Suggested title of dissertation:

Patterns and defects arising in nematic liquid crystal films

Dissertation supervisor:

Dr Georgy Kitavtsev

Description of the proposal:

Nematic liquid crystals can be described as special materials which interpolate mechanical and optical properties between isotropic liquids and crystalline solids. For high temperatures they flow like classical fluids but under a temperature quench has been applied their molecules exhibit orientation ordering and macroscopic patterns consisting of patches with different molecular polarisation are often observed in experiments. These patterns are characterised by a set of singular lines and point defects along which the polarisation field becomes discontinuous or is not properly defined. In turn, nematic liquid crystal films find nowadays important applications to development of LCD displays. Understanding and control of the structure of the polarisation patterns and corresponding defects in such films becomes therefore an extremely important question. From the mathematical point of view these patterns can be understood as the local minimisers of the Landau-de Gennes energy. In the recent study by Kitavtsev et al. 2016 the authors found numerically rich bifurcation diagrams of the minimisers depending on two control parameters: the diameter of the film and the magnitude of the elastic constant. The aim of this project is to revise these results both analytically and numerically for two important asymptotic limits of small and large elastic constants. Analysis oriented candidates can focus on derivation of asymptotic expansions for minimisers for one of the above limits. More computationally oriented candidates can focus their work on extending of the numerical optimisation algorithms developed in Kitavtsev et al. 16' to the limiting case of small elastic constants, when the Landau-de Gennes energy can be reduced to a simpler vector-field type energy.

Possible avenues of investigation:

- Derivation of reduced energies in the limits of large and small elastic constant

- Development and implementation of numerical optimisation algorithms
- Derivation and comparison of the asymtpotic expansions for critical points of the full and reduced energies for the above two limits
- Analysis of stability of the critical points

Pre-requisite knowledge:

Essential: Material from courses such as B5.2 Applied Partial Differential Equations and/or B6.2 Numerical Solution of Differential Equations II (see `https://courses.maths.ox.ac.uk/year/2018-2019`).

Useful: Depending on an avenue of investigation, candidates might find useful some of the material (but not everything) covered in other optional Part C courses, for example, Finite Element Method for PDEs (`https://courses.maths.ox.ac.uk/node/37016`), Perturbation Methods (`https://courses.maths.ox.ac.uk/node/36885`) and Numerical Linear Algebra (`https://courses.maths.ox.ac.uk/node/36963`).

Useful reading:

1. P. G. De Gennes and J. Prost. The Physics of Liquid Crystals, Oxford, 1993.
2. E. G. Virga. Variational Theories of Liquid Crystals, Chapman & Hall, 1994.

Further references:

1. G. Kitavtsev, J. Robbins, V. Slastikov, and A. Zarnescu. Liquid crystal defects in the Landau-de Gennes theory in two dimensions: Beyond the one-constant approximation, Math. Mod. Methods Appl. Sci 26 :2769- , 2016.
2. Mottram, N. J., and Newton, C. J. Introduction to Q-tensor theory. ArXiv preprint arXiv:1409.3542, 2014.
3. Hu, Y., Qu, Y., and Zhang, P. On the disclination lines of nematic liquid crystals. Communications in Computational Physics 19:354-379, 2016.