Examiners' Report: Final Honour School of Mathematics Part B Trinity Term 2025

October 23, 2025

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

  See Table 1.

	Numbers					Percentages %				
	2025	(2024)	(2023)	(2022)	(2021)	2025	(2024)	(2023)	(2022)	(2021)
I	52	(51)	(54)	(55)	(51)	41.27	(38.93)	(36.24)	(41.04)	(39.84)
II.1	50	(50)	(72)	(53)	(58)	39.68	(38.17)	(48.32)	(39.55)	(45.31)
II.2	18	(24)	(18)	(24)	(18)	14.29	(18.32)	(12.08)	(17.91)	(14.06)
III	-	(-)	(-)	(-)	(-)	-	(-)	(-)	(-)	(-)
P	-	(-)	(-)	(-)	(-)	-	(-)	(-)	(-)	(-)
F	-	(-)	(-)	(-)	(-)	-	(-)	(-)	(-)	(-)
Total	126	(130)	(149)	(134)	(157)	100	(100)	(100)	(100)	(100)

Table 1: Numbers and percentages in each class

- **Numbers of vivas and effects of vivas on classes of result.**

  As in previous years there were no vivas conducted for the FHS of Mathematics Part B.

- **Marking of scripts.**

  BOE Extended Essays, BSP Mathematical Modelling and Numerical Computation Structured Projects and coursework submitted for the History of Mathematics course were double marked.

  The remaining scripts were all single marked according to a pre-agreed marking scheme which was strictly adhered to. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**

  See Table 5.

B. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

C. Notice of examination conventions for candidates

The Notice to Candidates Offering Coursework was issued on 25 February 2025. The first Notice to Candidates was issued on 25 February 2025 and the second notice on 19 May 2025.

All notices and the examination conventions for 2025 are online at Examination conventions.

Part II

A. General Comments on the Examination

The examiners would like to record their thanks to all those who helped in the preparation, administering, and assessing of this year's examinations. The chair would like to thank Charlotte Turner-Smith, Waldemar Schlackow, Matt Brechin, Clare Donnelly and the rest of the academic administration team for their support of the Part B examinations.

In addition the internal examiners would like to express their gratitude to Professor Matt Tointon and Dr Ed Brambley for carrying out their duties as external examiners in such a constructive and supportive way during the year and for their thoughtful contributions during the final examiners' meetings.

For further comments on the examination process, see Section D below.

Standard of performance

The standard of performance was broadly in line with recent years. In setting the USMs, we took note of

- the Examiners' Report on the 2024 Part B examination, and in particular recommendations made by last year's examiners, and the Examiners' Report on the 2024 Part

A examination, in which the 2025 Part B cohort were awarded their USMs for Part A;

- the guidelines provided by the Mathematics Teaching Committee, including its recommendations on the proportion of candidates that might be expected in each class.

Setting and checking of papers and marks processing

The internal examiners initially divided between them responsibility for the units of assessment (that is, the exam papers and projects).

Following established practice, the questions for each paper were initially set by the course lecturer, with the lecturer of a related course involved as checker before the first draft of the questions was presented to the examiners. The course lecturers also acted as assessors, marking the questions on their course(s).

Requests to course lecturers to act as assessors, and to act as checker of the questions of fellow lecturers, were sent out early in Michaelmas Term, with instructions and guidance on the setting and checking process, including a web link to the Examination Conventions.

The internal examiners met at the beginning of Hilary Term to consider those draft papers on Michaelmas Term courses, and changes and corrections were agreed with the lecturers where necessary. Where necessary, corrections and any proposed changes were agreed with the setters. The revised draft papers were then sent to the external examiners. Feedback from external examiners was given to examiners and to the relevant assessor for response. The internal examiners at their meeting in mid Hilary Term considered the external examiners' comments and the assessor responses, making further changes as necessary before finalising the questions. The process was repeated for the Hilary Term courses, but necessarily with a much tighter schedule. Before questions were submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions.

Exams were held in-person in the Exams Schools. Papers were collected by the Academic Administration team and made available to assessors approximately half a day following the examination. Assessors were made aware of the marking deadlines ahead of time and all scripts and completed mark sheets were returned, if not by the agreed due dates, then at least in time for the script-checking process.

A team of graduate checkers, under the supervision of Clare Donnelly, Charlotte Turner-Smith, and Rosalind Mitchell sorted all the marked scripts for each paper of this examination, cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or incorrectly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way a number of errors were corrected,and each change was signed by one of the examiners who were present throughout the process.

Throughout the examination process, candidates were treated anonymously, identified only by a randomly-assigned candidate number.

Timetable

Examinations began on Monday 2 June and ended on Friday 20 June.

Consultation with assessors on written papers

Assessors were asked to submit suggested ranges for which raw marks should map to USMs of 60 and 70 along with their mark-sheets, and all did so. In most cases these were in line with the assignments given by the assessors.

Determination of University Standardised Marks

The Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average in each class over the last four years, together with recent historic data for Part B.

We followed the Department's established practice in determining the University standardised marks (USMs) reported to candidates. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration.

For details on the scaling process please see the Mathematical Institute's website where it is outlined in full: https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examiners-reports/scaling-algorithm.

This year a preliminary meeting of the internal examiners was held in advance of the final exam board meeting to compare the default settings produced by the algorithm alongside the reports from assessors. It was agreed that only a selection of scaling maps would be further reviewed at the final exam board, and that external examiners would be given an opportunity to review all maps prior to the meeting. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low. Where the examiners were in doubt as to the most appropriate scaling, the preliminary scalings were held over to the final exam board meeting, where the factors considered by those in the preliminary meeting were reviewed and weighed before a final decision was made.

Table 2 on page gives the final positions of the corners of the piecewise linear maps used to determine USMs.

In accordance with the agreement between the Mathematics Department and the Computer Science Department, the final USM maps were passed to the examiners in Mathematics & Computer Science. USM marks for Mathematics papers of candidates in Mathematics & Philosophy were calculated using the same final maps and passed to the examiners for that School.

Comments on use of Part A marks to set scaling boundaries

None.

Mitigating Circumstance Notice to Examiners

A subset of the examiners (the 'Mitigating Circumstances Panel') attended a pre-board meeting to band the seriousness of the individual notices to examiners. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate. See Section E for further details.

The full board of examiners considered all of the notices in the final meeting, along with a number of MCEs carried over from Part A. The examiners considered each application alongside the candidate's marks and the recommendations proposed by the Part A 2024 Exam board.

Table 2: Position of corners of the piecewise linear maps

Paper	P_1	P_2	P_3	P_4	Additional Corners	Difficulty Score
B1.1	6.8;37	12.46;59.5	26.38;68.5	38.19;79	50;100	5.54
B1.2	10.06;37	18.44;59.5	32.36;68.5	41.18;79	50;100	-0.44
B2.1	8.65;37	15.86;59.5	29.78;68.5	39.89;79	50;100	2.14
B2.2	9.12;37	16.72;59.5	30.64;68.5	40.32;79	50;100	1.28
B2.3	9.16;37	16.79;59.5	30.71;68.5	40.36;79	50;100	1.21
B3.1	11.85;37	21.73;59.5	35.65;68.5	42.83;79	50;100	-3.73
B3.2	8.68;37	15.91;59.5	29.83;68.5	39.92;79	50;100	2.09
B3.3	13.73;37	25.18;59.5	39.1;68.5	44.55;79	50;100	-7.18
B3.4	9.47;37	17.36;59.5	31.28;68.5	40.64;79	50;100	0.64
B3.5	12.2;37	22.36;59.5	36.28;68.5	43.14;79	50;100	-4.36
B4.1	9.09;37	16.67;59.5	30.59;68.5	40.3;79	50;100	1.33
B4.2	8.05;37	14.75;59.5	28.67;68.5	39.34;79	50;100	3.25
B4.3	10.57;37	19.37;59.5	33.29;68.5	41.65;79	50;100	-1.37
B5.1	9.65;37	17.7;59.5	31.62;68.5	40.81;79	50;100	0.3
B5.2	10.54;37	19.33;59.5	33.25;68.5	41.63;79	50;100	-1.33
B5.3	9.59;37	17.59;59.5	31.51;68.5	40.76;79	50;100	0.41
B5.4	9.63;37	17.65;59.5	31.57;68.5	40.79;79	50;100	0.35
B5.5	9.39;37	17.22;59.5	31.14;68.5	40.57;79	50;100	0.78
B5.6	10.69;37	19.6;59.5	33.52;68.5	41.76;79	50;100	-1.6
B6.1	12.75;37	23.38;59.5	37.3;68.5	43.65;79	50;100	-5.38
B6.2	11.65;37	21.35;59.5	35.27;68.5	42.64;79	50;100	-3.35
B6.3	12.18;37	22.33;59.5	36.25;68.5	43.13;79	50;100	-4.33
B7.1	9.4;37	17.24;59.5	31.16;68.5	40.58;79	50;100	0.76
B7.2	9.13;37	16.74;59.5	30.66;68.5	40.33;79	50;100	1.26
B7.3	8.61;37	15.79;59.5	29.71;68.5	39.86;79	50;100	2.21
B8.1	8.82;37	16.17;59.5	30.09;68.5	40.05;79	50;100	1.83
B8.2	10.24;37	18.78;59.5	32.7;68.5	41.35;79	50;100	-0.78
B8.3	12.32;37	22.58;59.5	36.5;68.5	43.25;79	50;100	-4.58
B8.4	13.37;37	24.51;59.5	38.43;68.5	44.22;79	50;100	-6.51
B8.5	10.57;37	19.37;59.5	33.29;68.5	41.65;79	50;100	-1.37
B8.6	14.35;37	26.3;59.5	40.22;68.5	45.11;79	50;100	-8.3
BSP	2000;100					0
SB1	7.24;37	32;59.5	43;68.5	66;100		10.48
SB1	34;100					10.48
SB2.1	14.57;37	26.71;59.5	40.63;68.5	45.32;79	50;100	-8.71
SB2.2	9.82;37	18.01;59.5	31.93;68.5	40.97;79	50;100	-0.01
SB3.1	7.86;37	14.41;59.5	28.33;68.5	39.17;79	50;100	3.59

B. Equality and Diversity issues and breakdown of the results by gender

Table 3: Breakdown of results by gender

Class	Number								
	2025			2024			2023		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	12	40	52	7	44	51	5	49	54
II.1	12	38	50	14	36	50	23	49	72
II.2	10	8	18	9	15	24	7	11	18
III	-	-	-	-	-	-	-	-	-
P	-	-	-	-	-	-	-	-	-
F	-	-	-	-	-	-	-	-	-
Total	34	97	131	37	112	149	40	93	134
Class	Percentage								
	2025			2024			2023		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	34.29	43.96	41.27	20.59	45.36	38.93	13.51	43.75	36.24
II.1	34.29	41.76	39.68	41.18	37.11	38.16	62.16	43.75	48.32
II.2	28.57	8.79	14.29	26.47	15.46	18.32	18.92	9.82	12.08
III	-	-	-	-	-	-	-	-	-
P	-	-	-	-	-	-	-	-	-
F	-	-	-	-	-	-	-	-	-
Total	100	100	100	100	100	100	100	100	100

Table 4: Rank and percentage of candidates with this or greater overall USMs

Av USM	Rank	Candidates with this USM and above	%
91	1	1	0.79
89	4	4	3.17
86	6	6	4.76
85	7	7	5.56
84	8	8	6.35
81	10	10	7.94
80	12	12	9.52
79	13	13	10.32
78	16	16	12.7
77	18	18	14.29
76	21	21	16.67
75	22	22	17.46
74	26	26	20.63
73	31	31	24.6
72	37	37	29.37
71	44	44	34.92
70	47	47	37.3
69	54	55	43.65
68	58	58	46.03
67	65	65	51.59
66	75	75	59.52
65	79	79	62.7
64	83	83	65.87
63	87	87	69.05
62	89	89	70.63
61	96	96	76.19
60	102	102	80.95
59	105	105	83.33
58	107	107	84.92
57	110	110	87.3
56	113	113	89.68
55	114	114	90.48
54	117	117	92.86
53	118	118	93.65
52	119	119	94.44
51	120	120	95.24
50	121	121	96.03
48	122	122	96.83
45	124	124	98.41
41	125	125	99.21
35	126	126	100

C. Detailed numbers on candidates' performance in each part of the examination

The number of candidates taking each paper is shown in Table 5.

Table 5: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
B1.1	47	20.6	9.48	62.96	11.3
B1.2	41	26.78	11.31	63.83	15.49
B2.1	29	27.9	13.38	66.59	19.33
B2.2	18	29.28	13.24	67.78	16.41
B2.3	11	31.55	12.86	70.27	20.14
B3.1	29	34.21	10.66	71.31	13.49
B3.2	20	28.75	9.65	68.05	12.26
B3.3	17	38.59	9.49	71.41	14.17
B3.4	21	30.38	9.45	67.86	12.65
B3.5	28	34.89	8.86	69.89	12.46
B4.1	43	29.67	9.78	69.49	13.05
B4.2	35	28.8	9.06	70.03	10.08
B4.3	15	35.13	7.25	71.93	7.71
B5.1	23	27.82	8.92	63.35	17.81
B5.2	50	30.24	9.86	67.44	11.3
B5.3	33	28.15	6.94	66.82	6.63
B5.4	31	28.45	8.72	66.39	10.77
B5.5	30	26.87	7.44	65.9	7.58
B5.6	21	28.9	6.2	65.67	5.44
B6.1	12	36.08	8.74	70.75	12.05
B6.2	16	27.56	6.61	63.19	6.89
B6.3	14	29.71	5.58	63.79	5.31
B7.1	25	29.88	8.34	69.2	8.49
B7.2	25	27.72	6.93	67.16	7.08
B7.3	12	26.5	12.12	65.33	16.21
B8.1	51	27.55	11.97	66.84	16.16
B8.2	29	32.34	10.99	70.31	14.91
B8.3	42	30.07	12.05	62.79	18.23
B8.4	34	32.88	10.36	64.79	15.99
B8.5	36	28.44	10.46	65.22	13.76
B8.6	4	34.5	9.54	65	11.46
BSP	21	1442.71	280.43	72.29	14.17
SB1	8	26.38	7.96	61.25	2.66
SB2.1	14	37.86	10	69.29	14.58
SB2.2	19	22.68	8.41	59	12.41
SB3.1	43	22.7	9.63	62.42	12.1

Individual question statistics for Mathematics candidates are shown below for those papers offered by no fewer than six candidates.

Paper B1.1: Logic

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.1	11.52	5.32	40	2
Q2	9.39	10.77	6.28	26	5
Q3	7.32	8.11	4.73	28	6

Paper B1.2: Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.96	14.77	6.72	22	5
Q2	9.88	9.88	4.89	26	0
Q3	14.36	15.18	6.74	34	2

Paper B2.1: Introduction to Representation Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.93	13.84	8.62	25	4
Q2	8.55	13.78	8.63	18	11
Q3	7.72	14.33	8.66	15	14

Paper B2.2: Commutative Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.54	15.54	6.79	13	0
Q2	15.56	15.56	5.35	16	0
Q3	12.67	12.67	9.54	6	0

Paper B2.3: Lie Algebras

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.5	17.5	6.04	10	0
Q2	14.27	14.27	8.83	11	0
Q3	15	15		1	0

Paper B3.1: Galois Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.87	16.87	6.95	23	0
Q2	17.59	18.04	5.34	28	1
Q3	11.33	14.14	6.96	7	2

Paper B3.2: Geometry of Surfaces

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.89	12.71	5.43	17	2
Q2	16.27	16.27	6.08	11	0
Q3	14.15	15	6.15	12	1

Paper B3.3: Algebraic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.79	17.79	6.12	14	0
Q2	20.57	20.57	5.09	14	0
Q3	19.83	19.83	2.32	6	0

Paper B3.4: Algebraic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.12	11.86	4.16	7	1
Q2	16	16.76	6.62	17	1
Q3	14.32	15	4.91	18	1

Paper B3.5: Topology and Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.55	18.55	4.57	22	0
Q2	16.73	17.19	5.41	21	1
Q3	15.12	16	4.15	13	3

Paper B4.1: Functional Analysis I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.79	14.03	5.32	38	1
Q2	11.24	13.31	6.13	13	4
Q3	16.29	16.29	5.26	35	0

Paper B4.2: Functional Analysis II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.11	14.11	4.77	27	0
Q2	14.69	15.06	5.26	31	1
Q3	11.73	13.33	5.82	12	3

Paper B4.3: Distribution Theory and Fourier Analysis: An Introduction

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.7	16.7	3.2	10	0
Q2	17.44	17.44	5.53	9	0
Q3	18.45	18.45	3.59	11	0

Paper B5.1: Stochastic Modelling and Biological Processes

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.55	15.55	5.35	20	0
Q2	13	14.42	6.04	12	2
Q3	9.57	10.67	4.47	12	2

Paper B5.2: Applied PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.26	13.5	3.99	32	2
Q2	16.77	16.96	7.05	47	1
Q3	10.52	13.48	5.57	21	12

Paper B5.3: Viscous Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.76	16.76	3.44	29	0
Q2	13.15	13.15	4.77	26	0
Q3	9.18	9.18	3.34	11	0

Paper B5.4: Waves and Compressible Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.6	13.6	5.62	20	0
Q2	14.74	14.74	4.9	27	0
Q3	13.44	14.13	5.18	15	1

Paper B5.5: Further Mathematical Biology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.55	12.1	5.82	10	1
Q2	11.29	12.17	4.76	24	4
Q3	14.85	15.12	4.13	26	1

Paper B5.6: Nonlinear Systems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.79	13.31	4.77	13	1
Q2	14.63	15.94	4.78	16	3
Q3	12.73	13.77	4.13	13	2

Paper B6.1: Numerical Solution of Differential Equations I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.62	16.62	4.24	8	0
Q2	19.67	19.67	4.69	9	0
Q3	15.38	17.57	9.65	7	1

Paper B6.2: Numerical Solution of Differential Equations II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.55	14.3	3.56	10	1
Q2	13.23	13.23	4.36	13	0
Q3	14	14	3.94	9	0

Paper B6.3: Integer Programming

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.36	14.36	2.53	14	0
Q2	14.92	14.92	4.27	13	0
Q3	21	21		1	0

Paper B7.1: Classical Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.23	14.38	7.12	16	6
Q2	15.5	15.5	4.94	22	0
Q3	13.14	14.67	5.6	12	2

Paper B7.2: Electromagnetism

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.92	11.92	3.67	24	0
Q2	11.1	13.14	6.44	7	3
Q3	15.29	16.58	5.82	19	2

Paper B7.3: Further Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.38	16.14	8.48	7	1
Q2	12.67	13.88	6.56	8	1
Q3	10.44	10.44	6.46	9	0

Paper B8.1: Martingales through Measure Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.51	13.44	7.8	36	3
Q2	10.41	10.73	4.84	26	1
Q3	15.45	16.05	6.3	40	2

Paper B8.2: Continuous Martingales and Stochastic Calculus

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.63	16.08	6.03	26	1
Q2	16.14	16.14	9.03	7	0
Q3	16.28	16.28	5.37	25	0

Paper B8.3: Mathematical Models of Financial Derivatives

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.89	13.89	5.8	27	0
Q2	13.44	13.44	7.53	25	0
Q3	17.25	17.25	6.01	32	0

Paper B8.4: Communication Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.7	16.36	6.09	25	2
Q2	14.89	15.47	6.33	17	1
Q3	16	17.15	6.76	26	2

Paper B8.5: Graph Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.35	14.35	4.21	34	0
Q2	14.18	14.59	6.64	27	1
Q3	11.92	12.91	8.33	11	1

Paper B8.6: High Dimensional Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.5	21.5	1	4	0
Q2	17	17	6.56	3	0
Q3	1	1		1	0

Paper SB1.1/1.2: Applied Statistics/Computational Statistics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.75	10.75	3.81	8	0
Q2	8.25	8.25	3.81	8	0
Q3	10.67	10.67	5.82	6	0
Q4	8	8	0	2	0
PR	23.75	23.75	6.07	8	0

Paper SB2.1: Foundations of Statistical Inference

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.38	22.45	7.75	11	2
Q2	16.2	16.2	6.05	10	0
Q3	17.29	17.29	6.58	7	0

Paper SB2.2: Statistical Machine Learning

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.08	10.91	6.73	11	1
Q2	9.94	10.56	4.74	16	1
Q3	12.25	12.91	5.01	11	1

Paper SB3.1: Applied Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.45	11.68	5.09	38	2
Q2	10.83	10.83	6.12	42	0
Q3	10.33	12.83	5.48	6	3

Assessors' comments on sections and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. The examiners have not included assessors' statements suggesting where possible borderlines might lie; they did take note of this guidance when determining the USM maps. Some statistical data which can be found in Section C above has also been removed.

B1.1: Logic

Question 1: Subpart (b)(iv) was correctly answered by only a few. A common mistake was to assume that if a partial valuation is inconsistent with Σ, then a single formula from Σ must be responsible. The compactness exercise in part (c) was often done well, but a distressing number invoked the compactness theorem to prove the trivial and irrelevant fact that a satisfiable set has a finite satisfiable subset. Part (d) on an incorrect strengthening of van der Waerden attracted a variety of answers, but only a handful put their finger on the key point that n' and k' might be nonstandard. The rest of the question was done well on the whole.

Question 2: In (b)(ii), many seemed not to understand that the point of the question was to find a model of Σ, as in the proof of completeness, and instead just equipped some structure with constants. In (b)(iii), which treated a version of the Tarski test, very few realised that they had to replace the constants in an $\mathcal{L}'$-sentence with unused variables in order to obtain an $\mathcal{L}$-sentence, but most of those who saw this could do it correctly. Part (c) saw the expected steep drop-off in correct answers as the question progressed, with (iii) attracting creative and sometimes correct solutions.

Question 3: There was a minor mistake in the formulation of (a): the formulas are referred to as sentences, but x_1 appears free in each. This does not affect the content of the question, and seems not to have confused anyone. A surprising number considered the equivalence in (iii) to be valid. Part (b) was mostly done well, though some wasted time by writing out a formal proof in full, when they were only asked to prove existence of a proof. The formalisation exercise in (c)(i) was done surprisingly badly, with many changing the language and trying to work with A and B as predicates, or even using second-order quantification. Few really attempted (c)(iii), and only one succeeded. In (c)(iv), many implicitly assumed consistency of the theory. In part (d), some got marks for progress in (i), but there was only one full solution. Few got anywhere with (ii), and only three gave real solutions, of which two were fully correct.

B1.2: Set Theory

Question 1 Most made decent progress. Part (b) was too easy, and should probably have replaced with a "you may assume without proof" in the later part using it. In (d), few were careful enough to properly explain the role of FC1 (non-emptiness).

Question 2 The bookwork in (a) was done surprisingly badly by many, perhaps because it is material from the end of the course. In (b)(iii), few really explained the role of AC in finding a family of witnesses to countability. Part (c) stumped most; many had the correct idea of considering the order on an ordinal in (c)(i), but missed that in the uncountable case, that ordinal must be precisely $\aleph_1$. There were only two correct solutions to (c)(ii);

many tried to derive a contradiction by applying (b)(iii) to f directly, which leads nowhere.

Question 3 The logical structure of (a) seemed to confuse many. It required showing two things: that no ordinal is both a successor and a limit, and that every ordinal is either a successor or a limit – but many attempted only one, and some tried to prove as separate statements that no successor ordinal is a limit ordinal, and that no limit ordinal is a successor ordinal, apparently without realising that these are equivalent. Parts (b) and (c) were often done well, though solutions were often messy, particularly in (b)(ii), and many attempted a transfinite induction in (c)(i) which went nowhere.

B2.1 Introduction to Representation Theory

Question 1 the most popular question, offering an alternative proof of Maschke's Theorem for complex representations of finite groups. It was pleasing to see a majority of candidates being able to construct the G-invariant inner product as required in Q1(b), although this task did elude a significant number of students. Part (c) was not done very well, but there were one or two correct alternative proofs, applying the Spectral Theorem to the Gram matrix of the G-invariant inner product from part (b). Nearly everyone got part (d) out. Part (e) was mostly fine, with the following lovely alternative solution being offered by a candidate: if $\langle -,-\rangle$ is a $\langle g\rangle$-invariant bilinear form and e_1, e_2 is the standard basis for $\mathbb{C}^2$, then $\langle e_1, e_2\rangle = \langle ge_1, ge_2\rangle = \langle e_1, e_1 + e_2\rangle$ implies $\langle e_1, e_1\rangle = 0$, so $\langle -,-\rangle$ cannot be positive definite.

Question 2 the least popular question, but still attempted by over half of the candidates. It was concerning to see that many people forgot how to do part (a) (which was on the problem sheets) —- a number of people tried to apply the Fixed Point Formula and use character theory, which is a very roundabout way to do the question. The bookwork part (b) was done very well. Part (c) proved to be challenging, and only one person realised that the one needs to look at subsets of G of the form HxH (the double cosets) to do part (d).

Question 3 a popular question. Part (a) was done very well; there were also several correct solutions for (b) using the Column Orthogonality Theorem. The character table in part (c) was correctly completed by about a third of the students who attempted Q3; it is difficult (but possible) to do this without using Sylow's Theorems. Only one or two students, however, managed to correctly explain why the values of the non-linear irreducible characters on the elements of order 11 cannot be real.

B2.2: Commutative Algebra

Question 1 was attempted by most students. There was a surprising number of mistakes in the statement of Zorn's lemma (the poset was not assumed to be non-empty, and totally ordered subsets were assumed to be countable chains), and many students failed to appreciate that one had to consider only proper ideals in part (a). In part (d), a very common mistake was the claim that all prime ideals of $\mathbb{Z}$ are of the form (p) where p is a prime, whereas of course (0) is also an example. Similarly, (0) was forgotten in K [x]. There was a variety of examples given in (d)(iii).

Question 2 was also very popular. Part (e) was difficult, with only a small number of correct solutions. Most students failed to notice how the Going- Up theorem is helpful here.

Question 3 was attempted by seven students. There was a strong bifurcation here: some solutions were very good, with hiccups only in part (e), whereas the other solutions were severely flawed already in the book-work part.

B2.3: Lie Algebras

Question 1 was the best-answered question, with the majority of candidates providing largely complete solutions. A common mistake in part (*b*) was to claim (sometimes implicitly) that the restriction of the adjoint representation of $\mathfrak{g}$ to $D(\mathfrak{g})$ was the adjoint representation of $D(\mathfrak{g})$, and hence to assert that its kernel was $\mathfrak{z}(D(\mathfrak{g}))$ rather than $\mathfrak{z}(\mathfrak{g}) \cap D(\mathfrak{g})$. In part (*c*), surprisingly many candidates had difficulty determining whether or not $D(\mathfrak{h})$ was nilpotent.

Question 2 was also generally well-answered. A number of candidates showed that $\mathfrak{g} = D(\mathfrak{g})$ when $\mathfrak{g}$ is semisimple by reducing to the case where $\mathfrak{g}$ is simple, which is a less efficient than simply noting $\mathfrak{g}/D(\mathfrak{g})$ must be abelian and semisimple. Part (*c*) was the most demanding, with some students seemingly failing to see that the condition that $p(\mathfrak{r}) = p(\mathfrak{g})$ is equivalent to $\mathfrak{g} = \mathfrak{r} + D^{\infty}(\mathfrak{g})$ and hence to $q(D^{\infty}(\mathfrak{g})) = q(\mathfrak{g})$ where $q\colon \mathfrak{g} \to \mathfrak{g}/\mathfrak{r}$.

Question 3 was the least popular, though it does not appear to have been more demanding than the other questions. Overall the vast majority of candidates were able to exhibit a good understanding of the course material in the answers they submitted.

B3.1: Galois Theory

Question 1 Most candidates performed well on part (a), though some were confused about the precise statement of the invariant factor decomposition for finitely generated abelian groups. In part (b), several candidates lost marks either for misinterpreting what was to be proved or for assuming non trivial facts about finite fields without justification. Nearly all candidates accurately reproduced the required bookwork in part (c)(i), and many succeeded in using it to prove part (c)(ii). In part (d), many candidates correctly identified the need for an inductive approach, but some struggled to carry it through.

Question 2 The bookwork in part (a) was generally handled well, but some marks were lost for incomplete definition of separability or for failing to verify that P(x) belongs to K[x]. Performance on part (b) was varied. Many candidates correctly answered (b)(i) and (b)(ii), but some omitted key steps in their reasoning — for instance, neglecting to verify irreducibility in (b)(ii). Part (b)(iii) proved more challenging, and only a small number successfully completed this challenging computation.

Question 3 Relatively few candidates attempted Question 3. Part (a) was generally answered well, although some candidates assumed the Fundamental Theorem of Symmetric Functions without proof, which was not admissible. Part(b)(i) proved more difficult while many realised that degrees were needed, some could not use this to provide a proof. Part (b)(ii) received fewer attempts, but those who did engage with it often performed well. Part (c) attracted many correct solutions.

B3.2: Geometry of Surfaces

Question 1 : The most popular question, though (c) proved rather difficult for many; overall this question was the least well done. The question mainly concerns geodesics, and in particular geodesic polar co-ordinates which lead to $E = 1$ and $F = 0$ in the first fundamental form. (a) was largely done well, though in (b) many missed that the half-line needs parameterizing as $(0, e^{\pm s})$ or equivalently noting that $\dot{y} = \pm y$ when arc length is used.

Part (c) sets up the geodesic polar co-ordinates in $\mathbb{H}$, centred at $z = i$. Whilst the resulting expressions are a little messy, the necessary algebra had been completed for candidates and could be assumed without justification. Setting $s = 0$ shows that all curves $\theta = c$ pass through i. In (ii), setting $\theta = c$ in the second expression for z and eliminating s shows that the curve is actually a semicircle with centre $(\cot c, 0)$ and radius $\csc c$.

For (iii), the hyperbolic distance of z from i can be seen to equal s (using the second expression for z) and consequently $E = 1$ as $\theta = c$ is parameterized by arc length. Finally in (iv) we fix s and eliminate c to find an equation connecting x and y. This is the equation of the Euclidean circle with centre $(0, \cosh s)$ and radius $\sinh s$. As the square of the distance between the circles' centres equals the sum of their radii, the circles in (iii) and (iv) are orthogonal and so the co-ordinate curves meet at right angles, or equally $F = 0$. No script successfully progressed this far.

Question 2 : The least popular of the three questions, though a number of close-to-perfect solutions were achieved; overall it was the best attempted question. (a) and (b) were done well in most attempts. Some solutions to (c) comment that there is a limit at ∞, showing continuity, but failed to change co-ordinates to show smoothness there. Many realized there was a minimum at 0 and a maximum at ∞, though further details were sometimes missing.

Derivations only appeared marginally in the lectures and notes, but (d) could be done using little more than the multivariate chain rule. The extended vector field has stationary points at 0 and ∞, each with index 1.

Question 3 : A popular question but there were only a few attempts achieving 20+ and many weak solutions. A good number of solutions forgot to mention that the Riemann-Hurwitz formula applies to *non-constant* holomorphic maps. In (b) only one person was able to show that $\mathbb{C}_\infty/\Gamma$ is homeomorphic to the sphere. One approach is to note that the upper unit semi-disc is a fundamental region and to appreciate how Γ identifies the semicircular boundary. The resulting quotient map then has degree 4 (the order of Γ) and 6 ramification points each with valency 2. In (c)(i) many correctly calculated the area of R to be 2π using the Local Gauss-Bonnet Theorem. In (iii) f is of the given form with $c = (1 - i)/2$. It was also important to check that the ends of a map to the ends of a'.

B3.3 Algebraic Curves

Question 1 (a) In the proof, when writing $p_i = [v_i]$ with $v_{n+2} = \lambda_1 v_1 + \cdots + \lambda_{n+1} v_{n+1}$ in $\mathbb{C}^{n+1}$ as $v_1, \ldots, v_{n+1}$ are a basis, most students forgot to use the general position condition to justify that $\lambda_i \neq 0$, before replacing v_i by $\lambda_i v_i$.

(d) The answer I wanted for 'Find the intersection points of C' and D'' was the explicit formula

$$\big[(\mu_1\lambda_2 - \lambda_1\mu_2)^{1/2}, \pm(\mu_2\lambda_0 - \lambda_2\mu_0)^{1/2}, \pm(\mu_0\lambda_1 - \lambda_0\mu_1)^{1/2}\big], \qquad (1)$$

or something equivalent to this. Very few students found this.

There was a minor mistake in the last part of (d): C' and D' can also intersect in 1 point, for example if $\lambda_0 = \mu_0 = 0$ and C', D' intersect in $[1, 0, 0]$ only. Only one student noticed this. There are 1, 2 or 4 points of intersection according to whether 1, 2 or 3 terms in (1) are nonzero.

Question 2 Was the most popular question and was mostly answered well.

Question 3 Students found part (c) and (e) difficult. For (c), one proof is to note that $\wp'(z)^{-1}\mathrm{d}\wp(z)$ is a meromorphic differential on $\mathbb{C}/\Lambda$ with no zeroes or poles. An alternative method is to take ω to be a meromorphic differential with canonical divisor $(\omega) = \kappa$, and use Riemann–Roch to show that $\ell(\kappa) = 1$. So $\mathcal{L}(\kappa) = \langle f \rangle_{\mathbb{C}}$, and then $0 = (f\omega)$ is a canonical divisor.

For (e), the answer I wanted was that $\left\{\wp(z)^k\wp'(z)^l : k \in \mathbb{N},\ l \in \{0,1\},\ 2k+3l \leq m\right\}$ is a basis for $\mathcal{L}(mp)$. Few students got the condition $l \in \{0,1\}$, without which the set is linearly dependent for $m \geq 6$.

B3.4: Algebraic Number Theory

Question 1 answered by 11 (out of 29) candidates; this is the question which the candidates found overall the most difficult part (d) in particular.

Question 2 was answered by 22 (out of 29) candidates, overall handled well.

Question 3 was answered by 25 (out of 29) candidates; parts (a) and (b) were answered very well; there was a range in quality of answer for part (c), with most students making partial progress, and there was difficulty for many students in giving complete answers to part (d).

B3.5 Topology and Groups

Question 1 (31 attempts): This question tested the understanding of cell complexes and homotopy equivalence. The general level of solutions was high. In (a)(i), many candidates failed to assume that the attaching map of the cell was continuous and there was little mention of the fact that the quotient topology was being used. In (a)(ii), few candidates mentioned that the 0-skeleton of a cell complex is endowed with the discrete topology. (b)(i) was typically fine. Many candidates attempted to do (b)(ii) by retracting each point along the unique edge path to a fixed root of the tree, without actually making sure that these homotopies fit together continuously. In (b)(iii), the assumption that X is path-connected was missing, but all candidates assumed this either implicitly or explicitly. Furthermore, there was a typo in the last line of the hint: The range of H is Y and not X. All candidates seemed to realise this was a typo.

Question 2 (33 attempts): This question tested knowledge of the fundamental group, homotopy retracts, and the Seifert–van Kampen theorem. The general level of solutions was good. Solutions for (a)(i) were typically correct. However, there were relatively few completely correct solutions for (a)(ii). Several candidates got the main idea. Solutions of (b)(i) were almost all correct. The majority of candidates got the key ideas for (b)(ii), however, the amount of technical detail was often lacking.

Question 3 (20 attempts): This question tested knowledge of the Cayley graph, the Cayley 2-complex, and covering spaces. The general level of solutions was mixed. In (a)(i), many candidates failed to mention that the graph was directed and that the edges were labelled by generators. Solutions for the rest of part (a) were typically correct. The description of the Cayley 2-complex is (b)(i) often lacked technical detail (one 2-cell for each group element), and few mentioned that it was the universal cover of the 2-complex associated with the presentation obtained by attaching 2-cells according to the relations to the bouquet corresponding to the generators. In (b)(ii), few candidates realised that one needs n 2-cells (likely stemming from the above issues in the definition of the Cayley 2-complex), and even fewer that the covering transformation not only rotates the 2-cells but also cyclically permutes them. In (b)(ii), again, few realised that there are two 2-cells for each small loop in the Cayley graph, forming a copy of S^2 each. Furthermore, very few managed to explain why this was the universal cover of $\mathbb{RP}^2 \vee \mathbb{RP}^2$.

B4.1: Functional Analysis I

Question 1 Almost all candidates attempted this question with variable degrees of success. Although key ideas were spotted, many candidates did not properly manage the delivery of their strategy, particularly in the easier part of the question. The following errors were common:

- In (a), while proving a sequence (f_n) converges uniformly to a function f, after establishing only pointwise convergence, it was claimed incorrectly that $\|f_n - g\| \to \|f - g\|$ by 'continuity of norm'.
- In (b)(i), it was incorrectly claimed that the function $x \mapsto |x|$ is not continuously differentiable in $[0, 1]$.
- In (b)(ii), after saying that (f_n) and (f_n') are Cauchy in X, it was claimed immediately and prematurely that (f_n) converges uniformly to some $f \in X$ and (f_n') converges uniformly to f' in X.
- In (b)(iii), it was incorrectly claimed that Z forms a sublattice of X.

Question 2 40% of the candidates attempted this question. Part (a) was done well by most candidates. In part (b), many candidates did not realise in (i) that the span of S_ε is the whole space, and these candidates had an issue with (ii) too. In part (c), most candidates spotted the relevant consequence of the Hahn-Banach theorem they should use, but a portion of the candidates either struggled with the proof of that statement or with how to use it to complete the question. In part (d), (i) and (ii) were handled relatively well. Very few candidates managed well part (d)(iii).

Question 3 80% of the candidates attempted this question. Parts (a), (b)(i), (b)(ii), (c)(i), (c)(ii) were handled reasonably well with minor exceptions. In part (b)(iii), many candidates did not manage to get A' in simple form and subsequently struggled somewhat with the computation of $\|A'\|$. In part (c)(iii), many candidates attempted to use Bessel's inequality, which over-complicates the problem. Not many candidates attempted (c)(iv) – only those who had some proper sense of (c)(iii) could get a sense of (c)(iv).

B4.2: Functional Analysis II

Question 1 Part a) of question 1 tested the understanding of basic material and I was glad to see that most of the students did very well on this. Part b i) and ii) are standard applications of concepts and results discussed in the course, and while i) was solved very well, quite a few students did not spot that ii) is related to the convergence of Fourier coefficients. Part b)iii) was designed to be more challenging, and while most students attempted to provide explicit functions f, many of these arguments were incomplete or incorrect as they did consider the cancellation effects coming from the oscillatory and/or symmetry behaviour of sin. These difficulties can be avoided by instead considering the operator norms and arguing via uniform boundedness principle and a few students did this successfully. Many students successfully proved the implication $(\alpha) \Rightarrow (\beta)$ in c) using a standard rescaling argument and quite a few students realised that the reverse direction could be approached using uniform boundedness principle, though the full proof via PUB was as expected quite challenging and only completed by a few students as the presence of the absolute valued makes it necessary to consider general linear combinations of the f_n with unit coefficients. Alternative arguments via either Baire's category theorem or by using the f_n to define a map into ℓ^1 were possible and used successfully by several students.

Question 2 The bookwork part of a) was done well, though quite a few students forgot to comment on how the open mapping theorem implies the inverse mapping theorem. Most students struggled to provide a correct example for a)ii), with only few realising that one can e.g. use a bijection whose algebraic inverse was unbounded. Parts b)i) and ii) covered very standard material on weak convergence and were very well solved and while iii) was a bit more challenging, this was solved quite well, though in a few cases through rather complicated arguments using dual operators rather than more directly via closed graph theorem and Hahn-Banach. Part c), in particular the implication that $(\beta) \Rightarrow (\alpha)$, was designed to be the most challenging part of the question and this was indeed the case, with only few complete solutions, though quite a few partial answers which correctly used Riesz representation theorem to deduce the easier direction.

Question 3 The early parts of a,) which covered basic material were solved well, but I was surprised to see how few students successfully solved a iii), given that orthogonal projections are well known examples of selfadjoint operators and that the easiest way of obtaining a strict inequality is to use non-zero operators whose composition is zero, such as projections onto orthogonal subspaces. The first part of b) was very well solved, with most students who solved question 3 successfully proving the compactness of the operator based on the Theorem of Arzela-Ascoli and then exploiting this to show that the spectrum only contains 0. The second part of b) was designed to be challenging and that indeed was the case, and while several students realised that Sf if Hölder continuous if $f \in L^2$ and used this to discuss the convergence of the Fourier series, they did not spot that this could also be used to solve (α). Part c) was then pretty standard and in general well solved, including (ii) for which some students constructed explicit counterexamples while others successfully translated this into a statement about the existence of operators whose approximate point spectrum does not agree with the point spectrum.

B4.3: Distribution Theory

Question 1 The general level of answers was good, even though no candidate got the full marks. Most candidates got full marks for those in Part(a), which were mostly bookwork and direct-application. Parts (b)–(c) were a combination of unseen, similar, and new riders, and required not only careful calculations but also a good understanding of the fundamental concepts/properties of distributions and their orders; these were largely done, even though some of the candidates struggled to provide examples for illustrations and to perform careful calculations.

Question 2 The general level of answers was slightly higher than Question 1. No candidate got the full marks, where in particular some marks were lost in Part (b), which concerned the extension of a positive distribution with compact support to a linear functional on the space of continuous functions with an upper-bound inequality, causing difficulties for some of the candidates. Part (a) was all bookwork, which was very well taken, while Part (c) was a new example and caused some struggles.

Question 3 Most of the candidates did better overall than Questions 1–2. Part (a) was either bookwork or new examples, but required good understanding of the concepts of the support, the singular support, and the derivative of distributions, which did considerably well, even though several candidates struggled for (iv). Part (b) concerned the solutions of several differential operators in the sense of distributions, which was done quite well overall; however, only very few attempted to verify that the differentiation is valid in the sense of distributions in the proof for (i).

B5.1: Stochastic Modelling and Biological Processes

Question 1 This was the most popular question, attempted by over 90% of students, with many achieving their best scores on this question. Some students struggled to write down the deterministic equations required for part (a) correctly, in particular, getting confused between deterministic rates, and stochastic intensities. This usually didn't have major knock-on effects in parts (b) and (c). In part (b), the question should not have said "$(x^*, y^*, z^*) \in \Delta_N$", since (x^*, y^*, z^*) was not necessarily an integer vector; but this did not seem to cause any confusion – students correctly interpreted this to mean only that $x^* + y^* + z^* = N$. Most could correctly write down the master equation, and found more or less tidy ways to show that the given distribution satisfied it. Inefficient approaches very likely cost some students quite a lot of time. When it came to calculating the normalising constant quite a few struggled with the combinatorics. Only a small fraction of students managed to do part (c) to a high standard, with many recognising, but struggling to calculate, the required sum of probabilities

Question 2 The question was attempted by just over 60% of students. Students fared slightly worse than on Question 1. In part (a), most correctly wrote down the auxiliary BVP, and correctly interpreted the given quantities as (scaled) fluxes. Solving the BVP in part (b) was essentially FSPDE content, and there were more or less quick ways to do it, leading to slightly different forms for the solution (which was only defined up to some normalising constant). It was easy to get mired in calculations, and only a minority of students who managed to solve the BVP went on to correctly show that the "survival probability" took the form given. Responses to part (c) split students quite clearly into those who had thought about stochastic simulations, and those who hadn't. Slightly tricky was that both the movement and the reproduction needed to be simulated. Several correctly identified the source of the systematic error in the simulations.

Question 3 The question was attempted by just over 60% of students. This question saw the lowest marks on average. Quite a few made errors writing down the SDE in part (a), which is core content. Most could solve for the mean in part (b), but several went wrong calculating the variance. In part (c), the key was to recognise that there was no need to solve any ODEs (this would be *very* time-consuming): it was only necessary to write down the algebraic equations for stationary values of the moments, and then solve these sequentially.

Many students made some progress on part (c), correctly calculating some moments, but a small minority completed the calculation of $\mathrm{Var}(A)/\langle A\rangle$. Several students at least partially understood the relationship between the model in part (d) and the models in previous parts of the question.

B5.2: Applied PDEs

Q2 was the most popular question by far; almost all candidates attempted this question. Candidates split roughly evenly across the other two questions.

Question 1 Part (a) was done quite well. Part (a)(i) and the scaling analysis in (ii) was completed by most students who attempted the question. Most students also formulated the correct ODE and conditions in similiarity variables (thought not in all cases the answers were complete), and many also got the general form for α and β. Some also got the solution for $n = 1$ as required in the (iii) case (except for the value of σ). Many candidates struggled with part (b). Often the formulation of the problem for the Green's function was wrong (i.e. not stating the *modified* GF that was required.) Determination of the GF was done by an even smaller group, as it differs in a number of details from the Dirichlet case.

Question 2 Generally well done by most candidates. In particular, most candidates got far with (a). Some stated a wrong or unproven condition for the causality of shocks. There were also a number of candidates who got the causality wrong for the examples in (iii) and (iv). Part (b) was also done well. Some found the final part challenging, which required the determination and solution of an ODE for the shock trajectory post merger with the rarefaction wave.

Question 3 (a) was generally done well. Some candidates struggled with getting the solution $q = 1$ of the initial value problem in (b), and determining and sketching the domain of definition was a stumbling stone for quite a few. Similarly for (c). Only very few succeeded in getting the essence of (d) right.

B5.3: Viscous Flow

Question 1 was attempted by the most candidates. The bookwork was generally well done, but some students forgot that the flux integral includes a factor of r in polar coordinates. A good number of candidates correctly found the solution for $u(r, t)$. However, numerous candidates failed to correctly work out the flux associated with this flow and take the small λ limit. No candidate found the phase shift $\alpha(\lambda)$ and only a couple of candidates managed to find the relationship between the parameters needed at the end.

Question 2 was also attempted by a large number of candidates. Some did not explain well the reason for the boundary layer. The cartesian version of the boundary-layer equations were generally well found, although many candidates incorrectly stated that the matching condition with the outer flow involved $V \to 0$ as $Y \to \infty$, and were penalised accordingly. In part (b), some candidates were defeated by algebraic manipulation. Some candidates confused velocity potential with streamfunction, and almost all failed to correctly find the matching condition for u as $Y \to \infty$; as a result, there were only a few candidates who correctly found the pressure gradient. A reasonable number of students incorrectly stated that the boundary layer would separate when $\mathrm{d}p/\mathrm{d}x = 0$, rather than when $\partial u/\partial Y = 0$. Sketches of the flow often didn't include an indication of the location of boundary layer on the sphere and that it persists in the flow once separated. For those who got to part (c), most explained that the leading-order boundary layer equations were the same and that the geometry only affects the slip velocity and pressure gradient. No candidates mentioned that the problems are fundamentally different at higher order.

Question 3 was attempted by fewer candidates. There was a superfluous time scaling listed as part of the nondimensionalisation; most students ignored it. Solutions were marred by algebraic manipulation errors, and some candidates laboured the calculations. No candidates correctly stated the boundary conditions that should be applied at $x' = 0, 1$ and $y' = 0, 1$, and only a couple attempted to find the conservation of mass relationship. Almost no candidates tried part (b); of those that did, only one managed to find the q'_{u_i}, no candidates scored any marks for part (ii), while no candidates attempted (iii). Some candidates jumped straight to the final part of (iv), and successfully found $\hat{\chi}$ as the solution to (1).

B5.4: Waves and Compressible Flow

Question 1 This question was quite popular but was found difficult by weaker candidates, who often struggled with basic algebraic manipulations. There was a notational ambiguity in part (c)(ii) as to whether the factor of $(1 + \epsilon\omega_1)$ should be in the numerator or the denominator. All candidates who got this far assumed it was in the denominator, and the given solution is correct in either case.

In part (a), the basic bookwork and sometimes even the definition of the entropy had not been well learned. In part (b), there were many fallacious proofs for $\boldsymbol{\nabla} \times \boldsymbol{u} = \boldsymbol{0}$. The derivation of the $\phi_{xxx} = 0$ boundary conditions was intended to be difficult, and in practice only the strongest candidates made any serious progress with it. There were many algebraic slips in part (c), and very few candidates correctly got the final formula for $|B|$.

Question 2 This was the most popular question, and there were many reasonably good solutions. In the final displayed equation in part (b), ϕ should have been ϕ_1, but no candidates appeared to be distracted by the typo.

There were some good derivations in parts (a) and (b), albeit with some confusion over which results hold generally and which rely on linearisation. There were few convincing derivations of the drag in part (b), with several candidates attempting to fallaciously appeal to Kelvin's Theorem or the Kutta-Joukowski Lift Theorem. The Fourier transform calculations in part (c) caused many problems, despite being almost identical to a problem sheet question. Basic errors included having the transformed potential $\hat{\phi}_1$ still depending on x, and trying to solve the resulting ODE with $\mathrm{e}^{-k|y|}$ instead of $\mathrm{e}^{-y|k|}$. There were few convincing solutions to part (c)(ii). Candidates often tried to evaluate $\partial\phi_1/\partial x(x, 0)$ (rather than appealing to parity directly) unwittingly ending up with undefined singular integrals.

Question 3 This was the least popular question, and there were not many First class marks.

In part (a), there was some carelessness over applying the given conditions to derive the PDE and the Rankine–Hugoniot conditions. Part (b) was generally OK though often overcomplicated. Many candidates struggled with the basic trigonometrical identities required for the first result in part (c). No-one successfully derived the final inequality, and maybe a hint could have been given to try minimising $\tan\delta$ first.

B5.5: Further Mathematical Biology

Question 1 Parts (a) and (b) contained mostly bookwork and were answered well by most candidates. Part (c) required a substitution that not many candidates spotted, however progress to part (d) was still possible given the solution to part (c) was in the question. Very few candidates made substantial progress in (d), despite the fact that the algebra was relatively simple after the required integral was computed.

Question 2 Parts (a)–(b) were well answered by the majority of those that attempted it, though many did not remember to explicitly state the boundary conditions in (b). Very few candidates attempted (c) and (d) by converting to a system of three first order equations and conducting linear stability analysis in the usual way, possibly as a result of the "show that" in (b).

Question 3 The majority of candidates attempted this question. Parts (a) and (b) were well answered. In part (c), many candidates did not find the constraints on the parameters required for the non-zero steady state. In (d) many candidates did not correctly assume that nutrient dynamics are quasi-steady, which prevented progress in analysing the linear stability.

B5.6: Nonlinear Systems

Candidates demonstrated a good understanding of the course material and received marks for the bookwork components of the questions. In the more complex parts, some candidates described general methods from the course rather than applying them to the specific dynamical systems in question. The most challenging parts required careful consideration of the best approach to derive solvable equations; a purely mechanical application of theory often resulted in unnecessarily complex calculations.

Question 1 focused on discrete-time dynamical systems (maps), with most candidates demonstrating the ability to find fixed points and assess their stability. The most challenging part was part (d), where some candidates attempted to compute the 3-cycles of the map F_1 directly by solving the equation $F_1(F_1(F_1(x))) = x$, but were unsuccessful. A more effective approach would have been to show that F_1 is conjugate to the logistic map, for which the 3-cycles can be computed by using the substitution $x_k = \sin^2 \theta_k$.

Question 2 covered material on continuous-time dynamical systems modeling chemical reaction networks. Most candidates were able to formulate the corresponding ODE model and identify its steady states. However, some struggled to draw the correct conclusion regarding the stability of the (non-hyperbolic) steady state at the origin. While most candidates correctly identified that the system undergoes a Hopf bifurcation at the bifurcation point $\mu = 2$, some lost marks for not determining whether the bifurcation is supercritical or subcritical.

Question 3 examined a system of three ODEs near the critical point at the origin. Most candidates began by analyzing the corresponding linearized system through its eigenvalues and eigenvectors. However, some lost marks for incorrectly using the extended center manifold to classify the bifurcation at the origin.

B6.1: Numerical Solution of Differential Equations I

Question 1 The question was concerned with the finite difference approximation of a selfadjoint two-point boundary-value problem. A few candidates expanded the differential operator by applying the product rule to the first term, and constructed a finite difference approximation of the resulting nonselfadjoint operator using a symmetric or nonsymmetric difference quotient for the first derivative in that differential operator. These approximations of the nonselfadjoint form of the differential operator however did not result in a system of linear algebraic equations with the desired symmetric matrix in the final part of the question.

Question 2 The question was concerned with the discrete maximum principle for the five-point finite difference approximation of a linear second-order nonselfadjoint elliptic boundary-value problem. The question was popular and was generally well done.

Question 3 The question was concerned with the stability analysis of the Lax–Wendroff finite difference approximation of a first-order linear hyperbolic PDE. This was a popular question and there were several almost complete answers. There was a typographical error in the second displayed line of part (c)(i), which should have read $|\lambda(k)|^2 = 1 - 4\mu^2(1 - \mu^2)\sin^4 \frac{k\Delta x}{2}$. Fortunately this didn't seem to confuse any of the candidates; those who obtained the correct expression for $\lambda(k)$ and/or $|\lambda(k)|^2$ were awarded full marks for part (c)(i). This part of the syllabus was clearly very well prepared for the exam by all candidates who attempted the question.

B6.2 Optimisation for Data Science

The students performed very well on the exam overall. The level of tackling each question was solid, showing good awareness of the course material. The choice of questions was relatively even, with questions 1 and 2 being preferred. A common occurrence was insufficient explanations or details of arguments, which were needed for full marks.

B6.3: Integer Programming

Overall the exam did an excellent job at separating candidates by ability.

Question 1 The first question was as attempted by almost all candidates, as it was largely bookwork and application of known techniques. Nevertheless a number still struggled with the correct setup and application of the simplex algorithm on what was a very simple instance. As expected, only a few students were able to correctly answer last part of the question as it required deep understanding of the material.

Question 2 was attempted by most candidates, no doubt drawn by the familiar territory of the set theoretic nature of the question. Many different proof techniques were employed, mostly correctly, for the third part of the question, despite the fact that this was bookwork. A surprising number of students either failed to correctly state the greedy algorithm in the final part of the question despite the fact that subsequent subparts of the question all but gave away the answer, or failed to notice the telescoping sum that was the only non-trivial step required to correctly answer the final subpart.

Question 3 Very few candidates attempted the final question, presumably because it contained material taught in the last week the course. Those that did generally answered the question very well, but there were some exceptions.

B7.1: Classical Mechanics

Question 1 This was a popular question and attracted some good answers. However, most candidates derived the effective potential incorrectly by substituting the conserved quantities back into the Lagrangian rather than into the energy thereby obtaining the wrong sign for the centrifugal terms. The $u = 1/r$ substitution was not attempted by many despite being in some questions and the angle of deflection was only found by a few.

Question 2 This question was also popular with some good solutions. The bookwork to obtain the Lagrangian was set out with more work than necessary by many, although the answers were generally good. For the last part, too may candidates did not think to differentiate the energy to obtain the second order equation needed to identify fixed points and their stability with very few candidates able to finish the question properly.

Question 3 Despite being on later and perhaps harder material, many candidates attempted this question and picked up respectable marks on the more routine earlier parts. There were also some good attempts on the later parts connecting the derivative of the action variable to the period and finding the equations of motion in the action angle variables.

B7.2: Electromagnetism

Question 1 The average mark on this question was low. Only the strongest candidates were clear on the strategy of solving the Poisson equation in different regions and then matching the solutions across the slab and sphere boundaries respectively. Recall that the electrostatic potential is continuous and the discontinuity of its derivative is proportional to the surface charge density, which vanishes except in part b)(ii).

In part a) the symmetry considerations were done well. In part a)(i) some candidates only considered the integrated Gauss law in regions that stretch all the way through the slab, and hence did not obtain the electric field inside the slab. In part a)(ii) many candidates tried to force the potential to be zero on both sides of the slab, which is inconsistent with the continuous differentiability of the solution of the Poisson equation.

In part b) the separation of variables was generally done well and the basic solution $r^n Y_{\ell,m}(\theta, \varphi)$ for appropriate n, ℓ, m was constructed. However, many candidates failed to take different linear combinations of these basic solutions inside and outside the sphere and then match them appropriately.

Question 2 In the computation of $\mathbf{m}$ some candidates started from the wrong expression for $\mathbf{J}(\mathbf{r})$, but almost all mistakenly treated $\mathbf{e}_\theta$ as a constant unit vector, whereas they should have realised that it is $(\cos\theta\cos\varphi, \cos\theta\sin\varphi, -\sin\theta)$. In reducing the Ampère-Maxwell equation to a differential equation for $F(r)$, some candidates made computational mistakes and obtained equations with some residual angle dependence. This leads to a contradiction, which many did not appreciate. Only the best candidates realised that outside the sphere the vector potential is exactly that of a magnetic dipole.

Question 3 The average mark on this question was high, especially on part a). Only a few candidates failed to treat the time derivative terms in the proof of the matching conditions appropriately. (Note that these are the same matching conditions that so few candidates implemented in Question 1.) In part b) it was cleanest to parametrise the wave and polarisation vectors with angles, but some candidates managed to get to enough equations by working in components. In some cases, the magnetic matching conditions were missed. Very few gave a proof of b)(ii), but then many were able to obtain the correct formula for the Brewster angle θ^+ in b)(iii).

B7.3: Further Quantum Theory

Question 1 This question was equally popular with the others. Bookwork was generally reasonably well done, though at the level of details many candidates missed marks. Parts (c) and (d) were applications of the Bohr–Somerfeld method for estimating energy levels and the variational method, respectively. Only a few candidates followed these to completion. For the WKB problem, there was an integral to wrangle with but the trick to simplify it was familiar from homework problems. Similarly, for the variational problem one should use the virial theorem to determine the expectation value of the kinetic energy operator in the given trial state, while evaluating the potential term required performing an integral. This was again similar to cases seen in the course. There were typos (in a sign and in the limits of integration of the second hint integral); the sign did not lead to any a serious issues, and the correct limit of integration was relatively clear in context. Candidates who got to the point where the limit of integration was relevant were awarded most of the available marks for that part.

Question 2 This question was equally popular with the others. The bookwork in part (a) of this problem was generally well done, though errors did arise and the asymptotic nature of the plane-wave solutions (in contrast to cases with a bounded interaction region) was only occasionally appreciated, despite an emphasis on this possibility in lectures. Part (b) mainly required a suitable change of variables of the time-independent Schrödinger equation, and candidates who noticed this mostly derived the correct Bessel-function solution for scattering states. The map between the coefficients of the solution in the Bessel function basis and the asymptotic plane-wave coefficients proved elusive in many cases, although by ignoring this subtlety one could incidentally produce the correct answer for the reflection and transmission coefficients. No candidates completed the final part, though a good deal of it could be done even if previous parts had not been completed, as it mostly required manipulating linear expression written in terms of values of Bessel functions and their derivatives (as specified in the question).

Question 3 This question was equally popular with the others. This perturbation theory problem proved challenging to candidates, though it had many similarities to a problem from two years ago. Most (though not all) candidates gave a good account of addition of angular momentum for the Hydrogen atom. There were already many errors in the treatment of the system where the perturbation was exactly diagonalisable, with the degeneracy of the system causing some issues. For the calculations of first-order corrections in part (c), many candidates made some progress but sloppy accounting regarding the states (inherited from issues in part (b)) often hindered the computations. Only a few candidates did a very good job of explaining their treatment of degenerate energy levels. Part (d) received very little attention.

B8.1: Probability, Measure and Martingales

Question 1 The overall quality of answers to Q1 varied significantly, with some students handling in perfect solutions and some only completing a very small part of the question. Q1(a) was bookwork-type problem on basic properties of measures. It was answered mostly well, some students could only argue one directions, others however got muddled up in their own arguments and wrote circular and/or involved arguments without actually proving the desired properties. Q1b(i) was simple bookwork but a non-negligible proportion of students forgot to say that the conditional expectation needed to be a $\mathcal{G}$-measurable random variable. Q1b(ii) was answered reasonably well. Some students did not realise it was enough to establish the defining property on the sets forming the partition, or used this but without giving any reasons. Individual marks were lost for forgetting to check/state the measurability and/or that a_k were well defined. Small proportion of the students struggled with this question and kept confusing conditional expectation with conditional probability seen in the earlier years. Q1c(i) was invariably correct. Q1c(ii) was done well by a majority of those who attempted it. The simplest argument used BC1 lemma. Some other arguments were also built, but some students either did not attempt the question or said it holds by the SLLN, ignoring that SLLN was not applicable and that $\mathbb{E}[X_k] = 0$, as the computed themselves a moment ago. Q1c(iii) was done correctly by a minority of students. Some used the natural contradiction argument, others directly checked the backwards martingale condition was not satisfied for some small n. Q1c(iv) was done correctly by a minority of students. Some students tried to write a generic formula without much progress. Those who checked the possible values quickly saw X_2 was in fact a function of S_3. Q1c(v) was correctly answered by a slightly higher proportion of students - this was bookwork setup of the proof of SLLN.

Question 2 The overall caused students more difficulty than the other questions. While vast majority of students correctly stated the definition of $\mathcal{F}_\tau$, comparably few of them went on to use this definition and answer Q2a(ii). Instead, many students were confused in this part and wrote things which the assessors were challenged to make any sense of. A common mistake was to consider $\mathcal{F}_{\tau\wedge n}$ as if it was a random variable (or a process indexed by n) and could be analysed on $\{\tau < n\}$ and on its complement, with students writing "$\mathcal{F}_{\tau\wedge n}\mathbf{1}_{\tau\geq n} = \mathcal{F}_n$", or similar, mixing σ-algebras and random variables in one expression. In Q2a(iii) often individual mark was lost either for not stating the results on martingale transforms correctly (e.g., forgetting $V > 0$) or, when doing a direct computation, for not checking the measurability and integrability. Q2b(i) most students had no troubles with. However many lost marks for forgetting to check measurability and/or integrability, or not giving even a gentle indication to what properties of conditional expectation they used. Even students who remembered to check U_n was adapted, sometimes carried on with the same argument and, falsely, asserted that product of integrable functions was integrable. Q2b(ii) caused serious difficulties. Many students could state the theorem correctly but either could not argue $(U_{n\wedge\tau^M})$ was bounded in L^1 or provided false arguments for this. Those who saw that $U_{n\wedge\tau^M} \geq -M$ usually got this part right. Q2b(iii) was mostly argued wrongly or incompletely. Similarly for Q2b(iv) were many tried to build involved arguments instead of relying on non-negativity of C_n and X_n. Very few students completed Q2c. Some used BC1 to argue $\sum_n B_n < \infty$ a.s. but failed to realise one needed also to show $\lim D_n > 0$ a.s.

Question 3 On the whole, this question was done best. Students on the whole completed the bookwork in part (a) well. Some errors involved mixing up the conditions of a λ-system from an alternative definition (which was not equivalent to the conditions being replaced). Most students identified that they needed to show the collection on which the two measures agreed is a λ-system. In part (b), many students struggled to specify the mode of convergence with justification. Some students recognised the example from lectures, but not many leveraged the characterisation of UI martingales (Theorem 8.32 in the lecture notes). Most students followed the hint for part (iii). However, if the convergence in (i) was not specified correctly, this necessarily made part (iii) more difficult. In such cases, some students chose to prove the Kolmogorov 0–1 law instead. For part (c), quite a few students forgot to check that the stochastic process is integrable and adapted when verifying that the process is a martingale, but otherwise (i) was reasoned well. In part (iii), many students followed the hint to show that N_n is a martingale bounded in L^2, but very few could convert this to a full, well-justified solution. Many recognised that the convergence of M_n is not in L^1 as the expectation of M_n was usually computed correctly in (i).

B8.2: Continuous Martingales and Stochastic Calculus

Question 1 This was the most popular question attempted by almost all the candidates. There was a wide range in the quality of answers but many candidates were able to score well. Part (a) was generally well done. For part (b) the Dambis-Dubins-Schwarz Theorem was often incorrectly stated. This meant that the candidates did not complete part (c)(iii) correctly. The final part of the question was more challenging to get right to the end. Most candidates could use Ito's formula. However, many did not write the evolution of the process W in terms of its real and imaginary parts correctly. The conclusion of the question was argued well by only a few candidates.

Question 2 This was the least popular question, though there were many very good attempts. Part (a) was bookwork which was reproduced with varying degrees of success. Part (b) was well done. The first part of (c) was found to be tricky by many and putting all the steps together in the right way to obtain the final inequality was only done by a few.

Question 3 This proved to be a popular question with a number of straight forward marks available. Part (a) was quite well done. Candidates lost marks for not checking all the conditions for M to be a martingale. Also a few thought that integrability implied the martingale was bounded and mistakenly applied the version of the martingale convergence theorem for UI martingales. Most recognised that this was an example of a martingale which is not UI. Part (b) was a version of a problem sheet question and the first section needed a careful argument using the optional stopping theorem. The connection with the uniform and exponential distributions was well done. The first sections of part (c) were reasonably well done. Many candidates lost marks for not checking the Gaussianity of the processes. The final part had a typo in the statement but this was usually recognised by candidates who got that far.

B8.3: Mathematical Models of Financial Derivatives

Question 1 This question is about the discrete-time binomial model, and was attempted by nearly all candidates. In Part (a), all the candidates mentioned the no-arbitrage principle. Only a small number did not clearly explain how an arbitrage opportunity would arise if that condition were violated. Part (b) tested the definition of Delta hedge. Most students handled this well, but some made algebraic or sign errors that led to an incorrect value for q. Part (c) is a new example not covered in the lecture slides but the calculation is relatively straightforward. When computing the hedge value in (ii), many candidates overlooked the distinction between n and $n+1$ in V, A and B, and lead to incorrect result. Moreover, the question asked for the cash value of the delta hedge, while some candidates only gave the number of assets held Δ_{t_n} without providing the corresponding cash value $\Delta_{t_n} S_{t_n}$. In Part (d), which required a brief discussion based on the terminal nodes, most students still performed detailed calculations, and many nonetheless arrived at the correct conclusion.

Question 2 This question is about the Black-Scholes PDE. It has been clearly noted that the direct use of Black-Scholes formula for the option price will attract no credit. A few candidates overlooked this instruction and, as a result, lost the majority of their marks. Part (a) is a straightforward extension of lecture material, and most of the candidates correctly identified both the spatial and time invariance. In Part (b), nearly everyone wrote down the correct Black-Scholes PDE, while some candidates made errors when differentiating the call price V^{call} with respect to the strike K, and therefore lead to incorrect results. In Part (c), only a relatively low proportion of candidates noticed that the asset itself S is a solution of the Black-Scholes PDE, and then provide the correct final answer. Part (d) involves the valuation impact of a discrete dividend at time T_D. This question is a bit problematic (I have also discussed it with the module director): There should be two cases with $t < T_D < T$ and $T_D < t < T$, but the provided mark scheme addresses only the first case. There is a few students rightly discussed both cases. However, no marks were deducted for treating only the first scenario.

Question 3 This question is about some exotic options. Part (a) mirrors the lecture material on the binary call, but here you price a binary put. In (i) most candidates wrote down the correct SDE, although a handful slipped up in applying Itô's lemma (even though they recalled the final form). In (ii) a modest number of students got the sign wrong. In (iii) almost everyone invoked put–call parity, but some stopped short of its simplest form by not using $-N(-d) = N(d)$. Part (b) also mirrors the lecture material on the perpetual American put. While the majority of students who chose this question set up the correct ODE, only a handful solved it correctly with the characteristic equation—those who merely memorised the solution without derivation still lost some marks. Part (c) was well answered by the majority with clear and brief discussions.

B8.4: Information Theory

Question 1 was attempted by most students. The question was well done, however some students were not clear in part bi on how to construct an optimal encoding, or succesfully performed a construction using Huffman's algorithm (but failed to comment on the fact this is generally optimal). Question bii had a variety of answers, with most students either suggesting a standard block code (which they then verified approached the lower bound), or an arithmetic code. Some students struggled a little with determining whether the binary expansion of X is a Markov chain (it is not), which becomes clear when you compute the probabilities explicitly.

Question 2 was attemped by about half of the students. Question 2aii caused some difficulties, with students failing to give definitions of some relevant terms. Part c also caused difficulties, where students were not clear in how to define the decoded (which should take an encoded message and return an estimate of the original message, not a codeword), and must produce a deterministic outcome (in terms of tie-breaking). This caused difficulties in part cii, where the minimal number of errors (2) leading to a worst-case error was not clearly presented.

Question 3 was attempted by most students. There was some clack of clarity in many of the arguments, with heuristic arguments used to justify inequalities for entropy stated in the place of explicit calculation, particularly in the proof of Fano's inequality. In part cii, many students either did not distinguish recognise that the channel gives bounds on $I(X;Y)$, or were not clear in how they used the data processing inequality to derive bounds on $I(f(X);g(Y))$.

B8.5: Graph Theory

Question 1 Although the average score was reasonably good, there was a tendency among students to make minor mistakes and incorrect deductions throughout this question. Part (a) was generally done well, but it was common in part (b) to assume the two paths neatly separate and join back up as they go between the two vertices. Another common mistake was to assume a closed walk must contain a cycle. In part (c) many students failed to note that $G - E(C)$ can't have isolated vertices, or proved that a path between adjacent leaves of T would create a longer cycle in (ii) without reference to the fact that we need this path not to intersect C. Generally there seemed to be a common assumption among students that paths and cycles never intersect at inconvenient places! Few students completed part (c) and I suspect many ran out of time. Also, very few students drew (useful) diagrams.

Question 2 This was generally done well except for the last part. Some students tried to use various properties associated with connectivity in (b), such as Menger's theorem, rather than note that it was just a constraint on the minimum degree. Part (c) was usually done well. Part (d) caused a lot of trouble as many students did not get that the vertices at largest distance from each other have degree at most 3. Several students deduced that the minimum degree is at most 4 and then tried to reproduce the Kempe Chain idea from part (c). This could in principle work, but needs a proof that the graph is planar.

Question 3 Relatively few students attempted this question. A common mistake in (a) was to state that all maximal flows are integral, rather than state that there exists one that is. Students either solved part (b) completely or not at all. Part (c) was generally done less well. A common mistake was to prove the triviality $c_-(T,S) \leq c_+(T,S)$ rather than $\leq c_+(S,T)$, or simply to not show that any such condition was necessary. Many students had trouble with the cut capacity calculation, as they tried to write everything as complicated double sums. It is helpful to use the $c_\pm(S,T)$ notation as this simplifies the algebra. On the other hand, converting the flow back to a circulation was almost always done correctly.

Summary: Overall, the distribution of marks was reasonable with similar averages for each question. However, I suspect many students took too long on Question 1 and then had little time for their second question. It should be noted that if a question has a long list of statements to prove, then each of them should have a short solution, and students should not spend too much time on each part. Another point to note is that, like last year, most students do questions 1 and 2, even though I believe question 3 was not harder.

B8.6: High Dimensional Probability

Question 1 was attempted by the most candidates. The bookwork was generally well done, but some students forgot that the flux integral includes a factor of r in polar coordinates. A good number of candidates correctly found the solution for $u(r, t)$. However, numerous candidates failed to correctly work out the flux associated with this flow and take the small λ limit. No candidate found the phase shift $\alpha(\lambda)$ and only a couple of candidates managed to find the relationship between the parameters needed at the end.

Question 2 was also attempted by a large number of candidates. Some did not explain well the reason for the boundary layer. The cartesian version of the boundary-layer equations were generally well found, although many candidates incorrectly stated that the matching condition with the outer flow involved $V \to 0$ as $Y \to \infty$, and were penalised accordingly. In part (b), some candidates were defeated by algebraic manipulation. Some candidates confused velocity potential with streamfunction, and almost all failed to correctly find the matching condition for u as $Y \to \infty$; as a result, there were only a few candidates who correctly found the pressure gradient. A reasonable number of students incorrectly stated that the boundary layer would separate when $\mathrm{d}p/\mathrm{d}x = 0$, rather than when $\partial u/\partial Y = 0$. Sketches of the flow often didn't include an indication of the location of boundary layer on the sphere and that it persists in the flow once separated. For those who got to part (c), most explained that the leading-order boundary layer equations were the same and that the geometry only affects the slip velocity and pressure gradient. No candidates mentioned that the problems are fundamentally different at higher order.

Question 3 was attempted by fewer candidates. There was a superfluous time scaling listed as part of the nondimensionalisation; most students ignored it. Solutions were marred by algebraic manipulation errors, and some candidates laboured the calculations. No candidates correctly stated the boundary conditions that should be applied at $x' = 0, 1$ and $y' = 0, 1$, and only a couple attempted to find the conservation of mass relationship. Almost no candidates tried part (b); of those that did, only one managed to find the q'_{u_i}, no candidates scored any marks for part (ii), while no candidates attempted (iii). Some candidates jumped straight to the final part of (iv), and successfully found $\hat{\chi}$ as the solution to (1).

BO1.1: History of Mathematics

Both the extended coursework essays and the exam scripts were blind double-marked. The marks for essays and exam were reconciled separately. The two carry equal weight when determining a candidate's final mark. The first half of the exam paper (Section A) consists of six extracts from historical mathematical texts, from which candidates must choose two on which to comment; the second half (Section B) gives candidates a choice of three essay topics, from which they must choose one. The Section B essay accounts for 50% of the overall exam mark; the answers to each of the Section A questions count for 25%.

Throughout the course, candidates were invited to analyse historical mathematical materials from the points of view of their 'context', 'content', and 'significance', and these were the three aspects that candidates were asked to consider when looking at the extracts provided in Section A of the exam paper. A number of candidates chose to use these as subheadings within their answers. The word 'significance' was used consistently throughout the course to capture a broad sense of where a given source sits within the historical development of mathematics. This usage was repeatedly stressed. Some candidates were penalised however for considering this only in the narrow sense of 'importance'. In connection with this, candidates were also penalised in places for being too 'presentist' in their approach to the extracts — the historical mathematics ought to have been treated on its own terms, rather than in (negative) comparison to how we do things nowadays. This last remark also applies to the extended essays (see below).

The Section A questions 1–6 were attempted by 8, 10, 9, 2, 8, and 3 candidates, respectively.

- *Question 1*: This was an extract connected with the development of symbolic notation *and* the solution of polynomial equations. Candidates who addressed only one of these strands were penalised. A number of candidates failed to describe *specifically* what was going on in the extract (the formation of a quartic equation from one quadratic and two linear factors, and the identification of the roots), while others omitted to mention key features, such as the preservation of homogeneity and the presence of an imaginary root. A misconception included by more than one candidate was that Thomas Harriot invented the $+$ and $-$ signs (he didn't).
- *Question 2*: On the whole, this question was done quite well, though some candidates strayed a little too far from the extract. This question was about the notion of limit that was nascent in Newton's *Principia*; the wider style and rigour of the *Principia* were certainly relevant, but answers needed to go beyond this. A common omission was Newton's debt to John Wallis. Several candidates asserted that the definition given in the extract concerns infinitesimals — this is certainly one way to interpret it, but this is probably not what Newton had in mind.
- *Question 3*: This is another question that was generally done well. Better marks were obtained by those candidates who recalled the context of the so-called 'challenge problems', and who were able to comment on the place and form of publication, as indicated in the reference given for the extract. A common pitfall was not being precise enough about Euler's two different definitions of a function, and his reasons from switching from one to the other.
- *Question 4*: This is a question that was quite tricky on the surface, but became doable if one realised that it concerned the history of determinants.
- *Question 5*: This was one of the more difficult questions on the paper, if only because the material needed to answer it was covered in just half a lecture. A common omission in answers to this question was a clear statement of the parallel postulate itself. Surveys of prior study of the postulate could have been more thorough. Some candidates misinterpreted the content of the extract: Lobachesky was not trying to *prove* the parallel postulate in its traditional form, but was asserting that it needed to be taken as an assumption if one is to prove such seemingly obvious results as the fact that the internal angles of a triangle add up to two right angles.
- *Question 6*: Some candidates misinterpreted this extract as concerning Dedekind cuts and the nature of numbers, whereas it is in fact about the definition of infinite sets.

The Section B questions 7–9 were attempted by 5, 12, and 3 candidates, respectively.

- *Question 7*: This was one of the more straightforward questions on the paper, with ample material having been supplied in lectures. The better answers were those that covered more than just tangents and quadrature.

- *Question 8*: Answers to this question were generally well done. Many marks could be attained for a straightforward narrative, but the better answers were those that included some depth of detail.
- *Question 9*: This was a tricky question that attracted some decent answers which included points that had not previously occurred to the assessors.

The standard of the extended essays was generally good, though candidates relied a little too heavily on secondary sources — the better essays were those that engaged thoroughly with the primary materials. In some cases, more care was needed over the use of quotations — not just providing references, but also making clear that particular passages *were* quotations (some candidates sailed a little close to the wind in this respect). Some candidates ought to have been more aware that *everything except the bibliography* counts towards the word count; in particular, footnotes and endnotes are both counted. With regard to content, common pitfalls were: not establishing the link between Robert Woodhouse and the Analytical Society firmly enough; not being sufficiently critical in approaching the 'declinist' narrative of Babbage and others; underestimating the role of inertia in the British retention of Newtonian calculus.

Statistics Options

Reports of the following courses may be found in the Mathematics & Statistics Examiners' Report.

SB1.1/1.2: Applied and Computational Statistics

SB2.1: Foundations of Statistical Inference

SB2.2: Statistical Machine Learning

SB3.1: Applied Probability

Computer Science Options

Reports on the following courses may be found in the Mathematics & Computer Science Examiners' Reports.

CS3a: Lambda Calculus & Types

CS4b: Computational Complexity

Philosophy Options

The report on the following courses may be found in the Philosophy Examiners' Report.

102: Knowledge and Reality

127: Philosophical Logic

D. Comments on performance of identifiable individuals

1.Aggregation of marks for the award of the classification on the successful completion of Parts A and B

Classification for a candidate was determined through the following method:

- 10 units at Part A (counting A2 as a double-unit and, for candidates offering 6 long options, two of the long option papers as half-units)
- 6 units (or equivalent) at Part B.

The two average USMs will be:

1. The relative weightings of the Parts is as follows:
   (a) The weighting of Part A is 40%.
   (b) The weighting of Part B is 60%.
2. The relative weightings of the Parts is as follows:
   (a) The weighting of Part A is 100%.
   (b) The weighting of Part B is 0%.

The first class Strong Paper Rule says that to get a first class degree the candidate must have:

(a) average USM ≥ 69.5;

(b) at least 6 units in Parts A and B with USMs ≥ 70;

(c) at least 2 units in Part B with USMs ≥ 70.

The analogous rules apply for II.1 and II.2 degrees. The examiners considered all candidates near each borderline who had been caught by the Strong Paper Rule, that is, who satisfied (a) but failed (b) or (c), and so were due to receive the lower degree class. For two such candidates at the I/II.1 borderline the examiners decided to suspend the examination conventions, and placed the candidates in the first class.

2. Prizes

Prizes were awarded as follows.

Gibbs Prize £500: Zhenyu Yang, Corpus Christi College
Gibbs Prize £200: Guoxi Liu, Trinity College

Part B Junior Mathematical Prize £200: Zizheng Fang, Exeter College
Part B Junior Mathematical Prize £200: Rebekah Glaze, Keble College

IMA Prize: Kira Patel, Mansfield College

F. Names of members of the Board of Examiners

- **Examiners:**

  Prof. Ben Green (Chair)
  Dr Neil Laws
  Prof. Radek Erban
  Prof. Xenia De La Ossa
  Prof. Alain Goriely
  Prof. Gui Qiang Chen

  Prof Matt Tointon (External)
  Dr Ed Brambley (External)

- **Assessors:**
  Prof. Andras Juhasz
  Prof. Andrea Mondino
  Prof. Andreas Muench
  Prof. Andrew Dancer
  Dr Catherine Wilkins
  Prof. Christopher Beem
  Prof. Christopher Breward
  Prof. Christopher Hollings
  Prof. Damian Rössler
  Prof. Dawid Kielak
  Prof. Dmitry Belyaev
  Prof. Dominic Joyce
  Prof. Endre Suli
  Prof. François Caron
  Prof. Ian Hewitt
  Prof. James Newton
  Prof. Jan Kristensen
  Prof. Jan Obloj
  Dr. Jaroslav Fowkes
  Dr. Jasmina Panovska-Griffiths
  Prof. Jason Lotay
  Prof. Jochen Koenigsmann
  Prof. Konstantin Ardakov
  Prof. Lionel Mason
  Prof. Luc Nguyen
  Dr. Lukas Brantner
  Prof. Mark Mezei
  Prof. Paul Balister
  Prof. Peter Howell
  Prof. Radek Erban
  Dr Robert Hinch
  Prof. Ruth Baker
  Prof. Sam Cohen
  Prof. Sam Howison
  Prof. Yuji Nakatsukasa
  Prof. Zhongmin Qian
  Dr Martin Bays
  Dr Jinhe Ye
  Dr Catherine Wilkins
  Prof. Melanie Rupflin
  Dr Murad Banaji
  Dr Richard Earl
  Dr Shiwei Liu
  Mr Edgar Sucar
  Prof. Alain Goriely
  Prof. Benjamin Hambly
  Prof. Christop Reisinger
  Prof. Coralia Cartis
  Prof. James Maynard
  Prof. Kevin McGerty
  Pro. Victor Flynn