Examiners' Report: Final Honour School of Mathematics Part B Trinity Term 2022

November 30, 2022

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

  See Table 1.

	Numbers					Percentages %				
	2022	(2021)	(2020)	(2019)	(2018)	2022	(2021)	(2020)	(2019)	
I	55	(51)	(73)	(59)	(58)	41.04	(39.84)	(46.5)	(39.07)	(38.16)
II.1	53	(58)	(66)	(67)	(67)	39.55	(45.31)	(42.04)	(44.37)	(44.08)
II.2	24	(18)	(13)	(20)	(25)	17.91	(14.06)	(8.28)	(13.25)	(16.45)
III	2	(1)	(4)	(4)	(2)	1.49	(0.78)	(2.55)	(2.65)	(1.32)
P	0	(1)	(0)	(0)	(2)	0	(0.64)	(0)	(0)	(1.52)
F	0	(0)	(1)	(0)	(0)	0	(0.66)	(0)	(0)	(0)
Total	134	(157)	(151)	(152)	(132)	100	100	(100)	(100)	(100)

Table 1: Numbers and percentages in each class

- **Numbers of vivas and effects of vivas on classes of result.**

  As in previous years there were no vivas conducted for the FHS of Mathematics Part B.

- **Marking of scripts.**

  BE Extended Essays and coursework submitted for the History of Mathematics course were double marked. Due to an unforeseen shortage of available markers, BSP projects were single-marked, but the submissions for each project were then reviewed by the assessor for another project to ensure the marks awarded were kept to a consistent standard.

  The remaining scripts were all single marked according to a pre-agreed marking scheme which was strictly adhered to[1]. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**

  See Table 5 on page 12.

B. New examining methods and procedure in the 2022 examinations

In light of the unusual circumstances in which this year's candidates for Part B had been taught and examined up to this point, a special committee was formed to consider how their examinations should be arranged. Its recommendation, made in September 2021, was that candidates should be permitted to bring a "summary sheet" with them into each of their examination. Candidates were thus permitted to use both sides of a sheet of A4 paper to This consisted of both sides of a sheet of A4 paper on which candidates could record whatever notes they wished on, and were free to consult this sheet while taking that paper. This had consequences both for the nature of questions that were set, and for the experience of in-person examinations that candidates had, but it is difficult to know what, if any, affect it had on results of the examination.

[1] In the case of one paper, the marking scheme on the paper given to candidates differed slightly from the pre-agreed scheme. The latter was used when marking.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

There were a number of typographical errors in mathematics examination papers which caused complications in assessing the work of candidates who offered those papers. In almost all of these errors, the correction required should have been evident to anyone with a basic knowledge of the material, but given that candidates should feel able to assume that their examination questions are correctly posed, even very able candidates could have spent time second-guessing their assessment that a question was posed incorrectly.

Had it been possible, as has previously been the case, for the assessor who wrote the paper (or someone with suitable knowledge of the subject acting as their deputy) to be present at the start of these examinations, it is likely that all of these errors would have been corrected, either by the assessor spotting the error themselves, or in response to a query from a candidate. It is unfortunate that the University Regulations currently do not permit this safety-net for errors which are more likely to occur in papers for technical subjects such as mathematics.

Unlike in the last two years, examinations this year did not have general provisions in place as a result of the pandemic, but it impact was nevertheless noticeable in some cases through MCE applications.

D. Notice of examination conventions for candidates

The first Notice to Candidates was issued on 25 March 2022 and the second notice on 25 May 2022.

All notices and the examination conventions for 2022 are on-line at http://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments.

Part II

A. General Comments on the Examination

The examiners would like to record their heartfelt thanks to all those who helped in the preparation, administering, and assessing of this year's examinations. We would like in particular to thank Elle Styler for her unflappable efficiency throughout the whole process, and Clare Sheppard, Charlotte Turner-Smith and Waldemar Schacklow, each of whom provided indispensable assistance.

In addition the internal examiners would like to express their gratitude to Professor John Hunton and Professor Anne Skeldon for carrying out their duties as external examiners in such a constructive and supportive way during the year (in particular for accommodating the tardy arrival of some of the draft examination papers) and for their input thoughtful contributions during the final examiners' meetings.

Standard of performance

The standard of performance was broadly in line with recent years. In setting the USMs, we took note of

- the Examiners' Report on the 2021 Part B examination, and in particular recommendations made by last year's examiners, and the Examiners' Report on the 2021 Part A examination, in which the 2022 Part B cohort were awarded their USMs for Part A;
- the guidelines provided by the Mathematics Teaching Committee, including its recommendations on the proportion of candidates that might be expected in each class.

It should also be noted however, that because each of the the last few years has presented examiners with its own highly exceptional set of circumstances, comparability between cohorts in those years has been more difficult to consider than it would usually be.

Setting and checking of papers and marks processing

The internal examiners initially divided between them responsibility for the units of assessment (that is, the exam papers and projects). It was noted that the research interests of this year's board were not distributed so as to allow the examiners to be responsible for topics in the general area of expertise.

Requests to course lecturers to act as assessors, and to act as checker of the questions of fellow lecturers, were sent out early in Michaelmas Term, with instructions and guidance on the setting and checking process, including a web link to the Examination Conventions.

Most assessors acted properly, though a small number failed to meet the stipulated deadlines by a considerable margin, and some papers needed significant revision from the draft first seen by the examiners. It might be useful to emphasise to lecturers in future years that it is helpful if they can make the examiners aware of any potential delays or difficulties in producing an examination paper as early as possible, to avoid placing unreasonable demands on the external examiners.

The internal examiners met at the beginning of Hilary Term to consider those draft papers on Michaelmas Term courses which had been submitted in time; consideration of the remaining papers had to be deferred. Where necessary, corrections and any proposed changes were agreed with the setters. The revised draft papers were then sent to the external examiners. Feedback from external examiners was given to examiners and to the relevant assessor for response. The internal examiners at their meeting in mid Hilary Term considered the external examiners' comments and the assessor responses, making further changes as necessary before finalising the questions. The process was repeated for the Hilary Term courses, but necessarily with a much tighter schedule.

A team of graduate checkers, under the supervision of Elle Styler, sorted all the marked scripts for each paper of this examination, cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or incorrectly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way a number of errors were corrected,and each change was signed by one of the examiners who were present throughout the process. A check-sum was also carried out to ensure that marks entered into the database were correctly read and transposed from the marks sheets.

Throughout the examination process, candidates were treated anonymously, identified only by a randomly-assigned candidate number.

Standard and style of papers

It was noted in the final meeting that B2.2 and B3.1 had both proved to be very challenging for candidates. It was noted that unexpectedly difficult (or easy) papers can present difficulties in assigning USMs as the scalings in such cases are often overly sensitive to small changes in raw marks.

Timetable

Examinations began on Monday 30 May and ended on Friday 17 June.

Consultation with assessors on written papers

Assessors were asked to submit suggested ranges for which raw marks should map to USMs of 60 and 70 along with their mark-sheets, and almost all did so. In most cases these were in line with the assignments given by the assessors, and where there were discrepancies, the examiners in general settled on boundaries which were somewhat more generous than the assessors recommendation.

Determination of University Standardised Marks

We followed the Department's established practice in determining the University standardised marks (USMs) reported to candidates. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part A performances of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage). Working with the data for this population, numbers N_1, N_2 and N_3 are first computed for each paper: N_1, N_2 and N_3 are, respectively, the number of candidates taking the paper who achieved in Part A average USMs in the ranges $[69.5, 100]$, $[59.5, 69.5)$ and $[0, 59.5)$.

The algorithm converts raw marks to USMs for each paper separately. For each paper, the algorithm sets up a map $R \to U$ (R = raw, U = USM) which is piecewise linear. The graph of the map consists of four line segments: by default these join the points $(100, 100)$, $P_1 = (C_1, 72)$, $P_2 = (C_2, 57)$, $P_3 = (C_3, 37)$, and $(0, 0)$. The values of C_1 and C_2 are set by the requirement that the number of I and II.1 candidates in Part A, as given by N_1 and N_2, is the same as the I and II.1 number of USMs achieved on the paper. The value of C_3 is set by the requirement that P_2P_3 continued would intersect the U axis at $U_0 = 10$. Here the default choice of *corners* is given by U-values of 72, 57 and 37 to avoid distorting nonlinearity at the class borderlines.

The results of the algorithm with the default settings of the parameters provide the starting point for the determination of USMs, and the Examiners may then adjust them to take account of consultations with assessors (see above) and their own judgement. The examiners have scope to make changes, either globally by changing certain parameters, or on individual papers usually by adjusting the position of the corner points P_1, P_2, P_3 by hand, so as to alter the map raw $\to$ USM, to remedy any perceived unfairness introduced by the algorithm. They also have the option to introduce additional corners. For a well-set paper taken by a large number of candidates, the algorithm yields a piecewise linear map which is fairly close to linear, usually with somewhat steeper first and last segments. If the paper is too easy or too difficult, or is taken by only a few candidates, then the algorithm can yield anomalous results—very steep first or last sections, for instance, so that a small difference in raw mark can lead to a relatively large difference in USMs. For papers with small numbers of candidates, moderation may be carried out by hand rather than by applying the algorithm.

Following customary practice, a preliminary, non-plenary, meeting of examiners was held in advance of the plenary examiners' meeting to compare the default settings produced by the algorithm alongside the reports from assessors. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low. Where the examiners were in doubt as to the most appropriate scaling, the preliminary scalings were held over to the plenary meeting, where the factors considered by those in the preliminary meeting were reviewed and weighed before a final decision was made.

Table 2 on page 9 gives the final positions of the corners of the piecewise linear maps used to determine USMs.

In accordance with the agreement between the Mathematics Department and the Computer Science Department, the final USM maps were passed to the examiners in Mathematics & Computer Science. USM marks for Mathematics papers of candidates in Mathematics & Philosophy were calculated using the same final maps and passed to the examiners for that School.

Comments on use of Part A marks to set scaling boundaries

None.

Mitigating Circumstance Notice to Examiners

A subset of the examiners (the 'Mitigating Circumstances Panel') attended a pre-board meeting to band the seriousness of the individual notices to examiners. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate.

The full board of examiners considered all of the notices in the final meeting, along with a number of MCEs carried over from Part A. The examiners considered each application alongside the candidate's marks and the recommendations proposed by the Part A 2020 Exam board.

Table 2: Position of corners of the piecewise linear maps

Paper	P_1	P_2	P_3	Additional Corners	N_1	N_2	N_3
B1.1	12.70;37	22.1;57	41.6;72	50;100	11	20	6
B1.2	13.61;37	21;57	40.2;72	50;100	19	22	8
B2.1	7.99;37	25;60	44;72	50;100	21	11	0
B2.2	2;30	15;60	24;70	50;100	13	7	1
B3.1	5.57;37	9.7;57	32;72	50;100	27	16	7
B3.2	18;50	31;57	41.5;72	50;102	13	5	3
B3.3	12.64;37	22;57	37;72	50;100	13	9	3
B3.4	10.00;37	17.4;57	35.4;72	50;100	18	13	4
B3.5	12.52;37	21.8;57	42.8;72	50;100	19	11	3
B4.1	7.64;37	16;50	30;70	50;100	31	14	5
B4.2	5;30	16;50	21;60	30;70, 50;100	25	12	4
B4.3	18.44;37	30;60	36.6;72	50;100	8	3	2
B4.4	30;60	35;70	50;100		4	2	1
B5.1	14;50	41.4;72	50;100		6	14	13
B5.2	10.86;37	18.9;57	44.4;72	50;100	14	29	9
B5.3	6.49;37	11.3;57	29;70	50;100	5	11	7
B5.4	13.27;37	23.1;57	42.6;72	50;100	4	9	6
B5.5	14.36;37	23;57	40;72	50;100	10	21	16
B5.6	14.70;37	25.6;57	46;70	50;100	7	20	9
B6.1	25;50	32;60	42;70	50;100	3	9	7
B6.2	13.10;37	22.8;57	43.8;72	50;100	6	12	7
B6.3	7.99;37	13.9;57	33;70	50;100	2	5	4
B7.1	15.51;37	27;57	42;72	50;100	8	6	5
B7.2	15.68;37	27.3;57	40.8;72	50;100	7	7	6
B7.3	13.56;37	23.6;57	36;70	50;100	6	2	0
B8.1	14;50	18.3;57	31.8;72	50;100	29	31	18
B8.2	13.39;37	23.3;57	36;70	50;100	15	14	7
B8.3	9;40	23;50	27;57	37;70, 50;100	18	37	19
B8.4	10.17;37	17.7;57	45;70	50;100	9	29	14
B8.5	12;37	28.5;57	46;70	50;100	6	31	9
BSP	2000;100				1	5	6
SB1	18.15;37	31.6;57	58.6;72	66;100	11	29	11
SB1	34;100				10	35	6
SB2.1	15.97;37	27.8;57	45;70	50;100	14	41	17
SB2.2	11.78;37	20.5;57	43;72	50;100	18	31	15
SB3.1	11;40	19.5;57	42;72	50;100	24	52	22
SB3.2	7.64;37	13.3;57	34;70	50;100	3	6	6

B. Equality and Diversity issues and breakdown of the results by gender

Table 3: Breakdown of results by gender

Class	Number								
	2022			2021			2020		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	5	50	55	13	38	51	18	55	73
II.1	19	34	53	22	36	58	28	38	66
II.2	15	9	24	4	14	18	3	10	13
III	1	1	2	1	0	1	1	3	4
P	0	0	0	0	1	1	0	0	0
F	0	0	0	0	0	0	0	1	1
Total	40	93	134	40	89	129	50	107	157
Class	Percentage								
	2022			2021			2020		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	12.5	53.19	41.04	32.5	42.70	39.53	36	51.4	46.50
II.1	47.5	36.17	39.56	55	40.45	44.96	56	35.51	42.04
II.2	37.5	9.57	18.32	10	15.73	13.95	6	9.35	8.28
III	2.5	1.06	5.88	2.5	0	0.78	2	2.8	2.55
P	0	0	0	0	1.12	0.78	0	0	0
F	0	0	0	0	0	0	0	0.93	0.64
Total	100	100	100	100	100	100	100	100	100

Table 3 shows the performances of candidates broken down by gender. It reveals a troubling feature of this year's results, which for that reason is important to highlight: the proportion of first-class degrees obtained by women dropped markedly this year, from 32.5% in 2021 to only 12.5% in 2022. This contrasts starkly with the proportion of first-class degrees awarded to male candidates, which jumps from 42.7% in 2021 to 53.19% in 2022, the highest in recent years (and presumably the highest ever). As a result, the overall proportion of first-class degrees awarded appears relatively stable, moving only slightly from 42.70% in 2021 to 41.04%.

The anonymity of the assessment process means that it is not easy to discern what factors may have contributed to this divergence in the performance of men and women in this year's exams.

Table 4: Rank and percentage of candidates with this or greater overall USMs

Av USM	Rank	Candidates with this USM and above	%
90	1	1	0.75
88	2	2	1.49
87	3	3	2.24
85	4	5	3.73
84	6	6	4.48
83	7	7	5.22
82	8	10	7.46
81	11	16	11.94
80	17	21	15.67
79	22	22	16.42
78	23	24	17.91
77	25	28	20.9
76	29	31	23.13
75	32	36	26.87
74	37	38	28.36
73	39	40	29.85
72	41	46	34.33
71	47	50	37.31
70	51	55	41.04
69	56	57	42.54
68	58	62	46.27
67	63	66	49.25
66	67	71	52.99
65	72	75	55.97
64	76	79	58.96
63	80	89	66.42
62	90	94	70.15
61	95	101	75.37
60	102	107	79.85
59	108	111	82.84
58	112	113	84.33
57	114	117	87.31
56	118	120	89.55
55	121	122	91.04
54	123	124	92.54
53	125	126	94.03
52	127	128	95.52
51	129	130	97.01
50	131	131	97.76
49	132	133	99.25
40	134	134	100

C. Detailed numbers on candidates' performance in each part of the examination

The number of candidates taking each paper is shown in Table 5. Details of papers with 5 or less candidates are not included.

Table 5: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
B1.1	35	35.71	8.76	70.51	12.68
B1.2	44	34.75	7.68	69.61	9.72
B2.1	33	38.03	10.19	74.28	13.61
B2.2	23	17.83	12.44	58.35	21.27
B3.1	51	25.2	12.76	67.63	16.52
B3.2	21	41.33	7.36	77.29	13.43
B3.3	25	34.2	7.56	71.24	10.41
B3.4	34	31.5	9.71	69.32	13.52
B3.5	33	38.24	7.8	71.64	10.15
B4.1	46	28.57	9.55	68.26	14.22
B4.2	40	30.28	10.76	72.05	14.83
B4.3	13	36.77	7.61	72.77	14.98
B4.4	7	40.14	7.06	80.29	14.12
B5.1	28	27.43	10.89	60.61	13.13
B5.2	40	34.05	8.45	66	5.85
B5.3	23	23.83	10	67	10.74
B5.4	19	31.79	9.34	64.68	12.26
B5.5	33	32.12	9.81	65.52	14.52
B5.6	29	39.24	9.36	71.45	16.19
B6.1	16	9.35	9.51	62.19	15.73
B6.2	19	31.05	9.35	62.11	10.14
B6.3	11	23.18	10.48	62.36	13.45
B7.1	19	35.74	7.52	67.58	11.26
B7.2	21	34.86	8.81	67.71	14.71
B7.3	9	34.44	7.89	70.11	11.95
B8.1	58	27.12	9.52	66.66	13.89
B8.2	31	32.48	9.74	67.74	15.75
B8.3	45	33.33	8.53	67.14	12.85
B8.4	40	31.12	11.03	64.75	12.54
B8.5	41	38	7.68	67.17	13.04
SB1	8	32.12	12.99	63.25	4.68
SB2.1	31	38.84	7.71	68.1	11.39
SB2.2	21	31.38	11.19	65.55	9.58
SB3.1	58	30.48	10.68	63.64	11.2
SB4	-	-		-	
BEE	8	-	-	81.13	14.87
BSP	TBC	TBC	TBC	TBC	TBC

Individual question statistics for Mathematics candidates are shown below for those papers offered by fewer than six candidates.

Paper B1.1: Logic

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.24	19.24	3.9	34	0
Q2	15.3	15.3	6	20	0
Q3	18.13	18.13	5.19	16	0

Paper B1.2: Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.21	17.46	6.76	26	2
Q2	20.41	20.41	3.72	39	0
Q3	10.8	12.13	5.03	23	6

Paper B2.1: Introduction to Representation Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.37	19.37	5.55	30	0
Q2	19.1	19.1	5.47	31	0
Q3	16.4	16.4	4.88	5	0

Paper B2.2: Commutative Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	9.9	9.9	7.78	21	0
Q2	0.6	1.5	1.34	2	3
Q3	8.65	9.05	5.98	22	1

Paper B3.1: Galois Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.37	12.91	7.49	44	2
Q2	7.88	9	5.93	33	6
Q3	14.37	16.8	7.89	25	5

Paper B3.2: Geometry of Surfaces

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.14	21.14	3.38	21	0
Q2	20.19	20.19	4.71	21	0
Q3	-	-	-	-	-

Paper B3.3: Algebraic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.8	13.95	3.91	19	1
Q2	17.94	18.53	4.02	17	1
Q3	19.2	19.64	4.28	14	1

Paper B3.4: Algebraic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.16	18.16	3.45	19	0
Q2	14.58	14.58	5.74	24	0
Q3	14.65	15.04	5.93	25	1

Paper B3.5: Topology and Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.17	18.86	7.64	21	3
Q2	17.04	17.44	3.29	25	2
Q3	20.32	21.5	5.26	20	2

Paper B4.1: Functional Analysis I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.80	14.8	4.53	41	0
Q2	14.58	14.58	5.94	36	0
Q3	13	13	4.24	14	0

Paper B4.2: Functional Analysis II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.69	15.69	6.12	39	0
Q2	14.57	14.93	5.81	29	1
Q3	12.92	13.83	6.92	12	1

Paper B4.3: Distribution Theory and Fourier Analysis: An Introduction

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.83	18.83	4.37	12	0
Q2	19.56	19.56	2.65	9	0
Q3	15.2	15.2	4.49	5	0

Paper B4.4: Fourier Analysis and PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21	21	2.28	6	0
Q2	17.5	17.5	5.97	4	0
Q3	21.25	21.25	3.86	4	0

Paper B5.1: Stochastic Modelling and Biological Processes

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.43	11.88	3.88	24	4
Q2	12.36	12.36	5.94	14	0
Q3	18.24	18.24	7.54	17	0

Paper B5.2: Applied PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.24	17.24	4.4	38	0
Q2	15.55	15.8	5.34	30	1
Q3	19.42	19.42	4.03	12	0

Paper B5.3: Viscous Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	5.88	5.88	4.97	8	0
Q2	15.57	15.57	3.9	21	0
Q3	9.42	10.24	6.22	17	2

Paper B5.4: Waves and Compressible Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.95	17.95	4.02	19	0
Q2	14.93	14.93	5.57	14	0
Q3	10.8	10.8	7.29	5	0

Paper B5.5: Further Mathematical Biology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.34	13.65	5.17	31	1
Q2	19.4	19.4	5.32	5	0
Q3	18	18	6.64	30	0

Paper B5.6: Nonlinear Systems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.09	20.1	6.43	21	2
Q2	17.58	17.92	5.54	25	1
Q3	22.33	22.33	3.52	12	0

Paper B6.1: Numerical Solution of Differential Equations I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.83	15.6	5.78	5	1
Q2	17.14	17.14	3.03	14	0
Q3	19.75	19.75	4.19	12	0

Paper B6.2: Numerical Solution of Differential Equations II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.05	15.05	5.84	19	0
Q3	16	16	4.33	19	0

Paper B6.3: Integer Programming

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.55	10.55	5.56	11	0
Q2	12.56	12.56	3.57	9	0
Q3	10.67	13	11.71	2	0

Paper B7.1: Classical Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.44	18.44	4.77	18	0
Q2	17.62	17.62	4.82	13	0
Q3	16.86	16.86	4.14	7	0

Paper B7.2: Electromagnetism

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.38	18.38	4.36	21	0
Q2	14.86	14.86	5.19	14	0
Q3	19.71	19.71	4.6	7	0

Paper B7.3: Further Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.57	17.57	5.19	7	0
Q2	17.17	17.17	3.13	6	0
Q3	16.8	16.8	6.3	5	0

Paper B8.1: Martingales through Measure Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.29	14.06	5.35	31	3
Q2	13.53	13.53	5.01	34	0
Q3	13.19	13.28	5.4	51	1

Paper B8.2: Continuous Martingales and Stochastic Calculus

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.87	16.87	5.19	23	0
Q2	11.76	13.24	6.4	17	4
Q3	16	17.91	6.95	2	3

Paper B8.3: Mathematical Models of Financial Derivatives

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.5	16.34	5.01	32	4
Q2	17.26	17.26	4.2	23	0
Q3	17.06	17.06	4.35	34	0

Paper B8.4: Communication Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.53	18.53	5.13	38	0
Q2	9.67	9.67	5.43	21	0
Q3	16.29	16.9	7.01	20	1

Paper B8.5: Graph Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	22.03	22.03	2.41	38	0
Q2	17.84	17.84	6.23	19	0
Q3	15.04	15.28	5.55	25	1

Paper SB1.1/1.2: Applied Statistics/Computational Statistics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13	13	1	3	0
Q2	15	15	2.16	4	0
Q3	12.25	16.33	9.71	3	0
Q4	8	9.5	7.94	2	0
PR	22.5	22.5	1	4	0

Paper SB2.1: Foundations of Statistical Inference

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.18	19.3	5.65	10	1
Q2	19.18	19.18	3.78	28	0
Q3	19.75	19.75	4.59	24	0

Paper SB2.2: Statistical Machine Learning

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.38	15.38	6.79	16	0
Q2	14.14	14.14	3.24	7	0
Q3	15.9	16.52	6.71	19	1

Paper SB3.1: Applied Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.11	13.56	5.92	43	2
Q2	13.92	13.92	3.13	38	0
Q3	13.18	13.18	5.21	11	0

Assessors' comments on sections and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. The examiners have not included assessors' statements suggesting where possible borderlines might lie; they did take note of this guidance when determining the USM maps. Some statistical data which can be found in Section C above has also been removed.

B1.1: Logic

Question 1 Almost everyone did this question fairly well. The last part (completeness of the new deductive system) seems to have been the most challenging - it was often overlooked that both the set of assumptions and the sentence to be derived from them are in the new language.

Question 2 Here the standard of solutions varied considerably. Part (c) was clearly the hardest. Very few candidates realized that in (c)(i) the isomorphism should also respect the interpretation of function symbols. And for (c)(ii) and (c)(iii) a good number of solutions showed no awareness of the fact that the Completeness Theorem could be used "for free" in this question.

Question 3 This question was rather popular among M&C students, but not at all among M&P's. In part (b)(ii) it was often shown that no finite subset of the given infinite set of axioms would suffice, instead of showing that no finite set of axioms at all would do the job. In (c)(iv) the relevance of the underlying language was generally not clearly seen, and so, in particular, the answer to the second (trick) part of the question went wrong in many cases.

B1.2: Set Theory

After two years of online teaching and online exams, this year things have gone back to normal, more or less (though the Covid-19 pandemic hasn't gone away). I am not in a position to speak for the students, but for this particular member of staff, the experience has been transformative. Perhaps these dreadful years have been a reminder not to take things for granted.

Two things seem to me to have changed.

The first is a vague impression, rather than an established fact. I found myself wondering whether two years of lockdowns and online exams have left people a little out of practice with in-person exams. There were failures to follow correct procedure, mostly trivial, but one serious: many candidates did not clearly indicate which of their working was rough.

My understanding of the rules is that the examiner must disregard any working that is crossed out or identified as rough, and must take account of any other working. Frequently I encountered what common sense said was rough working, but which the rules seemed to say must be marked. Fortunately I did not encounter any situations where doing so would have made me give a mark lower than the one I would have wanted to give. And there were instances where I felt (but of course cannot prove) that a candidate had been under time pressure, in a way that could have been avoided with more practice in exam technique.

The second is a deliberate innovation in procedure. From this year, candidates are allowed to take an A4 sheet of revision material into the exam. This raises the question of how much difference this makes to the quality of the answers given. My impression is (and it's only an impression), not much. If this is correct, then I can think of three possible explanations, which are not mutually exclusive. (1) You can't fit much on one A4 piece of paper. (2) Revision has always been about understanding and knowing how to use methods, rather than rote learning. Or (3): efficient ways to compress information into one A4 sheet of paper need to be identified, and taught.

Question 1. Question 1. is about the ordinals, and there were many good solutions to it.

Variations on part (a) have occurred on exam papers many times in the past, and candidates seemed on the whole to be well-prepared for it. The main difficulty in part (a) was vagueness or confusion about the rules of ordinal arithmetic; several candidates stated, for example, that ω^2 was equal to ω. They may have been confusing the systems of ordinal and cardinal arithmetic. Many candidates were completely successful in this part.

In part (b), again there were many successful solutions. In part (i), some candidates tried to use the Tarski Fixed Point Theorem, which does not apply since ω_1 is not equal to the powerset of any set. One or two, however, managed to make the necessary adjustments. In part (iii), if one is finding a sequence converging to the hoped-for joint fixed point for f and g, then both f and g have to be involved in the definitiion of that sequence; not everyone succeeded in doing this.

Part (c) proved rather tougher. In part (i), the fact that for many α, it might be the case that $f(\alpha)$ is strictly less than α, caused difficulty. In part (ii), the main task is to rule out the possibility that as α increases, $f(\alpha)$ approaches closer and closer to a limit without ever reaching it; many candidates did not appreciate this.

Question 2. Question 2. is about ordered triples, and about Zorn's Lemma.

Variations on part (a) have appeared many times before, and many good answers to it were given. A few people noticed that in (ii)iii., what is presented is, if a, b and c are all different, a strict linear order on the set $\{a, b, c\}$. The main difficulty in (ii)iii. is posed by triples in which at least two of the entries are equal.

Part (b) is about Zorn's Lemma. Part (ii) has appeared before on an exam paper. Many people completed it successfully but also quite a few did not, some due to confusion as to which set to apply Zorn's Lemma to, some through not checking correctly that Zorn's Lemma was applicable.

Part (iii) could be obtained directly from part (ii): for example, the set of all subsets B of the reals such that $B \cup \{p : p \text{ is prime}\}$ is linearly independent over the rationals, is of finite character. Most candidates, however, did a separate Zorn's Lemma argument, with a great deal of success. It proved difficult, though, to define precisely the right set to apply Zorn's Lemma to; some plausible alternatives turn out not to quite work for subtle reasons.

Question 3.

Question 3. is about cardinal arithmetic.

In part (a) (which is again a standard type of question for this paper), (i), (ii) and (iii) have all occurred before. Many correct answers were given. There were, however, some unusual errors, such as the assertion that the cardinality of the set of real numbers is $\aleph_0$. Several candidates implicitly assumed the Continuum Hypothesis (saying, for example, that because an uncountable set had size less than or equal to $2^{\aleph_0}$, that its size must be $2^{\aleph_0}$ precisely). Often in questions of this sort, topology is relevant to finding the cardinality of a set, and that is the case in part (iv). The key here is to recall that a monotonically increasing function on the reals is continuous except at countably many points, and so it is determined by what happens at the jumps, and by what happens at a countable dense set. These facts dramatically cut the number of possibilities. Part (v) proved difficult. There were a few successes, of which I believe just one used the solution that I had in mind: given a basis, you can multiply any subset of it by some non-zero rational different from 1 to get another basis, and thus there are very many bases.

In part (b), we explore the strangeness of cardinal arithmetic without the Axiom of Choice. This part could be seen as propaganda for the Axiom of

Choice, or it could be seen as an exploration of some interesting possibilities that it closes off.

Part (i) was done successfully by many candidates, despite the strangeness of the result obtained. Various successful strategies were then found for part (ii). Few, however, managed parts (iii) and (iv) (a version of part (iv) has appeared on previous papers), but many successfully completed part (v). The key to part (iv) is to use weirdness of a totally ordered set to define an injection into it from ω, thus obtaining a contradiction to part (ii).

0.1 B2.1 Introduction to Representation Theory

Question 1. Question 1 was very popular with the students with only four people not choosing to do this question. Part (a) was done very well; when calculating the character of the Hom(V,W) representation, those students who spotted the shortcut of using the basis of Hom(V,W) corresponding to the matrix units in this vector space, associated with a choice of eigenbases for V and W were spared the need to talk about tensor products. Part (b)(i) was more tricky; using the fact that the character table is square would have made it a bit easier. Part (b)(iii) was done very well indeed.

Question 2. Question 2 was again very popular, with only three people not choosing to do this question. For part (a)(iii), several people either forgot to give a counterexample, or gave a counterexample over a different field to the complex numbers — please read the question more carefully! Part (b)(v) was done fairly well, although it did catch out some students.

Question 3. Question 3 was the least popular. Part (a)(i) caused some trouble, and the injectivity required in part (b) was a bit tricky. Part (c) was harder still – although several people wrote down the correct map p, no-one was able to prove that it is an isomorphism.

B2.2: Commutative Algebra

Question 1: The ideals $p^(n)$ considered in this exercise also appear in the proof of Krull's principal ideal theorem described in the lectures, where some of the properties listed in (a) are mentioned (however it is not necessary to know the proof of this theorem to do this question). The ideal $p^(n)$ is called the n-th symbolic power of p in the literature. Not many candidates thought of using the properties of local Artin rings (local noetherian rings of dimension 0) in (b). In (a), many candidates got bogged down in complicated logical distinctions, due to the fact that their answer to (a) (i) was too complicated (even if correct).

Question 2: This question was almost never attempted and I therefore cannot provide any feedback on the solutions given by the candidates. The core idea of this question is that a property of a finitely generated module which holds locally - ie after localisation at a prime ideal p - often holds in a neighbourhood of p for the Zariski topology, in particular in a basic open set containing it.

Question 3: Part (a) can be reformulated as saying that a polynomial f with the properties listed must be in the radical of the ideal $(f_1, \ldots, f_k)$. The ring $Z[x_1, \ldots, x_r]$ is Jacobson, so the radical of $(f_1, \ldots, f_k)$ is its Jacobson radical. In combination with Q5 of Sheet 3, this quickly leads to the result. Very few candidates thought of using the Jacobson property. Part (b) was done correctly by most students. Part (c) is a straightforward application of (b) and the going-up theorem, once it is seen that it is sufficient to consider the situation of an inclusion of a domain A into a domain B, where B is integral over A. Many candidates struggled with the reduction of the argument to this case.

B3.1: Galois Theory

Question 1 This was the most popular question. For (a), note that the multiplicative group of non zero elements of a finite field F_{p^n} is cyclic of order $p^n - 1$. Hence F_{p^n} is the splitting field of $x^{p^n-1} - 1$. Most candidates answered (b) and (c) correctly. Part (d) was answered correctly by very few candidates. One way to approach (d) is to generalise the computation made in (c) (iii).

Question 2 This question was answered correctly by few candidates. For part (a), note that the result can be proven by induction on n, if one uses the fact that the product of all the Φ_d with $d|n$ equals $x^n - 1$. Part (b) follows from the fact that under the given assumptions $x^n - 1$ has multiple roots. Hence its derivative nx^{n-1} must vanish, ie $p|n$. For (d), it is sufficient by (c) to show that $\Phi_n(k)$ has infinitely many different prime factors as k varies. To show this, suppose for contradiction that there are only finitely many such factors, say $p_1, \ldots, p_l$. By (a), a prime factor of $\Phi_n(p_1 \ldots p_n)$ is not in the set $p_1, \ldots, p_l$, which is absurd.

Question 3. Part (a) was answered correctly by most candidates. In part (b), few candidates thought of using the hint and assumed irreducibility of T without proof. For (b) (ii), some candidates did not think of using (b) (i), which immediately shows that sigma has the right shape. In part (c), not everybody though of using Artin's lemma, which gives one direction in the equivalence.

B3.2: Geometry of Surfaces

All candidates did questions 1 and 2, and none attempted question 3, probably as it was longer and looked harder.

Question 1 was well done by most candidates. Question 1(d) caused the most difficulties: about half proved the $\binom{g+n-1}{n-1}$ count by a 'stars and bars' combinatorial argument, but noone gave a correct proof by induction on n.

Question 2 was more mixed. A large minority made calculational mistakes in 2(b), and some were then doomed, as they were trying to solve the wrong o.d.e.s in (c),(d). A distressing number of candidates, having found the principal curvature equation in the form $(a\kappa + b)(c\kappa + d) = 0$, rather than writing down $\kappa = -b/a, -d/c$, multiplied it out and used the formula for solutions of a quadratic, not necessarily correctly. For the 'prove carefully ...' in part (c), the best method was to verify that the given family of solutions satisfied the second order o.d.e. (easy), and realized all possible values $(0, \infty) \times \mathbb{R}$ of $(f(0), f'(0))$, and then appealing to properties of solutions of o.d.e.s (Picard's Theorem, though I gave full marks at this point to anyone who made clear that the o.d.e. and $(f(0), f'(0))$ determined f uniquely). Some candidates also more-or-less managed to integrate the o.d.e. directly.

B3.3 Algebraic Curves

Question 1: Question 1 was slightly more popular than the other two questions but was the least well answered. It involved relatively basic material from the start of the course, but much of it was not very similar to material seen before in problem sheets or past exam questions, and there were no solutions which were complete or even nearly complete.

Question 2: Question 2 contained fairly standard applications of Bezout's theorem and was mostly answered very well, though the very last part of the question was decidedly tricky.

Question 3: Question 3 on the Weierstrass $\wp$-function and related topics was slightly less popular than Questions 1 and 2, but almost all the answers were very competent and quite a number were very good. As with the other questions, the last part was hard and there were no perfect solutions, but some candidates got very close.

B3.4: Algebraic Number Theory

Question 1 was answered by 22 (out of 43) candidates and it was done to a high standard; parts (a) and (b) were well answered; only a few students managed to do part (c) completeley. Question 2 was answered by 31 (out of 43) candidates; many candidates found parts (b) and (f) challenging, and in particular not noticing how the results of parts (b) and (d) can help with part (f). Question 3 was answered by 33 (out of 43) candidates; for part (b), many candidates made an initial error in finding the Minkowski bound, and some also did not fully take into account that the field is of degree 4 when applying Dedekind's Theorem; part (d) was generally well answered.

B3.5 Topology and Groups

Question 1: Most solutions were of a high standard. In part (a), some candidates failed to conclude that the isomorphism depended on w via conjugation by the difference between the two choices of paths. In part (b), candidates typically realised that loops are freely homotopic if and only if they are conjugate, but some proofs were incorrect or lacking in detail. Most candidates found correct examples for part (c), which they took to be a wedge of two circles and two conjugate loops.

Question 2: Essentially all solutions for part (a) were correct. In part (b), many solutions failed to consider the case $n = 0$. In the case $n > 2$, there were various mistakes. The key idea is to remove two points from S^n and notice that this is homotopy-equivalent to S^{n-1}, then use the fundamental group to distinguish this from S^1. Part (c) proved to be difficult with few completely correct solutions. Candidates did not seem to realise the difference between a retraction and a deformation retraction and were looking for the latter. The correct solution is to radially retract $U \backslash \{x\}$ to a small circle about x, and consider the maps induced on the fundamental group by the embedding of the circle and the retraction, which compose to the identity.

Question 3: Generally, solutions were correct. A few candidates forgot to include in (a) the part of the Seifert-van Kampen theorem that the pushout maps are induced by the embeddings. In (c), some candidates constructed a cell decomposition of $K\backslash B$ that was more complicated than necessary (a wedge of two circles). In part (e), some of the solutions did not fully check that the maps defined on the generators satisfied the relations, and in a few cases they did not.

B4.1: Functional Analysis I

Question 1 was solved by most students. Part a) was very well solved and also b)i) was generally well solved, though quite a few students did not realise that the assumption on g and the continuity of g on the closed interval $[0, 1]$ implies that g is bounded from below by a positive constant. For the second part of b) it is best to construct a Cauchy sequence (f_n) that converges pointwise to a unbounded function f, e.g. by cutting off $f(x) = \frac{1}{\sqrt{x}}$, and students following that route generally did well on that part. A common mistake in part (iii) of b) was that students were claiming that the polynomials are dense in X equipped with the sup-norm thanks to Stone-Weierstrass. However this theorem does not apply as the interval is not closed, and a simple counterexample is given by $\sin(1/x)$. The very last part of b) was designed to be challenging, but was successfully solved by several students, mostly by combining Stone Weierstrass on a compact interval $[\delta, 1]$ with the assumption that $g(0) = 0$ and hence that $|g| \leq \epsilon$ on $[0, \delta]$ for suitably chosen δ. Part c) i) was essentially an example from the lecture where it was shown that the L^1 norm on bounded intervals can be controlled by the L^2 norm using Hölder's inequality, but this part was not solved well. The second part of c) was designed to be challenging and was successfully solved only a few students.

Question 2 was solved by many students. 2a) was a variation of bookwork and was solved very well. A common mistake in b)i) was that students were claiming that $\|f \circ h\|_{sup} \leq \|f\|_{sup} \cdot \|h\|_{sup}$, possibly because they were thinking of a multiplication operator instead. Other than that (i) was solved well and most students argued correctly in (ii) that if h is invertible then T_h is invertible. The reverse direction was less well solved. Not many students realised in (iii) that since for $h(x) = x$ the operator is simply the identity, the only point that could be in A is $\lambda = 1$. Most students who attempted (iv) spotted that the functional is not bounded with respect to the L^1 norm and hence that the claim must be wrong. Part c) was a question on Hahn-Banach, with (i) a straight forward application that was well solved, and (ii) easily obtainable from a corollary of Hahn-Banach. The last part was designed to be the most challenging part of the question. While several students got close to a solution, using that T_n is Cauchy and trying to extend $T_n - T_m$, only one student got a complete solution that dealt with the difficulty that the extension of an operator can be non-unique.

The third question was the least popular question. Parts (i)-(iii) of a) were well solved, the last part of a) was more challenging but several students realised that it suffices to construct an operator with spectrum S^1 and that the best way to go about this is to try and find an isometric isomorphism which has point spectrum that is dense in S^1. The easiest solution to b)(i) was to show that $\|T^n\|^{1/n} \to 0$, but it was also possible to solve the question directly. A surprisingly common mistake was that students left out the factors $\frac{1}{n}$ when they were trying to invert the operator, so instead ended up considering the right shift for which the spectrum is different. Part (ii) of b) was very well solved. The first part of c) was an application of a corollary of Hahn-Banach, though not many students realised this. Most students who solved c) observed correctly that the claim becomes false for a non-complete space as such a space can never be reflexive but can have a reflexive dual space e.g. if it is a dense subspace of a reflexive space.

B4.2: Functional Analysis II

Question 1: All but one candidates attempted this question. Part (a), (b)(i) and (ii) were handled generally well. In (b)(i) a number of candidates contemplate that $(Z, \|\cdot\|_A)$ may be incomplete if A is unbounded, without realising that condition ($\star$) does in fact imply that A is bounded. Part (b)(iii) was handled well for the most parts, though a number of candidates missed the subtle part about weak convergence in the graph of A vs. weak convergence in W. Those candidates who attempted (c) did well.

Question 2: About two thirds of the candidates attempted this question. Part (a), (b)(i) and (ii) were handled well with some minor exceptions. In (b)(iii), some candidates failed to see how (b)(ii) could be used to show that A is unitary, though most had no problem with the dichotomy. Most candidates who attempted (c) realised that A is isometric, but only about half of them saw that A is not surjective.

Question 3: About one third of the candidates attempted this question.

Part (a)(i), (ii) and (b)(i) were handled well for the most part. Those who attempted (a)(iii) and realised that A as an operator from H into $A(H)$ is invertible did well. In (b)(ii), a number of candidates claimed that $f(z) \mapsto \frac{1}{z}f(z)$ is the left inverse of A without realising that this map is not well-defined as an operator on H. Most of those who attempted (c) did well despite the difficulty they had in showing the last bit.

B4.3: Distribution Theory

The exam went without incidents and most candidates performed well, despite it not being an easy paper. **Question 1:** was attempted by almost all candidates and was probably also the easiest question on the paper. Very good solutions were obtained by many candidates and one got the full 25 marks. Part (a) (i)–(iii) were bookwork and only few marks were lost here. The calculation of distributional derivatives in (a)(iv) also didn't cause any difficulties. Part (b) was generally also done well, though a few candidates wanted to show that the Sobolev space $W^{1,1}$ is closed in the Lebesgue space L^1. Part (c) was done with various degrees of success–the requested example of a $W^{1,1}$ function on the plane without a continuous representative seems to have been more difficult than expected. But fortunately the last part of the question, about convergence in $W^{1,1}$-norm not implying uniform convergence, can be done without providing the explicit example and a few candidates obtained marks in that way.

Question 2: was attempted by most candidates. Part (a) was book work and was done well. Part (b) elaborates on an example related to one discussed in the lecture notes. Some candidates lost marks when determining the support of the distribution v_α. In part (c) the first question (i) is a variant of book work and the candidates who attempted it did ok. Marks were lost in the second question (ii), where a test function must be constructed to show that (for instance) v_1 from (b) yields a counter example. No candidate obtained full marks on this question.

Question 3: was the least popular question. It explores a variant of what in the literature is known as Kato's inequality. Another related and easier variant had been a question on a problem sheet. The solution involves showing in part (a) that a distribution of order 0 admits a unique extension to compactly supported continuous functions. While this is a variant of book work, most candidates found it difficult. The first part of (b) is similar to a calculation done on a problem sheet and went well for the candidates who attempted it. The second part was attempted by very few candidates and with limited success.

B4.4: Fourier Analysis and PDE's

The exam went without incidents and most candidates performed well, despite it not being an easy paper. **Question 1:** was attempted by most candidates. Part (a) was done by all and very few marks were lost here. Part (b) also went ok for most candidates, but all candidates lost marks on the last part by not explicitly observing that the functions h_n are nonzero and hence actually are eigenfunctions for the Fourier transform. Part (c) went quite well for the candidates who managed to get that far–there seems to have been no problem in recognizing the advantage of expressing the sine function in terms of complex exponentials and then use a translation rule to reduce the calculation to one that should be familiar from lectures/problem sheets.

Question 2: was attempted by most candidates. The first two parts of (a) were book work and variants thereof and as expected went well with very few losing any marks. The last part of (a) however caused problems for about half of the candidates who didn't manage to complete it. Part (b) was more challenging, even though the first part is similar to a question on a problem sheet. About half of the candidates got close to full marks for their attempts. The last part (c) is a bit more challenging, but about a third of the candidates managed to obtain close to full marks on this too.

Question 3: was attempted by very few candidates. It is however not a difficult question and the candidates who attmpted it did quite well.

B5.1: Stochastic Modelling and Biological Processes

Question 1 was attempted by all candidates, while Questions 2 and 3 were attempted by about a half of the candidates (52% and 58% of candidates for Questions 2 and 3, respectively). However, this does not mean that Question 1 could be characterized as "less difficult" than other questions, because the average raw mark of Question 1 was actually lower than the average raw marks of Question 2 or Question 3.

Question 1 covered the material discussed during the beginning of the course, which could explain its popularity. In Question 1, almost all candidates correctly formulated the corresponding chemical master equation, while only a few of them successfully got to the end of Question 1. Less successful candidates made either mistakes in deriving the ordinary differential equation for $G(z)$ or they did not attempt the later parts of Question 1 at all.

Interestingly, the candidates who attempted all three questions (15% of candidates) scored higher raw marks in Questions 2 and 3 than in Question 1, where they only attempted the bookwork part. In Question 2, most candidates were able to derive and solve equations for the mean and variances for one animal (*i.e.* for the first animal in part (a)), while some of them had difficulties to apply the concepts from the course to groups of N animals in parts (b) and (c).

Question 3 covered the material from the last third of the course. Candidates demonstrated good understanding of the course material in their solutions. Candidates used different techniques to calculate the inverse Laplace transform in part (a). Some of them get to the answer

$$\chi(\tau) = 3\exp[-\tau] - \exp[-3\tau],$$

while others left their answers in a number of equivalent forms, including

$$\chi(\tau) = 2\exp[-2\tau]\Big(\cosh(\tau) + 2\sinh(\tau)\Big)$$

or

$$\chi(\tau) = 2\exp[-2\tau]\,\cos(i\tau) - 4i\exp[-2\tau]\,\sin(i\tau).$$

Such answers can be further simplified to $\chi(\tau) = 3\exp[-\tau] - \exp[-3\tau]$. While the candidates did not lose any marks by leaving their answers in more complicated forms in any question, they could try to aim to derive as simplest final formulas as possible in their solutions, because such answers can give them more insights into the behaviour of the underlying mathematical model.

B5.2: Applied PDEs

Question 1: Q1 went smoothly. Most students got through a) i.e. setting up and solving Charpit's equation, with occasional mistakes due to poor algebra. b) was harder, with quite a few students failing to apply the Jacobian or envelope condition to determine the domain of dependence correctly. In c), many students got the correct answer, but some failed to recognize and process the Neumann conditions correctly.

Question 2: In Q2, many students got through (a) and (b) but some failed to apply the chain rule correctly to get the form (2.1) in polar coordinates, or respond sensibly to the question about why the solution vanishes for $r > t > 0$. In (b), some students failed to get a, b right or correctly deduce α, β. The main challenge was (c), getting the form for $f(\xi)$ and completing the final integrations to the final result.

Question 3: Q3 was only attempted by few students. Most of those who did got good answers in (a), with them main problems being incomplete answers (i.e. constants were not determined.) (b)(i) was done well, but with quite a few students giving no or incomplete answers for the case $\Omega = R^2$. Only few students attempted (b)(ii) and only some obtained the final answer i.e. h.

B5.3: Viscous Flow

Question 1: This question attracted few attempts, and very few candidates made significant progress.

For part (b) $x'_i = l_{ij}x_j$ becomes $\mathbf{x}' = L\mathbf{x}$ in matrix notation, so $L^{\mathsf{T}}\mathbf{x}' = L^{\mathsf{T}}L\mathbf{x} = \mathbf{x}$ as L is orthogonal. Hence $x_j = l_{ij}x'_i$. Using the chain rule,

$$\frac{\partial u'_i}{\partial x'_j} = \frac{\partial}{\partial x'_j}\left(l_{ip}u_p\right) = \frac{\partial}{\partial x_q}\left(l_{ip}u_p\right)\frac{\partial x_q}{\partial x'_j} = l_{ip}\frac{\partial u_p}{\partial x_q}l_{jq}.$$

Adding this result to the equivalent result with i and j swapped gives

$$E'_{ij} = l_{ip}E_{pq}l_{jq} = [L]_{ip}E_{pq}[L^{\mathsf{T}}]_{qj}.$$

Sums over adjacent indices are matrix multiplications, so $E' = L\,E\,L^{\mathsf{T}}$ in matrix notation. The velocity $\mathbf{u}$ is a vector, and ρ, p and μ are scalars, so $\Pi' = L\,\Pi\,L^{\mathsf{T}}$ in matrix notation.

Part (c) adapts a question on a Part A Fluids sheet for a non-ideal fluid. The only non-zero component of the matrix E is $E_{r\theta} = -\omega a^2/r^2$. Squaring the matrix E gives E^2, and hence the components of the tensor $\mathbf{E}^2$. Having found $\boldsymbol{\sigma}$, it is easiest to calculate $\boldsymbol{\Pi} = \rho(\omega a^2/r)^2\,\mathbf{e}_\theta\,\mathbf{e}_\theta - \boldsymbol{\sigma}$ and use the momentum equation in the form given at the end of the question. The θ component is automatically satisfied. The r and z components determine $\partial p/\partial r$ and $\partial p/\partial z$, which together give p and the free surface $p(r, z) = 0$. The free surface rises into $z > 0$ for $a < r < \sqrt{2\lambda}$ due to the $\mathbf{E}^2$ term in $\boldsymbol{\sigma}$.

Question 2: This was the most popular and best attempted question.

There was a mistake in the question. The axial viscous terms should have been $\partial^2 u/\partial x^2$ and $\partial^2 v/\partial x^2$. Part (a) was marked generously, and several candidates noted the mistake. The mistake had no effect on the rest of the question, as these terms do not appear in the leading-order dimensionless equations.

Many candidates specified a pressure scale from the beginning, rather than first introducing an arbitrary scale P and then determining P consistently from the momentum equations. In particular, many candidates incorrectly scaled the pressure to be asymptotically small to justify discarding it from the dimensionless momentum equations. The correct pressure scale is $P = \rho U^2$, so $\partial\hat{p}/\partial\hat{x}$ appears at leading order in the dimensionless x-momentum equation. The leading order dimensionless r-momentum equation is $\partial\hat{p}/\partial\hat{r} = 0$, so $\hat{p}$ only depends on $\hat{x}$. Considering the x-momentum equation for large r then determines that $\hat{p}$ is constant.

Many candidates spotted (as intended) that it is simplest to first use the far-field condition for $\hat{u} = 2\hat{x}^{p-q} f'(\eta)$ as $\eta \to \infty$ to establish that $p = q$. A few candidates left their partial derivatives of Ψ expressed using a mixture of all three of $\hat{r}$, $\hat{x}$ and η. It is necessary to eliminate either $\hat{r}$ or $\hat{x}$ to find the right similarity form of the ODE for $f(\eta)$. A few candidates found p and q by comparing their differential equation for f with the given answer, rather than by finding conditions on p and q for which Ψ was a similarity solution.

As $f = f' = 0$ on the boundary of the body, we expect f to be small close to the body. We can then linearise the ODE for f to $(\eta f'')' = 0$, and solve for $f = A(1 - \eta + \eta \log \eta)$ by applying these two boundary conditions on $\eta = 1$. Several candidates incorrectly argued instead that $\alpha \gg 1$ close to the body, but α is a fixed $O(1)$ parameter for the whole flow.

The solution $f(\eta) = A + B\eta$ corresponds to a uniform axial flow $\hat{u} = 2B$ and a radial inflow $\hat{v} = -A/r$ towards the axis.

Question 3: This was also popular, but less well attempted than Q2.

As in Q2, many candidates specified a pressure scale from the beginning, rather than determining the pressure scale self-consistently from the momentum equation.

In part (b) many candidates either just asserted that the pressure p must be continuous (which is a "show that") or asserted that the stress $\boldsymbol{\sigma}$ must be continuous. The correct condition is continuity of the normal stress $\boldsymbol{\sigma} \cdot \mathbf{n}$. The continuity of $\mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n}$ and $\mathbf{t} \cdot \boldsymbol{\sigma} \cdot \mathbf{n}$ gives the required results.

In part (c) many candidates jumped directly from expressions for $\hat{u}_1$ and $\hat{u}_2$ with arbitrary constants to the solution given in the question. Many forgot to impose the boundary condition that $\hat{u}_1 = \hat{u}_2$ on $\hat{z} = \epsilon\hat{\eta}$. The neatest approach writes expressions for $(1-\epsilon)\hat{u}_1$ and $(1+\epsilon)\hat{u}_2$ and imposes continuity of the tangential stress first.

The two fluids occupy the regions $-1 \leq \hat{z} \leq \epsilon\hat{\eta}$ and $\epsilon\hat{\eta} \leq \hat{z} \leq 1$. It is necessary to include the $\epsilon\hat{\eta}$ displacement of the interface in calculating the volume fluxes on either side of the interface. Each contribution contains $O(\epsilon)$ terms that cancel when added to compute the total volume flux.

The result in (e) follows from integrating $\partial\hat{u}/\partial\hat{x} + \partial\hat{w}_1/\partial\hat{z} = 0$ across the lower layer and using the kinematic boundary condition on $\hat{z} = \epsilon\hat{\eta}$.

The fluid velocity at the channel centre is $3\widehat{Q}/4$ for an unperturbed interface at $z = 0$, so it is natural to expect small disturbances to the interface to propagate with this speed.

B5.4: Waves and Compressible Flow

Question 1: This question was attempted by every candidate and was well done overall. The routine Stokes waves calculation in part (a)(i) caused some difficulties due to a lack of efficiency applying the boundary condition on the base or solving for the Fourier transform of η. While all candidates stated correctly the formulae for the phase and group velocities and many knew how to apply them to the tail of part (a)(ii), only a handful did so correctly in the limit in which $kL \gg 1$. In part (b)(i) many candidates identified correctly the dynamic boundary condition but did not state its physical significance; nearly all did not identify that the boundary conditions on the sides at $x = 0$ and $x = L$ correspond to the pressure being held at constant atmospheric pressure, despite precisely this boundary condition being imposed on a problem sheet question. Many made good progress separating the variables in part (a)(ii), but again a lack of efficiency in applying the boundary condition on the base resulted in many attempts getting bogged down in unnecessary algebra. The tail in part (b)(iii) was handled reasonably well.

Question 2: This question was attempted by about two-thirds of the candidates and was reasonably well done overall. The routine derivations in part (a) were generally well handled, though many made heavy work deriving the linearized version of the no-flux boundary condition. While the majority sketched correctly the characteristic diagram in part (b)(i), there were many sketchy applications of the principle of causality and only a handful received full marks for deriving the given formulae for ϕ in $y > 0$

and $y < 0$. This sketchiness continued in part (b)(ii) with many applying incorrectly the boundary conditions on $y = 0\pm$ or failing to translate correct ODEs for $F(x)$ and $G(x)$ in $x < a$, $|x| < a$ and $x > a$ into the correct solutions for ϕ in the six corresponding regions in the (x, y)-plane. There were some good solutions to part (b)(iii) with many candidates realizing that the wing is inclined to the oncoming flow.

Question 3: This question was attempted by about one-third of the candidates and was not well done overall, the average mark being distorted by one almost perfect solution. The routine derivation of the Rankine-Hugoniot conditions in part (a) was well done by two candidates, but left largely unattempted by the rest. Parts (b) and (c) concerned a simple model for the partial closing of a sluice gate in a uniformly flowing stream. While the overall setup was new to the candidates, its individual elements are similar to previously seen examples on the problems sheets and in recent past papers. Despite heavy signposting for the flow of information in parts (b)(i) and (ii), the integration of that information into a coherent solution caused difficulties for almost all candidates. This resulted in many fragmentary solutions to parts (b) and (c), despite part (c) being almost identical in structure and more straightforward than previously seen examples because the values of both h and c on the boundary of the domain are given in the question. The risk of attempting an unfamiliar looking question was perhaps unattractive compared to the more standard looking questions 1 and 2.

B5.5: Further Mathematical Biology

Question 1. This question was attempted by the majority of candidates. The assumptions underlying the model were generally not fully justified. The phase plane analysis was generally correct, though few candidates could properly sketch the phase plane. Very few could derive the stated form for the travelling wave speed, nor properly compare and contrast the model with the Fisher–KPP model.

Question 2. Very few candidates answered this question. Those that did generally made good progress as it was quite similar to examples seen in the lecture notes.

Question 3. The majority of candidates answered this question. The approaches required were standard up until part (d) and therefore the average mark on this question was high. Note that there was an error in stating the non-dimensional equations, and the mark scheme was adjusted accordingly.

B5.6: Nonlinear Systems

Question 1
This was a popular question and there were a lot of good answers, with a high average score. Most candidates coped well the finding the extended centre manifold, despite the extended system being 4-dimensional.

Question 2
This question was also very popular. There were a few very good answers, but in general the marks were lower than for Q1, with quite a wide spread. Typical mistakes included ignoring the question on stability in part (b) and only finding the bifurcation points, and attempting to use the chain rule in part (c) despite the system being discrete. There were also quite a number of algebraic errors.

Question 3
This was a very unpopular question, with fewer than half as many attempts as either Q1 or Q2. The candidates that did attempt this question gave very good answers, and the average mark for Q3 was higher than that for Q1 or Q2.

B6.1: Numerical Solution of Differential Equations I

Question 1
The question was concerned with the finite difference approximation of a boundary-value problem for a fourth-order linear differential equation, restated as a system of second-order differential equations, subject to homogeneous Dirichlet boundary conditions. There were eight attempts at the question, but only one was close to being complete. The majority of the arguments offered by the candidates as proofs of the required equality

$$\|u'\|^2_{L^2((0,1))} + \|w'\|^2_{L^2((0,1))} + \|w\|^2_{L^2((0,1))} = -(f, w)_{L^2((0,1))}$$

in part (a) of the question were convoluted, and several candidates failed to observe the trivial pair of inequalities

$$-(f, w)_{L^2((0,1))} \leq \|f\|_{L^2((0,1))}\|w\|_{L^2((0,1))} \leq \frac{1}{2}\|f\|^2_{L^2((0,1))} + \frac{1}{2}\|w\|^2_{L^2((0,1))},$$

(i.e. the Cauchy–Schwarz inequality followed by noting that $ab \leq \frac{1}{2}a^2 + \frac{1}{2}b^2$ for $a, b \in \mathbb{R}$), which would have then directly implied the desired inequality at the end of part (a) of the question; those candidates then also had difficulties with the analogous, and relatively simple, discrete counterpart of the argument in part (b).

Question 2
Almost all candidates attempted this question concerned with the finite difference approximation of the elliptic boundary-value problem

$$\Delta u - (1 + x^2 + y^2)u = f(x, y) \qquad \text{for } (x, y) \in \Omega := (0, 1)^2,$$

subject to the nonhomogeneous Dirichlet boundary condition $u|_{\partial\Omega} = B \leq 0$, with $f < 0$ on Ω. The answers offered to parts (a) and (b) of the question were mostly complete, as were most of the suggested proofs of the discrete minimum principle in part (d) of the question, but several candidates found the proof of the inequality in part (c) of the question, which bounds the discrete maximum norm of the global error by the discrete maximum norm of the consistency error, challenging.

Question 3
This question on the stability analysis of the explicit Euler finite difference approximation of the initial-value problem

$$\frac{\partial u}{\partial t} + u = a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^2 u}{\partial y^2}, \quad -\infty < x, y < \infty, \quad 0 < t \leq T,$$

subject to $u(x, y, 0) = u_0(x, y)$, in the discrete ℓ^2 norm via Fourier analysis and in the discrete maximum norm, was well done by most candidates. Most managed to produce almost complete answers, although there were also several attempts which, while conceptually correct, contained algebraic errors. Several candidates missed the fact that for a problem in two space dimensions there are two different wave numbers for the semidiscrete Fourier transform $\hat{U}$: $k_x \in [-\pi/\Delta x, \pi/\Delta x]$ and $k_y \in [-\pi/\Delta y, \pi/\Delta y]$, and that the value $U_{k,\ell}$ of the mesh function U at the mesh point $(x_k, y_\ell) := (k\Delta x, \ell\Delta y)$ is therefore

$$U_{k,\ell} = \frac{1}{(2\pi)^2} \int_{-\pi/\Delta x}^{\pi/\Delta x} \int_{-\pi/\Delta y}^{\pi/\Delta y} \hat{U}(k_x, k_y)\, \mathrm{e}^{\imath k_x x_k} \mathrm{e}^{\imath k_y y_\ell}\, \mathrm{d}k_x\, \mathrm{d}k_y, \quad k, \ell \in \mathbb{Z},$$

with $\imath := \sqrt{-1}$.

B6.2: Numerical Solution of Differential Equations II

The students did well in the exam, with Questions 1 and 3 being particularly popular with the students, but Question 2 was also addressed.

Question 1 dealt with the method of steepest descent and the derivation of its theoretical properties. Parts a)-c) addressed material that was mostly seen and concerned the theory in the case where a global Lipschitz bound on the objective function is a priori known. Part d) connected this theory with a tool the candidates had seen in connetion with Nesterov acceleration to deal with the situation in which the Lipschitz cnstant is merely known to exist but no value is at hand. This problem was generally well solved, but some students found Part d) challenging.

Question 2 was only attempted by 3 candidates, however all of whom did quite well. Candidates were asked to derive the coefficients of strong convexity and L-smoothness for a particular example of a quadratic objective, and to use this information in a comparison of the Heavy Ball Method and Nesterov Acceleration.

Question 3 was designed to query the candidates' understanding of the stochastic gradient descent method and techniques for reducing the noise floor. This problem was well solved with a comparable marks distribution to Question 1.

B6.3: Integer Programming

A total of 22 candidates sat the exam. All three problems were attempted, and judging by the achieved results, they seemed to have been of comparable difficulty level, although Questions 1 & 2 saw much higher uptake than Question 3. There was a good spread of marks, showing that the balance of book work and stretch material worked as intended.

Question 1 covered the modelling of a scheduling problem and the concept of total unimodularity in the form of matrices with the sequential ones property. Although the tested scheduling model had been seen in the course, candidates struggled getting all aspects right, partcularly the inclusion of big-M constraints as a tool to encode the condition that the starting times of different jobs correspond to a feasible schedule. Part b) concerned TU theory and was generally well solved. Part c) was an application of Part b), but most candidates missed this point and tried to prove the statements from first principles instead of relying on the theory established in Part b).

Question 2 concerned submodular optimisation. Part a) consisted of book work, but the second half was frequently incomplete. Part b) was generally well solved and covered an example that could either be solved via the technique suggested in Part a) or via a completely different technique that had been seen on one of the problem sheets. Part c) required combining existing knowledge from the theory of submodular optimisation with additional constraints on the sign of some of the variables. Most candidates missed the connection that allows to model the set of of independent subsets and in terms of the requirement that the indicator vector be an element of the submodular polyhedron.

Question 3 covered a simple example of the cutting stock problem with 3 different widths and queried theoretical knowledge about the delayed column generation method along the way. This question saw a lower take up and resulted in more polarised marks.

B7.1: Classical Mechanics

1. This question was very popular attempted by most candidates who were able to write down the Lagrangian efficiently and pick up good marks on the rest of the question. Most candidates were able to apply the normal frequencies and modes material Good marks were picked up by most candidates.

2. This question was also popular and showed that many candidates had been able to use the crib-sheet system efficiently to write out much of the theory accurately. The second half was more challenging although there were a number of good solutions.

3. This question, being on the last part of the course was less popular, but nevetheless attracted a good number of solid attempts.

B7.2: Electromagnetism

Question 1 was on electrostatics and the method of images, and was attempted by all candidates. Answers to part (a) sometimes lacked a clear explanation of how superposition leads to the integral formula, in particular when describing how the charge arises from the line charge density. Part (b) was generally very well answered. Most candidates made good progress through part (c), using the method of images, although there were sometimes inaccuracies in applying the method, and computational errors.

Question 2 was on magnetostatics, and answered by the majority of candidates. A common issue with part (a) was not proving that the divergence of **B** is zero everywhere, and a lack of explanation of how superposition leads to the given Biot-Savart law formula. Part (b) was generally answered well; while the integral can be done with a trigonometric substitution, this isn't necessary. Part (c) really differentiated the candidates: some realized that one could sum n contributions of the type given in part (b), with a little bit of geometry/trigonometry to work out how to modify the formula (the intended method), but others went back to first principles, which then typically didn't get far.

Question 3 was on electromagnetic waves, and was the least popular question. However, those who attempted it generally did well. A common error in part (b) was to not check all the Maxwell equations are satisfied. Answers to parts (c) and (d) were generally very good, with most candidates getting most of the steps correct, even if some steps were missing, and some formulas were slightly wrong.

B7.3 Further Quantum Theory

Question 1 This question was popular, and started with a bookwork discussion of Rayleigh quotients and the variational method that was answered well generally, though some candidates were not very clear in their discussion of the general use of the method. The next part was a calculation using a variational Ansatz that was the same as the ground state wave function of the Hydrogen-like atom. The expectation of the kinetic energy for this Ansatz could be computed quickly using the virial theorem by comparison with the Hydrogen case, but a number of candidates did not identify this fact. The expectation of the potential energy was a straightforward calculation that was performed by most candidates who tried. The subsequent part required a coherent interpretation of the previous result so as to find for what values of potential strength the variational Ansatz could be chosen to give a negative energy. It was important to optimise the right thing here, and this tripped up many candidates. The final part was a Bohr–Sommerfeld calculation that could be done quickly by specialising to the zero-energy case, but candidates who tried to calculate the general quantisation condition ran into an ungly integral. The description of the origin of the appropriate quantisation condition was also hit-and-miss for candidates who got to this point.

Question 2 This was a perturbation theory question, starting with bookwork which nevertheless required a careful discussion of the degenerate case in part (b), and this was not always well explained in candidates' answers. The last two parts were an analysis of a particular three-dimensional system, which included some degeneracy. While the first order energies were well computed, the treatement of degeneracy for the analysis of state corrections caused a lot of trouble, and this carried forward into the last part where there was very little success in analysing the second-order energy corrections for the degenerate states (this required tracking the ambiguities in states through first order).

Question 3 This was an angular momentum question, which started with a standard description of irreducible representations of angular momentum. The problem then progressed to an addition of angular momentum problem with intrinsic spin *one*. This was done fairly well in many cases. In the last parts there was a perturbative calculation that required evaluating some matrix elements of orbital angular momentum operators in added-basis states. This involved, in the first instance, a Clebsch–Gordan calculation that was done well by the few candidates who attempted it. Finally, there was an application of the Wigner–Eckhart theorem. Very few candidates got to the point of utilising this useful theorem to reproduce the results of the previous calculation, though a number reproduced the statement of the theorem correctly.

B8.1: Probability, Measure and Martingales

Question 1 was attempted by half of the students. Part (a).(i) was either done well or very poorly - in the former case the Monotone Class Theorem was used successfully, in the later candidates were confused about how to set up its application. Some candidates missed out on the fact that f was bounded in the first part but non-negative in the second, so the limiting procedure was both in the interval $[-N, N]$ and $min\{f, N\}$. Part (a).(ii) caused a lot of confusion: the candidates mainly simply said this followed from Fubini's theorem, while that result was stated for the product of two probability spaces (or for the Lebesgue measure in Part A Integration) as the problem emphasised. A correct solution employed monotone convergence theorem combined with Fubini for ξ^N (re-normalising Lebesque measure on $[-N, N]$). Parts (b).(i)-(b).(ii) were done very well. Part (b).(iii) was either done well using the results from part (a), or caused troubles with candidates trying to use a conditional Fubini with no proper justification. Part (b).(iv) was challenging and very few candidates showed it - the rest either run out of time, or worked with Y_n^λ instead of Z_n.

Question 2 was attempted by two thirds of candidates. The vast majority of candidates did part (a).(i) very well, but only about half found a correct argument for (a).(ii) (restricting either Y, Z or the function g to an interval $[-N, N]$ and taking $N \to \infty$). Solutions to part (b).(i) were also mixed, with roughly half of candidates being able to derive the identity by considering the two separate cases, $\bar{X}_{n+1} = \bar{X}_n$ and $\bar{X}_{n+1} = X_{n+1}$. Part (b).(ii) was done well, though a few candidates dropped a mark for not justifying why M^f is integrable and adapted to the filtration. Part (c).(i) was challenging and only completed by a small number of candidates. However, many candidates identified that $|X_n|$ was a submartingale or attempted to compute M^f or $F(x)$ when $f(x) = \ln^+(x)$. Most attempts for (c).(ii) tried to prove that the sequence was bounded, and so only a couple of candidates received marks for this part.

Question 3 was attempted by almost all candidates. Part (a) was usually done well but a number of candidates got confused and instead of using $V_n = \mathbf{1}_{\tau \geq n}$ to deduce (a).(ii) from (a).(i) took $n \pm 1$, or used "$\leq$" inequality. In (a).(iii) saying that "clearly" $(A_{n\wedge\tau})$ is predictable was not enough. Some candidates also tried to argue predictability separately for different (fixed) range of values for τ, instead of thinking of the process. Part (b) had a lot of easy material which some candidates got with no problem, even if many dropped a mark forgetting to check very simple properties (e.g., that $\mathbb{Q}(\emptyset) = 0$). However, those who were confused about what conditional expectation is found this part almost impossible. Part (b).(iv) was challenging and only a handful of candidates got full marks for it.

B8.2: Continuous Martingales and Stochastic Calculus

Question 1 was well done overall. Some students struggled slightly with bii, in particular they needed first to show that the stopped process M^T is convergent, so the lim sup and lim inf will agree whenever $T = \infty$. This allows the stated assumption to be used more easily. Part c is most easily addressed by applying the earlier results. Part ciii requires explicitly calculating the probabilities on the right and left hand side, in order to obtain an estimate for ϵ. Many students gave the lower bound on ϵ (which is much more interesting than the trivial upper bound, which was asked for due to a typo in the exam question).

Question 2 was generally found more difficult than the other two questions. Many students struggled with part ai, as they said that the space $\mathcal{H}^{2,c}_{\mathrm{loc}}$ is the L^2 bounded continuous local martingales, rather than the set of processes which are locally L^2-bounded martingales (which is the same as the space of all continuous local martingales). Part b caused some difficulty; the simplest approach is to assume the integral is well defined, then compute the quadratic variation of $\int HdW$. Taking the expectation and applying Fubini's theorem gives a simple integral, yielding the condition $\alpha > -1$. In part bii, one needs to show that H_t does not converge as $t \to 0$. For $\alpha \leq -1/2$, H_t is normally distributed with variance which does not go to zero, and so one can check that it's inf and sup near zero will differ. Blumenthal's 0-1 law was well applied overall. Part c was made more difficult if the covariance was not computed using the quadratic variation, which gives an easy calculation. For $s < T$ the variance is s, but in cii this is not sufficient to apply L'evy's characterization (as the variance is only the expected quadratic variation). The calculation in ciii is easily done through quadratic variation, by realizing that $W = \int 1dW$.

Question 3 was generally well don. In part ci many students omitted to show that $\tilde{\mathbb{P}}$ is nonnegative, which is a crucial property. In part cii, many attempted to use a multi-argument version of Ito's lemma, which leads to difficulties as it's easy to forget the covariation terms. An easier approach is to use Ito's product rule. In ciii, the key is to show that $Y_\tau = E[(N_\tau - \langle N, M\rangle_\tau)X_T|\mathcal{F}_\tau]$, after which the problem follows by optional stopping. The hint implicitly assumes that $N_0 = 0$, otherwise the right hand side of the inequality should be N_0.

B8.3: Mathematical Models of Financial Derivatives

Question 1 was attempted by most students, and was well done overall. Part a drew directly on material from the course, and was well answered. Part bi was well done. Part bii caused some difficulties – the method coming from the suggestion is to consider a portfolio with a decreasing number of options, the sale of these options being chosen to fund the required payments, and hence giving a self-financing portfolio (with nonnegative payoff, and sometimes strictly positive payoff), which has a postive price. Some student attempted to prove this by assuming the options had zero value (and hence no fees need to be paid), this argument struggles as the terminal value needs to be strictly positive (in some states of the world), and therefore the fees need to be accounted for. Many students simply did not consider whether this portfolio was self financing, and applied Ito's lemma without considering the financial setting. Part biii was well done overall, the simplest argument being that as the options have positive value, it's never worth exercising them when the exercise value is zero (even though there are fees to be paid). The derivation of the differential equation in part biv, and the use of the ansatz in part bv, were well done.

Question 2 was well done overall. Parts a and b were generally well done, being drawn fairly directly from the course. Part c caused more difficulties; in part ci the simplest thing is to argue, from part aiii, that the function C must be decreasing in K. Using the chain rule, this entails an inequality on V which simplifies to what is shown. In part cii, one needs to explain that the agent using Black–Scholes will be short Δ_{BS} stocks, however using the function C, in order to cancel out their sensitivity to the underlying, they should be short $\Delta_{BS} + K^{-1}\nu_{BS}V'(S/K)$ stocks. Therefore, if V is increasing, they need to short more stocks in order to reduce their instantaneous risk.

Question 3 was well done overall. Parts ai and aii were drawn from the course; part aiii required a some explanation of the fact that: "the dividend payment merely scales the geometric Brownian motion S, and so its timing has no effect on the terminal value of S, and hence no effect on the European option value" along with "for an American option, it might happen that the dividend causes the stock price to jump over the optimal exercise barrier, which would mean we would exercise before the dividend is paid, and hence the timing of the dividend will affect the option price". Part bi was well done, being a standard application of the reflection principle. Part bii was not well done – most students were not careful with defining the option pricing formula below the barrier in part bi, which meant that they failed to notice that the jump in the stock price could cause the barrier to be breached at the dividend time, even if the barrier was not breached immediately beforehand. This lead to difficulties in part biii, where the importance of the point $S = B/(1-A)$ was unclear, as this is the value which jumps to the barrier, causing a discontinuity. Some students (correctly) noticed that the discontinuity in the stock price automatically makes hedging difficult, independently of the barrier of the option.

B8.4: Information Theory

Question 1 was fundamental. It was taken by most students, and most students did well. Part (a) was mostly bookwork. Although some students proved equalities from definitions of entropy, and some of them also forgot to explain some special property used in their proofs, most answers were good enough. Part (b) has been mentioned in lecture without detailed example. But there were still wrong conclusion appeared. Part (c) is a famous inequality in textbook. Its proof is typical. Some student used an equality, which does hold in this case but not in general.

Question 2 is challenging. There is a similar question in homework with specific value of the parameter. Part (a) is standard and easy. Part (b) is not easy if you don't know how to improve the prefix code by swapping codewords. Part (c) has been explained for the question in homework. Part (d) is easy.

Question 3 is a balance of basic calculation and slight difficulty with hints. Part (a) is very easy, but some students forgot to express the condition on the matrix. Part (b) can be proved by several ways, some answers from students are quick and clear, and some are tedious (and hence costly in time) while correct. Part (c) is in fact not really hard with hints, which can be solved with brutal force or with a small trick which has been mentioned in tutorial classes.

Full marks appeared in all the three questions, and the average of total mark is about 32.2/50, which sounds proper.

B8.5: Graph Theory

Question 1 was attempted by almost all candidates, and was generally done well. In part (a) many candidates came up with rather convoluted methods of showing that a tree has at least two leaves, often using results from the course (e.g., edge count of tree being $n - 1$) that are arguably harder that the result to be proved. Parts (b) and (c) were usually done correctly. Part (d) caused the most trouble, with many candidates having difficulty counting the number of appropriate codes.

Question 2 was the least popular question. Parts (a) and (c) were usually done well, although some candidates failed to produce examples of 3-regular or almost 3-regular graphs giving the lower bound on the extremal number in (c). Part (b) was very much a case of you got it or you didn't. Most candidates got the correct result in (d), although sometimes the arguments as to why it was optimal were a bit vague. The easy way to do this is to argue that K_4 maximises the average degree of a vertex in any component of the graph.

Question 3 was probably the hardest of the questions. Many candidates failed to set up the correct network. For example, failing to put capacity constraints on the edges from X to Y (without this the 'subgraph' H could end up with multiple edges). Several candidates attempted to use vertex constraints, which again suffers from the same problem that it does not force the flow to correspond to a subgraph. Several candidates also failed to consider sufficiently general cuts, and instead only looked at cuts of a specific form.

BO1.1: History of Mathematics

Both the extended coursework essays and the exam scripts were blind double-marked. The marks for essays and exam were reconciled separately. The two carry equal weight when determining a candidate's final mark. The first half of the exam paper (Section A) consists of six extracts from historical mathematical texts, from which candidates must choose two on which to comment; the second half (Part B) gives candidates a choice of three essay topics, from which they must choose one. The Section B essay accounts for 50% of the overall exam mark; the answers to each of the Section A questions count for 25%.

Throughout the course, candidates were invited to analyse historical mathematical materials from the points of view of their 'context', 'content' and 'significance', and these were the three aspects that candidates were asked to consider when looking at the extracts provided in Section A of the exam paper. Two candidates chose to use these as subheadings within their answer.

The Section A questions 1-6 were attempted by 5, 1, 9, 1, 3 and 2 candidates, respectively (one candidate attempted three questions in Section A). Question 1 related to material from the very start of the course, namely the development of mathematical symbolism and the application of algebra to the study of geometry. Most candidates focused on Descartes use of the method of analysis, rather than considering how his work relied on the development of symbolism. Questions 2 and 3 were both related to the history of analysis, a core subject in the lecture course. Question 2 was the only specific extract that candidates had certainly seen before; although many students used content relating to this extract in Section B, it is possible that they did not attempt Question 2 as the extract itself does not invite much commentary on context. Question 3 was generally well done; there was a lot that could be said about this extract as it touched on functions, infinite series, and the notion of transcendental objects. Question 4 was perhaps only attempted by one student as it related to material of a single lecture. Question 5 looked at early group theory and was generally well done, though students could have thought more critically about why Galois' work was not immediately accepted for publication. Question 6 was perhaps the most difficult question in this section, requiring strong background knowledge in group theory to recognise what Cayley was working on. Frequently the responses to questions in Section A suffered from being a little too vague.

Questions 7-9 were attempted by 4, 4, and 2 candidates, respectively. In general, the essays were well structured with students ensuring that they referred back to the question regularly, and summarised their argument clearly in the conclusion. Question 7 pertained to plenty of the material in the lecture course, and there was a surprising variety in the arguments made by students. Question 8 was perhaps the most difficult question in this section, requiring students to think creatively about which material they drew upon in their essay. Question 9, chosen by the fewest candidates, considered an idea that was occasionally touched upon in lectures, but was never the focus of an essay or class discussion.

The standard of the extended essays was on the whole quite high, with good use of source materials and evidence of students completing independent reading around the subject. Many students proposed original arguments in their essays, demonstrating good historical understanding and innovative thinking. Overall, there was still a tendency to anachronistic language - for example, the word 'derivative' was frequently used without explanation or clarification, even though the development of different notations and language used in the calculus was a key component of the course. Candidates could also have taken more care to use a consistent referencing style, and ensured that page numbers were always included in citations where necessary.

Statistics Options

Reports of the following courses may be found in the Mathematics & Statistics Examiners' Report.

SB1.1/1.2: Applied and Computational Statistics

SB2.1: Foundations of Statistical Inference

SB2.2: Statistical Machine Learning

SB3.1: Applied Probability

SB3.2: Statistical Lifetime Models

Computer Science Options

Reports on the following courses may be found in the Mathematics & Computer Science Examiners' Reports.

CS3a: Lambda Calculus & Types

CS4b: Computational Complexity

Philosophy Options

The report on the following courses may be found in the Philosophy Examiners' Report.

122: Philosophy of Mathematics

D. Comments and Recommendations from the Examination Board

(i) The examiners, as noted above, would strongly support examination Regulations restoring the ability for assessors or suitable delegates to be present for the first 30 minutes of an examination, so that any typographical errors which manage to remain undetected before an examination can be corrected in that period. (It seems exceedingly likely that all of the errors in this year's papers would have been identified and corrected had this arrangement been permitted.)

(ii) There was some concern that checkers for individual papers had not been uniformly thorough in reviewing draft papers, both in relation to detecting errors and in monitoring the difficulty of the questions posed. The role of checker is especially important in cases where the overseeing examiner's own mathematical expertise is rather distant from the topic of the paper, as is likely to be the case for some papers in many years. The examiners would encourage Teaching Committee to consider ways in which the importance of the role could be emphasised.

(iii) There were also a small number of cases where papers were significantly delayed. It might be helpful to emphasize to lecturers in future years that it is helpful to make the examiners aware as early as possible of any potential delays or difficulties in producing an examination paper, so that the timetable agreed with the external examiners does not need to be adjusted on short notice.

(iv) It would be very helpful if examiners (likely through the Chair) received clearer and more timely information from the Proctors or the Disability Advisory Service (DAS) in relation to candidates who may need reasonable adjustments to be made to the manner in which they take their examinations. The nature of such adjustments should surely form part of the support plan for a student with a disability, and the earlier the examiners can be made aware of the need for adjustments to be made to exam papers the more time they have to consider how best to implement these (and if needed seek clarification on what is required).

It might also be reasonable to raise the possibility for the Chair of an examination board to be given latitude to make minor adjustments to examination arrangements where they receive such a request from a student, provided that request appears consistent with the student's support plan and the proposed adjustments are deemed reasonable by the Chair. At present it can take a considerable length of time to obtain approval for even quite modest proposals of this kind.

E. Names of members of the Board of Examiners

- **Examiners:**

  Prof Kevin McGerty (Chair)
  Prof Jochen Koenigsmann
  Dr. Neil Laws
  Prof. Luc Nguyen
  Prof. Jim (James) Oliver
  Prof. Damian Rossler

  Prof John Hunton (External)
  Prof Anne Skeldon (External)

- **Assessors:**
  Prof. Andreas Muench
  Prof. Andras Juhasz
  Dr Brigitte Stenhouse
  Prof. Christopher Beem
  Prof. Cora Cartis
  Prof. Damian Rossler
  Prof. Dominic Joyce
  Prof. Endre Suli
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  Prof. Jim (James) Oliver
  Prof. Jochen Koenigsmann
  Prof. Jon Chapman
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  Prof. Lionel Mason
  Prof. Luc Nguyen
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  Dr Paul Balister
  Prof. Paul Dellar
  Prof. Radek Erban
  Prof. Raphael Hauser

Dr Robin Knight
Prof. Ruth Baker
Prof. Sam Cohen
Prof. Victor Flynn