

Conformal nets

(1) Introduction to conformal nets. (André)

This talk will be an introduction to conformal nets, along with von Neumann algebra prerequisites. The basics of von Neumann algebras will include Tomita-Takesaki theory, and the structure of modules over type III factors.

(2) Definition and examples of conformal nets. (Yuhan)

General definitions can be found in [GF93], [CKLW18]. The notion of conformal net comes in two variants: Möbius covariant conformal nets, and diffeomorphism covariant conformal nets ([GF93] only treats Möbius covariant conformal nets). All important examples of conformal nets are in fact diffeomorphism covariant.

- Example 1: The Virasoro conformal nets [Carpi03]. The construction relies on estimates of Goodman and Walach discussed in [GoWa1], [GoWa2], and [TL01].

- Example 2: The Loop group conformal nets [GF93], [BDH13, Section 4.C].

An important generalisation of the concept of conformal net is that of a super-conformal net, also known as graded-local conformal nets [CGH25]. Examples of super-conformal nets include:

- Example 3: The free fermion conformal net [Hen14, p.49], [CGH25, Section 7].

- Example 4: The super-Virasoro conformal nets [CG25]

(3) Representation theory of conformal nets. The braiding. DHR theory. (Nathan)

The monoidal structure on the category of representations of a conformal net is called 'Connes fusion'. See [Gui20] for definitions and basic properties of Connes fusion. For generalities on DHR theory, and on the construction of the braiding, see [GF93]. For the equivalence between the category of representations of a conformal net and the DHR category, see [Gui20, Section 6].

The classification of representations of the loop group conformal nets is discussed in [GF93]. (It is the same as that of

integrable positive energy representations of the affine Lie algebra, discussed in [Hen20, pp 57–66].)

The proof of this equivalence is the content of Thm. II + Thm 3.40 of [Bin26], and is too hard to present.

(4) The Bisognano-Wichmann theorem, Haag duality, and the type III property. (Josep)

The Bisognano-Wichman theorem states that the vacuum Hilbert space of a conformal net is canonically isomorphic to the L^2 -space of the von Neumann algebra $A(I)$ associated to a semicircle I , and that the modular flow agrees under that isomorphism with the action of Möbius transformations fixing ∂I . Haag duality and the type III property are consequences of the Bisognano-Wichman theorem.

References: [GF93, Thm. 2.19], [BSM90], [DLR01, Thm. 2.1], [Bor92, Thm. II.9], or [Lon08, Thm. 6.2.3]).

(5) The Reeh-Schlieder theorem, Additivity. (Glen)

The Reeh-Schlieder theorem is the statement that the vacuum vector is cyclic and separating for the local algebras of the conformal net, see [Hen14. p.16-20] for a proof. Additivity is the statement that if two intervals I and J overlap, then $A(I)$ and $A(J)$ together topologically generate $A(I \cup J)$. See [FJ96] for a proof. A strengthening of additivity known as strong additivity can be proven under a certain finiteness assumption known as "rationality" of the conformal net [LX04].

(6) The Split property. (Adri)

From [MTW18]: "The split property is the statistical independence of local algebras associated to regions with a positive (spacelike) separation." Mathematically, the split property is the statement that if two intervals I and J are separated, then the von Neumann algebras $A(I)$ and $A(J)$ are not only commuting, but in fact generate a copy of the spatial tensor product $A(I) \otimes A(J)$. The main reference is [MTW18].

(7) VOAs $\rightarrow$ Conformal nets; Conformal nets $\rightarrow$ VOAs. (Nivedita)

It is conjectured that unitary vertex operator algebras are equivalent to conformal nets. The main reference for the VOAs $\rightarrow$ Conformal nets direction is [CKLW18]. The main reference for the Conformal nets $\rightarrow$ VOAs direction is [HT25].

(8) Construction of full CFTs in the functorial formalism (André).

This talk will describe ongoing research with James Tener in which we construct a fully extended 2d CFTs from conformal nets. When applied to the Virasoro conformal nets, we conjecture that the resulting full CFT is Liouville theory, which was recently constructed in [GKRV25].

References:

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<https://arxiv.org/pdf/math/0309366>

[Lon08] R. Longo. Lectures on conformal nets, part 2. https://www.mat.uniroma2.it/~longo/Lecture-Notes_files/LN-Part2.pdf, 2008.

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<https://arxiv.org/pdf/math/0106195>

[GoWa1] R. Goodman, N. R. Wallach, Structure and Unitary Cocycle Representations of Loop Groups and the Group of Diffeomorphisms of the Circle

[GoWa2] R. Goodman, N. R. Wallach, Projective Unitary Positive–Energy Representations of $\text{Diff}(S^1)$