UNIVERSITY OF OXFORD
Mathematical Institute

BA in MATHEMATICS
MMath in MATHEMATICS

SUPPLEMENT TO THE UNDERGRADUATE HANDBOOK – 2016 Matriculation

Prelims 2016-17
For examination in 2017
SYLLABUS and
SYNOPSES OF LECTURE COURSES

These details can also be found at:
http://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/handbooks-synopses

Issued October 2016

Syllabus and Synopses for the Preliminary Examination in Mathematics 2016–2017 for examination in 2017

Contents

1 Foreword

Syllabus

The syllabus here is that referred to in the *Examination Regulations 2016*[1] and has been approved by the Mathematics Teaching Committee for examination in Trinity Term 2017.

Examination Conventions can be found at: http://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions

A

The subject of the examination shall be Mathematics. The syllabus and number of papers shall be prescribed by regulation from time to time by the Mathematical, Physical and Life Sciences Board.

B

1. Candidates shall take five written papers. The titles of the papers shall be: Mathematics I, Mathematics II, Mathematics III, Mathematics IV, Mathematics V.

2. In addition to the five papers in cl 1, a candidate must also offer a practical work assessment.

3. Candidates shall be deemed to have passed the examination if they have satisfied the Moderators in all five papers and the practical assessment at a single examination or passed all five papers and the practical assessment in accordance with the proviso of cl 4.

4. A candidate who fails to satisfy the Moderators in one or two of papers I-V may offer those papers on one subsequent occasion; a candidate who fails to satisfy the Moderators in three or more of papers I-V may offer all five papers on one subsequent occasion; a candidate who fails to satisfy the Moderators in the practical work assessment may also offer the assessment on one subsequent occasion.

5. The Moderators may award a distinction to candidates of special merit who have passed all five written papers and the practical work assessment at a single examination.

6. The syllabus for each paper shall be published by the Mathematical Institute in a handbook for candidates by the beginning of the Michaelmas Full Term in the academic year of the examination, after consultation with the Mathematics Teaching Committee. Each paper will contain questions of a straight forward character.

7. The Chairman of Mathematics, or a deputy, shall make available to the Moderators evidence showing the extent to which each candidate has pursued an adequate course of practical work. In assessing a candidate's performance in the examination the Moderators shall take this evidence into account. Deadlines for handing in practical

[1] **Special Regulations for the Preliminary Examination in Mathematics**

work will be published in a handbook for candidates by the beginning of Michaelmas Full Term in the academic year of the examination.

Candidates are usually required to submit such practical work electronically; details shall be given in the handbook for the practical course. Any candidate who is unable for some reason to submit work electronically must apply to the Academic Administrator, Mathematical Institute, for permission to submit the work in paper form. Such applications must reach the Academic Administrator two weeks before the deadline for submitting the practical work.

8. The use of hand held pocket calculators is generally not permitted but certain kinds may be permitted for some papers. Specifications of which papers and which types of calculator are permitted for those exceptional papers will be announced by the Moderators in the Hilary Term preceding the examination.

Synopses

The synopses give some additional detail and show how the material is split between the different lecture courses. They include details of recommended reading.

Practical Work

The requirement in the *Examination Regulations* to pursue an adequate course of practical work will be satisfied by following the Computational Mathematics course and submitting two Computational Mathematics projects. Details about submission of these projects will be given in the Computational Mathematics handbook.

Notice of misprints or errors of any kind, and suggestions for improvements in this booklet, should be addressed to the Academic Administrator (academic.administrator@maths.ox.ac.uk) in the Mathematical Institute.

2 Syllabus

This section contains the Examination Syllabus.

2.1 Mathematics I

Sets: examples including the natural numbers, the integers, the rational numbers, the real numbers; inclusion, union, intersection, power set, ordered pairs and cartesian product of sets. Relations. Definition of an equivalence relation.

The well-ordering property of the natural numbers. Induction as a method of proof, including a proof of the binomial theorem with non-negative integral coefficients.

Maps: composition, restriction, injective (one-to-one), surjective (onto) and invertible maps, images and preimages.

Systems of linear equations. Expression as an augmented matrix (just understood as an array at this point). Elementary Row Operations (EROs). Solutions by row reduction.

Abstract vector spaces: Definition of a vector space over a field (expected examples $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{C}$). Examples of vector spaces: solution space of homogeneous system of equations and differential equations; function spaces; polynomials; $\mathbb{C}$ as an $\mathbb{R}$-vector space; sequence spaces. Subspaces, spanning sets and spans.

Linear independence, definition of a basis, examples. Steinitz exchange lemma, and definition of dimension. Coordinates associated with a basis. Algorithms involving finding a basis of a subspace with EROs. Sums, intersections and direct sums of subspaces. Dimension formula.

Linear transformations: definition and examples including projections. Kernel and image, rank nullity formula.

Algebra of linear transformations. Inverses. Matrix of a linear transformation with respect to a basis. Algebra of matrices. Transformation of a matrix under change of basis. Determining an inverse with EROs. Column space, column rank.

Bilinear forms. Positive definite symmetric bilinear forms. Inner Product Spaces. Examples: $\mathbb{R}^n$ with dot product, function spaces. Comment on (positive definite) Hermitian forms. Cauchy-Schwarz inequality. Distance and angle. Transpose of a matrix. Orthogonal matrices.

Introduction to determinant of a square matrix: existence and uniqueness and relation to volume. Proof of existence by induction. Basic properties, computation by row operations.

Determinants and linear transformations: multiplicativity of the determinant, definition of the determinant of a linear transformation. Invertibility and the determinant. Permutation matrices and explicit formula for the determinant deduced from properties of determinant.

Eigenvectors and eigenvalues, the characteristic polynomial. Trace. Proof that eigenspaces form a direct sum. Examples. Discussion of diagonalisation. Geometric and algebraic multiplicity of eigenvalues.

Gram-Schmidt procedure.

Spectral theorem for real symmetric matrices. Matrix realisation of bilinear maps given a basis and application to orthogonal transformation of quadrics into normal form. Statement of classification of orthogonal transformations.

Axioms for a group and for an Abelian group. Examples including geometric symmetry groups, matrix groups (GL_n, SL_n, O_n, SO_n, U_n), cyclic groups. Products of groups.

Permutations of a finite set under composition. Cycles and cycle notation. Order. Transpositions; every permutation may be expressed as a product of transpositions. The parity of a permutation is well-defined via determinants. Conjugacy in permutation groups.

Subgroups; examples. Intersections. The subgroup generated by a subset of a group. A subgroup of a cyclic group is cyclic. Connection with hcf and lcm. Bezout's Lemma.

Recap on equivalence relations including congruence mod n and conjugacy in a group. Proof that equivalence classes partition a set. Cosets and Lagrange's Theorem; examples. The order of an element. Fermat's Little Theorem.

Isomorphisms. Groups up to isomorphism of order 8 (stated without proof). Homomorphisms of groups. Kernels. Images. Normal subgroups. Quotient groups. First Isomorphism Theorem. Simple examples determining all homomorphisms between groups.

Group actions; examples. Definition of orbits and stabilizers. Transitivity. Orbits partition the set. Stabilizers are subgroups.

Orbit-stabilizer Theorem. Examples and applications including Cauchy's Theorem and to conjugacy classes. Orbit-counting formula.

The representation $G \to \mathrm{Sym}(S)$ associated with an action of G on S. Cayley's Theorem. Symmetry groups of the tetrahedron and cube.

2.2 Mathematics II

Real numbers: arithmetic, ordering, suprema, infima; real numbers as a complete ordered field. Countable sets. The rational numbers are countable. The real numbers are uncountable.

The complex number system. The Argand diagram; modulus and argument. De Moivre's Theorem, polar form, the triangle inequality. Statement of the Fundamental Theorem of Algebra. Roots of unity. De Moivre's Theorem. Simple transformations in the complex plane. Polar form, with applications.

Sequences of (real or complex) numbers. Limits of sequences of numbers; the algebra of limits. Order notation.

Subsequences; every subsequence of a convergent sequence converges to the same limit. Bounded monotone sequences converge. Bolzano–Weierstrass Theorem. Cauchy's convergence criterion. Limit point of a subset of the line or plane.

Series of (real or complex) numbers. Convergence of series. Simple examples to include geometric progressions and power series. Alternating series test, absolute convergence, comparison test, ratio test, integral test.

Power series, radius of convergence, important examples to include defintions of relationships between exponential, trigonometric functions and hyperbolic functions.

Continuous functions of a single real or complex variable. Definition of continuity of real valued functions of several variables.

The algebra of continuous functions. A continuous real-valued function on a closed bounded interval is bounded, achieves its bounds and is uniformly continuous. Intermediate Value Theorem. Inverse Function Theorem for continuous strictly monotonic functions.

Sequences and series of functions. The uniform limit of a sequence of continuous functions is continuous. Weierstrass's M-test. Continuity of functions defined by power series.

Definition of derivative of a function of a single real variable. The algebra of differentiable functions. Rolle's Theorem. Mean Value Theorem. Cauchy's (Generalized) Mean Value Theorem. L'Hôpital's Formula. Taylor's expansion with remainder in Lagrange's form. Binomial theorem with arbitrary index.

Step functions and their integrals. The integral of a continuous function on a closed bounded interval. Properties of the integral including linearity and the interchange of integral and limit for a uniform limit of continuous functions on a bounded interval. The Mean Value Theorem for Integrals. The Fundamental Theorem of Calculus; integration by parts and substitution.

Term-by-term differentiation of a (real) power series (interchanging limit and derivative for a series of functions where the derivatives converge uniformly).

2.3 Mathematics III

General linear homogeneous ODEs: integrating factor for first order linear ODEs, second solution when one solution is known for second order linear ODEs. First and second order linear ODEs with constant coefficients. General solution of linear inhomogeneous ODE as particular solution plus solution of homogeneous equation. Simple examples of finding particular integrals by guesswork.

Partial derivatives. Second order derivatives and statement of condition for equality of mixed partial derivatives. Chain rule, change of variable, including planar polar coordinates. Solving some simple partial differential equations (e.g. $f_{xy} = 0$, $f_x = f_y$).

Parametric representation of curves, tangents. Arc length. Line integrals.

Jacobians with examples including plane polar coordinates. Some simple double integrals calculating area and also $\int_{\mathbb{R}^2} e^{-(x^2+y^2)} dA$.

Simple examples of surfaces, especially as level sets. Gradient vector; normal to surface; directional derivative; $\int_A^B \nabla\phi \cdot d\mathbf{r} = \phi(B) - \phi(A)$.

Taylor's Theorem for a function of two variables (statement only). Critical points and classification using directional derivatives and Taylor's theorem. Informal (geometrical) treatment of Lagrange multipliers.

Sample space, algebra of events, probability measure. Permutations and combinations, sampling with or without replacement. Conditional probability, partitions of the sample space, theorem of total probability, Bayes' Theorem. Independence.

Discrete random variables, probability mass functions, examples: Bernoulli, binomial, Poisson, geometric. Expectation: mean and variance. Joint distributions of several discrete random variables. Marginal and conditional distributions. Independence. Conditional expectation, theorem of total probability for expectations. Expectations of functions of more than one discrete random variable, covariance, variance of a sum of dependent discrete random variables.

Solution of first and second order linear difference equations. Random walks (finite state space only).

Probability generating functions, use in calculating expectations. Random sample, sums of independent random variables, random sums. Chebyshev's inequality, Weak Law of Large Numbers.

Continuous random variables, cumulative distribution functions, probability density functions, examples: uniform, exponential, gamma, normal. Expectation: mean and variance. Functions of a single continuous random variable. Joint probability density functions of several continuous random variables (rectangular regions only). Marginal distributions. Independence. Expectations of functions of jointly continuous random variables, covariance, variance of a sum of dependent jointly continuous random variables.

Random samples, concept of a statistic and its distribution, sample mean as a measure of location and sample variance as a measure of spread.

Concept of likelihood; examples of likelihood for simple distributions. Estimation for a single unknown parameter by maximising likelihood. Examples drawn from: Bernoulli, binomial, geometric, Poisson, exponential (parametrized by mean), normal (mean only, variance known). Data to include simple surveys, opinion polls, archaeological studies, etc. Properties of estimators—unbiasedness, Mean Squared Error = (bias^2 + variance). Statement of Central Limit Theorem (excluding proof). Confidence intervals using CLT. Simple straight line fit, $Y_t = a + bx_t + \varepsilon_t$, with ε_t normal independent errors of zero mean and common known variance. Estimators for a, b by maximising likelihood using partial differentiation, unbiasedness and calculation of variance as linear sums of Y_t. (No confidence intervals). Examples (use scatter plots to show suitability of linear regression).

Linear regression with 2 regressors. Special case of quadratic regression $Y_t = a+bx_t+cx_t^2+\epsilon_t$. Model diagnostics and outlier detection. Residual plots. Heteroscedasticity. Outliers and studentized residuals. High-leverage points and leverage statistics.

Introduction to unsupervised learning with real world examples. Principal components analysis (PCA). Proof that PCs maximize directions of maximum variance and are orthogonal using Lagrange multipliers. PCA as eigendecomposition of covariance matrix. Eigenvalues as variances. Choosing number of PCs. The multivariate normal distribution pdf. Examples of PCA on multivariate normal data and clustered data. Clustering techniques; K-means clustering. Minimization of within-cluster variance. K-means algorithm and proof that it will decrease objective function. Local versus global optima and use of random initializations. Hierarchical clustering techniques. Agglomerative clustering using complete, average and single linkage.

2.4 Mathematics IV

Euclidean geometry in two and three dimensions approached by vectors and coordinates. Vector addition and scalar multiplication. The scalar product, equations of planes, lines and circles. Conics (normal form only), focus and directrix.

The vector product in three dimensions. Use of $\mathbf{a}, \mathbf{b}, \mathbf{a} \wedge \mathbf{b}$ as a basis. $\mathbf{r} \wedge \mathbf{a} = \mathbf{b}$ represents a line. Scalar triple products and vector triple products, vector algebra.

Orthogonal matrices. 2×2 orthogonal matrices and the maps they represent. Orthonormal bases in $\mathbb{R}^3$. Orthogonal change of variable; $A\mathbf{u} \cdot A\mathbf{v} = \mathbf{u} \cdot \mathbf{v}$ and $A\mathbf{u} \wedge A\mathbf{v} = \pm \mathbf{u} \wedge \mathbf{v}$. Showing the locus $Ax^2 + Bxy + Cy^2 = 1$ can be put in normal form via a rotation matrix; statement that a real symmetric matrix can be orthogonally diagonalized. Simple examples identifying some conics and quadrics not in normal form.

3×3 orthogonal matrices; $SO(3)$ and rotations; conditions for being a reflection. Isometries of $\mathbb{R}^3$.

Rotating frames in 2 and 3 dimensions. Angular velocity. $\mathbf{v} = \omega \wedge \mathbf{r}$.

Parametrised surfaces, including spheres, cones. Examples of coordinate systems including parabolic, spherical and cylindrical polars. Calculating normal as $\mathbf{r}_u \wedge \mathbf{r}_v$. Surface area.

Newton's laws, inertial frames, Galilean relativity. Dimensional analysis.

Forces, examples including gravity, electromagnetism, fluid drag. Conservative forces and the Newtonian gravitational potential. Energy and momentum.

Equilibria and the harmonic oscillator. Stability and instability via linearized equations, normal modes. Simple examples of equilibria in two variables via matrices.

Central forces, angular momentum and torque. Planar motion in polar coordinates, the effective potential, Kepler's laws and planetary motion.

Many particle systems, centre of mass motion. Rigid bodies, rotating frames and Coriolis force, inertia tensor, rigid body motion.

The Division Algorithm on Integers, Euclid's Algorithm including proof of termination with highest common factor. The solution of linear Diophantine equations.

Division and Euclid's algorithm for real polynomials. Examples.

Root finding for real polynomials. Fixed point iterations, examples. Convergence. Existence of fixed points and convergence of fixed point iterations by the contraction mapping theorem (using the mean value theorem).

Newton iteration. Quadratic convergence. Horner's Rule.

2.5 Mathematics V

Multiple integrals: Two dimensions. Informal definition and evaluation by repeated integration; example over a rectangle; properties. General domains. Change of variables. Examples.

Volume integrals: Jacobians for cylindrical and spherical polars, examples.

Recap on surface integrals. Flux integrals.

Scalar and vector fields. Vector differential operators: divergence and curl; physical interpretation. Calculation. Identities.

Divergence theorem. Example. Consequences: Greens 1st and second theorems. $\int_V \nabla\phi dV = \int_{\delta V} \phi dS$. Uniqueness of solutions of Poisson's equation. Derivation of heat equation. Divergence theorem in plane. Informal proof for plane.

Stokes's theorem. Examples. Consequences. The existence of potential for a conservative force.

Gauss' Flux Theorem. Examples. Equivalence with Poisson's equation.

Fourier series: Periodic, odd and even functions. Calculation of sine and cosine series. Simple applications concentrating on imparting familiarity with the calculation of Fourier coefficients and the use of Fourier series. The issue of convergence is discussed informally with examples. The link between convergence and smoothness is mentioned, together with its consequences for approximation purposes.

Partial differential equations: Introduction in descriptive mode on partial differential equations and how they arise. Derivation of (i) the wave equation of a string, (ii) the heat equation in one dimension (box argument only). Examples of solutions and their interpretation. D'Alembert's solution of the wave equation and applications. Characteristic diagrams (excluding reflection and transmission). Uniqueness of solutions of wave and heat equations.

PDEs with Boundary conditions. Solution by separation of variables. Use of Fourier series to solve the wave equation, Laplace's equation and the heat equation (all with two independent variables). (Laplace's equation in Cartesian and in plane polar coordinates). Applications.

Synopses of Lectures

3 Mathematics I

3.1 Introductory Courses

There are two short introductory courses within the first two weeks of Michaelmas term to help students adjust to University Mathematics. These are *Introduction to University Level Mathematics* and *Introduction to Complex Numbers.*

3.1.1 Introduction to University Level Mathematics — Prof. Alan Lauder — 8 MT

There will be 8 introductory lectures in the first two weeks of Michaelmas term.

Overview

Prior to arrival, undergraduates are encouraged to read Professor Batty's study guide "How do undergraduates do Mathematics?" https://www.maths.ox.ac.uk/system/files/attachments/study_public_1.pdf

The purpose of these introductory lectures is to establish some of the basic language and notation of university mathematics, and to introduce the elements of (naïve) set theory and the nature of formal proof.

Learning Outcomes

Students should:

(i) have the ability to describe, manipulate, and prove results about sets and functions using standard mathematical notation;

(ii) know and be able to use simple relations;

(iii) develop sound reasoning skills;

(iv) have the ability to follow and to construct simple proofs, including proofs by mathematical induction (including strong induction, minimal counterexample) and proofs by contradiction;

(v) learn how to write mathematics.

Synopsis

The natural numbers and their ordering. Induction as a method of proof, including a proof of the binomial theorem with non-negative integral coefficients.

Sets. Examples including $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$, and intervals in $\mathbb{R}$. Inclusion, union, intersection, power set, ordered pairs and cartesian product of sets.

Relations. Definition of an equivalence relation. Examples. Functions: composition, restriction; injective (one-to-one), surjective (onto) and invertible functions; images and preimages.

Writing mathematics. The language of mathematical reasoning; quantifiers: "for all", "there exists". Formulation of mathematical statements with examples.

Proofs and refutations: standard techniques for constructing proofs; counter-examples. Example of proof by contradiction and more on proof by induction.

Problem-solving in mathematics: experimentation, conjecture, confirmation, followed by explaining the solution precisely.

Reading

1. C. J. K. Batty, *How do undergraduates do Mathematics?*, (Mathematical Institute Study Guide, 1994) https://www.maths.ox.ac.uk/system/files/attachments/study_public_1.pdf.

Further Reading

1. G. Pólya. *How to solve it: a new aspect of mathematical method* (1945, New edition 2014 with a foreword by John Conway, Princeton University Press).
2. G. C. Smith, *Introductory Mathematics: Algebra and Analysis*, (Springer-Verlag, London, 1998), Chapters 1 and 2.
3. Robert G. Bartle, Donald R. Sherbert, *Introduction to Real Analysis*, (Wiley, New York, Fourth Edition, 2011), Chapter 1 and Appendices A and B.
4. C. Plumpton, E. Shipton, R. L. Perry, *Proof*, (MacMillan, London, 1984).
5. R. B. J. T. Allenby, *Numbers and Proofs*, (Butterworth-Heinemann, London, 1997).
6. R. A. Earl, *Bridging Material on Induction*, (Mathematics Department website).

3.1.2 Introduction to Complex Numbers — Dr Peter Neumann — 2 MT

This course of two lectures will run in the first week of Michaelmas Term.

Generally, students should not expect a tutorial to support this short course. Solutions to the problem sheet will be posted on Monday of Week 2 and students are asked to mark their own problems and notify their tutor.

Overview

This course aims to give all students a common background in complex numbers.

Learning Outcomes

Students will be able to:

(i) manipulate complex numbers with confidence;

(ii) understand geometrically their representation on the Argand diagram, including the nth roots of unity;

(iii) know the polar representation form and be able to apply it.

Synopsis

Basic arithmetic of complex numbers, the Argand diagram; modulus and argument of a complex number. Statement of the Fundamental Theorem of Algebra. Roots of unity. De Moivre's Theorem. Simple transformations in the complex plane. Polar form $re^{i\theta}$, with applications.

Reading

1. R. A. Earl, *Complex numbers* (https://www.maths.ox.ac.uk/study-here/undergraduate-study/bridging-gap
2. D. W. Jordan & P Smith, *Mathematical Techniques* (Oxford University Press, Oxford, 2002), Ch. 6.

3.2 Linear Algebra I — Dr Peter Neumann — 14 MT

Overview

Linear algebra pervades and is fundamental to algebra, geometry, analysis, applied mathematics, statistics, and indeed most of mathematics. This course lays the foundations, concentrating mainly on vector spaces and matrices over the real and complex number systems. The course begins with examples focussed on $\mathbb{R}^2$ and $\mathbb{R}^3$, and gradually becomes more abstract. The course also introduces the idea of an inner product, with which angle and distance can be introduced into a vector space.

Learning Outcomes

Students will:

(i) understand the notions of a vector space, a subspace, linear dependence and independence, spanning sets and bases within the familiar setting of $\mathbb{R}^2$ and $\mathbb{R}^3$;

(ii) understand and be able to use the abstract notions of a general vector space, a subspace, linear dependence and independence, spanning sets and bases and be able to formally prove results related to these concepts;

(iii) have an understanding of matrices and of their applications to the algorithmic solution of systems of linear equations and to their representation of linear maps between vector spaces.

Synopsis

Systems of linear equations. Expression as an augmented matrix (understood simply as an array at this point). Elementary Row Operations (EROs). Solutions by row reduction.

Abstract vector spaces: Definition of a vector space over a field (expected examples $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{C}$). Examples of vector spaces: solution space of homogeneous systems of equations and differential equations; function spaces; polynomials; $\mathbb{C}$ as an $\mathbb{R}$-vector space; sequence spaces. Subspaces, spanning sets and spans. (Emphasis on concrete examples, with deduction of properties from axioms set as problems).

Linear independence, definition of a basis, examples. Steinitz exchange lemma, and definition of dimension. Coordinates associated with a basis. Algorithms involving EROs to find a basis of a subspace.

Sums, intersections and direct sums of subspaces. Dimension formula.

Linear transformations: definition and examples including projections. Kernel and image, rank–nullity formula.

Algebra of linear transformations. Inverses. Matrix of a linear transformation with respect to a basis. Algebra of matrices. Transformation of a matrix under change of basis. Determining an inverse with EROs. Column space, column rank.

Bilinear forms. Positive definite symmetric bilinear forms. Inner Product Spaces. Examples: $\mathbb{R}^n$ with dot product, function spaces. Comment on (positive definite) Hermitian forms. Cauchy–Schwarz inequality. Distance and angle. Transpose of a matrix. Orthogonal matrices.

Reading

1. T. S. Blyth and E. F. Robertson, *Basic Linear Algebra* (Springer, London, 1998).
2. R. Kaye and R. Wilson, *Linear Algebra* (OUP, 1998), Chapters 1–5 and 8. [More advanced but useful on bilinear forms and inner product spaces.]

Alternative and Further Reading

1. C. W. Curtis, *Linear Algebra – An Introductory Approach* (Springer, London, 4th edition, reprinted 1994).
2. R. B. J. T. Allenby, *Linear Algebra* (Arnold, London, 1995).
3. D. A. Towers, *A Guide to Linear Algebra* (Macmillan, Basingstoke, 1988).
4. D. T. Finkbeiner, *Elements of Linear Algebra* (Freeman, London, 1972). [Out of print, but available in many libraries.]

5. B. Seymour Lipschutz, Marc Lipson, *Linear Algebra* (McGraw Hill, London, Third Edition, 2001).

3.3 Linear Algebra II — Prof. Alan Lauder — 8HT

Learning Outcomes

Students will:

(i) understand the elementary theory of determinants;

(ii) understand the beginnings of the theory of eigenvectors and eigenvalues and appreciate the applications of diagonalizability.

(iii) understand the Spectral Theory for real symmetric matrices, and appreciate the geometric importance of an orthogonal change of variable.

Synopsis

Introduction to determinant of a square matrix: existence and uniqueness. Proof of existence by induction. Proof of uniqueness by deriving explicit formula from the properties of the determinant. Permutation matrices. (No general discussion of permutations). Basic properties of determinant, relation to volume. Multiplicativity of the determinant, computation by row operations.

Determinants and linear transformations: definition of the determinant of a linear transformation, multiplicativity, invertibility and the determinant.

Eigenvectors and eigenvalues, the characteristic polynomial, trace. Eigenvectors for distinct eigenvalues are linearly independent. Discussion of diagonalisation. Examples. Eigenspaces, geometric and algebraic multiplicity of eigenvalues. Eigenspaces form a direct sum.

Gram-Schmidt procedure. Spectral theorem for real symmetric matrices. Quadratic forms and real symmetric matrices. Application of the spectral theorem to putting quadrics into normal form by orthogonal transformations and translations. Statement of classification of orthogonal transformations.

Reading

1. T. S. Blyth and E. F. Robertson, *Basic Linear Algebra* (Springer, London, 2nd edition 2002).
2. C. W. Curtis, *Linear Algebra – An Introductory Approach* (Springer, New York, 4th edition, reprinted 1994).
3. R. B. J. T. Allenby, *Linear Algebra* (Arnold, London, 1995).
4. D. A. Towers, *A Guide to Linear Algebra* (Macmillan, Basingstoke 1988).

5. S. Lang, *Linear Algebra* (Springer, London, Third Edition, 1987).

3.4 Groups and Group Actions — Dr Vicky Neale — 8 HT and 8 TT

Overview

Abstract algebra evolved in the twentieth century out of nineteenth century discoveries in algebra, number theory and geometry. It is a highly developed example of the power of generalisation and axiomatisation in mathematics. The *group* is an important first example of an abstract, algebraic structure and groups permeate much of mathematics particularly where there is an aspect of symmetry involved. Moving on from examples and the theory of groups, we will also see how groups *act* on sets (e.g. permutations on sets, matrix groups on vectors) and apply these results to several geometric examples and more widely.

Learning Outcomes

Students will appreciate the value of abstraction and meet many examples of groups and group actions from around mathematics. Beyond theoretic aspects of group theory students will also see the value of these methods in the generality of the approach and also to otherwise intractable counting problems.

Synopsis

HT (8 lectures)

Axioms for a group and for an Abelian group. Examples including geometric symmetry groups, matrix groups (GL_n, SL_n, O_n, SO_n, U_n), cyclic groups. Products of groups.

Permutations of a finite set under composition. Cycles and cycle notation. Order. Transpositions; every permutation may be expressed as a product of transpositions. The parity of a permutation is well-defined via determinants. Conjugacy in permutation groups.

Subgroups; examples. Intersections. The subgroup generated by a subset of a group. A subgroup of a cyclic group is cyclic. Connection with hcf and lcm. Bezout's Lemma.

Recap on equivalence relations including congruence mod n and conjugacy in a group. Proof that equivalence classes partition a set. Cosets and Lagrange's Theorem; examples. The order of an element. Fermat's Little Theorem.

TT (8 Lectures)

Isomorphisms, examples. Groups up to isomorphism of order 8 (stated without proof). Homomorphisms of groups with motivating examples. Kernels. Images. Normal subgroups. Quotient groups; examples. First Isomorphism Theorem. Simple examples determining all homomorphisms between groups.

Group actions; examples. Definition of orbits and stabilizers. Transitivity. Orbits partition the set. Stabilizers are subgroups.

Orbit-stabilizer Theorem. Examples and applications including Cauchy's Theorem and to conjugacy classes.

Orbit-counting formula. Examples.

The representation $G \to \mathrm{Sym}(S)$ associated with an action of G on S. Cayley's Theorem. Symmetry groups of the tetrahedron and cube.

Reading

1. M. A. Armstrong *Groups and Symmetry* (Springer, 1997)

Alternative Reading

1. R. B. J. T. Allenby, *Rings, Fields and Groups*, (Second revised edition, Elsevier, 1991)
2. Peter J. Cameron, *Introduction to Algebra*, (Second edition, Oxford University Press, 2007).
3. John B. Fraleigh, *A First Course in Abstract Algebra* (Seventh edition, Pearson, 2013).
4. W. Keith Nicholson, *Introduction to Abstract Algebra* (Fourth edition, John Wiley, 2012).
5. Joseph J. Rotman, *A First Course in Abstract Algebra* (Third edition, Pearson, 2005).
6. Joseph Gallian, *Contemporary Abstract Algebra* (8th international edition, Brooks/Cole, 2012).
7. Nathan Carter, *Visual Group Theory* (MAA Problem Book Series, 2009).

4 Mathematics II

4.1 Analysis I: Sequences and Series — Prof. Frances Kirwan — 15 MT

Overview

In these lectures we study the real and complex numbers, and study their properties, particularly completeness; define and study limits of sequences, convergence of series, and power series.

Learning Outcomes

Students will have:

(i) an ability to work within an axiomatic framework;

(ii) a detailed understanding of how Cauchy's criterion for the convergence of real and complex sequences and series follows from the completeness axiom for $\mathbb{R}$, and the ability to explain the steps in standard mathematical notation;

(iii) knowledge of some simple techniques for testing the convergence of sequences and series, and confidence in applying them;

(iv) familiarity with a variety of well-known sequences and series, with a developing intuition about the behaviour of new ones;

(v) an understanding of how the elementary functions can be defined by power series, with an ability to deduce some of their easier properties.

Synopsis

Real numbers: arithmetic, ordering, suprema, infima; the real numbers as a complete ordered field. Definition of a countable set. The countability of the rational numbers. The reals are uncountable. The complex number system. The triangle inequality.

Sequences of real or complex numbers. Definition of a limit of a sequence of numbers. Limits and inequalities. The algebra of limits. Order notation: O, o.

Subsequences; a proof that every subsequence of a convergent sequence converges to the same limit; bounded monotone sequences converge. Bolzano–Weierstrass Theorem. Cauchy's convergence criterion.

Series of real or complex numbers. Convergence of series. Simple examples to include geometric progressions and some power series. Absolute convergence, Comparison Test, Ratio Test, Integral Test. Alternating Series Test.

Power series, radius of convergence. Examples to include definition of and relationships between exponential, trigonometric functions and hyperbolic functions.

Reading

1. Lara Alcock, *How to Think About Analysis* (OUP, 2014) ISBN 9780198723530
2. Robert G. Bartle, Donald R. Sherbert, *Introduction to Real Analysis* (Wiley, Third Edition, 2000), Chapters 2, 3, 9.1, 9.2.
3. R. P. Burn, *Numbers and Functions, Steps into Analysis* (Cambridge University Press, 2000), Chapters 2–6. [This is a book of problems and answers, a DIY course in analysis.]
4. J. M. Howie, *Real Analysis*, Springer Undergraduate Texts in Mathematics Series (Springer, 2001) ISBN 1-85233-314-6.

Alternative Reading

The first four books take a slightly gentler approach to the material in the syllabus, whereas the last two cover it in greater depth and contain some more advanced material.

1. Mary Hart, *A Guide to Analysis* (MacMillan, 1990), Chapter 2.
2. J. C. Burkill, *A First Course In Mathematical Analysis* (Cambridge University Press, 1962), Chapters 1, 2 and 5.
3. Victor Bryant, *Yet Another Introduction to Analysis* (Cambridge University Press, 1990), Chapters 1 and 2.
4. G.C. Smith, *Introductory Mathematics: Algebra and Analysis* (Springer-Verlag, 1998), Chapter 3 (introducing complex numbers).
5. Michael Spivak, *Calculus* (Benjamin, 1967), Parts I, IV, and V (for a construction of the real numbers).
6. Brian S. Thomson, Judith B. Bruckner, Andrew M. Bruckner, *Elementary Analysis* (Prentice Hall, 2001), Chapters 1–4.

4.2 Analysis II: Continuity and Differentiability — Prof. Hilary Priestley — 16 HT

Overview

In this term's lectures, we study continuity of functions of a real or complex variable, and differentiability of functions of a real variable.

Learning Outcomes

At the end of the course students will be able to apply limiting properties to describe and prove continuity and differentiability conditions for real and complex functions. They will be able to prove important theorems, such as the Intermediate Value Theorem, Rolle's Theorem and Mean Value Theorem, and will continue the study of power series and their convergence.

Synopsis

Definition of the function limit. Examples and counter examples to illustrate when $\lim_{x\to a} f(x) = f(a)$ (and when it doesn't). Definition of continuity of functions on subsets of $\mathbb{R}$ and $\mathbb{C}$ in terms of ε and δ. Continuity of real valued functions of several variables.The algebra of continuous functions; examples, including polynomials. Continuous functions on closed bounded intervals: boundedness, maxima and minima, uniform continuity. Intermediate Value Theorem. Inverse Function Theorem for continuous strictly monotone functions.

Sequences and series of functions. Uniform limit of a sequence of continuous functions is continuous. Weierstrass's M-test for uniformly convergent series of functions. Continuity of functions defined by power series.

Definition of the derivative of a function of a real variable. Algebra of derivatives, examples to include polynomials and inverse functions. The derivative of a function defined by a power series is given by the derived series (proof not examinable). Vanishing of the derivative at a local maximum or minimum. Rolle's Theorem. Mean Value Theorem with simple applications: constant and monotone functions. Cauchy's (Generalized) Mean Value Theorem and L'Hôpital's Formula. Taylor's Theorem with remainder in Lagrange's form; examples of Taylor's Theorem to include the binomial expansion with arbitrary index.

Reading

1. Robert G. Bartle, Donald R. Sherbert, *Introduction to Real Analysis* (Wiley, Fourth Edition, 2011), Chapters 4–8.

2. R. P. Burn, *Numbers and Functions, Steps into Analysis* (Cambridge University Press, Second Edition, 2000). [This is a book of problems and answers, a DIY course in analysis]. Chapters 6–9, 12.

3. Walter Rudin, *Principles of Mathematical Analysis* (McGraw-Hill, Third Edition, 1976). Chapters 4,5,7.

4. J. M. Howie, *Real Analysis,* Springer Undergraduate Texts in Mathematics Series (Springer, 2001), ISBN 1-85233-314-6.

Alternative Reading

1. Mary Hart, *A Guide to Analysis* (MacMillan, Second Edition, 2001), Chapters 4,5.

2. J. C. Burkill, *A First Course in Mathematical Analysis* (Cambridge University Press, 1962), Chapters 3, 4, and 6.

3. K. G. Binmore, *Mathematical Analysis* A Straightforward Approach, (Cambridge University Press, Second Edition, 1982), Chapters 7–12, 14–16.

4. Victor Bryant, *Yet Another Introduction to Analysis* (Cambridge University Press, 1990), Chapters 3 and 4.

5. M. Spivak, *Calculus* (Publish or Perish, Fourth Edition, 2008), Part III.

6. Brian S. Thomson, Judith B. Bruckner, Andrew M. Bruckner, *Elementary Real Analysis* (classicalrealanalysis.com, Second Edition, 2008), Chapters 5–10 [Free Download].

4.3 Analysis III: Integration — Prof. Ben Green — 8 TT

Overview

In these lectures we define a simple integral and study its properties; prove the Mean Value Theorem for Integrals and the Fundamental Theorem of Calculus. This gives us the tools to justify term-by-term differentiation of power series and deduce the elementary properties of the trigonometric functions.

Learning Outcomes

At the end of the course students will be familiar with the construction of an integral from fundamental principles, including important theorems. They will know when it is possible to integrate or differentiate term-by-term and be able to apply this to, for example, trigonometric series.

Synopsis

Step functions, their integral, basic properties. Lower and upper integrals of bounded functions on bounded intervals. Definition of Riemann integrable functions.

The application of uniform continuity to show that continuous functions are Riemann integrable on closed bounded intervals; bounded continuous functions are Riemann integrable on bounded intervals.

Elementary properties of Riemann integrals: positivity, linearity, subdivision of the interval. The Mean Value Theorem for Integrals. The Fundamental Theorem of Calculus; linearity of the integral, integration by parts and by substitution.

The interchange of integral and limit for a uniform limit of continuous functions on a bounded interval. Term-by-term integration and differentiation of a (real) power series (interchanging limit and derivative for a series of functions where the derivatives converge uniformly).

Reading

Lecture notes will be provided

1. H. A. Priestley, *Introduction to Integration* (Oxford Science Publications, 1997)

2. W. Rudin, *Principles of Mathematical Analysis*, (McGraw-Hill, Third Edition, 1976).

Both of these books contain more material than is in the course.

5 Mathematics III

5.1 Introductory Calculus — Dr Cath Wilkins — 16 MT

Overview

These lectures are designed to give students a gentle introduction to applied mathematics in their first term at Oxford, allowing time for both students and tutors to work on developing and polishing the skills necessary for the course. It will have an 'A-level' feel to it, helping in the transition from school to university. The emphasis will be on developing skills and familiarity with ideas using straightforward examples.

Learning Outcomes

At the end of the course, students will be able to solve a range of ordinary differential equations (ODEs). They will also be able to evaluate partial derivatives and use them in a variety of applications.

Synopsis

General linear homogeneous ODEs: integrating factor for first order linear ODEs, second solution when one solution is known for second order linear ODEs. First and second order linear ODEs with constant coefficients. General solution of linear inhomogeneous ODE as particular solution plus solution of homogeneous equation. Simple examples of finding particular integrals by guesswork. [4]

Introduction to partial derivatives. Second order derivatives and statement of condition for equality of mixed partial derivatives. Chain rule, change of variable, including planar polar coordinates. Solving some simple partial differential equations (e.g. $f_{xy} = 0$, $f_x = f_y$). [3.5]

Parametric representation of curves, tangents. Arc length. Line integrals. [1]

Jacobians with examples including plane polar coordinates. Some simple double integrals calculating area and also $\int_{\mathbb{R}^2} e^{-(x^2+y^2)} dA$. [2]

Simple examples of surfaces, especially as level sets. Gradient vector; normal to surface; directional derivative; $\int_A^B \nabla\phi \cdot d\mathbf{r} = \phi(B) - \phi(A)$.[2]

Taylor's Theorem for a function of two variables (statement only). Critical points and classification using directional derivatives and Taylor's theorem. Informal (geometrical) treatment of Lagrange multipliers.[3.5]

Reading

M. L. Boas, *Mathematical Methods in the Physical Sciences* (Wiley, 3rd Edition, 2005).

D. W. Jordan & P. Smith, *Mathematical Techniques* (Oxford University Press, 3rd Edition, 2003).

E. Kreyszig, *Advanced Engineering Mathematics* (Wiley, 10th Edition, 2011).

K. A. Stroud, *Advanced Engineering Mathematics* (Palgrave Macmillan, 5th Edition, 2011).

5.2 Probability — Prof. James Martin — 16 MT

Overview

An understanding of random phenomena is becoming increasingly important in today's world within social and political sciences, finance, life sciences and many other fields. The aim of this introduction to probability is to develop the concept of chance in a mathematical framework. Random variables are introduced, with examples involving most of the common distributions.

Learning Outcomes

Students should have a knowledge and understanding of basic probability concepts, including conditional probability. They should know what is meant by a random variable, and have met the common distributions. They should understand the concepts of expectation and variance of a random variable. A key concept is that of independence which will be introduced for events and random variables.

Synopsis

Motivation, relative frequency, chance. Sample space, algebra of events, probability measure. Permutations and combinations, sampling with or without replacement. Conditional probability, partitions of the sample space, theorem of total probability, Bayes' Theorem. Independence.

Discrete random variables, probability mass functions, examples: Bernoulli, binomial, Poisson, geometric. Expectation: mean and variance. Joint distributions of several discrete random variables. Marginal and conditional distributions. Independence. Conditional expectation, theorem of total probability for expectations. Expectations of functions of more than one discrete random variable, covariance, variance of a sum of dependent discrete random variables.

Solution of first and second order linear difference equations. Random walks (finite state space only).

Probability generating functions, use in calculating expectations. Random sample, sums of independent random variables, random sums. Markov's inequality, Chebyshev's inequality, Weak Law of Large Numbers.

Continuous random variables, cumulative distribution functions, probability density functions, examples: uniform, exponential, gamma, normal. Expectation: mean and variance. Functions of a single continuous random variable. Joint probability density functions of several continuous random variables (rectangular regions only). Marginal distributions. Independence. Expectations of functions of jointly continuous random variables, covariance, variance of a sum of dependent jointly continuous random variables.

Reading

1. G. R. Grimmett and D. J. A. Welsh, *Probability: An Introduction* (Oxford University Press, 1986), Chapters 1–4, 5.1–5.4, 5.6, 6.1, 6.2, 6.3 (parts of), 7.1–7.3, 10.4.
2. J. Pitman, *Probability* (Springer-Verlag, 1993).
3. S. Ross, *A First Course In Probability* (Prentice-Hall, 1994).
4. D. Stirzaker, *Elementary Probability* (Cambridge University Press, 1994), Chapters 1–4, 5.1–5.6, 6.1–6.3, 7.1, 7.2, 7.4, 8.1, 8.3, 8.5 (excluding the joint generating function).

5.3 Statistics and Data Analysis — Dr Neil Laws and Prof. Jonathan Marchini — 16 TT

Overview

The course introduces the concept of likelihood for a probabilistic model and its use in estimating unknown model parameters. Models covered will include linear regression with one or two regressors. In many examples confidence intervals may be found by using the Central Limit Theorem (statement only). Model checking and outlier detection are core concepts that are broadly relevant across many aspects of mathematical modelling and will be explored here in the context of regression with one or two regressors. Regression models are an example of *supervised learning*, however a large part of statistics and data analysis can be classified as *unsupervised learning*, i.e. finding structure in data sets, e.g. data from financial markets, medical imaging, retail, population genetics, social networks. Techniques for finding structure in datasets are relevant to many parts of applied maths, specifically this course will cover principal components analysis and clustering techniques.

Learning Outcomes

Students should have an understanding of likelihood, the use of maximum likelihood to find estimators, and some properties of the resulting estimators. They should have an understanding of confidence intervals and their construction using the Central Limit Theorem. They should have an understanding of linear regression with one or two regressors, and of finding structure in data sets using principal components and some clustering techniques.

Synopsis

Random samples, concept of a statistic and its distribution, sample mean as a measure of location and sample variance as a measure of spread.

Concept of likelihood; examples of likelihood for simple distributions. Estimation for a single unknown parameter by maximising likelihood. Examples drawn from: Bernoulli, binomial, geometric, Poisson, exponential (parametrized by mean), normal (mean only, variance known). Data to include simple surveys, opinion polls, archaeological studies, etc. Properties of estimators—unbiasedness, Mean Squared Error = (bias^2 + variance).

Statement of Central Limit Theorem (excluding proof). Confidence intervals using CLT. Simple straight line fit, $Y_t = a + bx_t + \varepsilon_t$, with ε_t normal independent errors of zero mean and common known variance. Estimators for a, b by maximising likelihood using partial differentiation, unbiasedness and calculation of variance as linear sums of Y_t. (No confidence intervals). Examples (use scatter plots to show suitability of linear regression).

Linear regression with 2 regressors. Special case of quadratic regression $Y_t = a+bx_t+cx_t^2+\epsilon_t$. Model diagnostics and outlier detection. Residual plots. Heteroscedasticity. Outliers and studentized residuals. High-leverage points and leverage statistics. [2.5] Introduction to unsupervised learning with real world examples. Principal components analysis (PCA). Proof that PCs maximize directions of maximum variance and are orthogonal using Lagrange multipliers. PCA as eigendecomposition of covariance matrix. Eigenvalues as variances. Choosing number of PCs. The multivariate normal distribution pdf. Examples of PCA on multivariate normal data and clustered data. [3] Clustering techniques; K-means clustering. Minimization of within-cluster variance. K-means algorithm and proof that it will decrease objective function. Local versus global optima and use of random initializations. Hierarchical clustering techniques. Agglomerative clustering using complete, average and single linkage [2.5]

Reading

1. F. Daly, D. J. Hand, M. C. Jones, A. D. Lunn, K. J. McConway, *Elements of Statistics* (Addison Wesley, 1995). Chapters 1–5 give background including plots and summary statistics, Chapter 6 and parts of Chapter 7 are directly relevant.

2. G. James, D. Witten, T. Hastie, R. Tibshirani *An Introduction to Statistical Learning* (with Applications in R) (Springer 2013) - Chapter 3 on linear regression problems, Chapter 10 on unsupervised learning. This book is freely available online.

Further Reading

1. J. A. Rice, *Mathematical Statistics and Data Analysis* (Wadsworth and Brooks Cole, 1988).

2. S. Rogers and M. Girolami, *A First Course in Machine Learning* (CRC Press 2011)

6 Mathematics IV

6.1 Geometry — Dr Richard Earl — 15 MT

Overview

The course is an introduction to some elementary ideas in the geometry of euclidean space through vectors. One focus of the course is the use of co-ordinates and an appreciation of the invariance of geometry under an orthogonal change of variable. This leads into a deeper study of orthogonal matrices, of rotating frames, and into related co-ordinate systems.

Learning Outcomes

Students will learn how to encode a geometric scenario into vector equations and meet the vector algebra needed to manipulate such equations. Students will meet the benefits of choosing sensible co-ordinate systems and appreciate what geometry is invariant of such choices.

Synopsis

Euclidean geometry in two and three dimensions approached by vectors and coordinates. Vector addition and scalar multiplication. The scalar product, equations of planes, lines and circles. [3]

The vector product in three dimensions. Use of $\mathbf{a}, \mathbf{b}, \mathbf{a} \wedge \mathbf{b}$ as a basis. $\mathbf{r} \wedge \mathbf{a} = \mathbf{b}$ represents a line. Scalar triple products and vector triple products, vector algebra. [2]

Conics (normal form only), focus and directrix. Showing the locus $Ax^2 + Bxy + Cy^2 = 1$ can be put in normal form via a rotation matrix. Orthogonal matrices. 2×2 orthogonal matrices and the maps they represent. Orthonormal bases in $\mathbb{R}^3$. Orthogonal change of variable; $A\mathbf{u} \cdot A\mathbf{v} = \mathbf{u} \cdot \mathbf{v}$ and $A(\mathbf{u} \wedge \mathbf{v}) = \pm A\mathbf{u} \wedge A\mathbf{v}$. Statement that a real symmetric matrix can be orthogonally diagonalized. Simple examples identifying conics not in normal form. [3]

3×3 orthogonal matrices; $SO(3)$ and rotations; conditions for being a reflection. Isometries of $\mathbb{R}^3$. [2]

Rotating frames in 2 and 3 dimensions. Angular velocity. $\mathbf{v} = \omega \wedge \mathbf{r}$. [1.5]

Parametrised surfaces, including spheres, cones. Examples of coordinate systems including parabolic, spherical and cylindrical polars. Calculating normal as $\mathbf{r}_u \wedge \mathbf{r}_v$. Surface area. [3.5]

Reading

1. J. Roe, *Elementary Geometry* (Oxford Science Publications, 1992), Chapters 1, 2.2, 3.4, 4, 5.3, 7.1–7.3, 8.1–8.3, 12.1.

2. M. Lunn, *A First Course In Mechanics* (Oxford Science Publications, 1991) Chapter 4

6.2 Dynamics — Prof. James Sparks — 16 HT

Overview

The subject of dynamics is about how things change with time. A major theme is the modelling of a physical system by differential equations, and one of the highlights involves using the law of gravitation to account for the motion of planets.

Learning Outcomes

Students will be familiar with the laws of motion, including circular and planetary motion. They will know how forces are used and be introduced to stability in a physical system.

Synopsis

Newton's laws, inertial frames, Galilean relativity. Dimensional analysis [2.5]

Forces, examples including gravity, electromagnetism, fluid drag. Conservative forces and the Newtonian gravitational potential. Energy and momentum. [3]

Equilibria and the harmonic oscillator. Stability and instability via linearized equations, normal modes. Simple examples of equilibria in two variables via matrices. [2.5]

Central forces, angular momentum and torque. Planar motion in polar coordinates, the effective potential, Kepler's laws and planetary motion. [3]

Many particle systems, centre of mass motion. Rigid bodies, rotating frames and Coriolis force, inertia tensor, rigid body motion. [5]

Reading

1. David Acheson, *From Calculus to Chaos: an Introduction to Dynamics* (Oxford University Press, 1997), Chapters 1, 5, 6, 10.

Further Reading

1. M. Lunn, *A First Course in Mechanics* (Oxford University Press, 1991), Chapters 1–3, 5, 6, 7 (up to 7.3).
2. M. W. McCall, *Classical Mechanics: A Modern Introduction* (Wiley, 2001), Chapters 1–4, 7.

6.3 Constructive Mathematics — Prof. Andy Wathen — 8 TT

Overview

This course is an introduction to mathematical algorithms; that is procedures which one can carry out to achieve a desired result. Such procedures arise throughout mathematics both Pure and Applied.

Learning Outcomes

Students should appreciate the concept of an algorithm and be able to construct simple algorithms for the solution of certain elementary problems. Verification that certain procedures should work under appropriate conditions will give students good examples of the application of real analysis and implementation will require them to be able to make and run simple procedures in Matlab.

Synopsis

The Division Algorithm on Integers, Euclid's Algorithm including proof of termination with highest common factor. The solution of simple linear Diophantine equations. Examples.

Division and Euclid's algorithm for real polynomials. Examples.

Root finding for real polynomials. Fixed point iterations, examples. Convergence. Existence of fixed points and convergence of fixed point iterations by the contraction mapping theorem (using the mean value theorem).

Newton iteration. Quadratic convergence. Horner's Rule.

Reading

1. E. Süli and D. Mayers *An Introduction to Numerical Analysis*, CUP 2003 - Chapter 1

7 Mathematics V

7.1 Multivariable Calculus — Prof. Helen Byrne — 16 HT

Overview

In these lectures, students will be introduced to multi-dimensional vector calculus. They will be shown how to evaluate volume, surface and line integrals in three dimensions and how they are related via the Divergence Theorem and Stokes' Theorem - these are in essence higher dimensional versions of the Fundamental Theorem of Calculus.

Learning Outcomes

Students will be able to perform calculations involving div, grad and curl, including appreciating their meanings physically and proving important identities. They will further have a geometric appreciation of three-dimensional space sufficient to calculate standard and non-standard line, surface and volume integrals. In later integral theorems they will see deep relationships involving the differential operators.

Synopsis

Multiple integrals: Two dimensions. Informal definition and evaluation by repeated integration; example over a rectangle; properties. General domains. Change of variables. Examples. [2.5]

Volume integrals: Jacobians for cylindrical and spherical polars, examples. [1.5]

Recap on surface integrals. Flux integrals. [1.5]

Scalar and vector fields. Vector differential operators: divergence and curl; physical interpretation. Calculation. Identities. [2.5]

Divergence theorem. Example. Consequences: Greens 1st and second theorems. $\int_V \nabla\phi dV = \int_{\delta V} \phi dS$. Uniqueness of solutions of Poisson's equation. Derivation of heat equation. Divergence theorem in plane. Informal proof for plane. [4]

Stokes's theorem. Examples. Consequences. The existence of potential for a conservative force. [2]

Gauss' Flux Theorem. Examples. Equivalence with Poisson's equation. [2]

Reading

1. D. W. Jordan & P. Smith, *Mathematical Techniques* (Oxford University Press, 3rd Edition, 2003).

2. Erwin Kreyszig, *Advanced Engineering Mathematics* (Wiley, 8th Edition, 1999).

3. D. E. Bourne & P. C. Kendall, *Vector Analysis and Cartesian Tensors* (Stanley Thornes, 1992).

4. David Acheson, *From Calculus to Chaos: An Introduction to Dynamics* (Oxford University Press, 1997).

7.2 Fourier Series and PDEs — Prof. Jim Oliver — 16 HT

Overview

The course begins by introducing students to Fourier series, concentrating on their practical application rather than proofs of convergence. Students will then be shown how the heat equation, the wave equation and Laplace's equation arise in physical models. They will learn basic techniques for solving each of these equations in several independent variables, and will be introduced to elementary uniqueness theorems.

Learning Outcomes

Students will be familiar with Fourier series and their applications and be notionally aware of their convergence. Students will know how to derive the heat, wave and Laplace's equations in several independent variables and to solve them. They will begin the study of uniqueness of solution of these important PDEs.

Synopsis

Fourier series: Periodic, odd and even functions. Calculation of sine and cosine series. Simple applications concentrating on imparting familiarity with the calculation of Fourier coefficients and the use of Fourier series. The issue of convergence is discussed informally with examples. The link between convergence and smoothness is mentioned, together with its consequences for approximation purposes.

Partial differential equations: Introduction in descriptive mode on partial differential equations and how they arise. Derivation of (i) the wave equation of a string, (ii) the heat equation in one dimension (box argument only). Examples of solutions and their interpretation. D'Alembert's solution of the wave equation and applications. Characteristic diagrams (excluding reflection and transmission). Uniqueness of solutions of wave and heat equations.

PDEs with Boundary conditions. Solution by separation of variables. Use of Fourier series to solve the wave equation, Laplace's equation and the heat equation (all with two independent variables). (Laplace's equation in Cartesian and in plane polar coordinates). Applications.

Reading

1. D. W. Jordan and P. Smith, *Mathematical Techniques* (Oxford University Press, 4th Edition, 2003)

2. E. Kreyszig, *Advanced Engineering Mathematics* (Wiley, 10th Edition, 1999)

3. G. F. Carrier and C. E. Pearson, *Partial Differential Equations — Theory and Technique* (Academic Press, 1988)

8 Computational Mathematics

8.1 Computational Mathematics — Dr. Andrew Thompson — MT and HT

Michaelmas Term

Lectures: Week 2.

Demonstrating sessions: Weeks 3–8 of Michaelmas Term and weeks 1–2 of Hilary Term. Each student will have 4 two-hour sessions.

Hilary Term

Lectures: Weeks 1 and 2.

Demonstrating sessions: These continue into the beginning of Hilary Term in weeks 1 and 2.

Project drop–in sessions: Weeks 3–8 of Hilary Term. Extra sessions run during weeks 5 and 8 of Hilary Term ahead of the project submission deadlines.

Overview

Many mathematicians use general-purpose mathematical software which includes tools for symbolic and numerical computation and other features such as plotting, visualization and data analysis. Such software is used for solving linear and nonlinear equations, graphing the results, as a tool for exploring a mathematical concept, as a handbook of mathematical functions and integration rules, and as a technique for verifying the correctness of calculations done "by hand". In this introductory course, students will explore these ideas using the popular MATLAB software.

Learning Outcomes

Students will be introduced to MATLAB as a problem-solving environment and the use of computer programming to solve mathematical problems. Students will develop confidence and expertise in a tool which can be used in the later years of the mathematics course.

Synopsis

Using the MATLAB command line and working with ".m" files;

Plotting in two and three dimensions;

Works with lists, arrays, matrices and linear algebra;

Symbolic computing;

Logic, flow control and programming;

Solving problems in algebra, calculus, and applied mathematics.

Matlab

Students may access the system through college or individual computers; for the former they should consult the computing support at their own college. MATLAB may be installed and used on personally-owned computers under the University's site license. Information on downloading MATLAB can be found at http://www.maths.ox.ac.uk/members/it/software-personal-machines/matlab. To access this page students will need to login using their University account.

Teaching and Assessment

The course relies heavily on self-teaching through practical exercises. A manual for the course and examples to be worked through will be provided.

You will be timetabled for demonstrating sessions in the Mathematical Institute where you will have access to help and advice from demonstrators. You will need to bring a laptop to these sessions. To save time, please follow the instructions above to install MATLAB on your machine prior to the first session. If you are unable to bring a laptop to the sessions, please contact Nia Roderick, Academic Assistant (roderick@maths.ox.ac.uk) well ahead of your first session.

Your work in this course will be taken into account as part of the Preliminary Examinations. For further information, see the section on examinations in the Undergraduate Handbook.

Reading

1. *Computational Mathematics: Michaelmas Term Students' Guide*, available from reception and the course website.