PENROSE AT 95 TWISTORS at 60

St Antony’s College Oxford, September 2026

Bi-Twistor theory for general curved space-time – resolving the fundamental confusion of Twistor Theory!

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ENGELBERT SCHUCKING

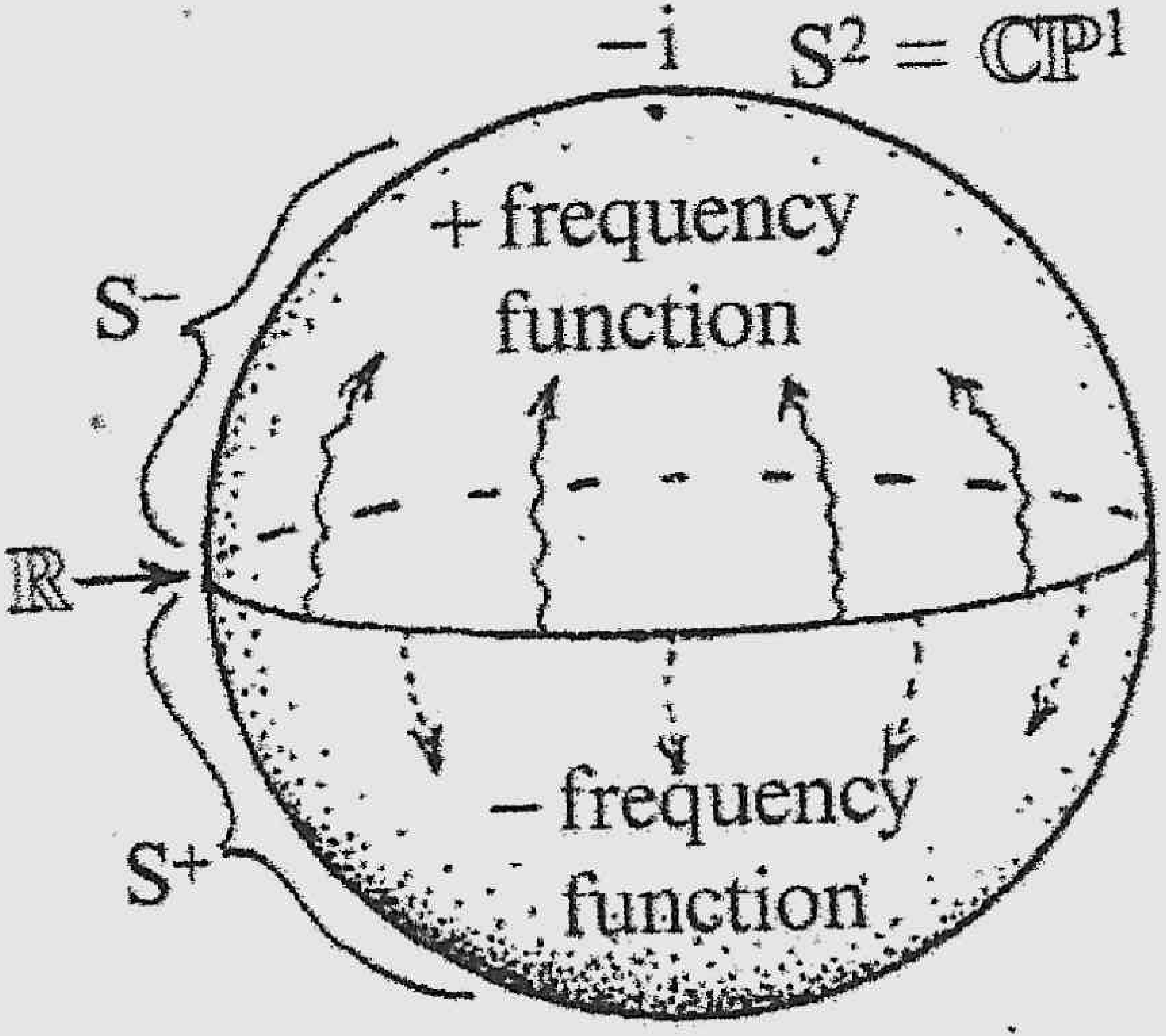
Drives underlying the origination of twistor theory

(1) Physics? (2) Geometry?

Insights from Engelbert Schücking Syracuse NY, USA, early 1961

(A) Conformal invariance of Maxwell's equations

(B) Fundamental significance, to quantum field theory of splitting field amplitudes into positive/negative frequency parts



$-i$
$S^2 = \mathbb{CP}^1$
+ frequency function
S^-
$\mathbb{R}$
− frequency function
S^+

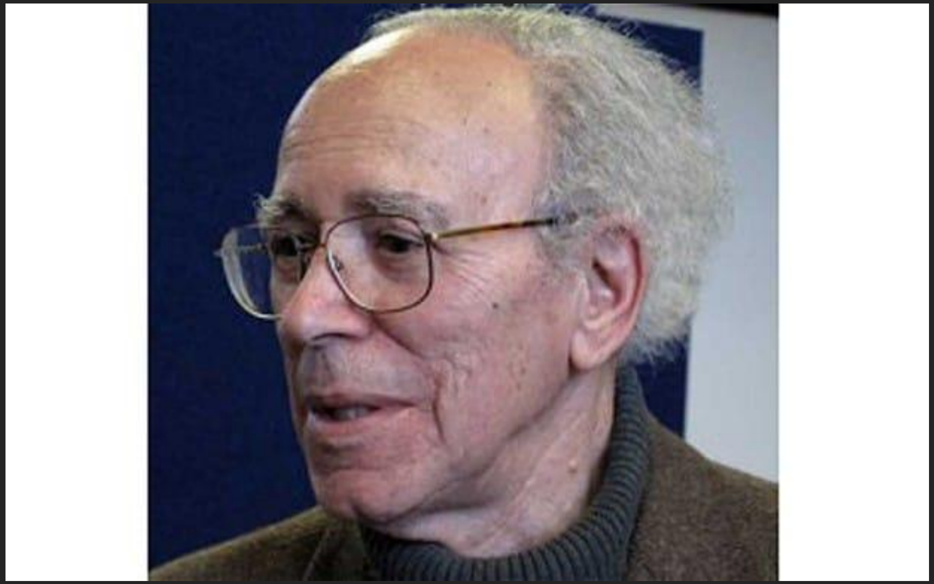
But how do we do this splitting for the whole of Minkowski space-time all at once, in a relativistically invariant way? (The “forward tube” is far too small!)

Answer: use light rays, not space-time points!

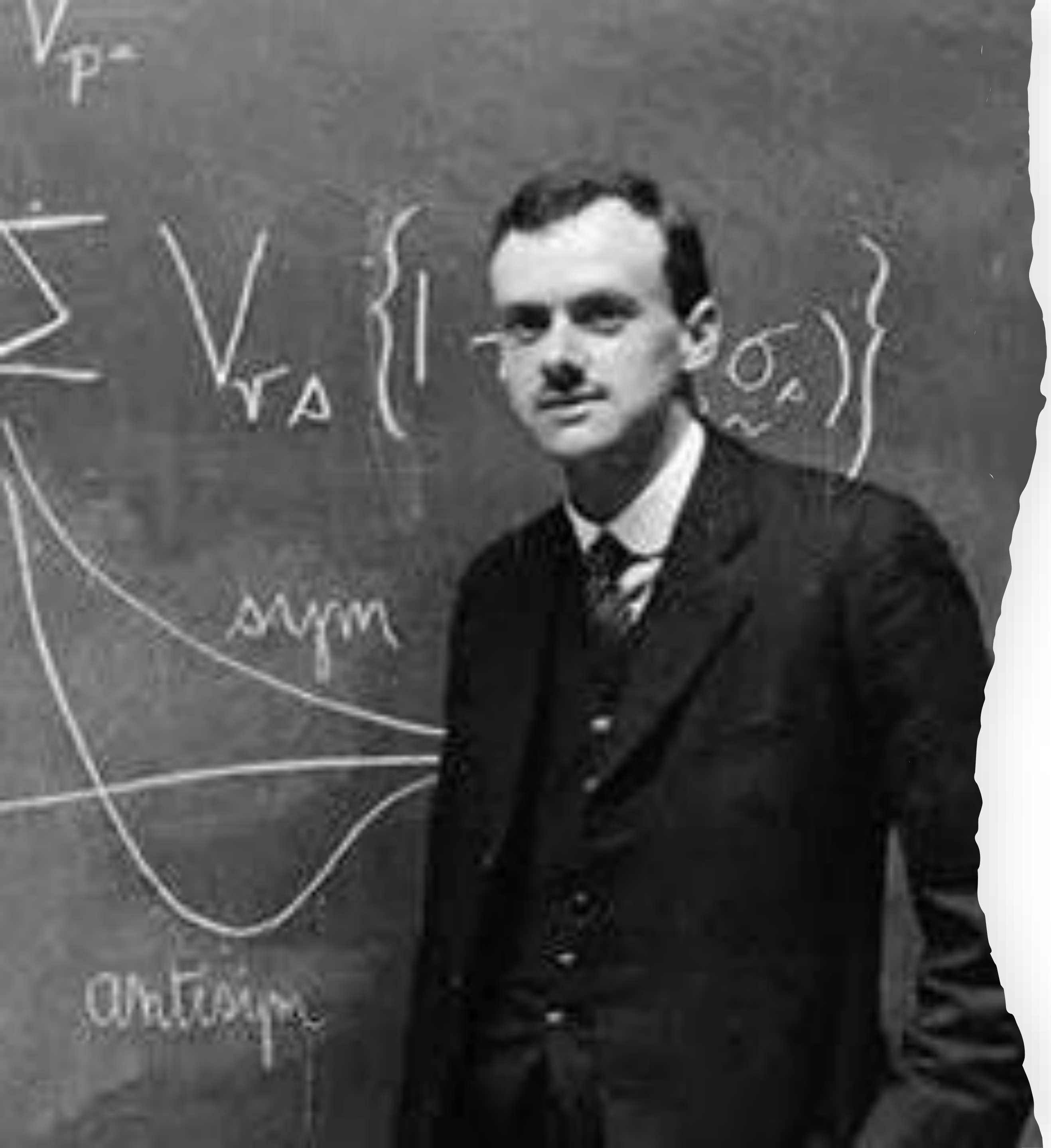
ROY KERR

RAY SACHS

HERMANN BONDI

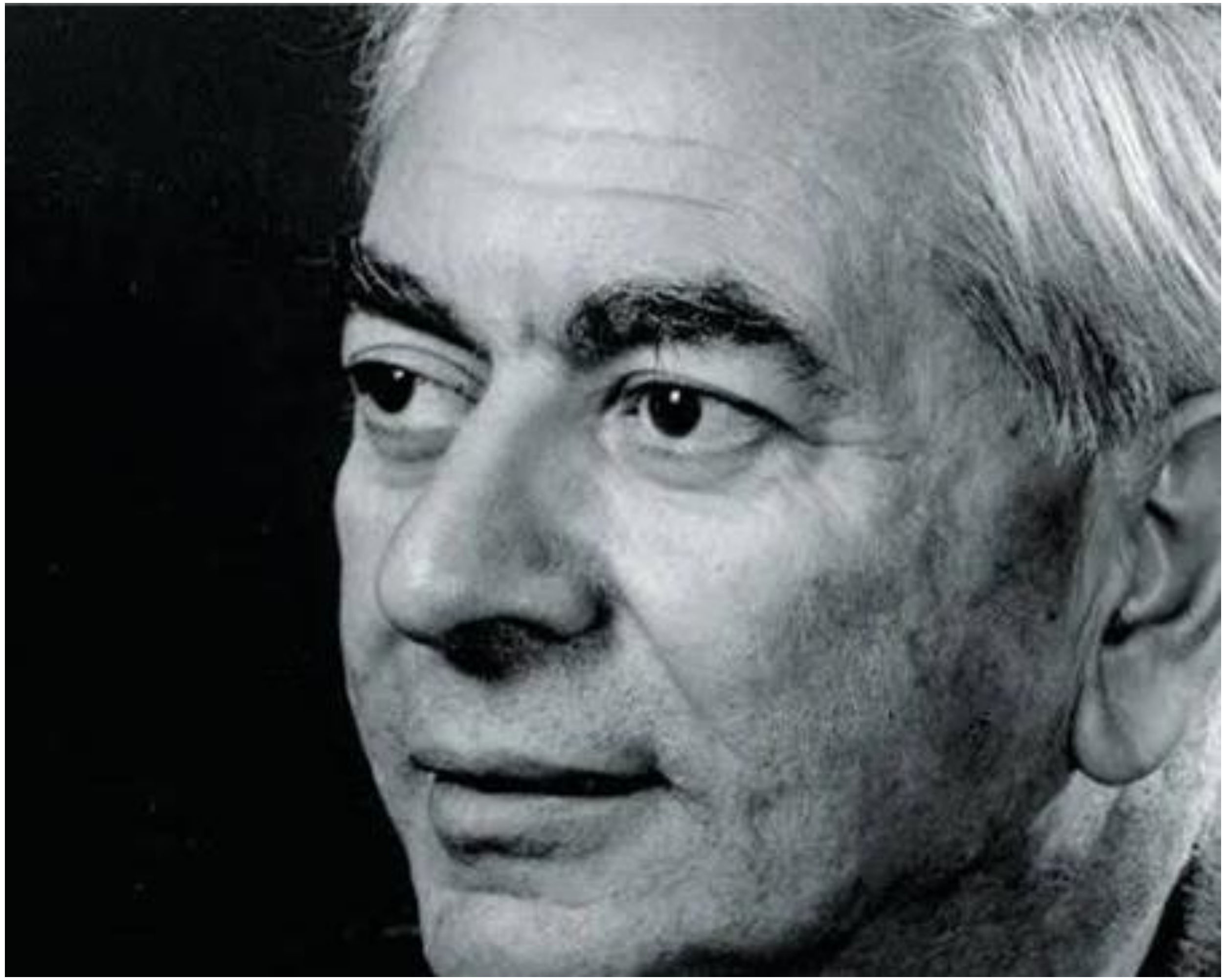


IVOR ROBINSON

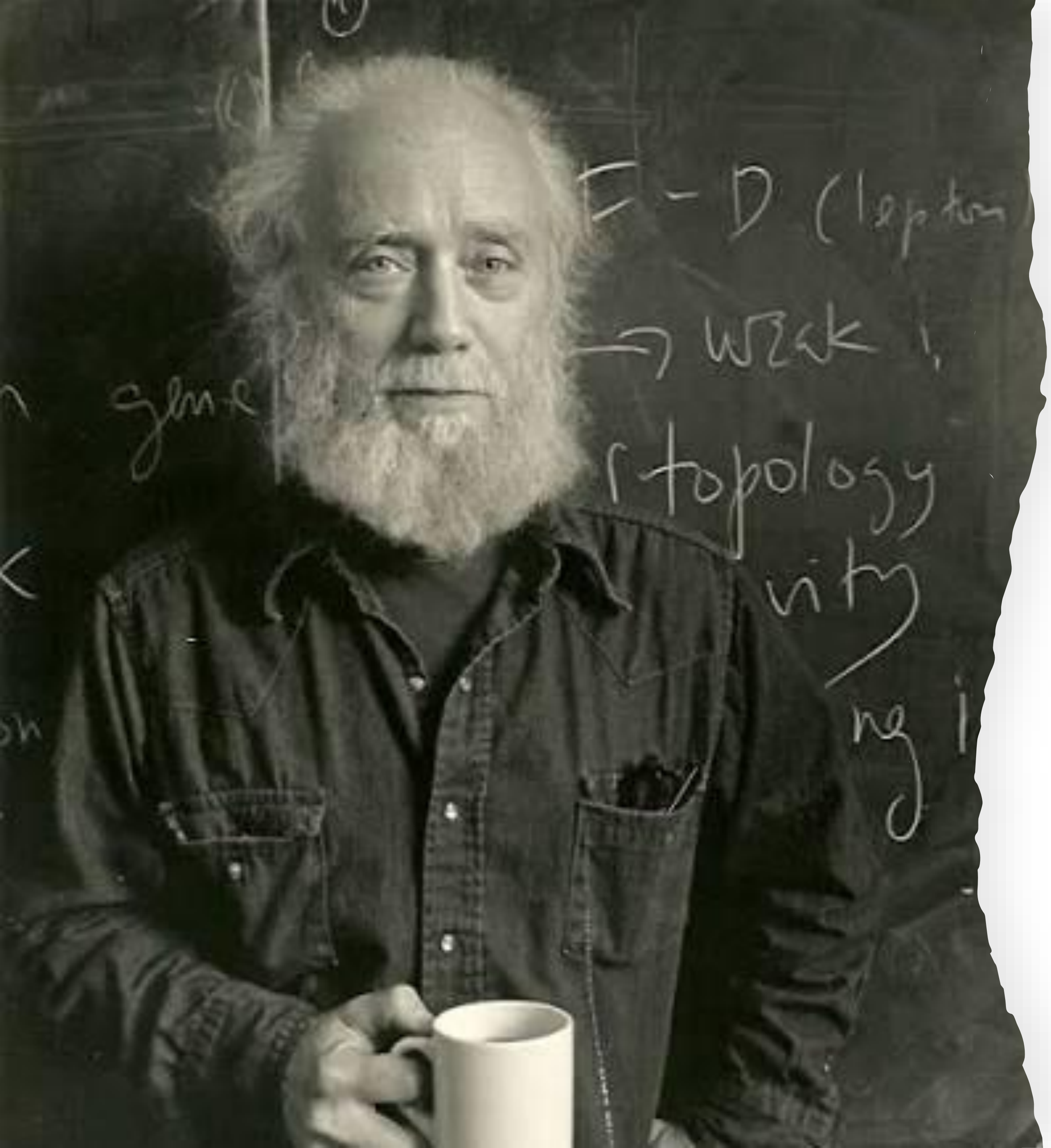


PAUL DIRAC

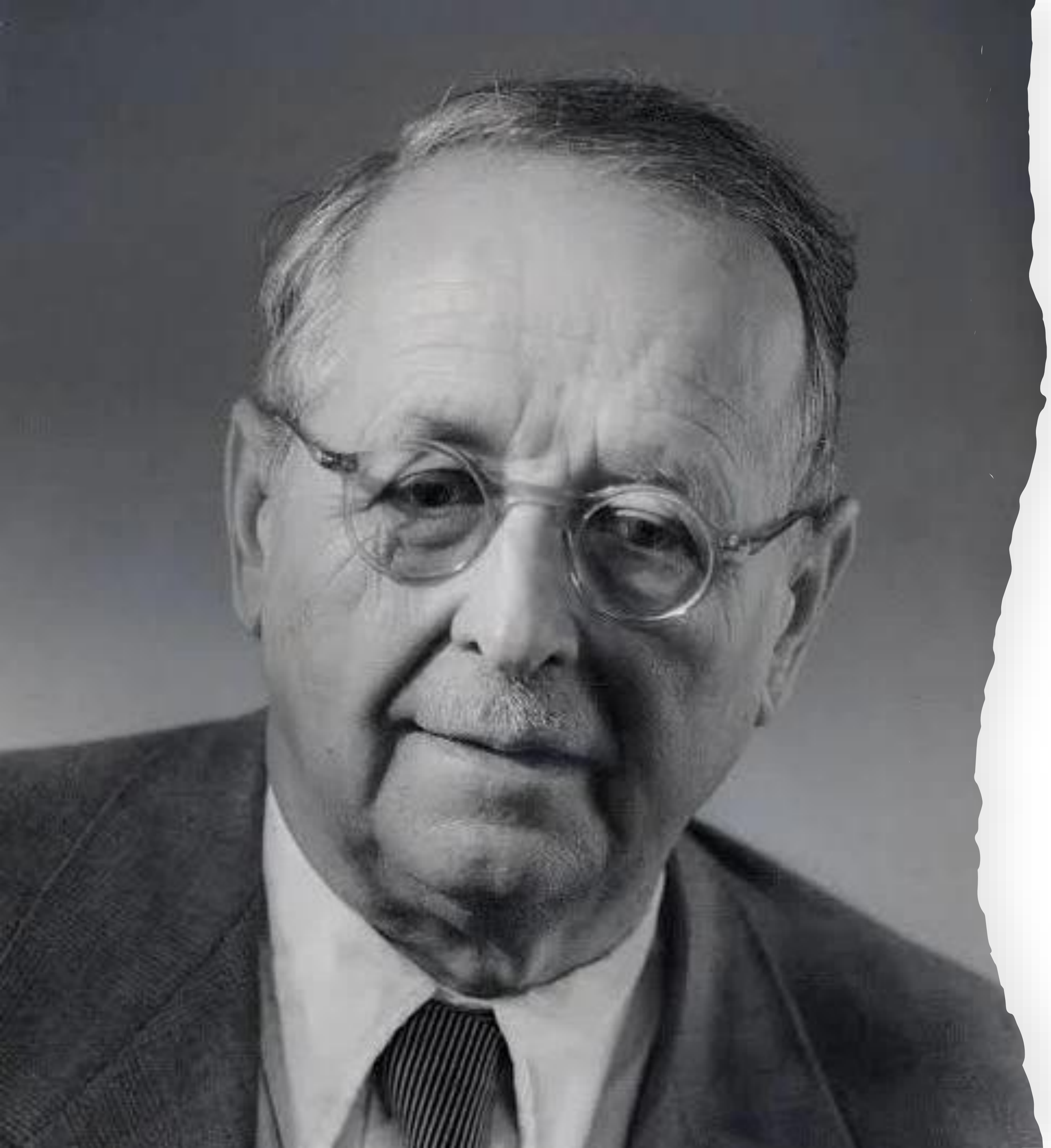
B L VAN DER WAERDEN



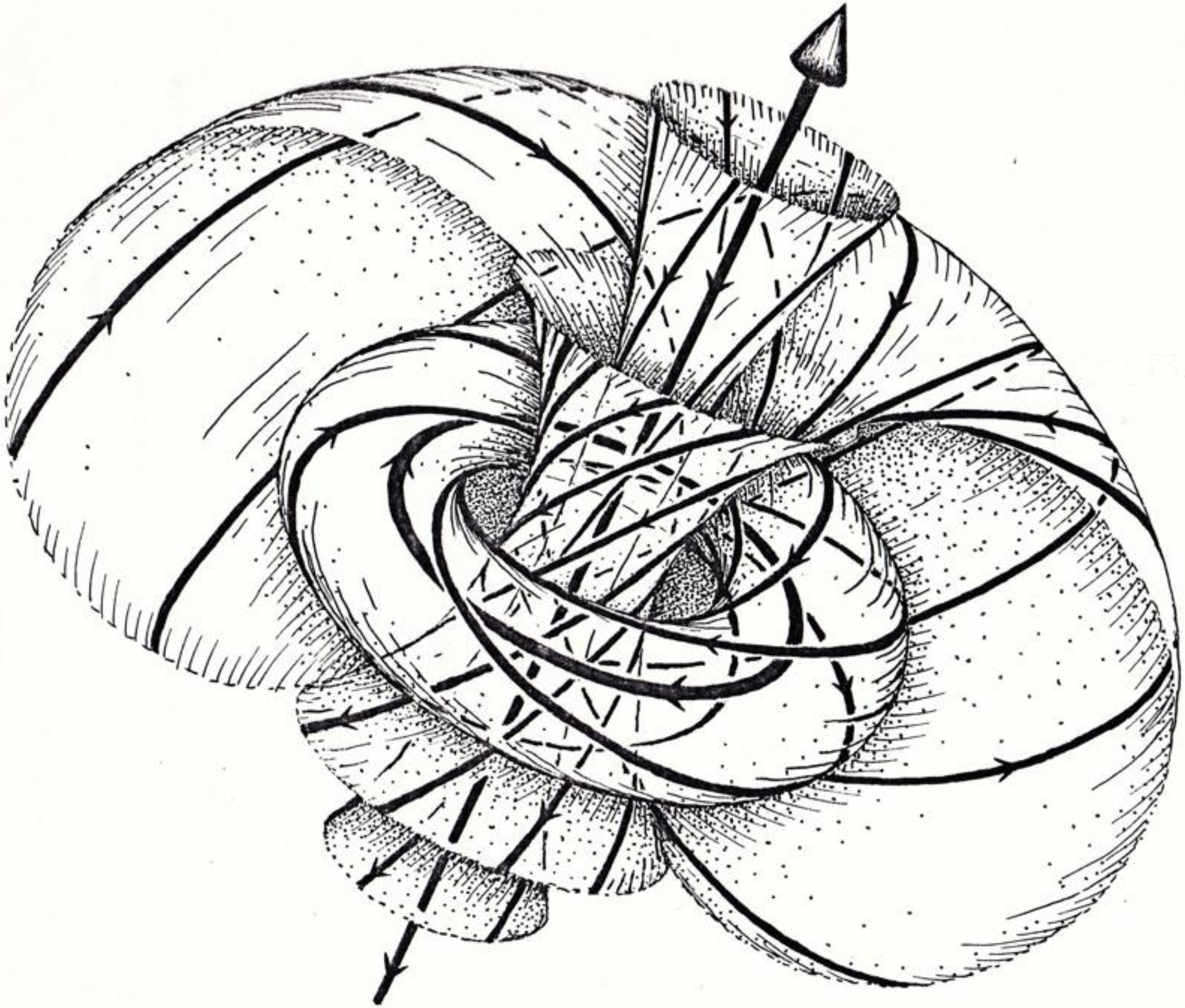
DENNIS SCIAMA

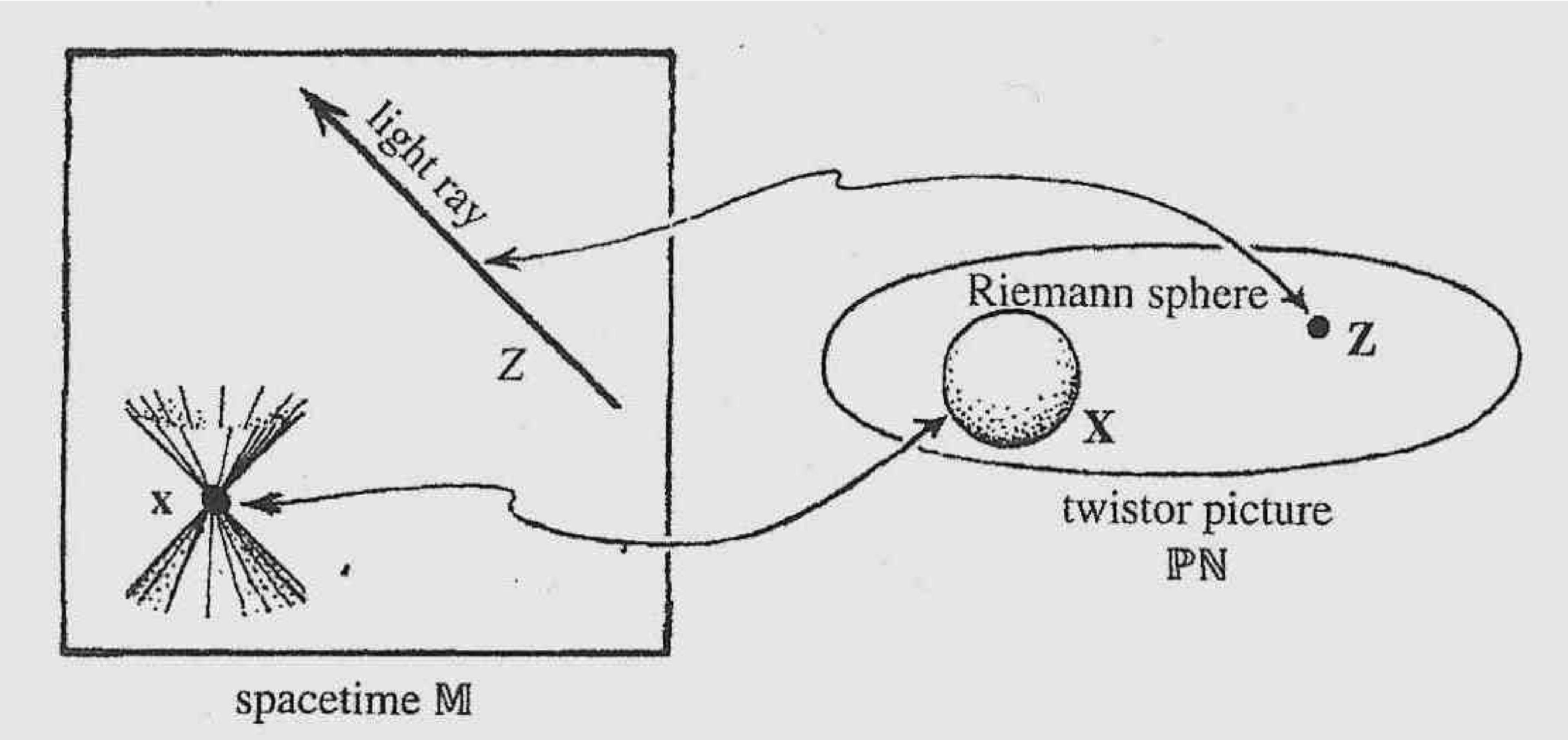


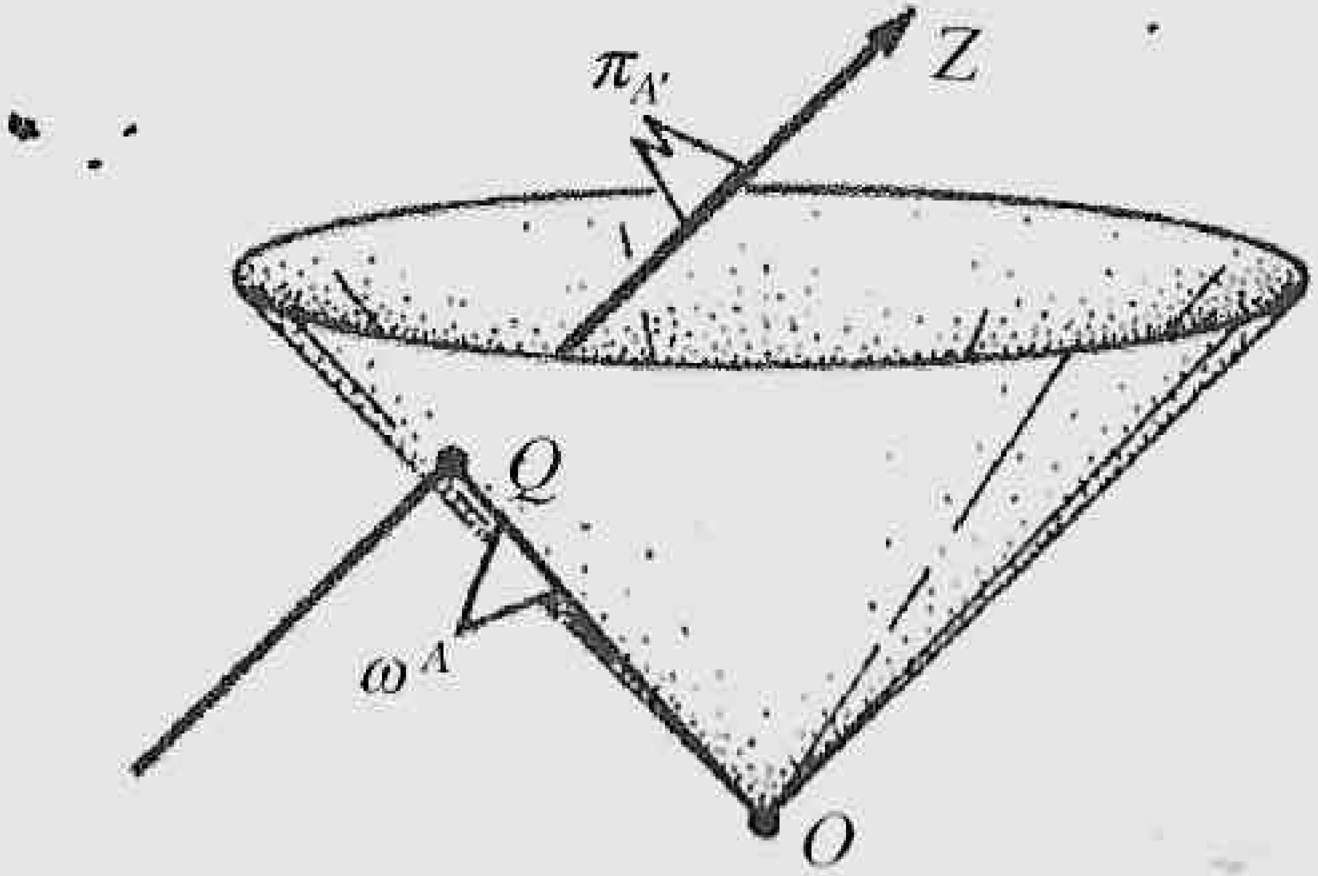
DAVID FINKELSTEIN



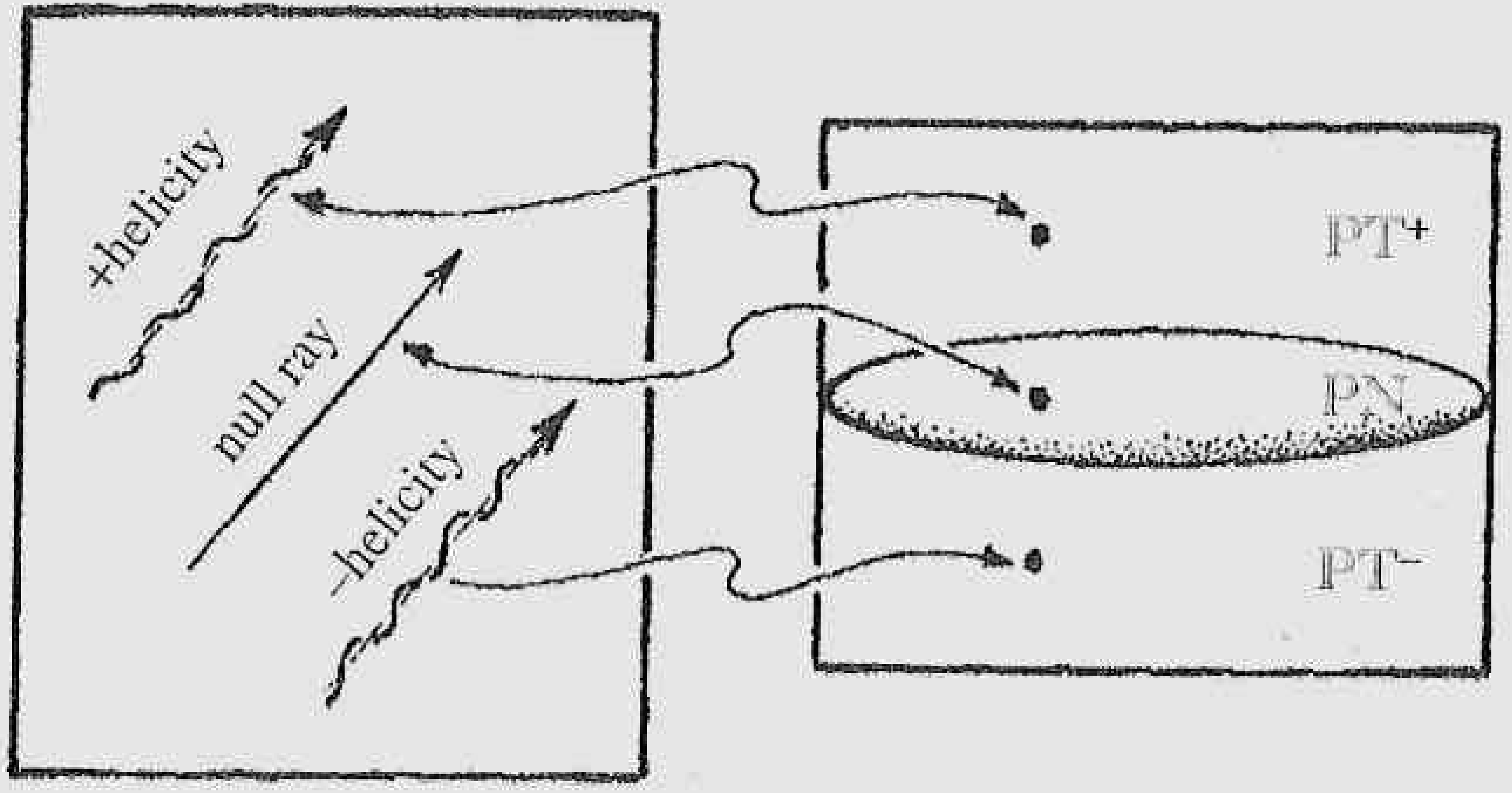
HERMANN WEYL




light ray
Z
x
Riemann sphere
Z
X
twistor picture
ℙℕ
spacetime 𝕄

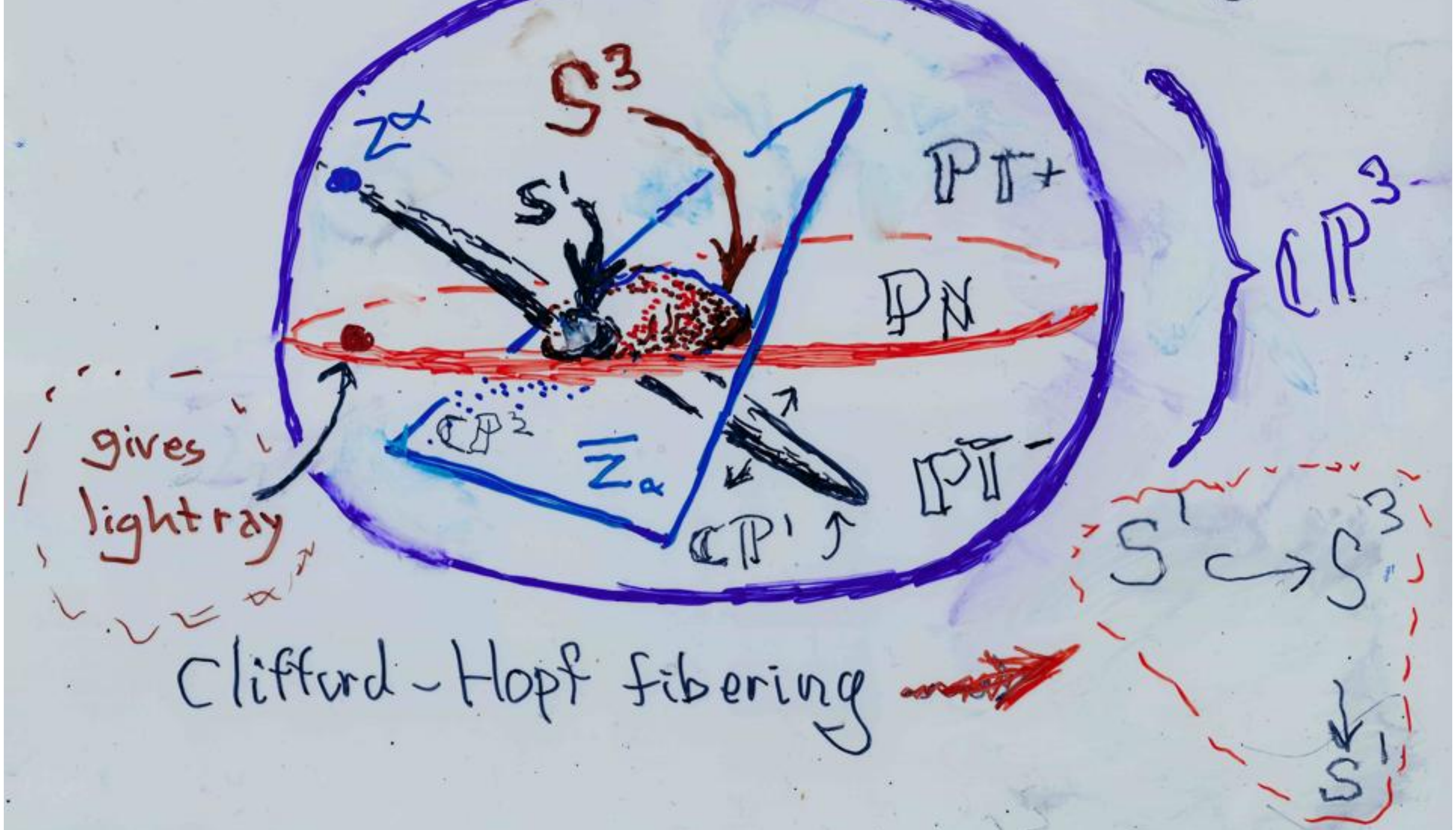


$\pi_{A'}$
Z
Q
ω^A
O


+helicity
null ray
–helicity
PT+
PN
PT-

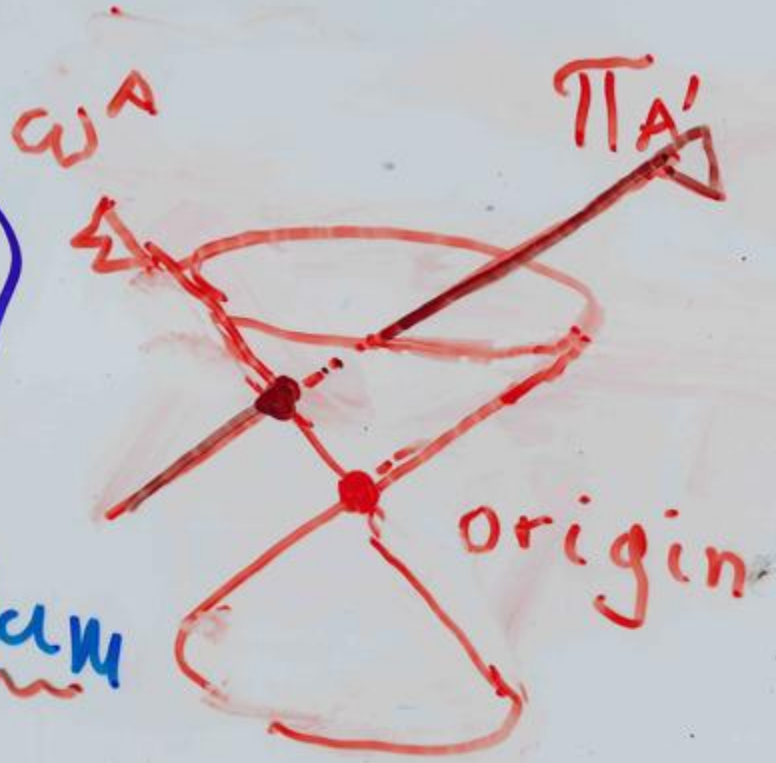
"Twistor"

from Robinson congruences



Twistors & angular momentum

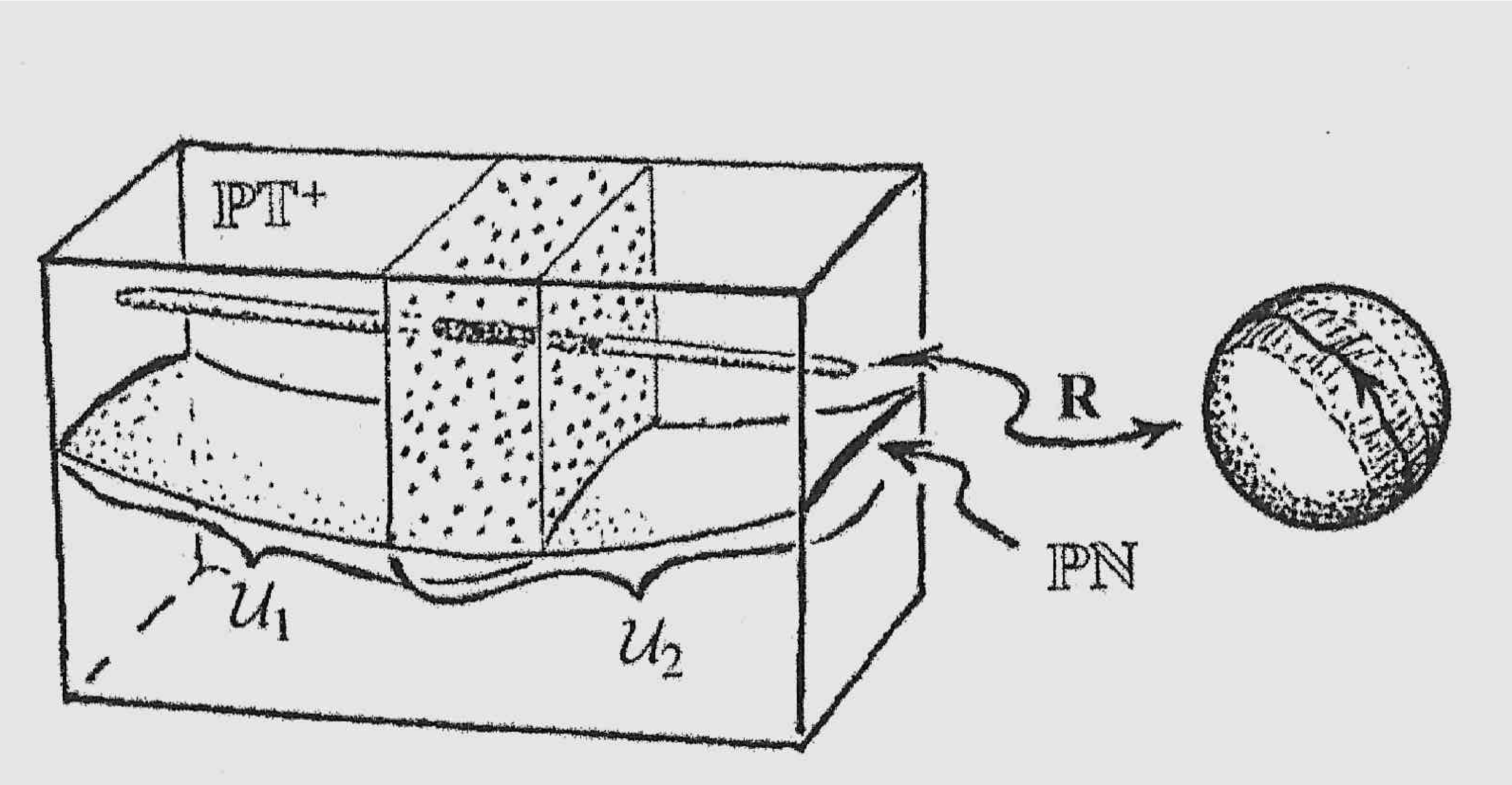
$$Z^\alpha = (\omega^A, \pi_{A'})$$



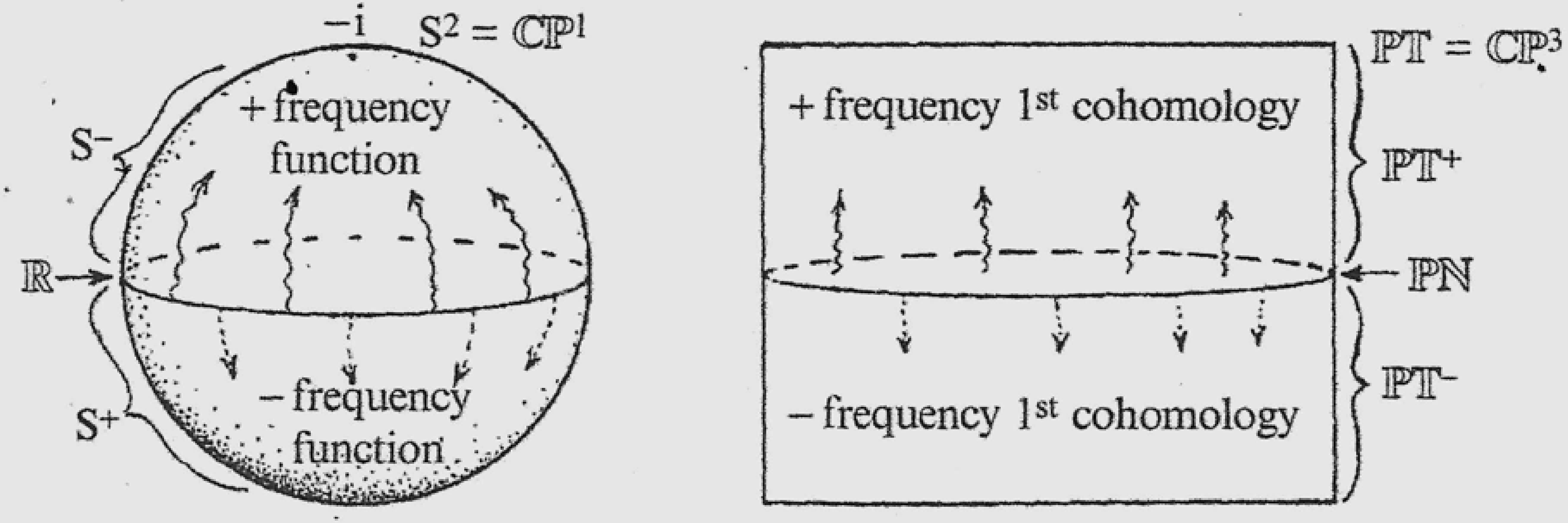
$\bar{\pi}_A \pi_{A'}$ is a momentum

$\therefore$ lower index

$$P_a = P_{AA'} = \bar{\pi}_A \pi_{A'}, \quad M^{ab} = i\omega^{(A}\bar{\pi}^{B)}\epsilon^{A'B'} - i\bar{\omega}^{(A'}\pi^{B')}\epsilon^{AB}$$



ℙ𝕋+
R
ℙℕ
$\mathcal{U}_1$
$\mathcal{U}_2$



−i
S² = ℂℙ¹
S⁻
+ frequency function
ℝ
S⁺
− frequency function
ℙ𝕋 = ℂℙ³
+ frequency 1st cohomology
ℙ𝕋⁺
ℙ𝕅
ℙ𝕋⁻
− frequency 1st cohomology

(a) (b)

The FUNDAMENTAL CONFUSION of twistor theory.

Does the splitting of twistor space into two halves refer to

± frequency or ± helicity?

At first, this seems like a relatively minor matter, but with the nonlinear graviton, it is crucial ("googly problem")!

$$(\omega^{A}, \pi_{A'}, \xi_{A}, \eta^{A'})$$

Twistor quantum theory (gives correct laws for momentum and angular momentum)

$$Z^{\alpha}\bar{Z}_{\beta} - \bar{Z}_{\beta}Z^{\alpha} = \hbar\delta^{\alpha}_{\beta}$$

$$Z^{\alpha}Z^{\beta} = Z^{\beta}Z^{\alpha}, \;\; \bar{Z}_{\alpha}\bar{Z}_{\beta} = \bar{Z}_{\beta}\bar{Z}_{\alpha}$$

Bi-twistor algebra, where $\mathbf{W}_\alpha$ behaves as $\bar{Z}_\alpha$

$$\mathcal{A}=A_\alpha\mathbf{Z}^\alpha+A^\alpha\mathbf{W}_\alpha,$$

$$\mathcal{B}=B_\alpha\mathbf{Z}^\alpha+B^\alpha\mathbf{W}_\alpha,$$

$$\mathcal{C}=C_\alpha\mathbf{Z}^\alpha+C^\alpha\mathbf{W}_\alpha$$

$$[\mathcal{A},\ \mathcal{B},\mathcal{C}]\ =\mathrm{i}\sum\nolimits_{\pm\circlearrowleft}\mathcal{A}\mathcal{B}\mathcal{C}$$

The Twistor Equation.

$$\nabla_{AA'}\,\omega^{B} = \varepsilon_{A}{}^{B}\,\pi_{A'}$$

from

$$\nabla_{A'(A}\,\omega_{B)} = 0$$

But this equation leads to inconsistencies in a conformally curved space-time.

For real bi-twistors we have

$$A^{\alpha} = \bar{A}^{\alpha}, \qquad B^{\alpha} = \bar{B}^{\alpha}, \qquad C^{\alpha} = \bar{C}^{\alpha}$$

in which case $[\mathcal{A}, \mathcal{B}, \mathcal{C}]$ is also real

$$Y_a = \Omega^{-1}\nabla_a\Omega = \nabla_a \log \Omega.$$

$$\nabla_{A'(A}\omega_{B)} = 0$$

$$\nabla_{A'A}\omega_B \mapsto \Omega\left(\nabla_{AA'}\omega_B - \Upsilon_{BA'}\omega_A + \Upsilon_{AA'}\omega_B\right)$$

$$\nabla_{AA'}\varphi_B \mapsto \tilde{\nabla}_{AA'}\tilde{\varphi}_B = \Omega^v\left(\nabla_{AA'}\varphi_B - \Upsilon_{BA'}\varphi_A + v\Upsilon_{AA'}\varphi_B\right)$$

$$\nabla_{AA'}\psi_{B'} \mapsto \tilde{\nabla}_{AA'}\psi_{B'} = \Omega^w\left(\nabla_{AA'}\psi_{B'} - \Upsilon_{AB'}\psi_{A'} + w\Upsilon_{AA'}\psi_{B'}\right).$$

$$\nabla_{AA'}\varphi_B \mapsto \tilde{\nabla}_{AA'}\tilde{\varphi}_B = \Omega^{v}\left(\nabla_{AA'}\varphi_B - \Upsilon_{BA'}\varphi_A + v\Upsilon_{AA'}\varphi_B\right)$$

$$\nabla_{AA'}\psi_{B'} \mapsto \tilde{\nabla}_{AA'}\psi_{B'} = \Omega^{w}\left(\nabla_{AA'}\psi_{B'} - \Upsilon_{AB'}\psi_{A'} + \mathrm{w}\Upsilon_{AA'}\psi_{B'}\right).$$

$$W_\alpha = (\xi_A, \eta^{A'}),$$

$$Z^\alpha W_\alpha = \omega^A \xi_A + \pi_{A'} \eta^{A'}$$

$$\pi_{A'} = -\frac{\mathrm{i}}{2} \nabla_{AA'} \omega^{A}$$

$$\pi_{A'} \mapsto \tilde{\pi}_{A'} = \pi_{A'} - \mathrm{i}\,\Upsilon_{AA'}\omega^{A}.$$

$$Z^{\alpha} = (\omega^{A}, \pi_{A'})$$

$$Z^{\alpha}\bar{Z}_{\alpha} = \omega^{A}\bar{\pi}_{A} + \pi_{A'}\overline{\omega}^{A'}$$

$$\bar{Z}_{\alpha} = (\bar{\pi}_{A}, \overline{\omega}^{A'}).$$

$$\eta^{A'} \mapsto \eta^{A'}, \qquad \xi_{\mathrm{A}} \mapsto \xi_{\mathrm{A}} + 2\mathrm{i}\,\Upsilon_{AA'}\eta^{A'}.$$

In the NLG construction, we immediately find that the *primed* spin-space is universally defined, owing to the presence of a *projection* of $\mathcal{T}$ down to a complex 2-dimensional vector space (the CP^2–{0} of Fig. 2 of [5]). This particular CP^2 refers uniformly to every line in $\mathbb{P}\mathcal{T}$ and therefore to every point of $\mathcal{G}$, thereby assigning an un-primed spin-space universally to the whole of $\mathcal{T}$. We may refer to a typical element of this 2-space as ξ_A.

On the other hand, the *primed* spin-space at P is a distinctly more delicate issue. For each p, the various elements of the primed spin-space would refer to the different "plane strips" through the line p, each of which is described by a smoothly propagated 2-plane element along the line p, all these containing the direction of p.
There is an integrability condition for this to provide a consistent primed spin-space for every p, arising from the vanishing of the appropriate Lie bracket, to provide the equation (4.2) of [5], namely

$$\eta_{A'}\eta^{B'}\nabla_{BB'}\eta^{A'} = 0,$$

If we make the replacement

$$\omega^A = \bar{\eta}^A$$

into this equation, we obtain

$$\omega^A \omega^B \nabla_{AA'} \omega_B = 0,$$

which will be referred to as the *weak twistor equation*

$$Z^{\alpha}\bar{Z}_{\alpha} = 0.$$

$$Z^{\alpha} W_{\alpha} = \frac{\mathrm{i}}{2} (\xi_A \nabla_{AA'} \omega^A - \pi_{A'} \nabla_{AA'} \eta^{A'}).$$

For the particular case of a null geodesic, this is expressed as

$$Z^{\alpha} \bar{Z}_{\alpha} = 0$$

$$\omega^{B}\nabla_{A'A}\omega_{B} = \mu_{A'}\omega_{A}$$

$$\pi_{A'} = \text{-i}\,\mu_{A'} = -\frac{\mathrm{i}}{2}\nabla_{AA'}\omega^{A}$$

$$W_\alpha = (\xi_A, \eta^{A'})$$

$$Z^\alpha W_\alpha = \omega^A \xi_A + \pi_{A'} \eta^{A'}$$

$$\pi_{A'} = -\frac{\mathrm{i}}{2} \nabla_{AA'} \omega^{A},$$

$$\xi_{A} = \frac{\mathrm{i}}{2} \nabla_{AA'} \eta^{A'}$$

$$\bar{\pi}_A \, \omega^A + \pi_{A'} \, \bar{\omega}^{A'} = 0$$

which is equivalent to the standard differential equation

$$n^a \nabla_a n^b = 0,$$

ENGELBERT SCHUCKING