# An Introduction to Optimal Control Theory

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**Aim:** These lectures are ideally intended for students in Mathematical Analysis, Differential Geometry, and Mechanics. However, because of the strong applicative potential of the addressed issues, also people from mathematically supported disciplines, like Engineering, Physics, Epidemiology, and Economics, might be interested in this course.

We aim at presenting crucial results in Optimal Control Theory of ODEs, with a special attention to necessary conditions for minima. It will be shown how the simple idea of 'set separation', including the corresponding separability criterions for (suitable) approximating cones, lie behind obtaining necessary condititions for minima (in the form of Maximum Priciples), in both elementary optimization problems and in nonlinear optimal control problems.

Time permitting, some connections with Differential Geometric Controllability or Hamilton-Jacobi equations will be treated as well.

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**Course requirements:** Basic calculus, basic Lebesgue measure theory. Other prerequisites –for instance, fixed point theorems, absolutely continuous maps, differential manifolds– will be recalled during the course.

**Main contents:**

1. Brower fixed point theorem and a parameterized version of Banach fixed point theoreom. A directional 'open mapping' theorem with low regularity. Set separation and cone separability

2. Review of ODE's with vector fields measurable in time: local and global existence, uniqueness, continuity and differentiability with respect to initial conditions.

3. An abstract constrained minimum problem.

4. The Pontryagin Maximum Principle (PMP) with end-point constraints, with applications.

5. Controllability of control systems, at the first or higher order (Lie brackets).

6. If time permits: basic elements of Hamilton-Jacobi PDE's.

**Teaching material:** The whole teaching material will available on Moodle Platform (to which students will be invited to subscribe), in the form of printed lecture notes.

**Online platform:** The course will be entirely delivered on Zoom platform.

**Schedule:** The following schedule might be changed after an unanimous decision by the students and the teacher. All lectures will begin at 9 a.m. Greenwhich time (i.e. at 10 a.m. Rome time) and will end at 10.45 a.m. Greenwhich time (i.e. at 11.45 a.m. Rome time), with a 5-10 minutes break after the first hour.

Dates:
*March 2021*: 2, 4, ,9, 11, 16, 18, 23, 25, 30;
*April 2021*: 8, 13,15.