Black holes in twistor space

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University of Edinburgh

Penrose at 95 & Twistors at 60

10 September 2026

with Araneda & Seet $\cup$ Sharma [2601.05037,2607.06236]

Let's get started...

We're here to celebrate an *incredible* intellectual legacy:

- conformal compactification
- spin-coefficient formalism
- singularity theorems
- energy extraction from black holes
- quasi-local mass
- twistor theory
- ⋮

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In this talk, focus our attention on one of Roger's many remarkable ideas

Twistor theory

For the uninitiated, what is **twistor theory**?

Quick answer: Physical information $\leftrightarrow$ Complex geometry

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Why should you care?

Quick answer: Can solve certain (highly non-linear) equations with free analytic data

Some housekeeping...

Before details, inevitable notational carnage:

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Let $Z^A = (\mu^{\dot{\alpha}}, \lambda_\alpha)$ homog. coords on $\mathbb{CP}^3$

$$\mathbb{PT} = \left\{ Z \in \mathbb{CP}^3 \,|\, \lambda_\alpha \neq 0 \right\}$$

So $\mathbb{PT} \cong \mathcal{O}(1) \oplus \mathcal{O}(1) \to \mathbb{CP}^1$

This will be our convention for 'flat' twistor space

i.e., twistor space of $\mathbb{M}$, complexified Minkowski space

Reminder

The *twistor correspondence* between $\mathbb{M}$ and $\mathbb{PT}$:

Points in $\mathbb{M} \leftrightarrow$ Twistor curves: $\mu^{\dot\alpha} = \mathsf{F}^{\dot\alpha}(\lambda)$ subject to

$$\bar\partial \mathsf{F}^{\dot\alpha} = 0\,, \qquad \mathsf{F}^{\dot\alpha}(b\,\lambda) = b\,\mathsf{F}^{\dot\alpha}(\lambda) \quad \forall b \in \mathbb{C}^*$$

Liouville's thm $\Rightarrow \mathsf{F}^{\dot\alpha} = \mathsf{F}^{\dot\alpha}(x,\lambda) = x^{\alpha\dot\alpha}\,\lambda_\alpha$

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Metric encoded in

$$\Sigma = \mathrm{d}_x \mathsf{F}^{\dot\alpha} \wedge \mathrm{d}_x \mathsf{F}_{\dot\alpha} = \lambda_\alpha\,\lambda_\beta\,\mathrm{d}x^{\alpha\dot\alpha} \wedge \mathrm{d}x^{\beta}{}_{\dot\alpha}$$

$\Rightarrow\; e^{\alpha\dot\alpha} = \mathrm{d}x^{\alpha\dot\alpha} \Leftrightarrow \mathrm{d}s^2 = \eta_{ab}\,\mathrm{d}x^a\,\mathrm{d}x^b$

More generally...

What is a twistor space in general?

A 3d complex manifold $(\mathbb{P}\mathscr{T}, \bar{\nabla})$ which:

- is a $\mathbb{C}$-deformation of $\mathbb{PT}$,
- has a complex structure $\bar{\nabla} : \Omega^{p,q}(\mathbb{P}\mathscr{T}) \to \Omega^{p,q+1}(\mathbb{P}\mathscr{T})$ of the form

$$\bar{\nabla} = \bar{\partial} + \frac{\partial \mathsf{h}}{\partial \mu_{\dot{\alpha}}} \frac{\partial}{\partial \mu^{\dot{\alpha}}}\,, \qquad \mathsf{h} \in \Omega^{0,1}(\mathbb{PT}, \mathcal{O}(2))$$

- which is integrable

$$\bar{\nabla}^2 = 0 \quad \Leftrightarrow \quad \bar{\partial}\mathsf{h} + \frac{1}{2}\frac{\partial \mathsf{h}}{\partial \mu_{\dot{\alpha}}} \wedge \frac{\partial \mathsf{h}}{\partial \mu^{\dot{\alpha}}} = 0$$

Who cares?

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- complex manifolds
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Great, but why do you care?

Time for one of Roger's Greatest Hits...

Non-linear Graviton Theorem

Theorem (Penrose)

There is a 1:1 correspondence between:

- *Vacuum, self-dual 4-manifolds* $\mathscr{M}$, *and*
- *twistor spaces* $\mathbb{P}\mathscr{T}$

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Refinement in Euclidean signature [Atiyah-Hitchin-Singer]

$$\mathbb{P}\mathscr{T} \cong \mathscr{M}_{\mathbb{R}} \times S^2 \quad \text{as real manifolds}$$

Quick reminder

What is a **vacuum, self-dual 4-manifold**?

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Weyl tensor decomposition

$$C_{abcd} = \epsilon_{\alpha\beta}\,\epsilon_{\gamma\delta}\,\widetilde{\Psi}_{\dot\alpha\dot\beta\dot\gamma\dot\delta} + \epsilon_{\dot\alpha\dot\beta}\,\epsilon_{\dot\gamma\dot\delta}\,\Psi_{\alpha\beta\gamma\delta}\,,$$

$\widetilde{\Psi}_{\dot\alpha\dot\beta\dot\gamma\dot\delta}/\Psi_{\alpha\beta\gamma\delta}$ are *self-dual*/*anti-self-dual* Weyl curvatures

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A metric is *vacuum self-dual* (i.e., *hyperkähler*) if:

$$R_{ab} = 0 \text{ and } \Psi_{\alpha\beta\gamma\delta} = 0$$

For Lorentzian-real 4-manifolds, $\widetilde{\Psi}_{\dot\alpha\dot\beta\dot\gamma\dot\delta} = \bar{\Psi}_{\dot\alpha\dot\beta\dot\gamma\dot\delta}$
$\Rightarrow$ non-trivial only in Euclidean or split signature, or for complexified metrics

Where is the HK metric encoded in $\mathbb{P}\mathscr{T}$?

Points $x \in \mathscr{M} \leftrightarrow$ holomorphic (wrt $\bar{\nabla}$) sections

$$(\mu^{\dot{\alpha}} = \mathsf{F}^{\dot{\alpha}}(x, \lambda),\ \lambda_\alpha) : \mathbb{CP}^1 \to \mathbb{P}\mathscr{T}$$

Holomorphic $\Rightarrow$

$$\bar{\partial}|_X \mathsf{F}^{\dot{\alpha}}(x, \lambda) = \left.\frac{\partial \mathsf{h}}{\partial \mu_{\dot{\alpha}}}\right|_X$$

HK metric

Build 2-form on $\mathscr{M} \times \mathbb{CP}^1$ [Gindikin]

$$\Sigma := \mathrm{d}_x \mathsf{F}^{\dot\alpha} \wedge \mathrm{d}_x \mathsf{F}_{\dot\alpha}\,, \qquad \text{obeys} \quad \bar\partial|_X \Sigma = 0$$

Liouville's thm $\Rightarrow \Sigma = \Sigma^{\alpha\beta}(x)\,\lambda_\alpha\lambda_\beta$

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$\Sigma \wedge \Sigma = 0 \Rightarrow \Sigma^{\alpha\beta} = e^{\alpha\dot\alpha} \wedge e^{\beta}{}_{\dot\alpha}$

- $e^{\alpha\dot\alpha}$ define metric $\mathrm{d}s^2 = \epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\, e^{\alpha\dot\alpha} \odot e^{\beta\dot\beta}$ on $\mathscr{M}$

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$\mathrm{d}_x\Sigma = 0 \Rightarrow \mathrm{d}\Sigma^{\alpha\beta} = 0$

- metric on $\mathscr{M}$ is HK $\Leftrightarrow$ vacuum SD!

Analogous construction for SD gauge fields over any SD $\mathscr{M}$

[Ward, Flaherty, Atiyah-Hitchin-Singer, Ward-Wells]

Generally, seems that twistor spaces give *self-dual* field configurations...

Can we get away from the SD sector in twistor space?

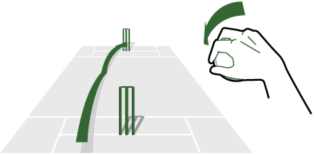
The Googly Problem

This is **the** central problem of twistor theory:

Is there a way to describe non-linear, non-SD fields which is:

1. given entirely on twistor space,
2. relies on holomorphic data,
3. does not explicitly solve field equations?

TMS Commentary



Wikipedia: 'Less skilled batters, or ones who have lost their concentration, can be deceived completely...'

The bad news...

Since 1977, **no** solution, at the fully non-linear level

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An alternative approach provided by *ambitwistor theory*, but field equations not easy

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Not all doom and gloom, though...

Public service announcement

Googly problem can be solved *perturbatively* [Witten, Mason, Sharma]

Central idea: Yang-Mills and GR admit pert. expansions around SD sector!

$$-\frac{1}{2\,\mathrm{g}^2}\int \mathrm{tr}(F\wedge \star F) \simeq \int \mathrm{tr}(B\wedge F^-) + \frac{\mathrm{g}^2}{4}\int \mathrm{tr}(B\wedge B)$$

[Chalmers-Siegel]

$$\frac{1}{2\,\kappa^2}\int \mathrm{d}^4x\,\sqrt{|g|}\,R \simeq \int \Sigma^{\alpha\beta}\wedge\left(\mathrm{d}\Gamma_{\alpha\beta} + \kappa^2\Gamma_\alpha{}^\gamma\wedge\Gamma_{\beta\gamma}\right)$$

[Plebanski]

Upshot: googly problem hasn't stopped twistor theory!

- Generating many/most classical integrable systems [Ward, Mason-Woodhouse, Costello-Yamazaki-Witten, Bittleston-Skinner,...]
- Complete tree-level S-matrices [Witten, Roiban-Spradlin-Volovich, Berkovits, Cachazo-Skinner,...]
- Amplitudes, Wilson loops & correlation functions [Mason-Skinner, Bullimore-Skinner, TA-Bullimore-Mason-Skinner, Koster-Mitev-Staudacher-Wilhelm,...]
- Bootstrap massless QCD amplitudes [Costello, Dixon-Morales,...]
- Asymp. flat holography [TA-Mason-Sharma, Costello-Paquette-Sharma, TA-Bu-Casali-Sharma,...]

(...plus tons of other work by people in the room!)

OK, but Googly Problem remains unsolved at fully non-perturbative level.

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Spoiler Alert: we are **not** going to solve googly problem in general.

Instead, look for one particularly interesting solution...

A mystery

Some long-standing facts:

- All black hole metrics are *conformally Kähler* [Flaherty, Aksteiner-Araneda]
- SD Kähler structures $\leftrightarrow$ quadric hypersurfaces in twistor spaces [Pontecorvo]

Is it possible to find a bridge between these two facts?

A remarkable paper

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Twistor quadrics and black holes

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Max Planck Institute for Gravitational Physics (Albert Einstein Institute),
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Rough idea:

1. Take a holmorphic quadric $\mathcal{Q} \subset \mathbb{PT}$
2. Induces a $\mathbb{C}$-structure and Kähler form (J, ω) on $\mathbb{M}$
3. Deform $\mathcal{Q}$ so that $J \to \tilde{J}$, $\omega \to \omega$
4. Demand compatibility between $(\tilde{J}, \omega)$, obtain curved, non-SD Kähler metric

Incredibly, the entire Plebanski-Demianski family can be obtained in this way! [Araneda]

Not quite a solution to the googly problem for BHs, though:

- Not clear if deformed quadrics live in a twistor space
- Emergence of field equations unclear: which deformations give conformally Einstein Kähler metrics?

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- Emergence of field equations unclear: which deformations give conformally Einstein Kähler metrics?

But clearly a smoking gun!

ASD Black Holes

To get to grips with this, want to understand a toy version of googly problem for BHs:

anti-self-dual black hole metrics

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ASD Black Holes

To get to grips with this, want to understand a toy version of googly problem for BHs:

anti-self-dual black hole metrics

Precise definition: any ASD vacuum Riemannian metric obtained as slice of complexified Plebanski-Demianski family

Only 3 ASD BHs: ASD Taub-NUT, Eguchi-Hanson, ASD Plebanski-Demianski

What we do **not** want to do:

Non-linear graviton on dual twistor space

Instead, want a *googly* description of these ASD BHs

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Non-linear graviton on dual twistor space

Instead, want a *googly* description of these ASD BHs

Basic idea: something to do with twistor quadrics

TWISTOR NEWSLETTER No. 34

Contents

Twistor quadrics

A holomorphic, non-degenerate quadratic variety $\mathcal{Q} \subset \mathbb{PT}$:

$$\mathcal{Q} = \left\{ Z \in \mathbb{PT} \,|\, Q_{AB}\, Z^A\, Z^B = 0 \right\}$$

for $Q_{AB} = Q_{BA}$, $Q_{AB} \neq C_{(A} D_{B)}$

Block decomposition

$$Q_{AB} = \begin{pmatrix} a_{\dot\alpha\dot\beta} & b^{\beta}{}_{\dot\alpha} \\ b^{\alpha}{}_{\dot\beta} & c^{\alpha\beta} \end{pmatrix}$$

for constant, symmetric $a_{\dot\alpha\dot\beta}$, $b^{\alpha}{}_{\dot\beta}$, $c^{\alpha\beta}$

Encode loads of nice data on $\mathbb{M}$, including [Tod-Ward, Sommers, Araneda]

$$Q|_X = K^{\alpha\beta}(x)\,\lambda_\alpha\,\lambda_\beta\,, \qquad \nabla^{(\alpha|\dot{\alpha}|}K^{\beta\gamma)} = 0$$

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$$Q|_X = K^{\alpha\beta}(x)\,\lambda_\alpha\,\lambda_\beta\,, \qquad \nabla^{(\alpha|\dot\alpha|}K^{\beta\gamma)} = 0$$

Let $o_{\dot\alpha} = (1,0)$, $\iota_{\dot\alpha} = (0,1)$ be spinor dyad.

Say that $\mathcal{Q}$ is *generic* if

$$o^{\dot\alpha}\,\frac{\partial Q}{\partial \mu^{\dot\alpha}} \neq 0$$

ASD Maxwell fields

Every generic twistor quadric corresponds to a null ASD Maxwell field:

$$f(Z) = -\mathrm{i}\left[Q\left(o^{\dot\alpha}\,\frac{\partial Q}{\partial\mu^{\dot\alpha}}\right)\right]^{-1} \in \check{H}^1(\mathbb{PT},\mathcal{O}(-4))$$

Penrose transform $\Rightarrow$

$$\phi_{\alpha\beta}(x) = \frac{1}{2\pi\mathrm{i}}\oint \mathrm{D}\lambda\,\lambda_\alpha\,\lambda_\beta\,f|_X = \frac{-\mathrm{i}\,A_\alpha\,A_\beta}{\langle B\,A\rangle\,(k\cdot\xi)^2}$$

obeys $\nabla^{\alpha\dot\alpha}\phi_{\alpha\beta} = 0 = \phi^{\alpha\beta}\,\phi_{\alpha\beta}$, where

$$K^{\alpha\beta} = A^{(\alpha}\,B^{\beta)}\,, \quad k^{\alpha\dot\alpha} = A^\alpha\,o^{\dot\alpha}\,, \quad \xi^{\alpha\dot\alpha} = \frac{2}{3}\,\nabla_\beta{}^{\dot\alpha}K^{\alpha\beta}$$

Cool result

Theorem (Tod)

Given any null ASD Maxwell field $\phi_{\alpha\beta}$, the metric

$$\mathrm{d}s^2 = \mathrm{d}x^{\alpha\dot{\alpha}}\,\mathrm{d}x_{\alpha\dot{\alpha}} + o_{\dot{\alpha}}\,o_{\dot{\beta}}\,\phi_{\alpha\beta}\,\mathrm{d}x^{\alpha\dot{\alpha}}\,\mathrm{d}x^{\beta\dot{\beta}}$$

is Kerr-Schild, vacuum ASD.

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is Kerr-Schild, vacuum ASD.

Upshot: generic twistor quadrics $\Rightarrow$ vacuum ASD Kerr-Schild metrics

Classifying quadrics

Up to translations, only 3 cases of generic twistor quadrics:

$$\text{Case A:} \qquad Q_{AB} = \begin{pmatrix} 0 & b^{\beta}{}_{\dot{\alpha}} \\ b^{\alpha}{}_{\dot{\beta}} & 0 \end{pmatrix}$$

$$\text{Case B:} \qquad Q_{AB} = \begin{pmatrix} a_{\dot{\alpha}\dot{\beta}} & 0 \\ 0 & 0 \end{pmatrix}$$

$$\text{Case C:} \qquad Q_{AB} = \begin{pmatrix} a_{\dot{\alpha}\dot{\beta}} & 0 \\ 0 & c^{\alpha\beta} \end{pmatrix}$$

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Additionally impose Euclidean reality conditions $\Rightarrow$ obtain Riemannian ASD Kerr-Schild metric of type D

So what?

Theorem (TA-Araneda-Seet)

There is a 1:1 correspondence between generic twistor quadrics and ASD black hole metrics, with

- *Case A* $\leftrightarrow$ *ASD Taub-NUT*
- *Case B* $\leftrightarrow$ *ASD Eguchi-Hanson*
- *Case C* $\leftrightarrow$ *ASD Plebanski-Demianski*

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Corollary

All ASD black hole metrics are Kerr-Schild

Proof:

1. Twistor quadrics encode tri-holomorphic Killing vector for associated KS metric
2. These metrics admit a Gibbons-Hawking form
3. Solve explicitly for diffeo from KS to Gibbons-Hawking coordinates in each case (hard!)
4. Match with standard expressions for ASD BH metrics in literature

Note: uncovered previously missed degenerate case of ASD Plebanski-Demianski in the process

What just happened?

Gave a googly construction for ASD black hole metrics!

- Entirely algebraic – no deformation problem
- Places type D condition on same footing as anti-self-duality
- Quadric encodes Kerr-Schild and Gibbons-Hawking forms of metrics

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- Quadric encodes Kerr-Schild and Gibbons-Hawking forms of metrics

Great, but we wanted a *non*-(A)SD black hole metric...

Old idea

Old intuitive picture:

$$\text{Schwarzschild `='} \ \text{SDTN} + \text{ASDTN}$$

[Hawking]

Old idea

Old intuitive picture:

$$\text{Schwarzschild ‘$=$’ SDTN} + \text{ASDTN}$$

[Hawking]

Explicit superposition principle using Kerr-Schild form of metrics [Kim]

The plan: superimpose SDTN with ASDTN in twistor space

Kerr-Schild form of SDTN [Kim, Desai-Herczeg-McNutt-Pezzelle]

$$
\begin{aligned}
\mathrm{d}s^2 = \mathrm{d}\tau^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 \\
- \frac{2M}{r}\left[\mathrm{d}\tau - \mathrm{i}\,\mathrm{d}z + \left(\frac{z+r}{x+\mathrm{i}\,y}\right)(\mathrm{i}\,\mathrm{d}x - \mathrm{d}y)\right]^2 \\
:= \delta - \frac{4\mathrm{i}\,M}{r\,(y-\mathrm{i}\,x)}\left(o_\alpha\,\alpha_{\dot\alpha}\,\mathrm{d}x^{\alpha\dot\alpha}\right)^2
\end{aligned}
$$

for

$$
\alpha_{\dot\alpha} = \sqrt{\frac{y-\mathrm{i}\,x}{2}}\left(o_{\dot\alpha} + \iota_{\dot\alpha}\,\frac{z+r}{x+\mathrm{i}\,y}\right), \qquad o_{\dot\alpha} = (1,0)\,, \quad \iota_{\dot\alpha} = (0,1)
$$

Twistor space

Non-linear graviton construction of SDTN in Kerr-Schild gauge
$+$
Euclidean reality conditions $\Rightarrow \mathbb{P}\mathscr{T} \cong \mathbb{R}^4 \times \mathbb{CP}^1$ with

$$\bar{\nabla} = \bar{\partial} - \frac{4\mathrm{i}\, M\, \langle o\, \lambda\rangle^2}{[\beta\, \alpha]}\, \frac{\alpha_{\dot{\alpha}}\, \alpha_{\dot{\beta}}}{\langle o|\, T\, |\alpha]^2}\, \frac{\hat{\lambda}_\alpha\, \hat{\lambda}^\beta}{\langle \lambda\, \hat{\lambda}\rangle^2}\, \mathrm{d}x^{\alpha\dot{\alpha}}\, \frac{\partial}{\partial x^{\beta\dot{\beta}}}$$

where $T^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} = \partial_\tau$,

$$\beta_{\dot{\alpha}} = \sqrt{\frac{y - \mathrm{i}\, x}{2}} \left(o_{\dot{\alpha}} + \iota_{\dot{\alpha}}\, \frac{z - r}{x + \mathrm{i}\, y} \right) , \qquad \hat{\lambda}_\alpha = T_{\alpha\dot{\alpha}}\, \bar{\lambda}^{\dot{\alpha}}$$

Obeys $\bar{\nabla}^2 = 0$ due to $\alpha^{\dot{\alpha}}\, \alpha^{\dot{\beta}}\, \nabla_{\alpha\dot{\alpha}}\alpha_{\dot{\beta}} = 0$

ASDTN

ASD Taub-NUT in Kerr-Schild form:

$$\mathrm{d}s^2 = \delta - 4\mathrm{i}\, M\, \frac{A_\alpha\, A_\beta\, o_{\dot\alpha}\, o_{\dot\beta}}{\langle B\, A\rangle\, \langle A|\, T\,|o]^2}\, \mathrm{d}x^{\alpha\dot\alpha}\, \mathrm{d}x^{\beta\dot\beta}$$

where $A_\alpha = \sqrt{2}\, T_\alpha{}^{\dot\alpha}\, \alpha_{\dot\alpha}$ and $B_\alpha = \sqrt{2}\, T_\alpha{}^{\dot\alpha}\, \beta_{\dot\alpha}$

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where $A_\alpha = \sqrt{2}\, T_\alpha{}^{\dot\alpha}\, \alpha_{\dot\alpha}$ and $B_\alpha = \sqrt{2}\, T_\alpha{}^{\dot\alpha}\, \beta_{\dot\alpha}$

From previous discussion, corresponds to quadric

$$Q(x, \lambda) := \langle A\, \lambda\rangle\, \langle B\, \lambda\rangle = 0$$

in the *flat* twistor space $\mathbb{PT} \cong \mathbb{R}^4 \times \mathbb{CP}^1$

Basic idea

Put quadric Q corresponding to ASDTN into $\mathbb{P}\mathscr{T}$ corresponding to SDTN

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Put quadric Q corresponding to ASDTN into $\mathbb{P}\mathscr{T}$ corresponding to SDTN

Obvious problem: Q *not* holomorphic in the complex structure of $\mathbb{P}\mathscr{T}$

$$\bar{\nabla} Q \neq 0$$

Coincidence locus

However, we can consider bits of Q which *are* holomorphic:

$$\chi = \left\{(x, \lambda) \in \mathbb{P}\mathscr{T} \mid Q = 0 = \bar{\nabla} Q\right\}$$

Call this χ the **coincidence locus**

Coincidence locus

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$$\chi = \left\{(x,\lambda) \in \mathbb{P}\mathscr{T} \mid Q = 0 = \bar{\nabla} Q\right\}$$

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Expect $\dim_{\mathbb{C}} \chi = 1$, but remarkably

$$\chi = \{(x,\lambda) \in \mathbb{P}\mathscr{T} \mid \langle A\,\lambda\rangle = 0\} \quad \Rightarrow \quad \dim_{\mathbb{C}} \chi = 2!$$

Coincidence locus inherits:

- a $\mathbb{C}$-structure $J : T_\chi \to T_\chi$ from $\mathbb{P}\mathscr{T}$

$$J = \mathrm{i}\left(\delta^{\dot\beta}_{\dot\alpha}\frac{B_\alpha A^\beta + B^\beta A_\alpha}{\langle A\, B\rangle} - 8\mathrm{i}M\frac{\alpha_{\dot\alpha}\,\alpha^{\dot\beta}\,B_\alpha\,B^\beta}{[\beta\,\alpha]\,\langle A\,B\rangle^2}\right)\frac{\partial}{\partial x^{\beta\dot\beta}}\otimes\mathrm{d}x^{\alpha\dot\alpha}$$

- a symplectic structure from Q

$$\omega := \frac{1}{2\pi}\oint\frac{\lambda_\alpha\,\lambda_\beta}{Q^2}\langle\lambda\,\mathrm{d}\lambda\rangle\wedge\mathrm{d}^2x^{\alpha\beta} = \mathrm{i}\,\frac{A_\alpha\,B_\beta}{\langle A\,B\rangle^3}\,\mathrm{d}^2x^{\alpha\beta}$$

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$$J = \mathrm{i}\left(\delta^{\dot\beta}_{\dot\alpha}\frac{B_\alpha A^\beta + B^\beta A_\alpha}{\langle A\, B\rangle} - 8\mathrm{i}M\frac{\alpha_{\dot\alpha}\,\alpha^{\dot\beta}\,B_\alpha\,B^\beta}{[\beta\,\alpha]\,\langle A\,B\rangle^2}\right)\frac{\partial}{\partial x^{\beta\dot\beta}}\otimes \mathrm{d}x^{\alpha\dot\alpha}$$

- a symplectic structure from Q

$$\omega := \frac{1}{2\pi}\oint \frac{\lambda_\alpha\,\lambda_\beta}{Q^2}\langle\lambda\,\mathrm{d}\lambda\rangle \wedge \mathrm{d}^2x^{\alpha\beta} = \mathrm{i}\,\frac{A_\alpha\,B_\beta}{\langle A\,B\rangle^3}\,\mathrm{d}^2x^{\alpha\beta}$$

Remarkable fact: these are *compatible* $\omega(J,J) = \omega$

Schwarzschild metric

Theorem (TA-Araneda-Seet-Sharma)

χ *inherits a Kähler metric:*

$$\begin{aligned} g_{\mathrm{K}} &= \frac{-1}{\langle A\,B\rangle^2}\left[\delta - \frac{4\mathrm{i}\,M}{[\beta\,\alpha]}\left(\frac{\alpha_{\dot\alpha}\,B_\alpha\,\mathrm{d}x^{\alpha\dot\alpha}}{\langle A\,B\rangle}\right)^2\right] \\ &= \frac{1}{r^2}\,g_{\mathrm{S}} \end{aligned}$$

where g_{S} *is the Schwarzschild metric in Kerr-Schild coordinates*

Conformal factor

Could worry we are identifying terms in the answer

However, r is in fact *the* conformal factor which makes g_{K} vacuum Einstein!

Follows from cool result in conformal Kähler geometry...

Theorem (Derdzinski)

Let $(g_{\rm K}, \omega)$ *be Kähler, non-SD, with non-constant scalar curvature* $R_{g_{\rm K}}$ *and* $\omega^{-1}(R_{g_{\rm K}}, \cdot)$ *a holomorphic Killing vector. Let* $\Omega := R_{g_{\rm K}}^{-1}$ *and* $\tilde{g} := \Omega^2 \, g_{\rm K}$*. If*

$$1 - 6\,\Delta\Omega = 0\,, \qquad \Delta := g_{\rm K}^{ab}\,\nabla_a\,\nabla_b$$

then $\tilde{g}$ *is vacuum Einstein.*

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Note: Derdzinski's actual statement much more general than this, and worded a bit differently, but this is good enough for us.

Taking g_{K} the Kähler metric on χ:

- ✓ non-self-dual
- ✓ $R_{g_{\mathrm{K}}} = \frac{24\,M}{r}$ non-constant
- ✓ $\omega^{-1}(R_{g_{\mathrm{K}}}, \cdot) = 24M\,\partial_\tau$, holomorphic & Killing

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- ✓ $1 - 6\,\Delta\Omega = 0$, $\tilde{g} = g_{\mathrm{S}}$

Upshot: obtaining Schwarzschild no accident!

Quick comment

Construction used Euclidean reality conditions for (A)SDTN, but...

reality conditions enter details of coincidence locus only through principal spinors

e.g., $\langle \hat{B}\, A \rangle = 0$

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These only require that $x, y, z \in \mathbb{R}$...can have $\tau \in \mathbb{R}$ *or* $\tau = \mathrm{i}\, t$

$\Rightarrow$ obtain Schwarzschild in Riemannian *or* Lorentzian signature!

Quick recap

What just happened?

1. Took SDTN in Kerr-Schild form
2. Constructed its twistor space
3. Put a quadric defined by ASDTN inside
4. Looked at holomorphic part of quadric, a 4-manifold
5. 4-fold inherits Kähler metric conformal to Schwarzschild!

Meets criteria for solving the googly problem (holomorphic data in twistor space, no explicit field equations)!

So what?

For normal people, why care?

- Twistor theory enables *exact* scattering calculations on SDTN [TA-Bogna-Mason-Sharma, Guevara-Kol, Guevara-Kol-Tran]
- Now, see that Schwarzschild $\subset$ SDTN twistor space (in appropriate coords)

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Crazy possibility: obtain information about scattering on *real* Schwarzschild from scattering on SDTN

Lots to do!

- Get other black holes
  - Reissner-Nordström – (mostly) done w/ Araneda, Seet, Sharma, Skinner
  - Kerr – surprisingly subtle, WIP w/ Araneda, Perry, Seet, Sharma
- Understand origin of Teukolsky system in the coincidence locus χ [Araneda]
- Beyond smooth BHs – Generic Plebanski-Demianski metrics? Generic gravitational instantons? [Chen-Teo, Aksteiner-Araneda,...]
- Twistor BHPT?

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But more generally....

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Twistor theory is in rude health!

- Incredible number of young people, doing excellent work – just look around the room

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Happy Birthday, Roger!