Examiners' Report: Final Honour School of Mathematics Part C Trinity Term 2024

October 15, 2025

Part I

A. STATISTICS

- **Numbers and percentages in each class**

  See Table 1, page .

- **Numbers of vivas and effects of vivas on classes of result.**
  As in previous years there were no vivas conducted for the FHS of Mathematics Part C.

- **Marking of scripts.**
  The dissertations and mini-projects were double marked. The remaining scripts were all single marked according to a pre-agreed marking scheme which was very closely adhered to. For details of the extensive checking process, see Part II, Section A1.

  For information on steps taken in response to the Marking and Assessments Boycott (MAB) please see Part I, Section B.

- **Numbers taking each paper.**
  See Table 7 on page .

Table 1: Numbers in each class (post-2021 classification)

	Number				Percentages %			
	2024	(2023)	(2022)	(2021)	2024	(2023)	(2022)	(2021)
Distinction	71	(61)	(46)	(60)	66.36	(64.89)	(58.23)	(60)
Merit	26	(21)	(19)	(20)	24.3	(22.34)	(24.05)	(20)
Pass	10	(12)	(14)	(18)	9.35	(12.77)	(17.72)	(18)
Fail	0	(0)	(0)	(2)	0	(0)	(0)	(20)
Total	107	(94)	(79)	(100)	100	(100)	(100)	(100)

Table 2: Numbers in each class (pre-2021 classification)

	Number					Percentages %				
	2020	(2019)	(2018)	(2017)	(2016)	2020	(2019)	(2018)	(2017)	(2016)
I	63	(58)	(53)	(48)	(44)	67.74	(57.43)	(56.99)	(57.14)	(50.57)
II.1	30	(40)	(26)	(23)	(31)	32.26	(39.6)	(27.96)	(27.38)	(35.63)
II.2	0	(2)	(13)	(12)	(9)	0	(1.98)	(13.98)	(14.29)	(10.34)
III	0	(1)	(1)	(1)	(3)	0	(0.99)	(1.08)	(1.19)	(3.45)
F	0	(0)	(0)	(0)	(0)	0	(0)	(0)	(0)	(0)
Total	93	(101)	(93)	(84)	(87)	100	(100)	(100)	(100)	(100)

B. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

C. Notice of examination conventions for candidates

The first notice to candidates was issued on 14th February 2024 and the second notice on 1st May 2024. These contain details of the examinations and assessments.

All notices and the examination conventions for 2024 examinations are on-line at http://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments.

Part II

A1. General Comments on the Examination

The examiners would like to convey their grateful thanks for their help and cooperation to all those who assisted with this year's examination, either as assessors or in an administrative capacity. The chair would like to thank Anwen Amos, Rosalind Mitchell, Charlotte Turner-Smith, Waldemar Schlackow, Matt Brechin and the rest of the academic administration team for their support of the Part C and OMMS examinations.

In addition the internal examiners would like to express their gratitude to Prof Alan Champneys and Prof Roger Moser for carrying out their duties as external examiners in a constructive and supportive way during the year, and for their valuable input at the final examiners' meetings.

Timetable

The examinations began on Monday 27th May and finished on Friday 7th June.

Mitigating Circumstances Notice to Examiners and other special circumstances

A subset of the examiners (the 'Mitigating Circumstances Panel') attended a pre-board meeting to band the seriousness of the individual notices to examiners. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate. See Section E for further details.

Setting and checking of papers and marks processing

Following established practice, the questions for each paper were initially set by the course lecturer, with the lecturer of a related course involved as checker before the first draft of the questions was presented to the examiners. The course lecturers also acted as assessors, marking the questions on their course(s).

The internal examiners met in early January to consider the questions on Michaelmas Term courses, and changes and corrections were agreed with the lecturers where necessary. The revised questions were then sent to the external examiners. Feedback from external examiners was given to examiners, and to the relevant assessor for each paper for a response. The internal examiners met a second time late in Hilary Term to consider the external examiners' comments and assessor responses. The cycle was repeated for the Hilary Term courses, with the examiners' meetings in the Easter Vacation. Before questions were submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions.

Exams were held in-person in the Exams Schools. Papers were collected by the Academic Administration team and made available to assessors approximately half a day following the examination. Assessors were made aware of the marking deadlines ahead of time and all scripts and completed mark sheets were returned, if not by the agreed due dates, then at least in time for the script-checking process.

A team of graduate checkers, under the supervision of Anwen Amos, Rosalind Mitchell and Charlotte Turner-Smith, reviewed the mark sheets for each paper of this examination, carefully cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Sub-totals for each part were also checked against the mark scheme, noting correct addition. In this way a number of errors were corrected, with each change approved by one of the examiners who were present throughout the process.

Determination of University Standardised Marks

The Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average in each class over the last four years, together with recent historic data for Part C, the MPLS Divisional averages, and the distribution of classifications achieved by the same group of students at Part B.

The examiners followed established practice in determining the University standardised marks (USMs) reported to candidates. This leads to classifications awarded at Part C broadly reflecting the overall distribution of classifications which had been achieved the previous year by the same students.

We outline the principles of the calibration method.

The Department's algorithm to assign USMs in Part C was used in the same way as last year for each unit assessed by means of a traditional written examination. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part B classification of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage).

The algorithm converts raw marks to USMs for each paper separately (in each case, the raw marks are initially out of 50, but are scaled to marks out of 100). For each paper, the algorithm sets up a map $R \to U$ (R = raw, U = USM) which is piecewise linear and automatically contains the points $(0, 0)$, $(100, 100)$. While the previous scaling process used in 2021 assigned two vertices for the preliminary scaling map using Part B marks, the algorithm used since 2022 only uses the number of firsts achieved at Part B to assign one vertex. This vertex, P_3, is placed at 72 USM with an associated raw mark that ensures that the number of 1st class Part C on the paper after scaling is the same as the number of Part C candidates taking the paper with a 1st class in Part B. The vertex P_3 is then joined to (0,40) by a line segment, with a further vertex, P_2, placed at 57 USM on this line segment. The vertex P_2 is then joined by a new line segment to (0,10), and an additional vertex, P_1, is placed at 37 USM on the new line segment. The points $(0, 0)$, P_1, P_2, P_3, $(100, 100)$ provide the piecewise linear map for each paper's preliminary scaling map.

The results of the algorithm with the above default settings of the parameters provide the starting point for the determination of USMs.

A preliminary meeting of the internal examiners was held two days ahead of the examiners' meeting to assess the results produced by the algorithm alongside the reports from assessors. The examiners reviewed each paper and assessors' reports, and discussed the preliminary scaling maps and the preliminary class percentage figures. The examiners have scope to make changes, usually by adjusting the position of the vertices P_1, P_2, P_3 by hand, so as to alter the map raw $\to$ USM, to remedy any perceived unfairness introduced by the algorithm, in particular in cases where the number of candidates is small. They also have the option to introduce additional vertices. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low.

Table 3: Vertex positions for the piecewise linear scaling maps, P_1, P_2, P_3 with the raw marks rescaled to be out of 50. The entries N_3, N_2, N_1 respectively give the number of incoming firsts, II.1s, and II.2s and below from Part B for that paper.

Paper	P_1	P_2	P_3	Additional corners	N_3	N_2	N_1
C1.1	13.67;37	20;57	43;72	50;100	0	7	4
C1.2	14.97;37	26.07;57	41;70	50;100	0	7	2
C1.3	10.09;37	17.57;57	32;72	50;100	0	13	8
C1.4	13.02;37	22;57	42;72	50;100	0	8	3
C2.2	11.72;37	20.4;57	38.4;72	50;100	0	7	10
C2.3	13.35;37	23.23;57	43.73;72	50;100	0	3	6
C2.4	9.77;37	17;57	32;72	50;100	0	8	2
C2.5	11.07;37	19.27;57	36.27;72	50;100	0	0	5
C2.6	8.46;37	14.73;57	27;72	50;100	0	3	5
C2.7	11.39;37	18;57	37.33;72	50;100	0	13	11
C3.1	10;50	21;65	32;72	50;100	0	12	12
C3.2	10;57	40;72	50;100		0	7	5
C3.3	11.72;37	20.4;57	36;72	50;100	0	8	9
C3.4	10.42;37	17.5;57	34.13;72	50;100	0	5	6
C3.5	10.74;37	18.7;57	35.2;72	50;100	0	3	4
C3.6	12.37;37	21.53;57	40.53;72	50;100	0	2	5
C3.7	11.72;37	20.4;57	38.4;72	50;100	0	7	10
C3.8	17;57	37;72	50;100		0	11	8
C3.10	11;50	25.5;72	50;100		0	9	8
C3.11	13.02;37	22.67;57	36;72	50;100	0	3	2
C3.12	10.09;37	17.57;57	33.07;72	50;100	0	2	4
C4.1	8.79;37	14.5;57	28.8;72	50;100	0	7	12
C4.3	10.42;37	18.13;57	34.13;72	50;100	0	1	6
C4.4	11.39;37	19.83;57	37.33;72	50;100	0	1	2
C4.6	10.74;37	18.7;57	35.2;72	50;100	0	1	5
C4.9	11.07;37	19.27;57	36.27;72	50;100	0	0	3
C5.2	14.97;37	26.07;57	38;70	50;100	0	8	2
C5.5	16;57	37.33;72	50;100		1	18	8
C5.6	12.37;37	21.53;57	37;72	50;100	0	6	5
C5.7	12.04;37	20.97;57	34;72	50;100	0	8	3
C5.9	14;37	22;57	44;72	50;100	0	12	2
C5.11	14;37	24.37;57	39;72	50;100	1	14	5
C5.12	14.32;37	24.93;57	44.5;72	50;100	1	19	6
C6.1	14.97;37	26.07;57	43;72	50;100	0	12	7
C6.2	12.04;37	19.5;57	38;72	50;100	0	11	6
C6.3	50;100				0	0	0
C7.4	16.28;37	28.33;57	46;70	50;100	0	3	0
C7.5	10.41;37	18.13;57	34.13;72	50;100	0	2	0
C7.6	22;50	36;70	50;100		0	2	0
C7.7	13.99;37	24.37;57	43;72	50;100	0	4	5
C8.1	10.09;37	17.57;57	31;72	50;100	0	7	13
C8.2	9.11;37	15.87;57	28.5;72	50;100	0	2	8
C8.3	12.04;37	20.97;57	39.47;72	50;100	0	16	11
C8.4	11.72;37	20;57	34;72	50;100	0	5	6
C8.6	13.35;37	23.23;57	36;72	50;100	0	1	5

Paper	P_1	P_2	P_3	Additional corners	N_1	N_2	N_3
SC1	13.02;37	19;57	40;72	50;100	1	28	7
SC2	12.04;37	20.97;57	37;72	50;100	0	16	8
SC4	8;37	14;50	22;65	32;72	1	19	14
SC5	4;37	20;57	38;72	50;100	0	17	12
SC6	9.77;37	17;57	36;72	50;100	0	3	6
SC7	13.02;37	22.67;57	42.67;72	50;100	0	9	9
SC9	9.44;37	16.43;57	30.93;72	50;100	0	7	9
SC10	50;100				0	0	0

B. Equality and Diversity issues and breakdown of the results by gender

Table 5: Breakdown of results by gender

Class	Number											
	2024			2023			2022			2021		
	Female	Male	Total	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	11	60	71	7	54	61	10	36	46	15	45	60
Merit	9	17	26	8	13	21	9	10	19	8	12	20
Pass	3	7	10	3	9	12	5	9	14	5	13	18
Fail	0	0	0	0	0	0	0	0	0	1	1	2
Total	23	84	107	18	76	94	24	55	79	29	71	100
Class	Percentage											
	2024			2023			2022			2021		
	Female	Male	Total	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	47.83	71.43	66.36	38.89	71.05	64.89	40	65.45	58.23	51.72	63.38	60
Merit	39.13	20.24	24.3	44.44	17.11	22.34	36	18.18	24.05	27.59	16.9	20
Pass	13.04	8.33	9.35	16.67	11.84	12.77	20	16.36	23.73	17.24	18.31	18
Fail	0	0	0	0	0	0	0	0	0	3.45	1.41	2
Total	100	100	100	100	100	100	100	100	100	100	100	100

Table 6: Breakdown of results by gender (pre-2021 classification)

Class	Number								
	2020			2019			2018		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	16	47	63	8	50	58	6	47	53
II.1	4	26	30	9	31	40	7	19	26
II.2	0	0	0	0	2	2	3	10	13
III	0	0	0	0	1	1	1	0	1
F	0	0	0	0	0	0	0	0	0
Total	20	73	93	17	84	101	17	76	93

Class	Percentage								
	2020			2019			2018		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	80	64.38	72.19	47.06	59.52	57.43	35.29	61.84	56.99
II.1	20	35.62	27.81	52.94	36.9	39.6	41.18	25	27.96
II.2	0	0	0	0	2.38	1.98	17.65	13.16	13.98
III	0	0	0	0	1.19	0.99	5.88	0	1.08
F	0	0	0	0	0	0	0	0	0
Total	100	100	100	100	100	100	100	100	100

C. Detailed numbers on candidates' performance in each part of the exam

Data for papers with fewer than six candidates are not included in the public versions of the examiners' report.

Table 7: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
C1.1	11	35.73	10.18	69.55	12.73
C1.2	9	38.22	6.55	71.44	13.28
C1.3	21	27.76	9.34	67.71	13.39
C1.4	11	36.91	8.96	71.64	15.56
C2.2	17	36.82	7.06	74.18	10.38
C2.3	9	40.11	9.41	75.33	15.74
C2.4	10	27.3	2.06	67.3	2.06
C2.5	-	-	-	-	-
C2.6	8	29.62	9.52	75.25	11.42
C2.7	24	34.46	3.68	70.38	4.03
C3.1	24	31.33	9.49	74.46	10.84
C3.2	12	33	6.45	69	3.95
C3.3	17	35.82	7.37	74.82	11.05
C3.4	11	32.64	10.66	72.27	15.99
C3.5	7	32	10.49	70.71	14.68
C3.6	7	39.71	4.82	74.57	10.26
C3.7	17	35.47	7.84	72.41	10.75
C3.8	19	32.84	8.71	71.21	9.53
C3.10	17	26.35	10.55	71.71	13.34
C3.11	-	-	-	-	-
C3.12	6	34	5.8	74.83	8.23
C4.1	19	31.21	9.7	75.63	12.49
C4.3	7	36.43	10.6	78	15.21
C4.4	-	-	-	-	-
C4.6	6	36.17	8.93	76.5	12.28
C4.9	-	-	-	-	-
C5.2	10	36	8.68	71.9	15.32
C5.5	23	28.61	7.04	65.87	4.99
C5.6	11	38.73	7.72	78.36	13.75
C5.7	11	29.82	9.63	67.45	14.45
C5.9	14	35.71	8.21	67.36	8.9
C5.11	17	36.59	6.48	72.24	10.08
C5.12	23	40.57	4.69	70.52	7
C6.1	14	43.29	6.08	79.36	10.33
C6.2	15	33.2	4.59	68.33	3.89
C7.4	-	-	-	-	-
C7.5	-	-	-	-	-
C7.6	-	-	-	-	-

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
C7.7	8	38.5	10.86	72	16.81
C8.1	20	31.65	8.7	73.75	11.79
C8.2	10	30.2	5.22	74.2	6.76
C8.3	21	37.43	5.15	72.43	8.08
C8.4	10	35.1	7.58	75.5	11.21
C8.6	6	43.67	6.09	87.67	11.48
SC1	17	35.76	5.38	69.29	4.41
SC2	8	33.62	7.98	71.25	11.63
SC4	14	28.43	5.21	69.64	5.11
SC5	6	36.17	7.81	74	11.76
SC7	-	-	-	-	-
SC9	11	29.36	10.4	70.82	14.61

The tables that follow give the question statistics for each paper for Mathematics candidates. Data for papers with fewer than six candidates are not included in the public version of the report.

Paper C1.1: Model Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19	23	9.27	4	1
Q2	14.63	14.63	6.55	8	0
Q3	18.4	18.4	4.55	10	0

Paper C1.2: Gödel's Incompleteness Theorems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.75	16.75	4.65	4	0
Q2	19	19	4.06	5	0
Q3	20.22	20.22	4.66	9	0

Paper C1.3: Analytic Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.8	15.8	3.35	5	0
Q2	14.4	14.4	5.39	20	0
Q3	12.71	12.71	6.31	17	0

Paper C1.4: Axiomatic Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	25	25	0	2	0
Q2	20.91	20.91	2.7	11	0
Q3	15.75	15.75	3.2	8	0

Paper C2.2: Homological Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19	19	3.03	16	0
Q2	18.3	18.3	3.92	10	0
Q3	17.38	17.38	7.27	8	0

Paper C2.3: Representation Theory of Semisimple Lie Algebras

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.88	21.88	3.52	8	0
Q2	19	21.5	7.16	6	1
Q3	16.4	14.25	8.41	4	1

Paper C2.4: Infinite Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.67	11.67	3.14	6	0
Q2	12.8	12.8	1.92	5	0
Q3	15.44	15.44	2.4	9	0

Paper C2.6: Introduction to Schemes

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.43	15.43	5.94	7	0
Q2	13	13	4.83	4	0
Q3	15.4	15.4	4.22	5	0

Paper C2.7: Category Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.34	17.38	2.55	16	0
Q2	16.12	16.19	2.71	16	1
Q3	17.12	18.13	4.57	16	1

Paper C3.1: Algebraic Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.65	14.65	5.16	23	0
Q2	16.46	16.46	5.63	24	0
Q3	8.5	20	8.7	1	3

Paper C3.2: Geometric Group Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.18	18.18	4.6	11	0
Q2	14.44	14.44	2.7	9	0
Q3	16.5	16.5	2.65	4	0

Paper C3.3: Differentiable Manifolds

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.44	19.07	4	15	1
Q2	18.14	18.14	3.24	7	0
Q3	16.33	16.33	5.65	12	0

Paper C3.4: Algebraic Geometry

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.33	18.33	5.77	3	0
Q2	14.78	14.78	5.14	9	0
Q3	17.1	17.1	5.72	10	0

Paper C3.5: Lie Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.14	16.14	6.23	7	0
Q2	14.33	14.33	6.11	3	0
Q3	17	17	4.08	4	0

Paper C3.6: Modular Forms

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.5	19.5	3.11	4	0
Q2	19.8	19.8	2.17	5	0
Q3	20.2	20.2	4.6	5	0

Paper C3.7: Elliptic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.2	16	6.46	6	4
Q2	18.92	18.92	4.09	13	0
Q3	17.4	17.4	4.14	15	0

Paper C3.8: Analytic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.82	17.82	4.86	17	0
Q2	14.36	14.36	5.26	14	0
Q3	17.14	17.14	3.63	7	0

Paper C3.10: Additive and Combinatorial Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.75	14.75	6.88	12	0
Q2	13.73	13.73	5.31	15	0
Q3	7	9.29	4.78	7	3

Paper C3.12: Low-Dimensional Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.83	16.83	3.31	6	0
Q2	17.17	17.17	3.66	6	0
Q3	-	-	-	-	-

Paper C4.1: Further Functional Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.88	13.67	4.73	15	1
Q2	13.57	14.33	6.85	6	1
Q3	17.76	17.76	5.74	17	0

Paper C4.3: Functional Analytical Methods for PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.5	16.5	7.78	2	0
Q2	19.29	19.29	6.85	7	0
Q3	17.4	17.4	3.78	5	0

Paper C4.6: Fixed Point Methods for Nonlinear PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.33	16.33	5.03	3	0
Q2	17.5	17.5	6.14	4	0
Q3	19.6	19.6	3.36	5	0

Paper C5.2: Elasticity and Plasticity

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.4	16.4	5.66	10	0
Q2	11	11		1	0
Q3	20.56	20.56	3.28	9	0

Paper C5.5: Perturbation Methods

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.95	14.22	6.38	18	2
Q2	14	14.29	4.35	21	1
Q3	11.78	14.57	6.36	7	2

Paper C5.6: Applied Complex Variables

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.75	15.75	2.99	4	0
Q2	19.44	19.44	5.48	9	0
Q3	20.89	20.89	4.46	9	0

Paper C5.7: Topics in Fluid Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.18	15.18	5.58	11	0
Q2	14.3	14.3	5.5	10	0
Q3	18	18		1	0

Paper C5.9: Mathematical Mechanical Biology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.89	16.25	5.18	8	1
Q2	19.23	19.23	4.49	13	0
Q3	17.14	17.14	6.01	7	0

Paper C5.11: Mathematical Geoscience

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16	16.63	3.04	8	1
Q2	18.46	18.46	2.76	13	0
Q3	18.36	19.15	5.97	13	1

Paper C5.12: Mathematical Physiology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.28	19.59	3.06	17	1
Q2	20.78	21.23	3.61	17	1
Q3	19.92	19.92	2.68	12	0

Paper C6.1: Numerical Linear Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.33	17.5	11.68	2	1
Q2	22.5	22.5	1.78	12	0
Q3	21.5	21.5	3.28	14	0

Paper C6.2: Continuous Optimization

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.13	17.13	2.23	15	0
Q2	14.86	14.86	3.85	7	0
Q3	17.13	17.13	3.48	8	0

Paper C7.7: Random Matrix Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.29	19.29	4.96	7	0
Q2	22.43	22.43	3.31	7	0
Q3	8	8	5.66	2	0

Paper C8.1: Stochastic Differential Equations

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.58	16.64	5.81	11	1
Q2	16.4	17.29	6.38	14	1
Q3	13.87	13.87	4.96	15	0

Paper C8.2: Stochastic Analysis and PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14	14	2.24	9	0
Q2	15.5	15.5	7.78	2	0
Q3	16.11	16.11	2.15	9	0

Paper C8.3: Combinatorics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.44	17.44	3.46	16	0
Q2	18.71	18.71	2.87	17	0
Q3	21	21	2.92	9	0

Paper C8.4: Probabilistic Combinatorics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.33	18.33	4.5	9	0
Q2	19	19		1	0
Q3	16.7	16.7	4.11	10	0

Paper C8.6: Limit Theorems and Large Deviations in Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.25	21.25	3.77	4	0
Q2	22.6	22.6	4.34	5	0
Q3	21.33	21.33	2.08	3	0

Paper SC1: Stochastic Models in Mathematical Genetics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.09	16.75	5.89	8	3
Q2	15.7	15.7	3.16	10	0
Q3	19.81	19.81	3.56	16	0

Paper SC2: Probability and Statistics for Network Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.29	18.29	5.35	7	0
Q2	16.4	16.4	4.62	5	0
Q3	13.4	14.75	6.11	4	1

Paper SC4: Statistical Data Mining and Machine Learning

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.92	15.58	4.11	12	1
Q2	10.67	10.75	2	8	1
Q3	15.63	15.63	2.5	8	0

Paper SC5: Advanced Simulation Methods

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.5	18.5	4.65	4	0
Q2	18.33	18.33	2.8	6	0
Q3	16.5	16.5	7.78	2	0

Paper SC9: Interacting Particle Systems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.89	14.89	4.59	9	0
Q2	9.75	9.75	10.72	4	0
Q3	16.67	16.67	3.08	9	0

D. Recommendations for Next Year's Examiners and Teaching Committee

E. Comments on papers and on individual questions

The following comments were submitted by the assessors.

C1.1: Model Theory

Question 1

This question received solid answers with 8 from 11 candidates gaining 18 marks of more. In part (b)(i), the theory T of infinite sets in the language with no non-logical symbols was often (correctly) given as an example where any f.s. Σ containing T is satisfied in all models, but in every other script the justification implicitly used quantifier elimination for T without even mentioning it.

Question 2

There was a wide range of marks for this question. Only two of 11 candidates choosing this question managed to provide in (a)(ii) an example of a complete theory where no countable model is $\aleph_0$-saturated, the canonical example being the thoery of real closed fields in the ring language. And in part (c), the three countable models of T were often acccurately described, but the proof that any countable model of T is isomorphic to one of them was mostly omitted, and only two candidates identified the $\aleph_0$-saturated model correctly.

Question 3

This was the most popular question, again with rather mixed performance. In part (b)(iii) one had to observe that if $\mathcal{B}$ is a model of $T \cup \mathrm{Th}_\forall(\mathcal{A})$ then $\mathrm{Th}(\mathcal{A}) \cup \mathrm{Th}_\exists(\mathcal{B})$ is satisfiable. In part (c), some candidates proved (i) and (ii) in one go, thus giving an alternative approach to the intended solution (which is perfectly fine), others followed the intended track and managed (i), but failed on (ii).

C1.2: Gödel's Incompleteness Theorems

Good solutions to all parts of the paper were submitted. I note below some of the common errors or omissions. It was difficult to spot any patterns in the errors except that, predictably, they were more common when more subtle questions of consistency or provability were in play.

In question 1, in part (a), many candidates either did not quote, or did not quote correctly, the required theorem, and this made other parts of the question more difficult for them.

Different parts of part (b) were attempted successfully or unsuccessfully by different candidates, to no pattern that it is easy for me to discern, except that part (iii) was, on the whole, easier than the others.

The main parts of part (c) were done quite well. The last part was more difficult than I expected; the crucial question about any provisional valuation is whether there are any formulae ϕ such that $v(\phi) = F$ and $v(\Box\phi) = T$. If there are, then we must have $v(\Box\psi) = T$ for all ψ.

In question 2, part (a) was done well, as were, on the whole, parts (b)(i) and (c)(i). The other parts of parts (b) and (c) were found easy or difficult, again, to no pattern that I found it easy to discern.

In question 3, part (a) was generally done well, and there were many good answers to part (b), though there were occasional confusions.

Many good attempts at part (c) were made, though few were completely successful; the most common omission was not ensuring that the union of the sets T_n was the whole of the theory of the natural numbers.

C1.3: Analytic Topology

Generally the standard of solutions was quite low with plenty of erroneous claims, technically incorrect definitions and lack of awareness of the results and techniques from the problem sheets.

Question 1 was unpopular although it was the least technically demanding question. Candidates who attempted this question generally got high marks. The common mistake in (a) was not verifying that what was claimed to be a basis for the topology of the Stone Dual was in fact a basis. In (c)(i) a common mistaken claim was that that the infimum is the intersection. However the intersection of open sets might of course not be open, so this is in fact the interior of the intersection. Candidates who realized this could then provide a counterexample.

Question 2(a) was well done, although not everyone highlighted where the Hausdorffness of the compactification is important. Part (b) was generally disappointing with candidates not having the technical abilities to navigate the parts. Particularly in (ii) students claimed without proof that the assumption generalizes to finitely many sets 'by induction' without even stating what they are proving i.e. whether the premise becomes '$A_1, \ldots, A_n$ with $\bigcap_{k \leq n} \overline{A}^Y \neq \emptyset$' or 'the Y-closures of $A_1, \ldots, A_n$ are pairwise non-disjoint' and similarly for the conclusion. Those who did attempt a proof ended up with a proof which would work without induction beyond using that a finite intersection of opens is open. No candidate managed to show that the extension of f in (γ) was continuous. Despite the forward direction of part (c) appearing on a problem sheet, only few students managed to do it and only few properly checked the condition in (b) for the backwards direction.

The bookwork in question 3 was generally done well, although in showing that every metric space has a σ-discrete basis some students took a σ-discrete refinement of $\{B_{1/n}(x) : x \in X, n \in \mathbb{N}\}$. This refinement typically won't be a basis. Only few candidates could do part (b)(i) although it is almost the same as the proof that Lindelöf regular spaces are normal which appeared on a problem sheet. Plenty claimed erroneously that $\overline{\bigcup_{n\in\mathbb{N}} W_n} = \bigcup_{n\in\mathbb{N}} \overline{W_n}$. In part (c) a large number of candidates attempted a proof to the effect that separability of the Sorgenfrey line implies Lindelöfness which would fail in the square of the Sorgenfrey line instead of following the hint and adapting the proof that the unit interval is compact in the Euclidean toplogy, i.e. given an open cover $\mathcal{U}$ consider $\alpha = \sup\{x \in [0,1] :$ there is a countable $\mathcal{V} \subseteq \mathcal{U}$ with $[0,x] \subseteq \bigcup \mathcal{V}\}$.

C1.4: Axiomatic Set Theory

There were good solutions submitted to all parts of the paper.

In question 1., part (a) was generally done well. So was part (i) of part (b). Solutions given to (b)(ii) In and (c) were generally correct; few complete, incorrect solutions were given.

In question 2., (a) was generally done well and (b) often was as well.

One of the commonest errors in part (c) (and actually elsewhere) was muddling ordinal and cardinal arithmetic. Another common error in part (c) was in assuming that certain operators were normal, such as, for example, the beth operator; or in failing to observe that while $\alpha \mapsto \mu_\alpha$ is normal, this isn't obvious.

In question 3., a few candidates gave wrong definitions of Σ_0 and Σ_1. The rest of part (a) was almost always correct.

In part (b), the biggest difficulties for many candidates were working out how to use the hint in (b)(i); and noticing that the statement "x and y have the same cardinality" is not Δ_1 (despite its innocent appearance).

C2.2: Homological Algebra

Q1: all candidates answered this question. Parts (a) and (b) were B/S and most candidates had no problems with these. Part (c) was new. Many candidates gave good solutions. Q2: the majority of candidates chose this question. Part (a) was B and most candidates gave god solutions. Part (b) was B/S. Many candidates gave good answers. Part (c) was S/N. Not many candidates gave full profs for (c)(i). Q3: some students chose this question. Part (a) was B and candidates gave good answers. Part (b) was N but quite straightforward. Part (c) was N. It is broken up to several steps and most candidates who attempted this question did quite well on it.

C2.3: Representation Theory of Semisimple Lie Algebras

All questions were generally well done. In Question 1(c)(i), the students often misunderstood the question to be asking for a classification of <u>irreducible</u> finite dimensional representations of $\mathfrak{g}$. But really what I wanted to see was a classification of <u>all</u> finite dimensional representations of $\mathfrak{g}$.

In Question 1(c)(i), some students used the fact that $\Phi_+ \subset \mathbb{N}\Pi$ to reach the desired conclusion, without realising that that's essentially equivalent to the question asked. So really, the intent of the question was to solve this without using the knowledge that $\Phi_+ \subset \mathbb{N}\Pi$.

In Question 2(b), the students often provided the correct argument which works for all $n \geq 3$, but failed to explain why their argument breaks down (as it should) when $n = 2$.

In Question 3(b), most students were very good at drawing root lattices and weight lattices, but some a small number of drawings were pretty disastrous. This might have disadvantaged students with poor drawing skills.

C2.4: Infinite Groups

Question 1. This question was attempted by more than half of the candidates. The new part did not require significant problem solving skills, it was a rather straightforward adaptation of arguments seen in lectures and classes. Surprisingly, the majority of the candidates were unable to advance significantly. In part (b), (i), the bounded generation argument did not always start by projecting on the given quotient Q, but rather by picking a representative for the coset of N containing the given arbitrary group element, which presented the risk of having to add an extra factor in N when applying the bounded presentability of Q.

In (b), (ii), it was not always clear when the equalities were in a free group or in its finitely generated quotient. Disappointingly few were able to answer (c), (ii), even if it was almost bookwork. As few were able to answer (c), (iii), which was a direct consequence of (b), (ii) and of (c), (ii).

Question 2. This question was attempted by about half of the candidates. A few candidates stated that the stabilizer of a complete flag is composed of unipotent matrices. In (c), (i) (again, almost bookwork), there was some confusion between being solvable and being nilpotent.

Question 3. This question was attempted by almost all candidates, with good results. The first two parts were in general well answered. Most candidates answered (c), (i), reasonably well, few answered (ii), and almost always provided an example that was not finitely generated. Even though this was not marked down, it was somewhat disappointing, as the example was always easy to embed into one of the finitely generated wreath products well studied in lectures and classes.

C2.5: Non-Commutative Rings

The submissions were generally of very good standard with errors mostly in the unseen parts of the questions.

Question 1 was attempted by most candidates but there was only one complete solution. Many got confused with technical computations in (c)(i).

Question 2 had one complete solution. Some candidates attempted to use the Artin Wedderburn theorem but were not able to investigate the zero divisors of R in (c) (ii)(iii) to complete the argument.

Question 3 had several good submissions. Few candidates thought to use Gabber's theorem in (c)(ii) and only one had a complete solution for that part.

C2.6 Introduction to Schemes

In problem 1, parts a) and b) were done well, the typical issue was that some students did not explain in a) ii) the compatibility with restriction maps in the definition of $\mathcal{O}$-module structure on the skyscraper sheaf. Part c) was done rather well with two possible mistakes appearing. In c) i), in the statement of the "Valuative criterion for properness" some students assumed that there could be considered different maps from a dvr to its fraction field. In c) ii) some students failed to explain the uniqueness of the constructed lift when applying valuative criterion. Part d) was a harder question, the solution consisted of 3 parts: 1) write down the excision sequence; 2) prove that embedding of the generic point is a flat map; 3) prove that flat pullback induced by this embedding gives a retraction in the excision sequence. Among students who tried to solve this problem, most students succeeded in doing 1), some students justified 2) and very few students figured out 3).

In problem 2, part a) was done very well, only few students forgot to sheafify the tensor product presheaf. Part b) was done mostly well, though many students did not simplify the expressions for coordinate rings of fibers, and because of that some students did not realize that there is a difference between fibers over {0} and {a} for non-zero scalars a. In Part c), students did very well c) i) but everyone failed to construct the isomorphism in c) ii) — seemingly, this part of the problem required a hint. Part d) was a harder question, the solution consisted of 3 parts: 1) construct a closed imersion; 2) explain why it is enough to show that the induced map of structure sheaves induces an isomorphism after applying pushforward to the base; 3) argue that a surjection of vector bundles of the same rank on an affine scheme is an isomorphism. Among students who tried to solve this problem, most students succeeded in doing 1), some students justified 3) and most students forgot to explain 2).

In problem 3, parts a) and b) were done well, the typical issue was that some students were not careful enough in a) ii) to distinguish abstract non-isomorphisms between $\mathbb{A}^n - 0$ and $\mathbb{A}^n$ and the map given by the standard embedding. Part c) caused surprising difficulties, especially c) i) — perhaps the students were not comfortable with considering fibers over subschemes different than points. Parts d) i) and d) ii) went well, except that many students forgot some conditions in the statement of "Independendence of Cech cohomology", such as the condition on affineness, which made it a wrong statement. Part d) was a harder question, the solution consisted of 3 parts: 1) write down the Cech complex; 2) compute H^1; 3) compute H^2. Among students who tried to solve this problem, most students succeeded in doing 1), some students wrote a vague explanation for 2), but very few students wrote down the correct answer in 3), although the computation in 3) is in fact easier than in 2).

C2.7 Category Theory

The three questions were roughly equally popular.

Question 1. The bookwork in part (a) was fine; some candidates didn't explain why all finite products existed (e.g., the final object is the product for the empty category). In (b) (i)-(iii), the solutions were generally correct. One mistake was the claim that the subcategory of finite sets is an exponential ideal. On the other hand, (b)(iv) was difficult. Not everyone attempted part (c), and in some cases the adjunction wasn't fully explained. No difficult ideas were needed for the solution in (c), but the currying argument needs a bit of care and organisation.

Question 2. The definitions and the proof of the Yoneda embedding in (a) were fine; some candidates did not explain why the embedding is the inverse of the projection in the Yoneda Lemma. The solutions to (b)(i) were correct. (b)(ii) was generally correct, but a few solutions erroneously constructed the equaliser of (e, e^2), rather than (e, id_y). (b)(iii) was difficult, with only a few complete (or close to complete) solutions. Part (c) also posed difficulties, but partial credit was given for the realisation of F as a colimit and the correct commutativity of colim with Hom.

Question 3. Part (a) was bookwork and almost all solutions were correct. Part (b) was similar to bookwork (split fork), and the solutions were correct; in very few cases, the candidates forgot to argue the unicity of the map in the universal property. Part (c) on the other hand saw a variety of solutions, with very few complete solutions, but lots of partial credit. The difficulty was in the definition of the second map for the objects on Set_2. In (c)(ii), often the candidates did not state what GF did on the morphisms (only on objects). In (c)(iii), the main thing was to extend the coequaliser to an object of Set_2, that is, to construct the second map in a unique way.

C3.1: Algebraic Topology

Some students erroneously answered that there was only one 0-cell in Question 1(a)(i), which has the effect of making subsequent computations significantly easier. Most students failed to recognise X in Question 1 as a torus. This make it particularly challenging to answer Question 1(c)(i).

Question 2 was done well by most students, with the exception of (b)(iii) which proved particularly tricky.

Question 3 was by far the most challenging, and the fact that the figure was missing made it essentially impossible to do.

C3.2 Geometric Group Theory

Question 1. This question was the most popular. Some candidates missed the requirement that A', B',C' should be pairwise distinct. Part (b) was well answered and displayed a sound knowledge of normal forms and structural properties of amalgamated products and their subgroups.

Question 2. This question was quite popular as well. Most candidates were able to produce an accurate construction of a free basis for a group acting freely on a tree. Part (b) was well answered too, with several smart answers, but also a few answers where some confusion between free groups and their bases, and linear spaces and their bases was perceptible.

Question 3. This question was attempted by less than half of the candidates, and those who answered it did not obtain high marks. Rather disappointingly, as it required mainly good knowledge of the theoretical material involved. In the end of (a), (ii), almost all candidates missed the fact that there were two cases to discuss. Parts (b) and (c), (ii), were answered well, question (c) (i) less so, even if it was very close to arguments seen.

C3.3: Differentiable Manifolds

Question 1. For (c), the answer I hoped for was along the lines of

$$\begin{aligned}\mathcal{L}_v(i_w(\alpha)) &= \frac{\mathrm{d}}{\mathrm{d}t}\big(\varphi_t^*(i_w(\alpha))\big|_{t=0} = \frac{\mathrm{d}}{\mathrm{d}t}\big(i_{\varphi_t^*(w)}(\varphi_t^*(\alpha))\big)\big|_{t=0}\\ &= \big(i_{\frac{\mathrm{d}}{\mathrm{d}t}\varphi_t^*(w)}(\varphi_t^*(\alpha)) + i_{\varphi_t^*(w)}(\frac{\mathrm{d}}{\mathrm{d}t}\varphi_t^*(\alpha))\big)\big|_{t=0}\\ &= i_{\mathcal{L}_v w}(\varphi_0^*(\alpha)) + i_{\varphi_0^*(w)}(\mathcal{L}_v\alpha) = i_{[v,w]}(\alpha) + i_w(\mathcal{L}_v\alpha),\end{aligned}$$

but few did this correctly. In (d), combining (c) and Cartan's formula gives an equation involving $+i_w \circ i_v(\mathrm{d}\alpha)$ rather than $-i_v \circ i_w(\mathrm{d}\alpha)$, and only minority noticed this and explained why they are equal. For (e), one should prove the result by induction on increasing k, which was often not completed.

Question 2. Part (c) was modelled on the computation of $H^*(\mathcal{S}^n)$ in the classes, but many candidates could not get to the end. In (c)(ii), the correct answer was $\epsilon = \eta_U\beta + \eta_V\gamma + \delta\wedge\mathrm{d}\eta_U$. Very few candidates got this, most wrote $\epsilon = \eta_U\beta + \eta_V\gamma$ and failed to notice that $\mathrm{d}\epsilon \neq \alpha$.

Question 3. The raw marks on this question varied between 0 and 25. Candidates who understood the material scored highly, others did poorly. Some candidates did not understand part (b) and attempted to answer (b)(i)–(iv) independently, rather than finding functions d_i satisfying all of (i)–(iv).

C3.4: Algebraic Geometry

Q1. Some students had difficulties with Part (c). This follows directly from the equivalence between dominant rational maps and the corresponding extensions of function fields, together with the existence of transcendance bases.

Part (d) was solved by very few candidates. Part (d) (i) follows from the fact that $f^{-1}(\{y\})$ is affine and complete. For Part (d) (ii), consider that by Noether's normalisation lemma, we may assume that $Y \simeq k^t$ for some $t \geq 0$. Applying (c), we obtain rational maps

$$X \dashrightarrow k^{t+n_0} \to k^t$$

factoring f. Using the comment, we conclude that the image of (some representative of) $X \dashrightarrow k^{t+n_0}$ contains an open subset of k^{t+n_0}. Since any translate of a non zero linear subspace of k^{t+n_0} meets this open subset in an infinite set, we conclude that $n_0 = 0$, otherwise f does not have finite fibres. Hence the transcendence degree of the function field of Y is t, and in particular $\dim(X) = \dim(Y)$.

Q2. Many students had difficulties with Part (f). One can easily see that

$$\mathrm{Z}(x_2x_0, x_1x_2, x_0x_1) = \{[0,0,1], [0,1,0], [1,0,0]\}$$

and one has to show that ϕ cannot extend past these three points. To see this, consider eg the line going through $[0,0,1]$ and $[1,0,1]$. On any point of this line, which is not $[0,0,1]$, the map ϕ has the constant value $[1,0,0]$. Similarly, on the line going through $[0,0,1]$ and $[0,1,1]$, the map ϕ has the constant value $[0,1,0]$. So ϕ cannot extend to $[0,0,1]$ by continuity. The reasoning for $[0,1,0]$ and $[1,0,0]$ is similar.

Q3. This question was quite popular. Only Part (e) proved difficult for some students. To establish Part (e), let U be an open subset of $X \times Y$. Let $y \in \pi_Y(U)$ and let $x \in X$ be such that $(x, y) \in U$. Let $\sigma_x : Y \to X \times Y$ be the map, which sends $z \in Y$ to (x, z). The set $\sigma_x^{-1}(\pi_X^{-1}(\{x\}) \cap U)$ is then an open neighbourhood of y in Y, which contained in $\pi_Y(U)$. Since y was arbitrary, this show that $\pi_X(U)$ is open.

C3.5: Lie Groups

Question 1. Part (a) was answered well by most students. Part (b) was also usually answered well, but some students did not include the maps in the correspondence. Part (c) was also answered well by most students, but some failed to show orthogonality to both X and Y (usually by not nothing that the Lie bracket is skew-symmetric), and others did not show that differentiating the Ad-invariance led to the answer. Part (d) had a little bit more mixed response, with common errors being the omission of why the inner product is symmetric or correct arguments for why it is real-valued. Part (e) had a very varied level of response, with errors occurring in all parts. In (e)(i), most students found difficulty in the 2-dimensional case. In (e)(ii), quite a few students did not give a correct example, often giving ones that are in fact embedded. In (e)(iii), the main issue was that students often did not see that they should use (a)(iii) to show the subgroups have to be compact. In (e)(iv), common errors were not to argue why conjugate Lie subalgebras lead to conjugate Lie subgroups and to not realise that conjugation will leave SU(2) invariant. This question was attempted by all students.

Question 2. Part (a) was usually done well. Part (b) had a more mixed response: common errors were not to show orthogonality of characters, and to not show that the ring structures are identified. Part (c)(i) was usually done well, but there were occasional errors in the calculation. Part (c)(ii) was more mixed: there were errors in the statement of the Weyl integration formula, and then lack of justification in its application. In part (d) most students knew that they were supposed to decompose χ_V^2 into sums of characters, but very few did this correctly. About half of the students attempted this question.

Question 3. Part (a)(i) was done well. Part (a)(ii) was mostly done well, with marks usually lost for not justifying why conjugation of tori leads to conjugation of the corresponding Weyl groups or for not stating that the Weyl group is the Weyl group of a maximal torus. Part (b) was done well. Part (c) had a mixed reponse. Part (c)(i) was done well but there were often errors in (c)(ii), the most common being unable to obtain the roots correctly (even if students knew what the answer should be). Part (d) was usually done quite well, but there were often key arguments missing in (d)(i) even if students knew the answer. About half the students attempted this question.

Summary. The exam seems to have gone well. There was a good range of marks and the questions appear to have been of roughly the same difficulty based on students' responses.

C3.6: Modular Forms

I feel the paper went well: all of the parts of the questions were successfully completely and there was a very broad range of marks.

Question 1: Attempted by most students. And most had a clear idea of the techniques needed for the different parts, with some even managing the more delicate analysis needed in part (c).

Question 2: More students attempted this than I expected, presumably attracted by the opening bookwork. I was impressed by how many managed all of the slightly fiddly arguments.

Question 3: Again a popular question, perhaps because of the amount of bookwork. There were a few quite elegant solutions to (a)(vi) which were more concise than my own.

C3.7: Elliptic Curves

Question 1 was the least popular question, but was still attempted by the majority of candidates. The first part, on Hensel's lemma and its application, was accessible to most candidates. Reducing to the case $a \in \mathbb{Z}_7^X$ was helpful; candidates who didn't do this often ran into difficulties. The second part of the question, about the group law on a curve not in Weierstrass form, was found challenging. Many candidates made algebraic errors determining the formula for doubling a point. Only a few candidates observed that the identity point (1 : -1 : 0) is a point of inflection, which justifies using the 'simplified' group law, or correctly determined the formula $(X : Y : Z) \mapsto (Y : X : Z)$ for the group inverse. With these facts in hand, parts b)ii) and iii) can be done without the doubling formula (e.g. by showing there are no points of order 4 in the reduction mod 7 of the curve), but no-one took this route.

Question 2 For full credit in part a), as specified in the question, candidates needed to recall a statement about formal groups rather than simply writing that the kernel of reduction is torsion free 'because of results on formal groups'. In part b) many candidates successfully computed the number of points on the curve mod 5 and mod p. In part c), a few candidates incorrectly applied part a) to the reduction mod p (a prime of bad reduction).

Question 3 was the most popular question. Many candidates did well at carrying out the 2-descent, although sign errors and missing cases affected some. Only a few found a point of infinite order in part c), but it was pleasing that those who did find the point (25, 120) gave good justifications that it had infinite order (by applying Nagell–Lutz, for example). The intention was that (25, -120) would be found as the image of the point (2, 20) on the dual curve, but it was more common for candidates to find the point by inspection, recalling the Pythagorean triple (7, 24, 25).

C3.8 Analytic Number Theory

Overall this years question was reasonably sucessful, producing a good spread of marks with all questions reasonably popular amongst candidates (question 3 was slightly less popular). A few candidates seemed to have suffered from time issues (particularly with question 3), but this did not appear to be too widespread. The questions succeeded in distinguishing between candidates, and almost all candidates were able to demonstrate at least a basic understanding of core concepts in the course.

In question 1 the bookwork part (a) was almost universally answered correctly. In part (b) many candidates struggled with demonstrating the inverse, which was not covered in lectures and so understandably harder. It was slightly surprising that despite many similar questions in examples sheets and revision sessions, a number of candidates didn't know the standard strategy for attempting (c).

Question 2 had the lowest average mark from candidates who attempted it. The most challenging part (c) did a good job of distinguishing between candidates, since at some points it required less obvious modifications of content covered in lectures.

Question 3 was the least popular question, but answered well by most candidates who attempted it. In hindsight the question was maybe slightly too long for the final question in an exam; several candidates didn't attempt part (d) even if this was less difficult in general. Several candidates didn't spot the relevance of (b)(i) to (b)(ii) despite the language used, which caused them to slip up slightly. It was pleasing that most candidates understood the overall strategy pretty well, and the technical execution of this distinguished between different candidates pretty well.

C3.10: Additive and Combinatorial Number Theory

Q1. Unexpectedly, a significant number of candidates answered part (c) in a different way to the one I anticipated, by mimicking the first stages of the proof of Roth's theorem from lectures in combination with the result of (b). This was much harder and longer than the official solution; perhaps candidates thought that part (c) must draw on the result of part (b), which the solution I had in mind does not.

Even more oddly, very few of these candidates could then answer part (e) at all correctly, even though they had basically done the work necessary already in their solution to part (c).

A significant minority of candidates failed to read the instruction before part (c), namely that for the rest of the question A is assumed to have size $N/10$. By ignoring this they were, for instance, able to trivialise part (d).

Q2 (a) (i) Many candidates lost a mark by not saying which sets they were applying Ruzsa covering with, which in a question as simple as this has to be counted as an omission.

(ii) Done fairly well by many candidates

(iii) There were almost no attempts to even try to compute the size of $(k-2)X$ for a fixed set X, which can be done in closed form as a binomial coefficient.

(b) A very similar question had appeared on an example sheet and it was clear that many candidates in some sense understood what was going on here, but very few were able to lay out a coherent and precise solution.

(c) Several candidates attempted to take A_n to be the set of primes up to n, quoting a substantial theorem from lectures to assert that $2A_n$ has positive density in $\{1, \dots, 2n\}$. Such attempts were awarded partial marks on the grounds that the statements are correct but the candidates were not told that they could quote such a result, and nor is it necessary.

Quite a few candidates managed this part of the question, which is to be expected as something almost identical was on an example sheet. There were one or two innovative solutions using digits in base 3 or 5.

Q3. This question was not well done, with a significant minority of candidates not even managing part (b), which involves nothing more than simple algebraic manipulation. No candidate got any marks for part (e), which was essentially bookwork.

C3.11: Riemannian Geometry

Question 1. Part (a) was done well. Part (b)(i) was usually done well, though some students decided to use an incorrect statement of the Koszul formula which led to difficulties. In part (b)(ii), students had the right idea but often lost marks for lack of justification (e.g. that the V_i are orthogonal). Part (c) proved challenging with computational errors and lack of justification. This was a popular question done by almost all students.

Question 2. Part (a) was done well. Part (b)(i) was done well, with the common error being not to note that the required distance is bounded above by $L - t$. Part (b)(ii) was done quite well, with marks typically lost for not recognising the significance of $|\alpha'| = 1$ (i.e. to get the correct parameterisation). Part (b)(iii) proved challenging, with students unable to construct a suitable variation. Part (c) elicited few attempts, but they were done well. About half the students attempted this question.

Question 3. Part (a) was done well by almost all students. Part (b) had a mixed response. As expected, the usual error in (b)(i) was to assume the metric was complete, whereas others correctly spotted the connection to hyperbolic space. Part (b)(ii) was usually done well. Part (b)(iii) had similar issues to (b)(i) again as expected, but again a good number of students spotted the connection to the round metric in higher dimensions. Part (b)(iv) was usually done well, with the most common error just to miss justification of completeness or that the product metric constructed as non-negative curvature. Part (b)(v) was also usually done well, with the only error begin lack of justification of a uniform lower bound on Ricci curvature. This was another popular question attempted by most students.

Summary. The exam went well with a good range of marks. Question 2 appeared to be a bit more challenging for the students, but not overly different from Questions 1 and 3 which seemed to be roughly equal difficulty.

C3.12: Low-Dimensional Topology and Knot Theory

Question 1 (7 attempts): This question tested knowledge of handle decompositions of smooth manifolds, especially surfaces, and their relationship to homology. The general level of solutions was high. In (b)(i), some candidates did not realise that the coefficients of the boundary map are given by the algebraic intersection number between the attaching circle and the belt circle. In (b)(ii), some candidates missed the half twist in the 1-handle. There were no complete solutions for (b)(iii), though several solutions made good progress.

Question 2 (7 attempts): This question tested knowledge of the linking number, Seifert form, and their relation to the intersection form of the Seifert surface. The general level of solutions was good. In (a)(i), some candidates forgot to say that one only considers crossings between K and K' in the formula for the linking number. In (a)(ii), some did not consider the oriented smoothing, hence potentially ending up with a non-orientable surface. Unfortunately, there is an absolute value missing from the definition of the determinant in the statement of part (b), which makes the determinant well defined only up to sign in (b)(i). No marks were deducted in (b)(i) related to any sign issues, and most candidates noticed the problem. There was no complete answer to (b)(ii), though there was one essentially complete solution. Some candidates considered projections of curves on the Seifert surface to the plane and counted crossings near the projection of their intersection point on the surface, but missed crossings not of this form.

Question 3 (1 attempt): This question tested knowledge of lens spaces, Dehn surgery, and their first homology and homotopy groups. There was one solution, of a good standard. Careful application of the Seifert–van Kampen formula in (a)(iii) and the Mayer–Vietoris exact sequence in (b)(ii) were the more challenging parts of the question.

C4.1: Further Functional Analysis

Problem 1. Students did pretty well this problem, especially parts (a) and (b) which were bookwork. Problem (c) involves creating some counter examples: many students successfully constructed counter-examples for (ii), but not for (iii) which was clearly seen much more difficult. Part (d) was completely new for students, and although it had a hint, not too many students gave a complete and correct proof.

Problem 2. Parts (a) and (b) are somehow standard and among the students who attempted solving this problems, most of them did this parts well. Most of the students who did (a)(i) knew that they have to apply Schauder's theorem for getting (a)(ii). For part (b), some students had some problems in pointing out a precise countable set which is dense in $T(X)$. Part (c) which was somehow new to students, gave difficulties to many of them.

Problem 3. In general students did very well on this problem comparing to the other two problems. There were some difficulties in obtaining a counterexample for (a)(ii) . For part (b), students did well (i) and (ii), but had found part (iii) a bit more difficult. It makes sense, since part (iii) involves the description of open neighbourhoods. For part (c), students did well on both parts, (i) and (ii). However, they had some difficulties for part (ii) when they had to correctly exploit the fact that the space X is reflexive.

C4.3: Functional Analytic Methods for PDEs

Question 1: About a third of the candidates attempted this question, and with variable degrees of success. Part (b)(ii) seemed somewhat challenging, but manageable with the hint. For (c), most candidates saw that they need to apply (b) to the first derivatives of u, but only about half managed to pull through the details.

Question 2: This question was tried by all candidates. In (a)(i), almost all candidates saw the desired convergence, but some could not show that the limits under the different modes of convergence were the same. Surprisingly (a)(ii) caused some trouble and only those who made use of an interpolation inequality could manage it. In (b) and (c), (b)(i) seemed to be challenging: while a good proportion of the candidates saw how to obtain a subsequence which weakly converges to 0, a much smaller proportion of the candidates were successful in using Reillich-Kondrachov's theorem to upgrade such weak convergence to strong convergence.

Question 3: About two thirds of the candidates attempted this question. In (a), a small number of candidates forgot the condition $u = 0$ on ∂B and/or mixed up a sign in the definition of $f + \operatorname{div} \mathbf{g}$. In the rest of the question, (c)(iv) and (v) seemed to be more challenging. In (c)(iv), those who tried to use (c)(i) did well. Fewer candidates attempted (c)(v) succesfully. A solution typically involved showing that any non-zero element of W must be either positive or negative in B.

C4.4 Hyperbolic Equations

The first question was attempted by all the candidates, the second question was attempted by three of them, and the third question was attempted by only one candidate.

In Question 1, Part (a)(i) was bookwork, which was perfectly done. Part (a)(ii) was unseen/similar and required a good understanding of the characteristics and a calculation of the minimal formation time of shock waves; every candidate worked out some part of them, and two candidates did quite well. Part (a)(iii) was unseen and new rider, which required not only truly careful calculations but also a good understanding of the characteristics and the shock wave speeds; two candidates got full marks, while the other candidates worked out some part of them. Part (b) was similar & unseen, one candidate did perfectly, while two of them did not make an attempt.

In Question 2, Parts (a) and (c) were bookwork/similar, but required some careful calculation, which were done well. Part (b) required some deep understanding of several properties of the Legendre-Fenchel conjugate of uniformly convex functions, which was done perfectly. Part (d) required a good understanding of the Lax-Oleinik formula for the entropy solutions and the property of the Legendre Fenchel conjugate of a convex function, as well as some careful calculations, for which one of them got the full marks.

Only one candidate did Question 3. Parts (a)–(b) were done almost perfectly, but no serious attempt was made for Part (c).

C4.6 Fixed Point Methods for Nonlinear PDEs

11 students took the exam of the course.

Question 1 was about the first part of the course regarding Retraction Principle, Brower's fixed point theorem, and a variant of Banach's fixed point theorem where the contraction is not uniform.

Part 1(a) was bookwork and was done perfectly (or almost perfectly) by everyone.

Part 1(b) was about 2 variants of the Retraction Principle. The first one was done well by half of the students, the second one was done well by only one student.

Part 1(c) was about a variant of Banach's fixed point theorem where the contraction is not strict. Subparts (i) and (ii) were standard and were done well by most students. Subpart (ii) was more challenging, no-one was able to completely solve it, however one script made good progress and arrived close to a complete solution.

The question was chosen by 7 students. 1 solution was at distinction level, 2 at upper intermediate level, 2 at lower internediate level, and 2 just above sufficiency.

Question 2 was about Schauder's fixed point theorem and applications to nonlinear PDEs, with the use of weak maximum principle. The question needed a good handling knowledge of preliminary material such as Sobolev embeddings and standard properties of the Laplacian.

Parts 2(a) and 2(b) were bookwork/standard and were done perfectly (or almost perfectly) by everyone.

Part 2(c) was a twist of a theorem proved during the course, it needed a good understanding of that proof as well as applying Sobolev embedding and Hölder inequality in the right way. It was done very well by half of the students, some missed the difference with the theorem proved in the lecture and basically just reproduced that proof without the necessary changes.

Part 2(d) was a direct application of weak maximum principle and was done well by most students.

Part 2(e): 3 students gave the right answer, 2 gave a quite convincing argument, but no-one gave a complete proof.

The question was chosen by 5 students. 2 solutions at distinction level (almost perfect), 1 at intermediate level, and 2 just above sufficiency.

Question 3 was about monotone operators and Perron's method of sub/supersolutions to solve non-linear elliptic PDEs. As in question 2, also here it was necessary to have a good handling knowledge of preliminary material such as Sobolev embeddings, weak maximum principle and standard properties of the Laplacian.

Part 3(a) (i) and (ii) were bookwork/standard and were done perfectly (or almost perfectly) by everyone. 3(a)(iii) was new and was done well by 3 students.

Part 3(b) was bookwork/standard. Common mistakes have been: not to use traces, but just restrict a Sobolev function to the boundary of a smooth open set; missing to specify boundary conditions in defining sub/super-solutions.

Part 3(c) was a twist of a theorem proved during the course, it needed a good understanding of that proof as well as applying Sobolev embedding and H¨older inequality in the right way. It was done very well by half of the students, some missed the difference with the theorem proved in the lecture and basically just reproduced that proof without the necessary changes, 2 did not attempt this part.

The question was chosen by 10 students. 4 solutions were at distinction level (1 almost perfect), 1 at upper intermediate level, 3 at lower intermediate level, 1 third level, and 1 at pass level.

C4.9: Optimal Transport & Partial Differential Equations

Question 1: Parts (a) and (b) were straightforward and were in general answered quite well. Some candidates found part (c) more challenging. Part (d) was mostly well done too, although the equality case of the estimate was giving them some troubles. Most of the students understood how to produce couplings based on optimal plans to estimate transport distances between probability measures.

Question 2: Parts (a) and (b) were reasonably straightforward bookwork. Candidates found part (c) more difficult and they struggle to construct a transference plan. Part (d) was a simple consequence of previous parts and spotted well.

Question 3: Parts (a) and (b) were generally well answered, although some students found difficulties in writing the good definition of weak solutions for the PDE. Part (c) was satisfactorily answered by half of the candidates. Some of them did more work than needed. Actually, instead of using directly the course content, they reproved certain parts making their answers longer than needed.

Summary: This year's exam seems to have worked quite well to distinguish between the excellent and the very good students in the course. Overall, I am quite happy with the spread of marks, showing the high level of achievement of part of the cohort. Each question was attempted this year by the same number of students showing the spread of knowledge and work on the material over the cohort.

C5.2: Elasticity and Plasticity

Question 1: This question was attempted by all candidates. The bookwork in part (a) was well done overall though there was some confusion about the orientation of the moment at the right-hand end of the small control segment of the beam and about the imposition of the boundary conditions on the force applied to each end of the beam. In part (b) all candidates derived the solution to the linearized problem, but many struggled with the subtle scaling argument. There were many nice solutions to part (c), though several attempts failed to seek a perturbation solution or ran into difficulties with the algebraic manipulations.

Question 2: This question was attempted by one candidate.

Question 3: This question was attempted by all but one candidate. The bookwork in part (a) was well done overall, though only a minority took the most efficient route. Part (b) was very well done overall, with many candidates scoring close to full marks, the main difficulties being the derivation of the boundary conditions on the normal stress, the efficient solution of systems of linear equations and the tail of part (b)(ii).

C5.5: Perturbation Methods

Question 1. Many candidates attempted this question. The early parts of (a) were well answered, but the majority of candidates did not use a formal proof by induction to establish the full asymptotic expansion. In (b) the majority of candidates struggled with establishing the steepest descent contours, though most recognised that the dominant contribution to the integral comes from the region close to the saddle point and could correctly simplify the problem to get the stated result.

Question 2. Many candidates attempted this question. There was a mistake in this question, the term $\partial^2 u/\partial x^2$ should have read $\partial^3 u/\partial x^3$. This did not affect solutions to part (a), and this was very well-answered. The typo affected parts (b)-(d) – all candidates were given marks for making all possible progress.

Question 3. A smaller subset of candidates answered this question. In (a) some marks were lost for not fully justifying where the boundary layer lies. In (b) many candidates struggled to solve the equation for f_0^{outer}, and then solve for the full outer solution. Part (c) was relatively well answered, but very few candidates could properly demonstrate use of an intermediate variable to match the inner and outer solutions in (d).

C5.6: Applied Complex Variables

Question 1

This was by far the least popular question, attempted by only a quarter of candidates, with the lowest average mark. Parts (a) and (b), which were bookwork, were handled relatively well. A few candidates managed part (c), though some did not realise $\bar{\zeta} = 1/\zeta$ when $|\zeta| = 1$ so that $\mathrm{Re}(\zeta) = \mathrm{Re}(1/\zeta)$. No candidate completed part (d), though some managed to get the blowup time, while others managed the volume calculation.

Question 2

This was a popular question, attempted by 88% of candidates. The material was straightforward, but the question a little unfamiliar (certainly for part (a)). Candidates who applied what they had learned, rather than trying to remember calculations, did well. Part (b) was handled better than part (a) in general.

Question 3

This was also a popular question, attempted by 88% of candidates, and had the highest average mark. The material was more challenging than Q2, but the format of the question was familiar, and there were some very good answers. Most mistakes were algebraic. One commom mistake was for candidates to write that the residue of $1/(2k + i)$ at $k = -i/2$ was 1 rather than 1/2.

C5.7: Topics in Fluid Mechanics

This section will be published in the Examiners' Report. Please include comments on each of the questions, summarising the standard of answers and noting any common difficulties encountered by the candidates.

The paper produced a wide spread of marks, enabling clear differentiation between the stronger and weaker examination scripts for the relatively small cohort.

Q1 A very popular question, considered by every candidate. The bookwork in part (a) was implemented well in general, with b(i) also tackled well by almost all candidates. In part b(ii) the need to expand with respect to the Capillary number started to cause difficulties in terms of navigating a path through the question, but this ultimately enabled a very extensive simplification. A very small minority of solutions made it past this point to secure essentially all of the available marks.

Q2 Overall, the question was tackled well, with the bookwork in part (a) answered well in essentially all solutions. Solving Laplace's equation in part (b) caused difficulty in a small number of solutions, but more so than expected. Part (c) was largely well implemented. However, part (d) with involved algebra and the need for carefully considering the boundary conditions at $r = 2a$, while implementing approximations, was only navigated by a very small number of solutions.

Q3 This was not a popular question in the examination. Of the attempts parts (a), (b) were implemented well. In contrast, in almost all cases, the torque calculation in part (c) caused difficulty and progress in part (d) was very limited. The marks for these candidates on this question were nonetheless very similar to their marks for Q1, indicating an appropriate level of difficulty.

C5.9: Mechanical Mathematical Biology

Q1. This question was attempted by more than half of the candidates. Parts (a) and (b) were generally well done. There was some small confusion with the minus sign relating u

and $\theta'(S)$, which also depended on which orientation was defined as positive θ, but such small discrepancies were not penalised. For parts (c) and (d), the most common mistake was failing to apply the appropriate boundary conditions at the tip of the trunk. In particular, in (d)(i) the mass creates a boundary condition on n_z, but no body force, while the stick in (d)(ii) creates a torque, thus the boundary condition on θ at $S = L$ required taking the appropriate cross-product between the force of the stick and the $\mathbf{d}_3$ vector.

Q2. This question was attempted by almost all candidates, and was generally very well done, with a number of candidates scoring 20 or higher. The geometry bookwork in (a) was straightforward, but full marks required justification of each step in computing the matrix $L = G^{-1}K$, or at least the diagonal components. The statistical mechanics in part (b) presented a simpler version of the challengin Flicker spectroscopy calculation presented in lectures, and was handled well on the most part. Some candidates lost marks for not being careful with tracking variables through the steps, or not completing the final part, perhaps due to a lack of time.

Q3. This question was attempted by bit fewer than half the candidates, and results were a bit hit or miss, with some candidates struggling even with the bookwork components, which followed almost identically to examples worked in the lectures and problem sheets. Candidates who performed well had little trouble with parts (a) through (c), including the final part of (c), which required recognising that at the stable state $b = b^*$, the fact that $r = b^*$ is unstable means that material appearing at the boundary due to growth then moves inwards. Part (d) was more challenging in terms of knowing how to get started, and was only succesfully completed by one of two candidates. The key was to solve exactly the growh equation applied at $r = b$, from which one gets an exact expression for α and subsequently t_θ at $r = b$, showing that t_θ diverges to $-\infty$, i.e. infinite compressive hoop stress.

C5.11: Mathematical Geoscience

Question 1

This was the least popular question, with the lowest average mark. Most candidates managed part (a) well. Very few candidates managed part (b) well. There was a typo in part (b)(ii)—the inequality was the wrong way round, so it should have read $A_0 < 16k/(4+k)^2$. This did not adversely affect any candidate—only one candidate got near this result, and they identified the mistake.

Question 2

This was the most popular question, with the highest average mark. Most candidates did well on part (a), and quite a few did well on part (b). Very few candidates managed to get part (c), although there were some good attempts.

Question 3

In contrast to the other two questions, the scores on this question were more bimodal—there we some very good answers but a few poor answers. Parts (a) and (b) were largely straightforward and handled well. Many candidates also made a reasonable effort at part (c), although only a couple managed it completely. There was a small typo in part (c)—the lower case h should have been an upper case H in the equation $-\rho_m b = \rho h$. This caused no confusion to any candidate, and only one candidate commented on it.

C5.12: Mathematical Physiology

There was an even spread of candidates attempting questions 1, 2 and 3 and each question received a similar average mark by the candidates who attempted the question.

Question 1 Part (a) was generally completed well by all. Some candidates forgot to draw the graph of v versus t.

Part (b)(i) was well completed by all.

Part (b)(ii) approximately half of the candidates did not realize that they should equate powers of v to find the solution. Approximately half of the candidates who did realize this made algebraic mistakes that led to the wrong answer. Most candidates realized that there was something wrong as the answers did not makes sense, but they did not spot what.

Part (b)(iii) many candidates struggled with. Many candidates knew the ingredients but were unable to make the concrete connection to specific values for w^*, v_1 and v_2 and how these would be determined

Question 2 Part (a) was well completed by all.

Part (b), the majority of candidates did not define the variables that they were using for the equations. Candidates were not penalized for this. Many wrote the incorrect law of mass action equation for s_2.

Part (c) was straightforward and well completed by all.

Part (d) many candidates forgot the lower bound on α.

Part (e) was challenging for the majority of candidates. The majority of candidates knew what they needed to do, and articulated this in words even if they were unable to perform the associated mathematical manipulations. Many candidates did not expand locally around the points, which led to the wrong answers.

Question 3 Part (a), some candidates forgot to say that the valves are closed in the isovolumetric stages.

Parts (b)–(d) were well completed by all.

Part (e) candidates found challenging. Many candidates left terms involving δ in the leading-order equations, which led to the incorrect answers.

Part (f) very few managed to complete. Many did not attempt. For those who did, many spelled out in words what they wanted to do but could not supply the accompanying mathematics to support this.

C6.1: Numerical Linear Algebra

Question 1 had the fewest attempts. Students who attempted this question generally did well, with the most challenging part being the more elaborate proof of the search direction orthogonality.

Question 2 had unusually high scores due in part to the similarity between parts a) and c) which seemed to cause part c) to be more readily solved than anticipated. Part c) was anticipated to be challenging as Jacobi for eigen-values was only covered briefly in lecture and as it makes use of an unusual variant of Givens rotations; that said students performed remarkably on this part which was impressive.

Question 3 part a) on Householder transforms was standard and solved well by most students with the main omission being not showing the matrices are unitary. Part b) on the singular value decomposition was solved admirably with the main issue being eigenvalues which were zero and completing the matrices with orthogonal columns to span the space. Part c) on the power method was generally well solved, though few students were careful in considering the case of repeated eigenvalues and the eigenvector subspace being greater than one-dimensional.

C6.2: Continuous Optimisation

Question 1 was undertaken by all students, with strong answers showing good understanding. There was an essentially equal split between candidates attempting Question 2 and 3, with Question 2 being perceived as slightly more difficult. Overall, I have found the students have done well.

C6.3 Approximation of Functions

There was just one script. The attempts were solid particularly on bookwork problems. The understanding of the connection between analyticity and convergence of Chebyshev series and interpolation was delightfully advanced. Attempts at problems requiring new thinking were also solid, though not always correct.

C6.5: Theories of Deep Learning

The students generally produced compelling reports discussing a topic of their choice within the broad envelope of listed topics. Reports were typically of a high quality with an appropriate academic tone, excellent literature reviews, some computational experiments, and a rich discussion of the mathematical aspect of deep learning being considered.

C7.4: Introduction to Quantum Information

Question 1: This question was the most popular among the students, and those who attempted it performed very well. Parts (a), (b) and (c) posed no significant problems. In part (d), most students demonstrated a solid understanding of quantum circuit analysis, including the phase kick-back mechanism. Some even connected the parity check to the Bernstein-Vazirani problem discussed during lectures. Part (e) did not pose a problem for those who understood the phase kick-back mechanism, but some students struggled to explicitly specify the state of the first register at the output. The final part required interpreting maximally mixed states as an equally weighted statistical mixture of any two orthogonal states, specifically $|+\rangle$ and $|-\rangle$. Most students succeeded in this and correctly identified the probability of the inconclusive answer. However, the second part of (f), in which a can be any binary string of length three, allowed for some interpretation. Some students assumed the second register could be measured, while others assumed it could not. Both assumptions led to different probabilities of the inconclusive outcome, and both answers were accepted if properly justified. Overall, this question was relatively easy.

Question 2: This question was attempted by 23 out of 49 candidates. Overall, the students demonstrated a solid understanding of the core concepts of quantum error correction, including the origin of syndromes and the conditions for detecting and correcting errors. Consequently, nearly all of them provided reasonable answers to all parts of the question. However, due to frequent minor mistakes, only one person achieved full marks. The common mistakes are: missing minus sign for YY, confusing the role of checks and data qubits, not being able to identify the logical X and Z operators and confusing error detection with error correction.

Question 3: Apart from a few small points, most students who attempted this question did very well on the first few parts. Quite a few lost points in (b) for not actually proving that the new Kraus operators did indeed satisfy the orthogonality condition necessary to be Kraus operators. Part (c) was maybe the hardest for most, with few students successfully arguing why the Kraus operators had to be proportional to the identity; some used a clever commutativity argument, and others noticed the idea given in the model solutions. Part (e) was maybe poorly worded, since a lot of students did not justify why sigma' was in general distinct from sigma. As for parts (g) and (h), students tended to either get full marks (often drawing pictures of orthogonal isometries into a larger Hilbert space as non-overlapping subsets) or few marks at all; quite a few just neglected to define the measurements asked for in (g), and some tried to argue for applying V^{-1} instead of $V^{\dagger}$.

C7.5: General Relativity I

The most common combination of questions was Q2 with one of Q1 or Q3, the latter had almost the same number of attempts.

Q1: Part a (i) and (ii) were generally well done being purely book work and things seen in class. There were a few students who struggled with a (ii) but by far the majority got full marks for this part of part a. On the whole a (iii) was poorly done. Only a few students managed to find all three Killing vectors, though most managed to find the obvious one by inspection. Many were confused about how to begin despite being given the equation for a Killing vector in two different forms and some of those that managed to set up the differential equations then struggled to find solutions from these.

b) The vast majority managed to get 7/7 for part b. A common issue amongst those who did not get full marks was not being able to recognise how to combine the derivative on the RHS at the end. This was less a problem with the knowledge of the course and an oversight on the Leibniz rule for derivatives.

c) Only the top students managed to get full marks on this part of the question, though many students picked up a few marks. This looks like a nasty question at first sight but by using part b the student should recognise that b(i) fixes the connection up to the weight w. Some students managed to see this but then did not input the weight w into their answer. If the student managed to get c(i) extending to c(ii) was largely trivial, though a few who did get c(i) did not get c(ii). Some students obtained marks in c(ii) without having done c(i) correctly by observing how to extend c(i) to c(ii).

Q2: This was the most popular question by far and generally the best performance of students came in this question.

a) Generally well done. a(i) was almost always done correctly. A few students struggled to derive the geodesic equations while a few decided to derive the affinely parametrised equations instead despite it being stated to find the non-affinely parametrised one. For a(iii) a few students just stated to make the parameter the proper time without showing anything. Others just stated that if they made $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\lambda} = 0$ it would be affinely paramtrised, this does not answer the question and so they picked up only 1 mark.

b) This was done well by most students. Almost everyone got full marks for b(i). Some lost a mark for b(ii) because they did not justify why they can take the initial conditions that they chose, saying spherical symmetry was sufficient but they instead said 'assume'. The plots were generally correct, though sometimes the interpretation was patchy. The energy needs to be greater than the potential for motion to makes sense, a few students allowed for these types of trajectories.

c) Only a handful of students managed to get full marks. For c(i) none noticed that the light would be reflected from the boundary as it gets there in finite time. Some also solved for the affine parameter rather than the time. c(ii) should have been obvious from their plots in part b and was added to test their understanding. A radial timelike geodesic never reaches $r = \infty$ as can be seen from part b. Despite this, quite a few students managed to find a finite time for the timelike curve to reach $r = \infty$. This included students who had correctly plotted the potentials and correctly identified the motion.

Q3: Quite a few people forgot parts of Birkhoff's theorem and so did not get full marks. For part b) this was seen in the problem sheets. A number of students started trying to solve the geodesic equations despite being told not to assume a geodesic. Other students were on the correct track but then could not form the inequalities correctly and manipulate them. For example they identified correctly that $f(r)$ was negative but then when multiplying by $f(r)$ the inequality remained the same. They would still end up at the correct answer but it was clear that they had fudged the result to match the given answer.

c) Most got 5/5 for this question.

d) Very few managed to perform the coordinate transformation in d(i). Some ended up with a metric that did not make sense since it was degenerate on the horizon which they stated later was not the case. They should have realised this here. (ii) were free marks and most got 2/2. (iii) Not everyone explained this well. They needed to state in the original coordinates one has $U < 0, V > 0$ however in the new metric there is no singularity at $U = 0$ or $V = 0$ and so can be extended to $U, V \in \mathbb{R}$. On the whole this was done well. A few students did not manage to label the diagram correctly and the black hole region and exterior were flipped or in the part where there was no spacetime. These students were the ones who did not score well in on this question and should have used part (ii) to double check their labelling.

C7.6: General Relativity II

Q1 was the most popular, followed by Q2 and then Q3 though there was very little between the numbers.

Q1: Part (a) was done well by most candidates, though a few mixed up the definitions of a(ii) and a(iii).

Many got most marks in part (b). A common mistake in b (i) amongst those who did not get the full marks offered was just to show that a null vector remains null after a conformal transformation making no mention of geodesics. Part b(iv) confused a number of people despite the conserved charge being the same in the Killing vector case, not all of those who gave the conserved charge also said it needed to be a null geodesic.

Part (c) was not done very well. Very few managed to write down the Penrose diagram. There was a worrying number of students who were unable to identify that the radial coordinate was finite in this case. It behaves in much the same way that the θ behaves which should have made them think.

Q2: Parts (a) and (b) were done well overall. Part c was mixed. The strong students managed to do this without issue while the weaker students scored very few marks on this question.

Q3: Part (a) was probably too easy as most who attempted it scored 8/8. There were a few who lost marks because they did not define $h^{\mu\nu}$. Part (b) really separated the stronger candidates from the weaker ones. A number of candidates did not realise that they could use their results from part a to simplify the computations despite the hint to do so.

C7.7: Random Matrix Theory

Question 1 was well done by the students who attempted it. In (a) (iii), few students mentioned the fact that the moments are easily bounded here. Few students appealed to the Wick's theorem which does not apply here, as the distribution is not Gaussian. Part (b) is essentially bookwork, but some students were not precise enough when describing the correspondence with graphs in (i). Parts (i) and (ii) were well done in general. Common mistakes in Part c include the wrong development of the square. The explanation in (ii) of the $1/n^2$ were quite well done. In (iii), perhaps surprisingly, few students computed that the average is 2, using the number of Dyck paths or the Catalan number. The application of Borel-Cantelli was well understood.

Question 2 was attempted by most students. The average was quite high as the amount of bookwork or previously seen material was quite high. In Part a, most students saw the connection with the vandermonde determinant and showed orthogonality of the functions. Most students cited Gaudin's lemma correctly and got the right form for the probability density. Part b was well done, as most spacing got the correct scaling. In Part c, some students did not apply the correlation function to the difference of the angle and that gave them the wrong scaling. Perhaps surprisingly, few students were able to calculate the asymptotic behaviour of the density for small x even though the density was provided in the answer.

Question 3 was attempted by few students (around 10). This is perhaps because it involved Dyson Brownian motion which was covered in lectures but perhaps less than the first two questions. The question was a twist on a problem on the first problem sheet and a good student should have seen the connection. All students that attempted the question got Part (a) as it was simple linear algebra. No student realized that the gap process has a simple form, 2D Bessel process as seen in lecture. Few students applied the Itô's formula correctly. Most students got that the SDE is ill-defined if one starts at 0. Nobody attempted Part (c) even though the density of the OU process was provided. It is easy to write down the integral for the expectation from the density. It would have gotten full mark but nobody attempted it. The PDF is a similar computation. No information on Part (b) was necessary.

Summary: The exam was well done in general. There was a good amount of bookwork and most students did well on this. Some (very few) students struggled with the questions even though they were close to questions done in classes or in lectures.

C8.1: Stochastic Differential Equations

Question 1

Candidates choose this questions less often than the other two. Most of them tried to prove the martingale property in part (a) via Ito calculus and some boundedness consideration, which saved some work for part (b). The faster approach was via the semigroup property of the transition density $P(t, x)$. Those who attempted the rest of part (a) usually got it right, included the point (iii) which require a computation of probability based on the explicit form of the density Z. Part (b) was straightforward via Ito formula and the definition of stochastic exponential, bookwork related to Girsanov's transform. Some candidates didn't recognized that Z does not have a bounded variation part and therefore identified a wrong process L which was not a local martingale. Point (iv) of part (b) needed an argument to show that the solution of the SDE coincide in law with the conditioned process, so both reach 0 at time T. This can be concluded via pathwise uniqueness for the SDE. Part (c)(i) was correctly carried over by many, some applied wrongly the Ito formula. Part (c)(ii) posed problem to many which didn't recognized the Gaussian law of the process obtained.

Question 2

Part (a)(i,ii) was done correctly by most of the candidates which attempted it. It should be noted that some candidates estimated the stochastic integral as a standard Lebesgue integral failing to apply basic ideas of stochastic calculus. Part (a)(iii) which required some more computations, but which followed the same pattern as the previous point, was attempted by approx. half of the candidates. Part (b) required some care in handling the Ito formula for a non-smooth function using stopping times to avoid the region where the function was not regular. Anyway most of the students which applied, sometime formally, the Ito formula, then managed to proved the required boundedness properties and conclude. Part (b)(iii) also needed some care in the arguing the comparison result. Anyway most of the candidates which attempted had a correct intuition and got some points. Some candidated failed to note that (b)(iv) was a direct consequence of the previous points, trying to use, without success, different arguments based on Gronwall or other ideas. Many candidates gave the Tanaka SDE as example in Part (c), but failed to argue correctly about existence of solutions (via an auxiliary Brownian motion) while noticing correctly the non-uniqueness in law.

Question 3

Part (a)(i,ii) was done correctly by most of the candidates which attempted it. Some of the candidates didn't argued with precision about the continuity of the local time which allows to conclude the point (ii). Some candidates skipped some steps in the part (a)(iii) while applying Gronwall lemma to conclude. Part (a)(iv) was straightforward given the previous points and the candidates saw that. Few candidates understood that Part (b)(ii) depended on Part (a) and positivity could be concluded by comparison. Part (b)(iii) also created problems to many. In particular while most of the candidates identified the new Brownian motion which drives the SDE, almost none gave a precise argument to prove that. Part (b)(iv) was argued heuristically by many, but few applied part (a) to conclude the uniqueness in law via pathwise uniqueness. Part (c)(i) was done correctly by most of the candidates which attempted it, identifying in the continuity the crucial property which allow to identify the exponential from the multiplicative property. Part (c)(ii) was done in some meaningful way only by an handful of candidates. It required to apply Ito formula and identify a PDE from the vanishing of the bounded variation part. Of those which got the idea right some applied Ito formula not in the correct way, forgetting for example the explicit time dependence of the function $u(t, x)$.

C8.2: Stochastic Analysis and PDEs

Most candidates did well on part a) in each question. For some reason the question, Queston 2, about duality was by far the least popular although it was fairly straightforward; probably the use of random subsets in b) of that question was less familiar to many candidates.

Question 1 was well done by most candidates, although few managed to get far in the last part about finding the generator (which as a directly consquence of the cadlage Ito formula).

Queston 3 was also well done, though very few students managed to make substantional progress on part c) even despite the hint.

C8.3: Combinatorics

Question 1 Candidates knew the bookwork very well, almost everyone did correctly part (a). The proof for Sauer-Shelah (b), was also mostly correct (also bookwork). Although most candidates decided to present the proof using induction and not with compression, which could have helped them in the last item (as was also hinted in the last item). Candidates who presented a correct proof received full marks (regardless of the method of proof). The next item (c) was also simple bookwork. The final item (d) was splitted into 3. In (d)(i) candidates were hinted to use compression. The compression needed here is almost identical to the one used to prove Sauer-Shelah (from item (b)). Some candidates used the wrong compression (deleting and adding elements from the set, instead of just deleting as in Sauer-Shelah), which didn't give the required outcome of the question (a downset of the same size with diameter non-increased). Those who used the correct compression (deleting one item in each step), were almost always able to complete this argument correctly. Item (d)(ii) was very short and most candidates were able to solve it. The final item (d)(iii) was the most challenging one, more than half of the candidates did correctly the special case of the small sets as was hinted in this item. Only few candidates were able to prove the general case.

Question 2 The first part of item (a) was bookwork and most candidates wrote the definitions correctly. They were then required to prove a short statement regarding the size of a poset. This statement follows directly from Dilworth's Theorem from the lectures. Only few of the candidates used the theorem to show the statement (which in this case is a 3 lines proof). Some of the candidates used induction and some basically re-proved parts of the Dilworth's theorem, which made the argument much longer. Candidates who answered this correctly received full mark (regardless of the method). Some candidates tried to prove this by using the fact that every antichain intersect with a chain at most once. However this gives the opposite bound then what needed to solve this problem (this is how we proved one direction of Dilworth's theorem, but it is the opposite direction from what needed here). In item (b) the candidates were ask to state Dilworth's Theorem, which the all did correctly. In item (c) most candidates defined the partial order correctly (as the divisibility order). Once they did this the answer was almost immediate. In the final step they were expected to use what was proved about the size of a poset in item (a). Most of them did it as expected, some re proved it using Dilworth's theorem, and few tried to re-proved it from scratch. The final item (d) was the most challenging with only few candidates able to define the correct partial order. Many defined a partial order on the edges (instead of the vertices), which indeed gives a partial order, but the chains and antichains it defines does not corresponds to those needed in order to solve the question.

Question 3 In item (a)(i) almost all candidates stated Kleitman's Theorem correctly, and also applied it to show the new inequality. In the next item (a)(ii), most candidates understood the hint and used it. However for some the next step was incorrect or missing. Several defined the upset incorrectly in such a way that the intersection between the upset and the downset didn't contain the original family. And thus even when applying the statement from the first item correctly, the outcome didn't give any information regarding the original family. Some didn't define an upset and tried to work only with the downset from the hint. Item (b) was bookwork and most candidates proved the theorem correctly, as we did in the lectures. Item (c) was an immediate application of the theorem proved in item (b). Item (d) was slightly more challenging, where candidates were expected to apply the statement proved in item (c). Some candidates were able to prove only the special case mentioned in the hint. Some candidates approached the general case correctly, but failed to define the parameters b_i.

C8.4: Probabilistic Combinatorics

Question 1 was mostly reasonably well done; the (fairly substantial) part (b)(iii) was the main discriminating factor. For (a)(iv) and (b)(v) something brief was expected, though for the former it does not suffice to say that the events being counted are not independent.

Question 2 was (as always with this section of the course - but it is examinable, and that is emphasized in lectures each year) the least popular. Those who did attempt it did fairly well. The main part is only a very minor modification of bookwork.

Question 3 (a) was mostly well done, though a number of candidates proved the first form of Janson's inequality, which the question asks you to assume. And a number weren't clear that the random choice of a subset of events to consider happens in a different probability space from the events themselves. (You don't need to explicitly say this, but do need to apply Janson 1 to a fixed subset S of the events; and then/separately argue that a subset S exists with suitable parameters μ_S and Δ_S by a random argument.) (b) was generaly well done. (c) proved trickier; partly many candidates failed to correctly use the results of part (b) in the calculation.

C8.6: Limit Theorems and Large Deviations in Probability

Question 1. The question was attempted by most of candidates and received good solutions for the standard book work part. Candidates generally knew the basic definitions but had difficulty to apply them to answer (a)(iii) which was a slight variation of a basic fact of weak convergence. Candidates generally knew one should prove (c) by using Kolmogorov's theorem, but failed to produce a simple estimate to argue.

Question 2. Overall the question received good solutions. The candidates who attempted this question did well for part (a) and part (b). Part (c) requires the candidate to collect a few basic facts about the rate function appearing in the Cramer's large deviation principle, and to work out the rate function explicitly. Surprisingly to me, a few candidates even could not write done the distribution of a Poisson distribution, and thus could not complete this part.

Question 3. This is a less attracting question for candidates, so thos who attempted answered quite well for part (a) and part (b), which were mainly developed from standard (book work type) material in the lecture. Candidates generally knew that one should answer part (c) by using Varadhan's contraction principle, while few candidates could carry out the proof properly.

Statistics Units

Reports on the following courses may be found in the Mathematics and Statistics examiners' report.

SC1 - Stochastic Models in Mathematical Genetics SC2 - Probability and Statistics for Network Analysis SC4 - Advanced Topics in Statistical Machine Learning SC5 - Advanced Simulation Methods SC7 - Bayes Methods SC9 - Interacting Particle Systems

Computer Science

Reports on the Computer Science courses may be found in the Mathematics and Computer Science examiners' report.

F. Comments on performance of identifiable individuals

1. Prizes

The following prizes were awarded:

- Gibbs Prize. Anubhab Ghosal, St Edmund Hall
- Gibbs Prize. Mario Marcos Losada, Brasenose College
- Gibbs Dissertation Prize. Samuel Moore, Merton College Oliver Perree, Balliol College
- Junior Mathematics Prize. Samuel Ketchell, St Hugh's College
- IMA Prize. Orson Hart, Pembroke College

G. Names of members of the Board of Examiners

- **Examiners:**
  Prof. Jason Lotay (Chair)
  Prof. Eamonn Gaffney
  Prof. Emmanuel Breuillard
  Prof. Ehud Hrushovski Prof. Philip Maini
  Prof. Alan Champneys (External)
  Prof. Roger Moser (External)

- **Assessors**
  Prof. Konstantin Ardakov
  Prof. Louis-Pierre Arguin
  Prof. Ruth Baker
  Prof. Paul Balister
  Dr. Martin Bays
  Dr. Philip Beeley
  Dr. Lukas Brantner
  Prof. Helen Byrne
  Prof. Jose Carrillo
  Prof. Coralia Cartis
  Prof. Jon Chapman
  Prof. Gui-Qiang Chen
  Prof. Dan Ciubotaru
  Dr. Chris Couzens
  Prof. Andrew Dancer
  Prof. Chris Douglas
  Dr. Daniel Drimbe
  Prof. Cornelia Drutu
  Prof. Artur Ekert
  Prof. Radek Erban

Dr. Karin Erdmann
Dr. Antonio Esposito
Dr. Paz Fink Shustin
Dr. Kathryn Gillow
Prof. Christina Goldschmidt
Prof. Ben Green
Prof. Ian Griffiths
Prof. Peter Grindrod
Prof. Massimiliano Gubinelli
Prof. Ben Hambly
Prof. Heather Harrington
Prof. Raphael Hauser
Prof. André Henriques
Prof. Ian Hewitt
Dr. Christopher Hollings
Dr. Aleksander Horawa
Prof. Peter Howell
Dr. Sam Hughes
Dr. Desi Ivanova
Prof. Dominic Joyce
Prof. Andras Juhasz
Prof. Jon Keating
Prof. Peter Keevash
Prof. Dawid Kielak
Prof. Robin Knight
Prof. Jochen Koenigsmann
Dr. Brett Kolesnik
Dr. Andrey Kormilitzin
Prof. Yakov Kremnitzer
Prof. Jan Kristensen
Dr. Gal Kronenberg
Prof. Renaud Lambiotte
Dr. Tim LaRock
Prof. Alan Lauder
Prof. Terry Lyons
Prof. James Maynard
Prof. Kevin McGerty
Dr. Andrew McLeod
Prof. Andrea Mondino
Prof. Derek Moulton
Prof. Andreas Muench
Prof. Simon Myers
Prof. Yuji Nakatsukasa
Prof. Vidit Nanda
Prof. James Newton
Prof. Luc Nguyen
Prof. Geoff Nicholls
Prof. Nikolay Nikolov

Prof. Harald Oberhauser
Prof. James Oliver
Dr. Charles Parker
Dr. Nicola Pedreschi
Dr. Harry Petyt
Prof. Zhongmin Qian
Prof. Gesine Reinert
Prof. Christopher Reisinger
Prof. Oliver Riordan
Prof. Alexander Ritter
Prof. Damian Rossler
Dr. Emilio Rossi Ferruci
Prof. Melanie Rupflin
Prof. Tom Sanders
Prof. Alex Scott
Dr. Davide Spriano
Prof. Rolf Suabedissen
Dr. Saifuddin Syed
Prof. Jared Tanner
Dr. Robin Thompson
Prof. Dominic Vella
Prof. Sarah Waters
Prof. Andrew Wathen
Dr. Maria Yakerson
Dr. Jinhe Ye