent homology: barcodes and stability, applications todata analysis, generalisations. (4 lectures)

*Topic B:*Sheaf cohomology and applications to quantum non-locality and contextuality.Sheaf-theoretic representation of quantum non-locality and contextuality asobstructions to global sections. Cohomological characterizations and proofs of contextuality.(6 lectures)

Reading List

H. Edelsbrunner and J.L. Harer, *Computational Topology -An Introduction*,AMS (2010).

See also, U. Tillmann, Lecture notes for CAT 2012, in http://people.maths.ox.ac.uk/tillmann/CAT.html

Topic A:

G. Carlsson, *Topology and data*, Bulletin A.M.S.46 (2009), 255-308.

H. Edelsbrunner, J.L. Harer, *Persistent homology: A survey*, Contemporary Mathematics 452 A.M.S. (2008), 257-282.

S. Weinberger, *What is ... Persistent Homology?*, Notices A.M.S. 58 (2011), 36-39.

P. Bubenik, J. Scott, *Categorification of Persistent Homology*, Discrete Comput. Geom. (2014), 600–627.

Topic B:

S. Abramsky and Adam Brandenburger, The Sheaf-Theoretic Structure Of Non-Locality and Contextuality. In *New Journal of Physics*, 13(2011), 113036, 2011.

S. Abramsky and L. Hardy, Logical Bell Inequalities, Phys. Rev. A 85, 062114 (2012).

S. Abramsky, S. Mansfield and R. Soares Barbosa, The Cohomology of Non-Locality and Contextuality, in *Proceedings of Quantum Physics and Logic 2011*, Electronic Proceedings in Theoretical Computer Science, vol. 95, pages 1–15, 2012.

2.21 C4.1: Functional Analysis — Dr David Seifert — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Part A Topology, Part B Banach Spaces and Hilbert Spaces

Overview

This course builds on B4.1 and B4.2, by extending the theory of Banach spaces and operators. As well as developing general methods that are useful in operator theory, we shall look in more detail at the structure and special properties of "classical" sequence spaces and function spaces.

Synopsis

Normed spaces and Banach spaces; dual spaces, subspaces, direct sums and completions; quotient spaces and quotient operators.

Baire's Category Theorem and its consequences (review).

Hahn–Banach extension and separation theorems; the bidual space and reflexivity.

Smoothness and uniform convexity of norms; classical Banach spaces and their duals.

Compact sets and compact operators. Ascoli's theorem.

Operators with closed range; Fredholm operators.

Weak and weak* topologies. The Banach–Alaoglu theorem and Goldstine's theorem. Weak compactness.

Schauder bases; examples in classical spaces. Gliding-hump arguments.

Reading

1. M. Fabian et al., *Functional Analysis and Infinite-Dimensional Geometry* (Canadian Math. Soc, Springer 2001)

2. N.L. Carothers, *A Short Course on Banach Space Theory* (LMS Student Text, Cambridge University Press 2004).

Further Reading

1. H. Brezis, *Functional Analysis, Sobolev Spaces and Partial Differential Equations* (Springer 2011)

2.22 C4.2 Linear Operators — Prof. Charles Batty — 16HT

Level: M-level **Method of assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Essential: B4.1, B4.2. Useful: C4.1

Overview

Many of the linear operators that arise in mathematical physics and models from other sciences are not bounded operators. Typically they are defined on a dense subspace of a Banach or Hilbert space. They may be closed operators, but sometimes it is necessary to find the appropriate closed extension of the operator and the domain of the extension may be unclear. This course describes some of the theory of unbounded operators, particularly spectral properties of closed operators and ways to convert them into bounded operators.

Synopsis

Review of bounded operators and spectrum.

Unbounded operators; closed and closable operators; adjoints, spectrum.

Operators on Hilbert space; symmetric, self-adjoint, essentially self-adjoint. Spectral theorem and functional calculus. Quadratic forms, simple differential operators.

Semigroups of operators, generators, Hille-Yosida theorem, dissipative operators.

Reading

E.B. Davies, Linear operators and their spectra, CUP, 2007

P. Lax, Functional Analysis, Wiley, 2002

Further Reading

M. Reed & B. Simon, Methods of modern mathematical physics I,II, Academic Press, 1972, 1975

E.B. Davies, Differential operators and spectral theory, CUP, 1995

2.23 C4.3 Functional Analytic Methods for PDEs — Prof. Yves Capdeboscq — 16MT

Level: M-level **Method of assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Part A Integration. There will be a 'Users' Guide to Integration' on the subject website and anyone who has not done Part A Integration can read it up over the summer vacation. In addition some knowledge of functional analysis, in particular Banach spaces (as in B4) and compactness (as in Part A Topology), is useful. We will however recall the relevant definitions as we go along so these prerequisites are not strictly needed.

Overview

The course will introduce some of the modern techniques in partial differential equations that are central to the theoretical and numerical treatments of linear and nonlinear partial differential equations arising in science, geometry and other fields.

It provides valuable background for the Part C courses on Calculus of Variations, Fixed Point Methods for Nonlinear PDEs, and Finite Element Methods.

Learning Outcomes

Students will learn techniques and results about Lebesgue and Sobolev Spaces, distributions and weak derivatives, embedding theorems, traces, weak solution to elliptic PDE's, existence, uniqueness, and smoothness of weak solutions.

Synopsis

Why functional analysis methods are important for PDE's?

Revision of relevant definitions and statements from functional analysis: completeness, seperability, compactness, and duality.

Revision of relevant definitions and statements from Lebesgue integration theory: sequences of measurable functions, Lebesgue and Riesz theorems.

Lebesgue spaces: completeness, dense sets, linear functionals and weak convergence.

Distributions and distributional derivatives.

Sobolev spaces: mollifications and weak derivatives, completeness, Friedrichs inequality, star-shaped domains and dense sets, extension of functions with weak derivatives.

Embedding of Sobolev spaces into Lebesgue spaces: Poincare inequality, Reillich-Kondrachov-Sobolev theorems on compactness.

Traces of functions with weak derivatives.

Dirichlet boundary value problems for elliptic PDE's, Fredholm Alternative (uniqueness implies existence).

Smoothness of weak solutions: embedding from Sobolev spaces into spaces of Hölder continuous functions, interior regularity of distributional solutions to elliptic equations with continuous coefficients.

Reading

Lawrence C. Evans, *Partial differential equations*, (Graduate Studies in Mathematics 2004), American Mathematical Society

Elliott H. Lieb and Michael Loss, *Analysis*, 2nd Edition, (Graduate Studies in Mathematics 2001), American Mathematical Society

Further Reading

E. Kreyszig, *Introductory Functional Analysis with Applications*, Wiley (revised edition, 1989)

P.D. Lax *Functional analysis* (Wiley-Interscience, New York, 2002).

J. Rauch, *Partial differential equations*, (Springer–Verlag, New York, 1992).

2.24 C4.6: Fixed Point Methods for Nonlinear PDEs — Prof. Melanie Rupflin — 16HT

Level: M-level **Method of assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

C4.3: Functional Analytic Methods for PDEs. Some knowledge of functional analysis, in particular Banach spaces (as in B4) and compactness (as in Part A Topology), is useful.

Overview

This course gives an introduction to the techniques of nonlinear functional analysis with emphasis on the major fixed point theorems and their applications to nonlinear differential equations and variational inequalities, which abound in applications such as fluid and solid mechanics, population dynamics and geometry.

Learning Outcomes

Besides becoming acquainted with the fixed point theorems of Banach, Brouwer and Schauder, students will see the abstract principles in a concrete context. Hereby they also reinforce techniques from elementary topology, functional analysis, Banach spaces, compactness methods, calculus of variations and Sobolev spaces.

Synopsis

Examples of nonlinear differential equations and variational inequalities. Contraction Mapping Theorem and applications. Brouwer's fixed point theorem, proof via Calculus of Variations and Null-Lagrangians. Compact operators and Schauder's fixed point theorem. Applications of Schauder's fixed point theorem to nonlinear elliptic equations. Variational inequalities and monotone operators. Applications of monotone operator theory to nonlinear elliptic equations (p-Laplacian, stationary Navier-Stokes)

Reading

1. Lawrence C. Evans, *Partial Differential Equations,* Graduate Studies in Mathematics (American Mathematical Society, 2004).
2. E. Zeidler, *Nonlinear Functional Analysis I & II* (Springer-Verlag, 1986/89).
3. M. S. Berger, *Nonlinearity and Functional Analysis* (Academic Press, 1977).
4. K. Deimling, *Nonlinear Functional Analysis* (Springer-Verlag, 1985).
5. L. Nirenberg, *Topics in Nonlinear Functional Analysis,* Courant Institute Lecture Notes (American Mathematical Society, 2001).
6. R.E. Showalter, *Monotone Operators in Banach Spaces and Nonlinear Partial Differential Equations,* Mathematical Surveys and Monographs, vol.49 (American Mathematical Society, 1997).

2.25 C4.8: Complex Analysis: Conformal Maps and Geometry — Prof. Dmitry Belyaev — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

The only necessary prerequisite is the basic complex analysis covered in Part A: analytic functions, Taylor series, contour integration, Cauchy theorem, and residues. Integration option is recommended but not necessary.

Overview

The aim of the course is to teach the principal techniques and methods of analytic and geometric function theory. This is a beautiful subject on its own right but it also have many applications in other areas of mathematics: potential theory, analytic number theory, probability. In the recent years the theory of Loewner equation became a crucial tool in the study of statistical physics lattice models.

This course is a continuation of the basic undergraduate complex analysis course but has much more geometric emphasis. Our main subject will be the theory of conformal maps, their analytical and geometrical properties.

Learning Outcomes

Students will have been introduced to ideas and techniques of geometric function theory that play important role and have a lot of applications in other areas of analysis. In particular, they will learn the proof of the Riemann mapping theorem and the concept of conformal invariants.

Synopsis

- Riemann mapping theorem. The main goal will be to prove Riemann's theorem which tells us that any non-trivial simply-connected domain can be conformally mapped onto the unit disc. This will be the key result for the entire course since it will allow as to connect the geometry of the domain with the analytical properties of the map which sends this domain to the unit disc. Within this section we will discuss
  - Maximum principle and Schwarz lemma, hyperbolic metric and Möbius transformations
  - Normal families, Hurwitz theorem
  - Proof of Riemann uniformization theorem
  - Constructive uniformization: Christoffel-Schwarz mappings and zipper algorithm (no proofs)
  - Uniformization for multiply-connected domains (sketch of the proof)
  - Applications: Dirichlet problem
- Theory of univalent functions. Univalent function is another term for one-to-one analytical map. We will be mostly interested in their boundary behaviour and how it is related to the geometry of the boundary. This section will cover
  - Area theorem and coefficient estimates
  - Koebe 1/4 theorem, distortion theorems
  - Conformal invariants: extremal length and its applications

Reading

1. L. Ahlfors, *Complex analysis.* This is a very good advanced textbook on Complex analysis. If you are a bit rusty on the basic complex analysis, then you might find everything you need (and a bit more) in Chapters 1–4. We will cover some of the material from chapters 5–6.

2. L. Ahlfors, *Conformal Invariants.* We will cover some topics from Chapters 1–6.

3. Ch. Pommerenke, *Univalent functions.* This book is mostly for further reading. We will discuss some of the results that are covered in Chapters 1,5,6, and 10.

4. Ch. Pommerenke, *Boundary behaviour of conformal maps.* This is an updated version of the previous book. We will be interested in Chapters 1,4, and 8.

5. P. Duren, *Univalent functions.* This is an excellent book about general theory of the univalent functions. We are mostly interested in the first three chapters.

6. G. Goluzin, *Geometric Theory of Functions of a Complex Variable.* This book contains vast amount of information about the geometric function theory. We will cover some of the results from the first four chapters.

2.26 C5.1: Solid Mechanics — Dr Angkana Ruland — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

There are no formal prerequisites. In particular it is not necessary to have taken any courses in fluid mechanics, though having done so provides some background in the use of similar concepts. Use is made of (i) elementary linear algebra in (e.g., eigenvalues, eigenvectors and diagonalization of symmetric matrices, and revision of this material, for example from the Prelims Linear Algebra course, is useful preparation); and (ii) some 3D calculus (mainly differentiation of vector-valued functions of several variables).

Overview

Solid mechanics is a vital ingredient of materials science and engineering, and is playing an increasing role in biology. It has a rich mathematical structure. The aim of the course is to derive the basic equations of elasticity theory, the central model of solid mechanics, and give some interesting applications to the behaviour of materials.The course is useful preparation for C5.2 Elasticity and Plasticity. Taken together the two courses will provide a broad overview of modern solid mechanics, with a variety of approaches.

Learning Outcomes

Students will learn basic techniques of modern continuum mechanics, such as kinematics of deformation, stress, constitutive equations and the relation between nonlinear and linearized models. The emphasis on the course is on the structure of the models, but some applications are also discussed.

Synopsis

1. Nonlinear and linear elasticity

   Lagrangian and Eulerian descriptions of motion, analysis of strain (polar decomposition, Cauchy–Green tensors, local state of strain, invariants). Balance laws of continuum mechanics (conservation of mass, linear momentum, angular momentum, Cauchy stress, Piola–Kirchhoff stress). Nonlinear elasticity (frame indifference, constitutive equations, material symmetries, isotropy). Linear elasticity as a linearization of nonlinear elasticity.

2. Exact solutions in elastostatics.

   Incompressibility and models of rubber. Universal deformations for compressible materials. Exact solutions for incompressible materials, e.g. the Rivlin cube, simple shear, inflation of a balloon. Cavitation in polymers.

3. Phase transformations in solids

   Martensitic phase transformations, twins and microstructure.

   Austenite-Martensite interfaces, the shape-memory effect.

Reading

1. R. J. Atkin and N. Fox, *An introduction to the theory of elasticity* (Courier Corporation, 2013)
2. M. E. Gurtin, *A introduction to continuum mechanics*, (Academic Press, 1982).

Further Reading

1. S. S. Antman, *Nonlinear Problems of Elasticity*, vol 107 of Applied Mathematical Sciences (Springer, 2015).
2. J. E. Marsden and T.J.R. Hughes, *Mathematical Foundations of Elasticity* (Courier Corporation, 1994)
3. P. G. Ciarlet, *Mathematical Elasticity*, Studies in Mathematics and its Applications; v. 20, 27, 29 (North–Holland, 1988).
4. K. Bhattacharya, *Microstructure of martensite: why it forms and how it gives rise to the shape-memory effect* (Oxford University Press, 2003)

2.27 C5.2: Elasticity and Plasticity — Prof. Dominic Vella — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Familiarity will be assumed with Part A Complex Analysis, Differential Equations 1 and 2 and Calculus of Variations. A basic understanding of stress tensors from either B5.3 Viscous Flow or C5.1 Solid Mechanics will also be required. The following courses are also helpful: B5.1 Techniques of Applied Mathematics, B5.2 Applied Partial Differential Equations, C5.5 Perturbation Methods, C5.6 Applied Complex Variables.

Overview

The course starts with a rapid overview of mathematical models for basic solid mechanics. Benchmark solutions are derived for static problems and wave propagation in linear elastic materials. It is then shown how these results can be used as a basis for practically useful problems involving thin beams and plates. Simple geometrically nonlinear models are then introduced to explain buckling, fracture and contact. Models for yield and plasticity are then discussed, both microscopically and macroscopically.

Synopsis

Review of tensors, conservation laws, Navier equations. Antiplane strain, torsion, plane strain. Elastic wave propagation, Rayleigh waves. Ad hoc approximations for thin materials; simple bifurcation theory and buckling. Simple mixed boundary value problems, brittle fracture and smooth contact. Perfect plasticity theories for granular materials and metals.

Reading

1. P. D. Howell, G. Kozyreff and J. R. Ockendon, *Applied Solid Mechanics* (Cambridge University Press, 2008).
2. S. P. Timoshenko and J. N. Goodier, *Theory of Elasticity* (McGraw-Hill, 1970).
3. L.D. Landau and E.M. Lifshitz, *Theory of Elasticity* (Pergamon Press, 1986).

2.28 C5.3: Statistical Mechanics — Prof. Andrew Fowler — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

A familiarity with classical mechanics, probability and fluid mechanics will be helpful.

Overview

Statistical mechanics is a subject which has fundamental and powerful connections with probability, mechanics, stochastic processes, fluid mechanics, thermodynamics, quantum mechanics (though we avoid this), and even philosophy. It is also notoriously inaccessible to applied mathematicians. This course will endeavour to trace a rational path towards classical statistical mechanics, beginning with classical mechanics, and then developing the concepts of thermodynamics through study of the Boltzmann equation. In passing, we derive the Navier-Stokes equations, before developing a mechanically-based formulation of thermodynamics and its famous second law concerning entropy. The latter parts of the course develop a variety of applications of current interest.

Learning Outcomes

Students will have developed a sound knowledge and appreciation of some of the tools, concepts, and computations used in the study of statistical mechanics. They will also get some exposure to some modern research topics in the field.

Synopsis

Classical mechanics: Newton's second law, D'Alembert's principle, Lagrange's equations, Hamilton's equations. Chaos. Probability: probability density functions, moment generating function, central limit theorem. Fluid mechanics: material derivative, Euler and Navier-Stokes equations, energy equation. Random walks, Brownian motion, diffusion equation. Loschmidt's paradox.

Liouville equation, BBGKY hierarchy, Boltzmann equation. The collision integral for a hard sphere gas. Boltzmann H theorem. Maxwellian distribution. Definition of entropy and temperature. Gibbs and Helmholtz free energies. Thermodynamic relations.

Classical statistical mechanics. Ergodic theorem, equiprobability. Microcanonical ensemble for the hard sphere gas, entropy. Canonical ensemble. Grand canonical ensemble.

Selected applications and extensions: for example, chemical potential, phase change, binary alloys, surface energy, radiative transfer, polymer solution theory, Arrhenius kinetics, nucleation theory, percolation theory, renormalisation.

Reading

1. David Chandler, *Introduction to Modern Statistical Mechanics* (Oxford University Press 1987)
2. M. Kardar, *Statistical Physics of Particles* (Cambridge University Press 2007)
3. F. Schwabl, *Statistical Mechanics* 2nd ed. (Springer-Verlag 2006)
4. J.P. Sethna, *Statistical Mechanics: Entropy, Order Parameters, and Complexity* (Oxford University Press 2006) [available online at http://pages.physics.cornell.edu/sethna/StatMech]

2.29 C5.4: Networks — Dr Heather Harrington — 16HT

Level: M-level **Method of Assessment:** Mini-project (see section 1.2)

Weight: Unit

Recommended Prerequisites

None [in particular, C5.3 (Statistical Mechanics) is not required], though some intuition from modules like C5.3, the Part B graph theory course, and probability courses (at the level that everybody has to take anyway) can be useful. However, everything is self-contained, and none of these courses are required. Some computational experience is also helpful, and ideas from linear algebra will certainly be helpful.

Overview

This course aims to provide an introduction to network science, which can be used to study complex systems of interacting agents. Networks are interesting both mathematically and computationally, and they are pervasive in physics, biology, sociology, information science, and myriad other fields. The study of networks is one of the "rising stars" of scientific endeavors, and networks have become among the most important subjects for applied mathematicians to study. Most of the topics to be considered are active modern research areas.

Learning Outcomes

Students will have developed a sound knowledge and appreciation of some of the tools, concepts, models, and computations used in the study of networks. The study of networks is predominantly a modern subject, so the students will also be expected to develop the ability to read and understand current (2016) research papers in the field.

Synopsis

1. Introduction and Basic Concepts (1-2 lectures): nodes, edges, adjacencies, weighted networks, unweighted networks, degree and strength, degree distribution, other types of networks

2. Small Worlds (2 lectures): clustering coefficients, paths and geodesic paths, Watts-Strogatz networks [focus is on modelling and heuristic calculations]

3. Toy Models of Network Formation (2 lectures): preferential attachment, generalizations of preferential attachment, network optimization

4. Additional Summary Statistics and Other Useful Concepts (2 lectures): modularity and assortativity, degree-degree correlations, centrality measures, communicability, reciprocity and structural balance

5. Random Graphs (2 lectures): Erdős-Rényi graphs, configuration model, random graphs with clustering, other models of random graphs or hypergraphs; application of generating-function methods [focus is on modelling and heuristic calculations; material in this section forms an important basis for sections 6 and 7]

6. Community Structure and Mesoscopic Structure (2 lectures): linkage clustering, optimization of modularity and other quality functions, overlapping communities, other methods and generalizations

7. Dynamics on (and of) Networks (3-4 lectures): general ideas, models of biological and social contagions, percolation, voter and opinion models, temporal networks, other topics

8. Additional Topics (0-2 lectures): games on networks, exponential random graphs, network inference, other topics of special interest to students [depending on how much room there is and interest of current students]

Reading

(most important are [2] and [3]):

1. A. Barrat et al, *Dynamical Processes on Complex Networks*, Cambridge University Press, 2008

2. M. E. J. Newman, *Networks: An Introduction*, Oxford University Press, 2010

3. Various papers and review articles (see the Math C5.4 blog at http://networksoxford.blogspot.co.uk for examples). The instructor will indicate a small number of specific review articles that are required reading, and other helpful (but optional) articles will also be indicated.

2.30 C5.5: Perturbation Methods — Prof. Jim Oliver — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Part A Differential Equations and Core Analysis (Complex Analysis). B5 courses are helpful but not officially required.

Overview

Perturbation methods underlie numerous applications of physical applied mathematics: including boundary layers in viscous flow, celestial mechanics, optics, shock waves, reaction-diffusion equations, and nonlinear oscillations. The aims of the course are to give a clear and systematic account of modern perturbation theory and to show how it can be applied to differential equations.

Synopsis

Introduction to regular and singular perturbation theory: approximate roots of algebraic and transcendental equations. Asymptotic expansions and their properties. Asymptotic approximation of integrals, including Laplace's method, the method of stationary phase and the method of steepest descent. Matched asymptotic expansions and boundary layer theory. Multiple-scale perturbation theory. WKB theory and semiclassics.

Reading

1. E.J. Hinch, *Perturbation Methods* (Cambridge University Press, 1991), Chs. 1–3, 5–7.

2. C.M. Bender and S.A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers* (Springer, 1999), Chs. 6, 7, 9–11.

3. J. Kevorkian and J.D. Cole, *Perturbation Methods in Applied Mathematics* (Springer-Verlag, 1981), Chs. 1, 2.1–2.5, 3.1, 3.2, 3.6, 4.1, 5.2.

2.31 C5.6: Applied Complex Variables — Prof. Peter Howell — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

The course requires second year core analysis (complex analysis). It continues the study of complex variables in the directions suggested by contour integration and conformal mapping. A knowledge of the basic properties of Fourier Transforms is assumed. Part A Waves and Fluids and Part C Perturbation Methods are helpful but not essential.

Overview

The course begins where core second-year complex analysis leaves off, and is devoted to extensions and applications of that material. The solution of Laplace's equation using conformal mapping techniques is extended to general polygonal domains and to free boundary problems. The properties of Cauchy integrals are analysed and applied to mixed boundary value problems and singular integral equations. The Fourier transform is generalised to complex values of the transform variable, and used to solve mixed boundary value problems and integral equations via the Wiener-Hopf method.

Learning Outcomes

Students will learn advanced techniques in complex analysis and use them to solve free and mixed boundary value problems, integral equations and differential equations.

Synopsis

Review of core complex analysis, analytic continuation, multifunctions, contour integration, conformal mapping and Fourier transforms.

Riemann mapping theorem (in statement only). Schwarz-Christoffel formula. Solution of Laplace's equation by conformal mapping onto a canonical domain; applications including inviscid hydrodynamics; Free streamline flows in the hodograph plane. Unsteady flow with free boundaries in porous media.

Application of Cauchy integrals and Plemelj formulae. Solution of mixed boundary value problems motivated by thin aerofoil theory and the theory of cracks in elastic solids. Reimann-Hilbert problems. Cauchy singular integral equations. Complex Fourier transform. Contour integral solutions of ODE's. Wiener-Hopf method.

Reading

1. G. F. Carrier, M. Krook and C.E. Pearson, *Functions of a Complex Variable*(Society for Industrial and Applied Mathematics, 2005.) ISBN 0898715954.

2. M. J. Ablowitz and A. S. Fokas, *Complex Variables: Introduction and Applications* (2nd edition, Cambridge University Press, 2003). ISBN 0521534291.

3. J. R. Ockendon, S. D. Howison, A. A. Lacey and A. B. Movichan, *Applied Partial Differential Equations: Revised Edition* (Oxford University Press, 2003). ISBN 0198527713. Pages 195–212.

2.32 C5.7: Topics in Fluid Mechanics — Prof. Andreas Muench — 16MT

Level: M-level **Method of Assessment:** Written examination,

Weight: Unit

Recommended Prerequisites

B5.3 Viscous Flow, B5.4 Waves and Compressible Flow.

Overview

The course will expand and illuminate the 'classical' fluid mechanics taught in the third year B5.3 and B5.4 courses, and illustrate its modern application in a number of different areas in industry and geoscience.

Synopsis

Thin film flows: coatings and foams. Lubrication theory: gravity flows, Marangoni effects. Droplet dynamics, contact lines, menisci. Drying and wetting.

Flow in porous media: Darcy's law; thermal and solutal convection; gravity-driven flow and carbon sequestration.

Multiphase flows.

Reading

1. L.G Leal, *Advanced Transport Phenomena*,(Cambridge University Press, Cambridge, 2007).

2. O.M. Phillips, *Geological Fluid Dynamics*,(Cambridge University Press, Cambridge, 2009).

3. J.S. Turner, *Buoyancy Effects in Fluids*, (Cambridge University Press, Cambridge, 1973).

4. A.C. Fowler, *Mathematical Models in the Applied Sciences*, (Cambridge University Press, 1997).

Further Reading

1. G.K. Batchelor, H.K. Moffatt and M.G. Worster (eds.), *Perspectives in Fluid Dynamics* (Cambridge University Press, Cambridge, 2000).

2.33 C5.9: Mathematical Mechanical Biology — Prof. Eamonn Gaffney — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Fluid Mechanics: Part A Waves and Fluids and Part B Viscous flow is recommended. Solid mechanics: One of the Part C courses (Solid Mechanics or Elasticity/Plasticity is recommended). Mathematical biology or physiology is desirable but not necessary as the material for a particular biological system will be part of the course.

Overview

The course will be motivated by outstanding problems in physiology and biology but the emphasis is on the mathematical tools needed to answer some biologically relevant problems. The course is divided into modules and three modules will be given during a term but these modules can change from one year to the next.

Learning Outcomes

The goal of this course is to learn the physical background and mathematical methods behind many problems arising in mechanical biology from the cellular level and upwards. Students will familiarise themselves with key notions used in modern research in bio-physics and mechano-biology.

Synopsis

1. 1D Biological Mechanics. Bio-Filaments (2 1/2 weeks)
   (a) Introduction: bio-molecules (actin, microtubules, DNA,...)
   (b) Randomly fluctuating chains (statistical mechanics)
   (c) Continuous filaments (neurons, stems, roots, plants)
   (d) Differential geometry of curves: Kirchhoff rod theory and beam theory

2. 2D Biological Mechanics. Bio-Membranes (2 1/2 weeks)
   (a) Introduction: lipid bilayer, cell membranes
   (b) Differential geometry of surfaces: curvatures, Gauss–Bonnet theorem

(c) Fluid membranes: shape equation, fluctuating membranes
(d) Solid membranes: hyperelastic isotropic materials, shells. Application to the cell, plants and microbes.

3. 3D Biological Mechanics.
   (a) Low Reynolds number flows: Scallop theorem, Cell Motility, Ciliary Pumping.
   (b) Introduction to nonlinear 3D elasticity for soft tissues
   (c) Coupling low Reynolds fluids and non-linear elastic solids, with application to poro-elastic tissue.

   The following modules will not be taught in 2016-17.

4. Bio-Fluids (3 weeks)
   (a) Low Reynolds Number: Motility, Scallop theorem.
   (b) Complex biofluids: active and non-Newtonian fluids
   (c) Circulation: Blood flow, microcirculation, networks

5. Multiphase/Multiphysics methods (3 weeks)
   (a) Coupling fluids and solids: poro-elastic tissue
   (b) Coupling fluid, solids and chemistry: tissue swelling
   (c) A general thermodynamics approach
   (d) Application to tissue engineering, wound healing.

6. Bio-solids and growth (3 weeks)
   (a) Introduction: nonlinear elasticity for soft tissues
   (b) one-dimensional growth theory
   (c) volumetric growth: multiplicative decomposition
   (d) application to neuronal growth, tumour

Reading

1. Physical Cell Biology, second ed. Rob Phillips et al. Garland Science.
2. Cardiovascular solid mechanics. Cells, tissues, and organs, Humpherey, 2002, Springer.
3. Nonlinear Solid Mechanics: A Continuum Approach for Enineering: A Continuum Approach for Engineering, G. Holzapfel, 200, Wiley.
4. The hydrodynamics of swimming microorganisms, Lauga and Powers, Rep Prog Phys 72, 2009.

2.34 C5.11: Mathematical Geoscience — Prof. Ian Hewitt — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

B5.2 Applied Partial Differential Equations and B5.3 Viscous Flow recommended.

Overview

The aim of the course is to illustrate the techniques of mathematical modelling in their particular application to environmental problems. The mathematical techniques used are drawn from the theory of ordinary differential equations and partial differential equations. The course requires a willingness to become familiar with a range of different scientific disciplines. In particular, familiarity with the concepts of fluid mechanics will be useful.

Learning Outcomes

Students will have developed a sound knowledge of some of the models studied in mathematical geoscience. They will also get exposure to some modern research topics in the field.

Synopsis

Applications of mathematics to environmental or geophysical problems involving the use of models with ordinary and partial differential equations. Examples to be considered are:
Climate dynamics (radiative balance, greenhouse effect, ice-albedo feedback, carbon cycle)
River flows (conservation laws, flood hydrographs, St Venant equations, sediments transport, bed instabilities)
Glacier dynamics (non-Newtonian flow, mass balance, hydrology, glacier surges)

Reading

1. A. C. Fowler, *Mathematical Geoscience* (Springer, 2011).
2. J. T. Houghton, *The Physics of Atmospheres* (3rd ed., Cambridge University Press., Cambridge, 2002).
3. K. Richards, *Rivers* (Methuen, 1982).
4. K. M. Cuffey and W. S. B. Paterson, *The Physics of Glaciers* (4th edition, Butterworth-Heinemann, 2011).

2.35 C5.12: Mathematical Physiology — Prof. Sarah Waters — 16 MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

B5.5 Mathematical Ecology and Biology highly recommended.

Overview

The course aims to provide an introduction which can bring students within reach of current research topics in physiology, by synthesising a coherent description of the physiological background with realistic mathematical models and their analysis. The concepts and treatment of oscillations, waves and stability are central to the course, which develops ideas introduced in the more elementary B5.5 course. In addition, the lecture sequence aims to build understanding of the workings of the human body by treating in sequence problems at the intracellular, intercellular, whole organ and systemic levels.

Learning Outcomes

Students will have developed an understanding of mathematical modelling or physiological systems and will have demonstrable knowledge or the mathematical theory necessary to analyse such models.

Synopsis

Trans-membrane ion transport: Hodgkin–Huxley and Fitzhugh–Nagumo models.

Excitable media; wave propagation in neurons.

Calcium dynamics; calcium-induced calcium release. Intracellular oscillations and wave propagation.

The electrochemical action of the heart. Spiral waves, tachycardia and fibrillation.

Discrete delays in physiological systems. The Glass–Mackey model of respiration. Regulation of stem cell and blood cell production.

Reading

The principal text is:

1. J. Keener and J. Sneyd, *Mathematical Physiology* (Springer-Verlag, 1998). First edition or Second edition Vol I: Chs. 2, 7. Vol II: Chs. 11, 13, 14. (Springer-Verlag, 2009)]

Subsidiary mathematical texts are:

1. J. D. Murray, *Mathematical Biology* (Springer-Verlag, 2nd ed., 1993). [Third edition, Vols I and II, (Springer-Verlag, 2003).]

2. L. Glass and M. C. Mackey, *From Clocks to Chaos* (Princeton University Press, 1988).

3. P. Grindrod, *Patterns and Waves* (oup, 1991).

General physiology texts are:

1. R. M. Berne and M. N. Levy, *Principles of Physiology* (2nd ed., Mosby, St. Louis, 1996).
2. J. R. Levick, *An Introduction to Cardiovascular Physiology* (3rd ed. Butterworth–Heinemann, Oxford, 2000).
3. A. C. Guyton and J. E. Hall, *Textbook of Medical Physiology* (10th ed., W. B. Saunders Co., Philadelphia, 2000).

2.36 C6.1 Numerical Linear Algebra — Prof. Andy Wathen — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Only elementary linear algebra is assumed in this course. The part A Numerical Analysis course would be helpful, indeed some swift review and extensions of some of the material of that course is included here.

Overview

Linear Algebra is a central and widely applicable part of mathematics. It is estimated that many (if not most) computers in the world are computing with matrix algorithms at any moment in time whether these be embedded in visualization software in a computer game or calculating prices for some financial option. This course builds on elementary linear algebra and in it we derive, describe and analyse a number of widely used constructive methods (algorithms) for various problems involving matrices.

Numerical Methods for solving linear systems of equations, computing eigenvalues and singular values and various related problems involving matrices are the main focus of this course.

Synopsis

Common problems in linear algebra. Matrix structure, singular value decomposition. QR factorization, the QR algorithm for eigenvalues. Direct solution methods for linear systems, Gaussian elimination and its variants. Iterative solution methods for linear systems.

Chebyshev polynomials and Chebyshev semi-iterative methods, conjugate gradients, convergence analysis, preconditioning.

Reading

L. N. Trefethen and D. Bau III, *Numerical Linear Algebra* (SIAM, 1997).

J. W. Demmel, *Applied Numerical Linear Algebra* (SIAM, 1997).

A. Greenbaum, *Iterative Methods for Solving Linear Systems* (SIAM, 1997).

G. H. Golub and C. F. van Loan, *Matrix Computations* (John Hopkins University Press, 3rd edition, 1996).

H. C. Elman, D. J. Silvester and A. J. Wathen, *Finite Elements and Fast Iterative Solvers* (Oxford University Press, 1995), only chapter 2.

2.37 C6.2 Continuous Optimization — Prof. Coralia Cartis — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

None.

Overview

The solution of optimal decision-making and engineering design problems in which the objective and constraints are nonlinear functions of potentially (very) many variables is required on an everyday basis in the commercial and academic worlds. A closely-related subject is the solution of nonlinear systems of equations, also referred to as least-squares or data fitting problems that occur in almost every instance where observations or measurements are available for modelling a continuous process or phenomenon, such as in weather forecasting. The mathematical analysis of such optimization problems and of classical and modern methods for their solution are fundamental for understanding existing software and for developing new techniques for practical optimization problems at hand.

Synopsis

Part 1: Unconstrained Optimization
Optimality conditions, steepest descent method, Newton and quasi-Newton methods, General line search methods, Trust region methods, Least squares problems and methods.

Part 2: Constrained Optimization
Optimality/KKT conditions, penalty and augmented Lagrangian for equality-constrained optimization, interior-point/ barrier methods for inequality constrained optimization. SQP methods.

Reading

Lecture notes will be made available for downloading from the course webpage.
A useful textbook is J.Nocedal and S.J.Wright, *Numerical Optimisation*, (Springer, 1999 or 2006).

2.38 C6.3 Approximation of Functions — Prof. Nick Trefethen — 16MT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites:

None.

Overview

How can a function f(x) be approximated over a prescribed domain by a simpler function like a polynomial or a rational function? Such questions were at the heart of analysis in the early 1900s and later grew into a mature subject of approximation theory. Recently they have been invigorated as problems of approximation have become central to computational algorithms for differential equations, linear algebra, optimization and other fields. This course, based on Trefethen's new text in which results are illustrated by Chebfun computations, will focus in a modern but still rigorous way on the fundamental results of interpolation and approximation and their algorithmic application.

Synopsis

Chebyshev interpolants, polynomials, and series. Barycentric interpolation formula. Weierstrass approximation theorem. Convergence rates of polynomial approximations. Hermite integral formula and Runge phenomenon. Lebesgue constants, polynomial rootfinding. Orthogonal polynomials. Clenshaw-Curtis and Gauss quadrature. Rational approximation.

Reading

1. L. N. Trefethen, *Approximation Theory and Approximation Practice*

This course will be based on the textbook by Nick Trefethen, *Approximation Theory and Approximation Practice*, published by SIAM in 2013. All students taking the course are recommended to have a copy of this book. The lectures and examination will be closely tied to the book, and the problems assigned will be largely taken from the book. Trefethen's text provides references to many other books and articles that can be read to expand understanding of the course material so a longer reading list is not included here.

2.39 C6.4 Finite Element Methods for Partial Differential Equations — Prof. Patrick Farrell — 16HT

Level: M-Level. **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

No formal prerequisites are assumed. The course builds on elementary calculus, analysis and linear algebra and, of course, requires some acquaintance with partial differential equations such as the material covered in the Prelims Multivariable Calculus course, in particular the Divergence Theorem. Part A Numerical Analysis would be helpful but is certainly not essential. Function Space material will be introduced in the course as needed.

Overview

Computational algorithms are now widely used to predict and describe physical and other systems. Underlying such applications as weather forecasting, civil engineering (design of structures) and medical scanning are numerical methods which approximately solve partial differential equation problems. This course gives a mathematical introduction to one of the more widely used methods: the finite element method.

Synopsis

Finite element methods represent a powerful and general class of techniques for the approximate solution of partial differential equations. The aim of this course is to introduce these methods for boundary value problems for the Poisson and related elliptic partial differential equations.

Attention will be paid to the formulation, the mathematical analysis and the implementation of these methods.

Reading

H. Elman, D. Silvester & A. Wathen, *Finite Elements and Fast Iterative Solvers.* Second edition. OUP, 2014. [Mainly Chapters 1 and 3].

or

H. Elman, D. Silvester & A. Wathen, *Finite Elements and Fast Iterative Solvers.* OUP, 2005. [Mainly Chapters 1 and 5].

Further Reading

S.C. Brenner & L.R. Scott, *The Mathematical Theory of Finite Element Methods.* Springer, 2nd edition, 2002.[Chapters 0,1,2,3; Chapter 4: Secs. 4.1–4.4, Chapter 5: Secs. 5.1–5.7].

C. Johnson, *Numerical Solution of Partial Differential Equations by the Finite Element Method.* CUP, 1990. [Chapters 1–4; Chapter 8:Secs. 8.1–8.4.2; Chapter 9: Secs. 9.1–9.5].

Typed lecture notes covering a previous version of the entire course (and more):

Endre Süli, *Finite Element Methods for Partial Differential Equations.* Mathematical Institute, University of Oxford, 2011.

are available from the course material webpage.

Some of the introductory material is covered in

Endre Süli & David Mayers, *An Introduction to Numerical Analysis*,CUP 2003; Second Printing 2006. [Chapter 11 and in particular Chapter 14].

2.40 C7.1: Theoretical Physics — Prof. Essler and Dr Haisch — 24MT and 16HT

Note: This double unit is offered by the Physics Department (syllabus for C6)

Level: M-level **Method of Assessment:** Written examination.

Weight: Double unit only.

Recommended Prerequisites

Part A Quantum Theory, Part A Classical Mechanics, B7.1 Classical Mechanics, B7.2 Electromagnetism.

Overview

This course is intended to give an introduction to some aspects of many-particle systems, field theory and related ideas. These form the basis of our current theoretical understanding of particle physics, condensed matter and statistical physics. An aim is to present some core ideas and important applications in a unified way. These applications include the classical mechanics of continuum systems, the quantum mechanics and statistical mechanics of many-particle systems, and some basic aspects of relativistic quantum field theory.

Synopsis

1. Path Integrals in Quantum Mechanics

   - Mathematical tools for describing systems with an infinite number of degrees of freedom: functionals, functional differentiation; Multi-dimensional Gaussian integrals.

   - Quantum mechanical propagator as a path integral. Semiclassical limit. Free particle.

   - Quantum statistical mechanics in terms of path integrals. Harmonic oscillator.

   - Perturbation theory for non-Gaussian functional integrals. Anharmonic oscillator. Feynman diagrams.

2. Quantum Many-Particle Systems

   - Second Quantization: bosons and fermions, Fock space, single-particle and two-particle operators.

   - Applications to the Fermi gas, weakly interacting Bose condensates, magnons in (anti)ferromagnets, and to superconductivity.

   - quantum field theory as a low-energy description of quantum many-particle systems.

3. Classical Field Theory

   - Group theory and Lie algebra primer: basic concepts, SU(N), Lorentz group.

   - Elements of classical field theory: fields, Lagrangians, Hamiltonians, principle of least action, equations of motion, Noether's theorem, space-time symmetries.

- Applications: scalar fields, spontaneous symmetry breaking, U(1) symmetry, Goldstone's theorem, SU(2)U(1) symmetry, vector fields, Maxwell's theory, scalar electrodynamics.

4. Canonical Quantisation

   - Free real and complex scalar fields: Klein-Gordon field as harmonic oscillators, Heisenberg picture.

   - Propagators and Wick's theorem: correlators, causality, Green's functions.

   - Free vector fields: gauge fixing, Feynman propagator.

5. Interacting Quantum Fields

   - Perturbation theory: classification of interactions, interaction picture, Feynman diagrams.

   - Applications: tree-level decay and scattering processes of scalar and U(1) gauge fields.

   - Path integrals: effective action, Feynman diagrams from path integrals.

6. Statistical Physics, Phase Transitions and Stochastic Processes

   - Transfer matrices: one-dimensional systems in classical statistical mechanics. Transfer matrices in $D = 2$ and their relation to path integrals.

   - Phase transition in the 2D Ising model: Peierls argument.

   - Landau Theory of phase transitions: phase diagrams, first-order and continuous phase transitions. Landau-Ginzburg-Wilson free energy functionals. Examples including liquid crystals. Critical phenomena and scaling theory.

   - Stochastic processes: the Langevin and Fokker-Planck equation. Brownian motion of single particle.

Reading

The lecturers are aware of no book that presents all parts of this course in a unified way and at an appropriate level. For this reason, detailed lecture notes will be made available.

2.41 C7.4: Introduction to Quantum Information — Prof. Artur Ekert — 16HT

It is not possible to take C7.4 to examination in Part C in 2017 if it was taken as part of the Part B examination in 2015

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites:

Quantum Theory.

The course material should be of interest to physicists, mathematicians, computer scientists, and engineers. The following will be assumed as prerequisites for this course:

- elementary probability, complex numbers, vectors and matrices;
- Dirac bra-ket notation;
- a basic knowledge of quantum mechanics especially in the simple context of finite dimensional state spaces (state vectors, composite systems, unitary matrices, Born rule for quantum measurements);
- basic ideas of classical theoretical computer science (complexity theory) would be helpful but are not essential.

Prerequisite notes will be provided giving an account of the necessary material. It would be desirable for you to look through these notes slightly before the start of the course.

Overview

The classical theory of computation usually does not refer to physics. Pioneers such as Turing, Church, Post and Goedel managed to capture the correct classical theory by intuition alone and, as a result, it is often falsely assumed that its foundations are self-evident and purely abstract. They are not! Computers are physical objects and computation is a physical process. Hence when we improve our knowledge about physical reality, we may also gain new means of improving our knowledge of computation. From this perspective it should not be very surprising that the discovery of quantum mechanics has changed our understanding of the nature of computation. In this series of lectures you will learn how inherently quantum phenomena, such as quantum interference and quantum entanglement, can make information processing more efficient and more secure, even in the presence of noise.

Synopsis

1. Bits, gates, networks, Boolean functions, reversible and probabilistic computation
2. “Impossible” logic gates, amplitudes, quantum interference

3. One, two and many qubits
4. Entanglement and entangling gates
5. From interference to quantum algorithms
6. Algorithms, computational complexity and Quantum Fourier Transform
7. Phase estimation and quantum factoring
8. Non-local correlations and cryptography
9. Bell's inequalities
10. Density matrices and CP maps
11. Decoherence and quantum error correction

Reading

Beyond the Quantum Horizon by D. Deutsch and A. Ekert, Scientific American, Sep 2012.

Less reality more security by A. Ekert, Physics World, Sep 2009.

The Limits of Quantum Computers, by S. Aaronson, Scientific American, Mar 2008.

A Do-It-Yourself Quantum Eraser by R. Hillmer and P. Kwiat, Scientific American, May 2007.

Quantum Seeing in the Dark by P. Kwiat et al, Scientific American, Nov 1996

Physical Limits of Computation by C.H. Bennett and R. Landauer, Scientific American, Jul 1985.

2.42 C7.5: General Relativity I — Dr Andreas Braun — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Part A Special Relativity, Part B Classical Mechanics and Electromagnetism.

Overview

The course is intended as an elementary introduction to general relativity, the basic physical concepts of its observational implications, and the new insights that it provides into the nature of space time, and the structure of the universe. Familiarity with special relativity and electromagnetism as covered in the Part A and Part B courses will be assumed. The lectures will review Newtonian gravitation, tensor calculus and continuum physics in special relativity, physics in curved space time and the Einstein field equations. This will suffice for an account of simple applications to planetary motion, the bending of light and the existence of black holes.

Learning Outcomes

This course starts by asking how the theory of gravitation can be made consistent with the special-relativistic framework. Physical considerations (the principle of equivalence, general covariance) are used to motivate and illustrate the mathematical machinery of tensor calculus. The technical development is kept as elementary as possible, emphasising the use of local inertial frames. A similar elementary motivation is given for Einstein's equations and the Schwarzschild solution. Cosmological solutions are discussed.

The learning outcomes are an understanding and appreciation of the ideas and concepts described above

Synopsis

Review of Newtonian gravitation theory and problems of constructing a relativistic generalisation. Review of Special Relativity. The equivalence principle. Tensor formulation of special relativity (including general particle motion, tensor form of Maxwell's equations and the energy momentum-tensor of dust). Curved space time. Local inertial coordinates. General coordinate transformations, elements of Riemannian geometry (including connections, curvature and geodesic deviation). Mathematical formulation of General Relativity, Einstein's equations (properties of the energy-momentum tensor will be needed in the case of dust only). Planetary motion, the bending of light, introduction to black hole solutions and the Schwarzschild solution. The introduction to cosmology including cosmological principles, homogeneity and isotropy, and the Friedman–Robertson–Walker solutions.

Reading

1. S. Carroll, *Space Time and Geometry: An Introduction to General Relativity* (Addison Welsey, 2003)

2. L.P. Hughston and K.P. Tod, *An Introduction to General Relativity,* LMS Student Text 5 (London Mathematical Society, Cambridge University Press, 1990), Chs 1–18.

3. N.M.J. Woodhouse, *Notes on Special Relativity*, Mathematical Institute Notes. Revised edition; published in a revised form as *Special Relativity, Lecture notes in Physics m6* (Springer-Verlag, 1992), Chs 1–7

Further Reading

1. B. Schutz, *A First Course in General Relativity* (Cambridge University Press, 1990).

2. R.M. Wald, *General Relativity* (Chicago, 1984).

3. W. Rindler, *Essential Relativity* (Springer-Verlag, 2nd edition, 1990).

2.43 C7.6: General Relativity II — Prof. Xenia de la Ossa — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

C7.5 General Relativity I

Overview

In this, the second course in General Relativity, we have two principal aims. We first aim to increase our mathematical understanding of the theory of relativity and our technical ability to solve problems in it. We apply the theory to a wider class of physical situations, including gravitational waves and black hole solutions. Orbits in the Schwarzschild solution are given a unified treatment which allows a simple account of the three classical tests of Einstein's theory. This leads to a greater understanding of the Schwarzschild solution and an introduction to its rotating counterpart, the Kerr solution. We analyse the extensions of the Schwarzschild solution show how the theory of black holes emerges and exposes the radical consequences of Einstein's theory for space-time structure.

Synopsis

Mathematical background, the Lie derivative and isometries. The Einstein field equations with matter; the energy-momentum tensor for a perfect fluid; equations of motion from the conservation law. Linearised general relativity and the metric of an isolated body. Motion on a weak gravitational field and gravitational waves. The Schwarzschild solution and its extensions; Eddington-Finkelstein coordinates and the Kruskal extension. Penrose diagrams and the area theorem. Stationary, axisymmetric metrics and orthogonal transitivity; the Kerr solution and its properties; interpretation as rotating black hole.

Reading

1. S. Carroll, *Space Time and Geometry: An Introduction to General Relativity* (Addison Welsey, 2003)
2. L. P. Hughston and K. P. Tod, *An Introduction to General Relativity*, LMS Student Text 5, CUP (1990), Chs.19, 20, 22-26.
3. R. M. Wald, *General Relativity*, Univ of Chicago Press (1984).

Further Reading

1. B. Schutz, *A First Course in General Relativity* (Cambridge University Press, 1990).
2. R.M. Wald, *General Relativity* (Chicago, 1984).
3. W. Rindler, *Essential Relativity* (Springer-Verlag, 2nd edition, 1990).
4. S. Hawking and G. Ellis, *The Large Scale of the Universe*, (Cambridge Monographs on Mathematical Physics, 1973).

2.44 C8.1: Stochastic Differential Equations — Prof. Harald Oberhauser — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Part A integration, B8.1 Martingales Through Measure Theory and B8.2 Continuous Martingales and Stochastic Calculus, is expected.

Overview

Stochastic differential equations have been used extensively in many areas of application, including finance and social science as well as in physics, chemistry. This course develops the theory of Itô's calculus and stochastic differential equations.

Learning Outcomes

The student will have developed an appreciation of stochastic calculus as a tool that can be used for defining and understanding diffusive systems.

Synopsis

Recap on Brownian motion, quadratic variation, Ito's calculus: stochastic integrals with respect to local martingales, Ito's formula.

Lévy's characterisation of Brownian motion, exponent and Cameron-Martin martingales, exponential inequality, Burkholder-Davis-Gundy inequalities, Girsanov's Theorem, the Martingale Representation Theorem, Dambis-Dubins-Schwarz.

Local time, motion and Tanaka's formula.

Stochastic differential equations: strong and weak solutions, questions of existence and uniqueness, diffusion processes. Discussion of the one-dimensional case, a comparison theorem. Numerical schemes.

Conformal invariance of Brownian motion.

Reading

1. Prof Oberhauser's online notes:

2. M. Yor and D. Revaz, *Continuous Martingales and Brownian Motion* (Springer).

3. R. Durrett, *Stochastic Calculus* (CRC Press).

Further Reading

1. N. Ikeda & S. Watanabe, *Stochastic Differential Equations and Diffusion Processes* (North–Holland Publishing Company, 1989).
2. I. Karatzas and S. E. Shreve, *Brownian Motion and Stochastic Calculus,* Graduate Texts in Mathematics 113 (Springer-Verlag, 1988).
3. L. C. G. Rogers & D. Williams, *Diffusions, Markov Processes and Martingales Vol 1 (Foundations) and Vol 2 (Ito Calculus)* (Cambridge University Press, 1987 and 1994).
4. H. P. McKean, *Stochastic Integrals* (Academic Press, New York and London, 1969).
5. B. Oksendal, *Stochastic Differential Equations: An introduction with applications* (Universitext, Springer, 6th edition). Chapters II, III, IV, V, part of VI, Chapter VIII (F).

2.45 C8.2: Stochastic Analysis and PDEs — Prof. Ben Hambly — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Part A integration, B8.1 Martingales Through Measure, B8.2 Continuous Martingales and Stochastic Calculus and C8.1 Stochastic Differential Equations. B4.1 is useful but not essential.

Overview

Stochastic analysis and partial differential equations are intricately connected. This is exemplified by the celebrated deep connections between Brownian motion and the classical heat equation, but this is only a very special case of a general phenomenon. We explore some of these connections, illustrating the benefits to both analysis and probability.

Learning Outcomes

The student will have developed an understanding of the deep connections between concepts from probability theory, especially diffusion processes and their transition semigroups, and partial differential equations.

Synopsis

Feller processes and semigroups. Resolvents and generators. Hille-Yosida Theorem (without proof). Diffusions and elliptic operators, convergence and approximation. Stochastic differential equations and martingale problems. Dynkin's formula. Duality. Speed and scale for one dimensional diffusions. Green's functions as occupation densities. The Feynman-Kac formula. Semilinear equations and branching processes. Examples from genetics.

Reading

A full set of typed notes will be supplied.

Important references:

1. O. Kallenberg. *Foundations of Modern Probability.* Second Edition, Springer 2002. This comprehensive text covers essentially the entire course, and much more, but should be supplemented with other references in order to develop experience of more examples.
2. L.C.G Rogers & D. Williams. *Diffusions, Markov Processes and Martingales*; Volume 1, Foundations and Volume 2, Itô calculus. Cambridge University Press, 1987 and 1994. These two volumes have a very different style to Kallenberg and complement it nicely. Again they cover much more material than this course.

Supplementary reading:

1. S.N. Ethier & T.G. Kurtz. *Markov Processes: characterization and convergence.* Wiley 1986. It is not recommended to try to sit down and read this book cover to cover, but it is a treasure trove of powerful theory and elegant examples.
2. S. Karlin & H.M. Taylor. *A second course in stochastic processes.* Academic Press 1981. This classic text does not cover the material on semigroups and martingale problems that we shall develop, but it is a very accessible source of examples of diffusions and things one might calculate for them.

A fuller list of references will be included in the typed notes.

2.46 C8.3: Combinatorics — Prof. Alex Scott — 16MT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Part B Graph Theory is helpful, but not required.

Overview

An important branch of discrete mathematics concerns properties of collections of subsets of a finite set. There are many beautiful and fundamental results, and there are still many basic open questions. The aim of the course is to introduce this very active area of mathematics, with many connections to other fields.

Learning Outcomes

The student will have developed an appreciation of the combinatorics of finite sets.

Synopsis

Chains and antichains. Sperner's Lemma. LYM inequality. Dilworth's Theorem.

Shadows. Kruskal-Katona Theorem.

Intersecting families. Erdos-Ko-Rado Theorem. Cross-intersecting families.

VC-dimension. Sauer-Shelah Theorem.

t-intersecting families. Fisher's Inequality. Frankl-Wilson Theorem. Application to Borsuk's Conjecture.

Combinatorial Nullstellensatz.

Reading

1. Bela Bollobás, *Combinatorics*, CUP, 1986.
2. Stasys Jukna, *Extremal Combinatorics*, Springer, 2007

2.47 C8.4: Probabilistic Combinatorics — Prof. Oliver Riordan — 16HT

Level: M-level **Method of Assessment:** Written examination.

Weight: Unit

Recommended Prerequisites

Part B Graph Theory and Part A Probability. C8.3 Combinatorics is not as essential prerequisite for this course, though it is a natural companion for it.

Overview

Probabilistic combinatorics is a very active field of mathematics, with connections to other areas such as computer science and statistical physics. Probabilistic methods are essential for the study of random discrete structures and for the analysis of algorithms, but they can also provide a powerful and beautiful approach for answering deterministic questions. The aim of this course is to introduce some fundamental probabilistic tools and present a few applications.

Learning Outcomes

The student will have developed an appreciation of probabilistic methods in discrete mathematics.

Synopsis

First-moment method, with applications to Ramsey numbers, and to graphs of high girth and high chromatic number.
Second-moment method, threshold functions for random graphs.
Lovász Local Lemma, with applications to two-colourings of hypergraphs, and to Ramsey numbers.
Chernoff bounds, concentration of measure, Janson's inequality.
Branching processes and the phase transition in random graphs.
Clique and chromatic numbers of random graphs.

Reading

1. N. Alon and J.H. Spencer, *The Probabilistic Method* (third edition, Wiley, 2008).

Further Reading

1. B. Bollobás, *Random Graphs* (second edition, Cambridge University Press, 2001).

2. M. Habib, C. McDiarmid, J. Ramirez-Alfonsin, B. Reed, ed., *Probabilistic Methods for Algorithmic Discrete Mathematics* (Springer, 1998).

3. S. Janson, T. Luczak and A. Rucinski, *Random Graphs* (John Wiley and Sons, 2000).

4. M. Mitzenmacher and E. Upfal, *Probability and Computing : Randomized Algorithms and Probabilistic Analysis* (Cambridge University Press, New York (NY), 2005).

5. M. Molloy and B. Reed, *Graph Colouring and the Probabilistic Method* (Springer, 2002).

6. R. Motwani and P. Raghavan, *Randomized Algorithms* (Cambridge University Press, 1995).

2.48 CCD : Dissertations on a Mathematical Topic

Level : M-level

Weight : Double-unit (10,000).

Students may offer a double-unit dissertation on a Mathematical topic for examination at Part C. A double-unit is equivalent to a 32-hour lecture course. Students will have approximately 8 hours of supervision for a double-unit dissertation distributed over Michaelmas and Hilary terms. In addition there are lectures on writing mathematics and using LaTeX in Michaelmas and Hilary terms. See the lecture list for details.

Students considering offering a dissertation should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* available at:

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects.

Application

Students must apply to the Mathematics Projects Committee for approval of their proposed topic in advance of beginning work on their dissertation. Proposals should be addressed to the Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room S0.20, Mathematical Institute and are accepted from the end of Trinity Term. All proposals must be received before 12noon on Friday of Week 0 of Michaelmas Full Term. For CD dissertations candidates should take particular care to remember that the project must have substantial mathematical content. The application form is available at
https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects.
Once a title has been approved, it may only be changed by approval of the Chairman of the Projects Committee.

Assessment

Each project is blind double marked. The marks are reconciled through discussion between the two assessors, overseen by the examiners. Please see the *Guidance Notes on Extended Essays and Dissertations in Mathematics* for detailed marking criteria and class descriptors.

Submission

THREE copies of your dissertation, identified by your candidate number only, should be sent to the Chairman of Examiners, FHS of Mathematics Part C, Examination Schools, Oxford, to arrive no later than **12noon on Monday of week 10, Hilary Term 2017**. An electronic copy of your dissertation should also be submitted via the Mathematical Institute website. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics.*

3 Other Units

3.1 Statistics Options

Students in Part C may take units drawn from Part C of the Honour School of Mathematics and Statistics. For full details of these units see the syllabus and synopses for Part C of the Honour School Mathematics and Statistics, which are available on the web at http://www.stats.ox.ac.uk/current_students/bammath/course_handbooks/

The Statistics units available are as follows:

- Stochastic Models in Mathematical Genetics
- Probability and Statistics for Network Analysis
- Graphical Models
- Data Mining and Machine Learning
- Bayes Methods
- Advanced Simulation (if teaching resources allow)

3.2 Computer Science Options

Students in Part C may take units drawn from Part C of the Honour School of Mathematics and Computing. For full details of these units see the Department of Computer Science's website (http://www.cs.ox.ac.uk/teaching/courses/)

Please note that these three courses will be examined by mini-project (as for MSc students). Mini-projects will be handed out to candidates on the last Monday or Friday of the term in which the subject is being taught, and you will have to hand it in to the Exam Schools by noon on Monday of Week 1 of the following term. The mini-project will be designed to be completed in about four to five days. It will include some questions that are more open-ended than those on a standard sit-down exam. The work you submit should be your own work, and include suitable references.

Please note that the Computer Science courses in Part C are 50% bigger than those in earlier years, i.e. for each Computer Science course in the 3rd year undergraduates are expected to undertake about 10 hours of study per week, but 4th year courses will each require about 15 hours a week of study. Lecturers are providing this extra work in a variety of ways, e.g. some will give 16 lectures with extra reading, classes and/or practicals, whereas others will be giving 24 lectures, and others still will be doing something in between. Students will need to look at each synopsis for details on this.

The Computer Science units available are as follows:

- CCS1 Categories, Proofs and Processes
- CCS2 Quantum Computer Science
- CCS3 Automata, Logics and Games
- CCS4 Computer Animation [1]

3.3 Other Options

3.3.1 Philosophy: Double Units

Students in Part C may take options, all double units, drawn from Part C of the Honour School of Mathematics and Philosophy. For full details of these double units see the Faculty of Philosophy's website http://www.philosophy.ox.ac.uk/undergraduate/course_descriptions

Students interested in taking a Philosophy double unit are encouraged to contact their college tutors well in advance of term, to ensure that teaching arrangements can be made.

The Philosophy units available are as follows:

- 180 The Rise of Modern Logic (Double unit)

This course will be examined by a three-hour exam and a submitted essay of up to 5000 words.

3.3.2 COD : Dissertations on a Topic Related to Mathematics

Level : M-level

Weight : Double-unit (10,000 words).

Students may offer a double-unit dissertation on a Mathematically related topic for examination at Part C. For example, applications of mathematics to another field (eg Maths in Music), historical topics, topics concentrating on the analysis of statistical data, or topics concentrating on the production of computer-generated data are acceptable as topics for an OD dissertation. (Topics in mathematical education are not allowed.)

A double-unit is equivalent to a 32-hour lecture course. Students will have approximately 8 hours of supervision for a double-unit dissertation distributed over Michaelmas and Hilary terms. In addition there are lectures on writing mathematics and using LaTeX in Michaelmas and Hilary terms. See the lecture list for details.

[1] Some programming experience is recommended and a willingness to develop animations using a modern animation program like Blender.

Candidates considering offering a dissertation should read the *Guidance Notes on Extended Essays and Dissertations in Mathematics* available at:

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects.

Application

Students must apply to the Mathematics Projects Committee for approval of their proposed topic in advance of beginning work on their dissertation. Proposals should be addressed to the Chairman of the Projects Committee, c/o Mrs Helen Lowe, Room S0.20, Mathematical Institute and are accepted from the end of Trinity Term. All proposals must be received before 12noon on Friday of Week 0 of Michaelmas Full Term. The application form is available at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects.

Once a title has been approved, it may only be changed by approval of the Chairman of the Projects Committee.

Assessment

Each project is blind double marked. The marks are reconciled through discussion between the two assessors, overseen by the examiners. Please see the *Guidance Notes on Extended Essays and Dissertations in Mathematics* for detailed marking criteria and class descriptors.

Submission

THREE copies of your dissertation,identified by your candidate number only, should be sent to the Chairman of Examiners, FHS of Mathematics Part C, Examination Schools, Oxford, to arrive no later than **12noon on Monday of week 10, Hilary Term 2017**. An electronic copy of your dissertation should also be submitted via the Mathematical Institute website. Further details may be found in the *Guidance Notes on Extended Essays and Dissertations in Mathematics.*

4 Language Classes: French and German

Language courses in French and German or Spanish (in alternate years) are offered by the University Language Centre.

Students in the FHS Mathematics may apply to take language classes. In 2016-2017, French and German language classes will be run in MT and HT. We have a limited number of places but if we have spare places we will offer these to joint school students , Mathematics and Computer Science, Mathematics and Philosophy and Mathematics and Statistics.

Two levels of French courses are offered, a lower level for those with a good pass at GCSE, and a higher level course for those with A/S or A level. Acceptance on either course will depend on satisfactory performance in the Preliminary Qualifying Test usually held in Week 1 of Michaelmas Term (Monday, 17.00-19.00 at the Language Centre). Classes at both levels will take place on Mondays, 17.00-19.00. A single class in German or Spanish at a lower or higher level will be offered on the basis of the performances in the Preliminary Qualifying Test, held at the same time as the French test. Classes will also be held on Mondays, 17-00-19.00.

Performance on the course will not contribute to the class of degree awarded. However, upon successful completion of the course, students will be issued with a certificate of achievement which will be sent to their college.

Places at these classes are limited, so students are advised that they must indicate their interest at this stage. If you are interested please contact Nia Roderick (roderick@maths.ox.ac.uk or tel. 01865 615205), Academic Assistant in the Mathematical Institute, as soon as possible.

Aims and rationale

The general aim of the language courses is to develop the student's ability to communicate (in both speech and writing) in French, German or Spanish to the point where he or she can function in an academic or working environment in a French-speaking, German-speaking or Spanish-speaking country.

The courses have been designed with general linguistic competence in the four skills of reading, writing, speaking and listening in mind. There will be opportunities for participants to develop their own particular interests through presentations and assignments.

Form and Content

Each course will consist of a thirty-two contact hour course, together with associated work. Classes will be held in the Language Centre at two hours per week in Michaelmas and Hilary Terms.

The content of the courses is based on coursebooks together with a substantial amount of supplementary material prepared by the language tutors. Participants should expect to spend an additional two hours per week on preparation and follow-up work.

Each course aims to cover similar ground in terms of grammar, spoken and written language and topics. Areas covered will include:

Grammar:

- all major tenses will be presented and/or revised, including the subjunctive
- passive voice
- pronouns
- formation of adjectives, adverbs, comparatives
- use of prepositions
- time expressions

Speaking

- guided spoken expression for academic, work and leisure contact (e.g. giving presentations, informal interviews, applying for jobs)
- expressing opinions, tastes and preferences
- expressing cause, consequence and purpose

Writing

- Guided letter writing for academic and work contact
- Summaries and short essays

Listening

- Listening practice of recorded materials (e.g. news broadcasts, telephone messages, interviews)
- developing listening comprehension strategies

Topics (with related readings and vocabulary, from among the following)

- life, work and culture
- the media in each country
- social and political systems
- film, theatre and music
- research and innovation
- sports and related topics
- student-selected topics

Teaching staff

The courses are taught by Language Centre tutors or Modern Languages Faculty instructors.

Teaching and learning approaches

Each course uses a communicative methodology which demands active participation from students. In addition, there will be some formal grammar instruction. Students will be expected to prepare work for classes and homework will be set. Students will also be given training in strategies for independent language study, including computer-based language learning, which will enable them to maintain and develop their language skills after the course.

Entry

Two classes in French and one in German or Spanish (probably at Basic and Threshold levels) will be formed according to level of French/German/Spanish at entry. The minimum entry standard is a good GCSE Grade or equivalent. Registration for each option will take place at the start of Michaelmas Term. A preliminary qualifying test is then taken which candidates must pass in order to be allowed to join the course.

Learning Outcomes

The learning outcomes will be based on internationally agreed criteria for specific levels (known as the ALTE levels), which are as follows:

Basic Level (corresponds to ALTE Level 2 "Can-do" statements)

- Can express opinions on professional, cultural or abstract matters in a limited way, offer advice and understand instructions.
- Can give a short presentation on a limited range of topics.
- Can understand routine information and articles, including factual articles in newspapers, routine letters from hotels and letters expressing personal opinions.
- Can write letters or make notes on familiar or predictable matters.

Threshold Level (corresponds to ALTE Level 3 "Can-do" statements)

- Can follow or give a talk on a familiar topic or keep up a conversation on a fairly wide range of topics, such as personal and professional experiences, events currently in the news.
- Can give a clear presentation on a familiar topic, and answer predictable or factual questions.
- Can read texts for relevant information, and understand detailed instructions or advice.
- Can make notes that will be of reasonable use for essay or revision purposes.
- Can make notes while someone is talking or write a letter including non- standard requests.

Assessment

There will a preliminary qualifying test in Michaelmas Term. There are three parts to this test: Reading Comprehension, Listening Comprehension, and Grammar. Candidates who have not studied or had contact with French or German/Spanish for some time are advised to revise thoroughly, making use of the Language Centre's French, German or Spanish resources.

Students' achievement will be assessed through a variety of means, including continuous assessment of oral performance, a written final examination, and a project or assignment chosen by individual students in consultation with their language tutor.

Reading comprehension, writing, listening comprehension and speaking are all examined. The oral component may consist of an interview, a presentation or a candidate's performance in a formal debate or discussion.