The first celestial holographer

Tim Adamo
University of Edinburgh

From Good Cuts to Celestial Holography

10 July 2025

Motivation

Celestial (and Carrollian) holography: describe (some class of) QFTs in asymptotically flat spacetimes in terms of dual theory on (part of) conformal infinity

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Slightly less ambitious: How much of an asymptotically flat spacetime (the bulk) can be reconstructed from data at infinity (the boundary)?

This question would have been familiar to some researchers 60 years ago!

Today:

- A bit of history
- Pay tribute to some heroes
- 'Old' ideas and their connection to this meeting

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Parental Advisory: largely a story (not mine!), but still, I hope, illuminating

A bit of history

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Appointed faculty at Syracuse in 1947

Bergmann made many contributions:

- Kaluza-Klein theory [Einstein-Bergmann]
- First GR textbook (1942)
- Canonical formulation of GR [Bergmann, Anderson-Bergmann]
- Observables and symmetries in GR
- Scalar-tensor theory
- Quantum measurements [Aharonov-Bergmann-Lebowitz]

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...but perhaps most important was as a mentor/supervisor!

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Over next 40 years, supervised 40+ PhD students, with freakish success

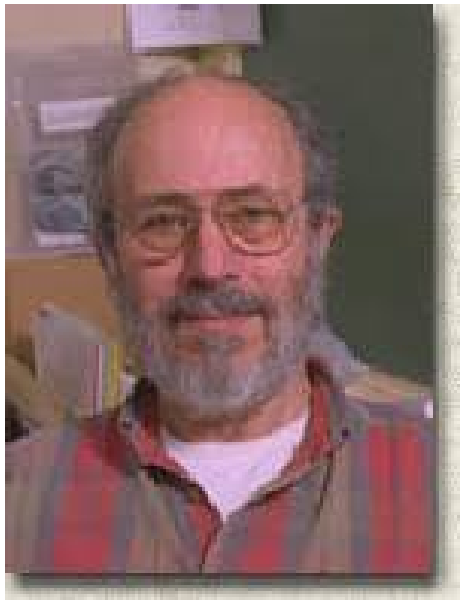
Bergmann's students

Joshua N. Goldberg (1925 – 2020)
PhD 1952

Bergmann's students

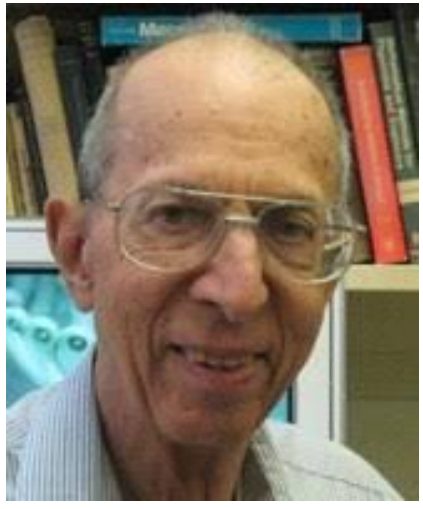
Joel Lebowitz
PhD 1956

Bergmann's students



Ezra T. Newman (1929 – 2021)
PhD 1956

Bergmann's students



Allan Janis (1930 – 2024)
PhD 1957

Bergmann's students

Rainer K. Sachs (1932 – 2024)
PhD 1958

Bergmann's students

John Boardman (1932 – 2025)
PhD 1962

Bergmann's students

Jeffrey Winicour
PhD 1964

Just in GR, a staggering list of achievements:

- **Goldberg-Sachs** theorem
- Spin-weighted spherical harmonics
- Bondi-Metzner-**Sachs** group
- Bondi-**Sachs** expansion
- **Newman**-Penrose formalism
- Taub-**Newman**-Unti-Tamburino (NUT) metric
- Kerr-**Newman** metric
- **Newman**-Unti group
- **Newman-Janis** trick
- **Janis-Newman-Winicour** naked singularity
- Kantowski-**Sachs** metric
- **Sachs**-Wolfe effect
- Ehlers-Geren-**Sachs** theorem

Bergmann's record in GR surpassed only by Sciama & Wheeler

Many of these students' work links with **celestial holography**!

Today: focus on contributions of Ted Newman

- Some less (globally) famous ideas closely linked with CH
- Understand where title of this meeting comes from
- Review some ideas which are still crying out for an application

Ezra ‘Ted’ Newman

(1929 – 2021)

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Career: NYU (BA 51) $\rightarrow$ Syracuse (PhD 56) $\rightarrow$ Pitt (56)

Wright-Patterson AFB (57), Syracuse (60-61), KCL (64-65)

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(NP formalism, Taub-NUT, Kerr-Newman,...)

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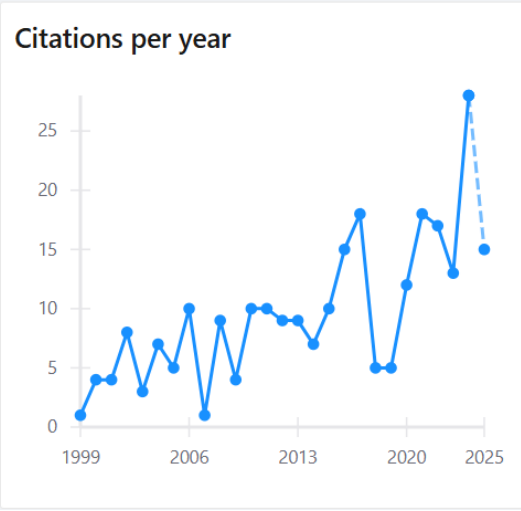
...but also some which are less known in this room

Space-time perspective of Schwarzschild lensing

Simonetta Frittelli (Duquesne U. and Pittsburgh U.), Thomas P. Kling (Pittsburgh U.), Ezra T. Newman (Pittsburgh U.)
Nov, 1999

22 pages
Published in: *Phys.Rev.D* 61 (2000) 064021
e-Print: gr-qc/0001037 [gr-qc]
DOI: 10.1103/PhysRevD.61.064021
View in: AMS MathSciNet, ADS Abstract Service

pdf cite claim reference search 257 citations



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No. 4986 May 22, 1965 NATURE 811

PHYSICS

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...but unable to make this connection precise – variety of reasons

Nevertheless, Newman made a **key** contribution which *does* underpin much of what we do in celestial holography

Heaven and Its Properties

EZRA T. NEWMAN

Department of Physics, University of Pittsburgh, Pittsburgh, Pennsylvania 15213

General Relativity and Gravitation, Vol. 7, No. 1 (1976), pp. 107–111

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Heaven and Its Properties

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Famous for some of us, unknown to others

(before this week! – talks by Freidel, Kmec)

Basic question:

How much of an asymptotically flat spacetime can be reconstructed from its data at infinity?

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To see how Newman tried to answer this question, need to set the stage with some formalism

Bondi coords

Usually describe future of asymp. flat spacetimes in *Bondi coords* $(u, r, z, \bar{z})$

- u retarded time
- r parameter along outgoing, constant-u null hypersurfaces
- $(z, \bar{z})$ stereographic coords on S^2, $z = \mathrm{e}^{\mathrm{i}\phi} \cot(\theta/2)$

E.g., Minkowski spacetime

$$\mathrm{d}s^2_{\mathrm{flat}} = 2\,\mathrm{d}u\,\mathrm{d}r + \mathrm{d}u^2 - 4\,r^2\,\frac{\mathrm{d}z\,\mathrm{d}\bar{z}}{(1+|z|^2)^2}$$

These coords are annoying as they break Lorentz invariance:

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$$(z, \bar{z}) \text{ not global on } S^2$$

Resolution: introduce *homogenous Bondi coords*

[Sparling, Eastwood-Tod]

Homog. coords

Instead, work with

$$(u, r, \lambda_\alpha, \bar{\lambda}_{\dot{\alpha}}) \sim (|b|^2 u, |b|^{-2} r, b\lambda_\alpha, \bar{b}\bar{\lambda}_{\dot{\alpha}})$$

for all $b \in \mathbb{C}^*$

$[\lambda_\alpha, \bar{\lambda}_{\dot{\alpha}}]$ now homog coords on $\mathbb{CP}^1 \cong S^2$

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Spin/conformal weighted functs $\leftrightarrow$ sections of $\mathcal{O}(p, q)$:

$$f(|b|^2 u, |b|^{-2} r, b\lambda, \bar{b}\bar{\lambda}) = b^p\, \bar{b}^q\, f(u, r, \lambda, \bar{\lambda})$$

$s = (p - q)/2$, $w = -(p + q)/2$

Package these coordinates into

$$x^{\alpha\dot{\alpha}} = \frac{u\, t^{\alpha\dot{\alpha}}}{\langle \lambda\, \hat{\lambda} \rangle} + r\, \lambda_{\alpha}\, \bar{\lambda}_{\dot{\alpha}}$$

for $t^{\alpha\dot{\alpha}}$ time-like, $\hat{\lambda}_{\alpha} := t_{\alpha\dot{\alpha}}\, \bar{\lambda}^{\dot{\alpha}}$

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Minkowski metric now

$$\mathrm{d}s^2_{\mathrm{flat}} = 2\,\mathrm{d}u\,\mathrm{d}r + \frac{\mathrm{d}u^2}{\langle \lambda\, \hat\lambda\rangle^2} - 2\, r^2\, \mathrm{D}\lambda\, \mathrm{D}\bar\lambda$$

for $\mathrm{D}\lambda := \langle \lambda\, \mathrm{d}\lambda\rangle$

Get standard/affine coords on patch

$$t^{\alpha\dot\alpha} = \frac{1}{\sqrt{2}}\,\mathrm{diag}(1,1)\,, \qquad \lambda_\alpha = \frac{2^{1/4}}{\sqrt{1+|z|^2}}\,(1,\, z)$$

Bondi-Sachs expansion

In the far future, an asymp. flat spacetime admits expansion

$$\mathrm{d}s^2 = 2\,\mathrm{d}u\,\mathrm{d}r - r^2\left(2\,\mathrm{D}\lambda\,\mathrm{D}\bar{\lambda} - \frac{\boldsymbol{\sigma}^0}{r}\,\mathrm{D}\lambda^2 - \frac{\bar{\boldsymbol{\sigma}}^0}{r}\,\mathrm{D}\bar{\lambda}^2\right)$$

$$+\left(\frac{1}{\langle\lambda\,\hat{\lambda}\rangle^2} - \frac{2\,m_{\mathrm{B}}}{r}\right)\,\mathrm{d}u^2 - \mathrm{d}u\left(\eth\bar{\boldsymbol{\sigma}}^0\,\mathrm{D}\bar{\lambda} + \bar{\eth}\boldsymbol{\sigma}^0\,\mathrm{D}\lambda\right) + O(r^{-2})$$

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Here:

- $\boldsymbol{\sigma}^0(u,\lambda,\bar{\lambda}) \in \mathcal{O}(-3,1)$ – asymp. shear of constant-u hypersurfaces
- $m_{\mathrm{B}}(u,\lambda,\bar{\lambda}) \in \mathcal{O}(-3,-3)$ – Bondi mass aspect
- $\eth : \mathcal{O}(p,q) \to \mathcal{O}(p-2,q)$

$$\eth := \frac{\hat{\lambda}_\alpha}{\langle\lambda\,\hat{\lambda}\rangle}\,\frac{\partial}{\partial\lambda_\alpha}$$

Conformal compactification

Let $R := r^{-1}$, consider

$$\mathrm{d}\hat{s}^2 := R^2\,\mathrm{d}s^2 = -2\,\mathrm{d}u\,\mathrm{d}R - \mathrm{D}\lambda\,\mathrm{D}\bar{\lambda} + O(R)$$

so limit as $R \to 0$ gives conformal boundary $\mathscr{I}^+$:

$$\mathrm{d}\hat{s}^2_{\mathscr{I}^+} = 0 \times \mathrm{d}u^2 - \mathrm{D}\lambda\,\mathrm{D}\bar{\lambda}$$

Topologically, $\mathscr{I}^+ \cong \mathbb{R} \times S^2$

$\mathscr{I}^+$ and BMS

Homog. coords on $\mathscr{I}^+$ are

$$(u, \lambda_\alpha, \bar{\lambda}_{\dot{\alpha}}) \sim (|b|^2 u, b\lambda_\alpha, \bar{b}\bar{\lambda}_{\dot{\alpha}}) \quad \forall b \in \mathbb{C}^*$$

Realises $\mathscr{I}^+$ as total space of $\mathcal{O}_{\mathbb{R}}(1,1) \to \mathbb{CP}^1$, u coordinate on fibre

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Note: u affine – no preferred origin, so can shift

$$u \to u + f(\lambda, \bar{\lambda}), \qquad f(\lambda, \bar{\lambda}) \in \mathcal{O}_{\mathbb{R}}(1,1)$$

These are *supertranslations*!

Combine with Möbius on $\mathbb{CP}^1$ base to get BMS group

Back to the basic question: how much of an AF spacetime can be constructed from data at $\mathscr{I}^+$?

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Start with an easier question:

How to reconstruct Minkowski spacetime at $\mathscr{I}^+$?

Light cone cuts

Consider light cone with vertex at $x \in \mathbb{R}^{1,3}$

This intersects $\mathscr{I}^+$ in spherical *cut*:

$$u = G(\lambda, \bar{\lambda}) = x^{\alpha\dot{\alpha}}\, \lambda_\alpha\, \bar{\lambda}_{\dot{\alpha}}$$

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Can characterise these cuts as solutions of

$$\frac{\partial^2 G}{\partial \bar{\lambda}^{\dot{\alpha}}\, \partial \bar{\lambda}^{\dot{\beta}}} = 0 \qquad \Leftrightarrow \qquad \bar{\eth}^2 G = 0$$

for $G(\lambda, \bar{\lambda})$ valued in $\mathcal{O}_{\mathbb{R}}(1,1)$

Shear free cuts

On the other hand, consider a generic spherical cut $u = G(\lambda, \bar{\lambda})$ of $\mathscr{I}^+$

Generated by intersection of a *shear free* null hypersurface iff

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Upshot: $\mathbb{R}^{1,3}$ = set of shear free cuts of $\mathscr{I}^+$

Next step:

Natural first guess for AF spacetime:

Points in 'bulk' $\leftrightarrow$ shear free cuts of $\mathscr{I}^+$

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Nope!

Generic AF spacetime has **no** shear-free NGCs!
(Goldberg-Sachs thm)

Next best thing:

Look for *asymptotically* shear free cuts of $\mathscr{I}^+$:

$\boldsymbol{\sigma}^0(u,\lambda,\bar{\lambda})$ is asymptotic shear in given Bondi frame, then cut $u = G(\lambda,\bar{\lambda})$ is ASF iff:

$$\frac{\partial^2 G}{\partial\bar{\lambda}^{\dot{\alpha}}\partial\bar{\lambda}^{\dot{\beta}}} = \bar{\lambda}_{\dot{\alpha}}\,\bar{\lambda}_{\dot{\beta}}\,\bar{\boldsymbol{\sigma}}^0 \quad \Leftrightarrow \quad \bar{\eth}^2 G = \bar{\boldsymbol{\sigma}}^0$$

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Highly non-linear PDE ; Newman called this

the **good cut equation**

What do we know about the GCE?

At this point, very little!

- Does it have solutions?
- If so, how many?
- Does the solution space correspond to the AF spacetime we started with?

Complexification

Fall at first hurdle!

$$G \text{ real, } \bar{\boldsymbol{\sigma}}^0 \text{ complex} \Rightarrow \text{no real solutions!}$$

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Newman's key idea: consider (small) partial complexification $\mathscr{I}^+ \to \mathscr{I}_{\mathbb{C}} \cong \mathbb{C} \times \mathbb{CP}^1$

Now $G(\lambda, \bar{\lambda})$ is *complex*, valued in $\mathcal{O}(1,1)$

So what?

On $\mathscr{I}_{\mathbb{C}}$:

- should be solutions, for sufficiently small $\bar{\boldsymbol{\sigma}}^0$
- but how many?
- and what do they correspond to in the bulk?

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Twistor theory to the rescue!

Twistor space to $\mathscr{I}_{\mathbb{C}}$

Recall (lectures by Bittleston, Sharma):

twistor space of complexified Minkowski space is
$\mathbb{PT} = \mathcal{O}(1) \oplus \mathcal{O}(1) \to \mathbb{CP}^1$

Let $(\mu^{\dot{\alpha}}, \lambda_\alpha) \sim b\,(\mu^{\dot{\alpha}}, \lambda_\alpha)\ \forall b \in \mathbb{C}^*$ be homog. coords on $\mathbb{PT}$

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There is a natural projection $p : \mathbb{PT} \to \mathscr{I}_{\mathbb{C}}$

$$p : (\mu^{\dot\alpha},\ \lambda_\alpha) \mapsto (u = [\mu\,\bar\lambda],\ \lambda_\alpha,\ \bar\lambda_{\dot\alpha})$$

Asymptotic twistor space

Integrate $\bar{\boldsymbol{\sigma}}^0$ to get

$$\bar{\Sigma}(u, \lambda, \bar{\lambda}) = \int^u \mathrm{d}s \, \bar{\boldsymbol{\sigma}}^0(s, \lambda, \bar{\lambda})$$

valued in $\mathcal{O}(2, -2)$

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Define $h = \mathrm{D}\bar{\lambda} \, p^* \bar{\Sigma} \in \Omega^{0,1}(\mathbb{PT}, \mathcal{O}(2))$, then

$$\bar{\nabla} = \bar{\partial} + \mathrm{D}\bar{\lambda} \, \bar{\lambda}_{\dot{\alpha}} \, p^* \bar{\boldsymbol{\sigma}}^0 \, \frac{\partial}{\partial \mu_{\dot{\alpha}}} \quad \text{obeys} \quad \bar{\nabla}^2 = 0$$

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Upshot: data $\bar{\boldsymbol{\sigma}}^0$ defines a 'curved' twistor space $\mathbb{P}\mathscr{T}$

So what?

By non-linear graviton theorem, $\mathbb{P}\mathscr{T}$ corresponds to a vacuum SD 4-manifold $\mathcal{M}$ [Penrose]

Points/metric constructed from *holomorphic rational curves*, X (cf., Skinner's talk):

$$\mu^{\dot\alpha} = F^{\dot\alpha}(X, \lambda)\,, \qquad \bar\partial|_X F^{\dot\alpha} = \bar\lambda^{\dot\alpha}\, \mathrm{D}\bar\lambda\, p^* \bar{\boldsymbol{\sigma}}^0$$

Kodaira $\Rightarrow$ 4-parameter family of these curves, labeled by $x^{\alpha\dot\alpha} \in \mathcal{M}$

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Kodaira $\Rightarrow$ 4-parameter family of these curves, labeled by $x^{\alpha\dot\alpha} \in \mathcal{M}$

If $u = F^{\dot\alpha}(x, \lambda)\, \bar\lambda_{\dot\alpha} \equiv G(\lambda, \bar\lambda)$, then can show that

$$\frac{\partial^2 G}{\partial \bar\lambda^{\dot\alpha} \partial \bar\lambda^{\dot\beta}} = \bar\lambda_{\dot\alpha}\, \bar\lambda_{\dot\beta}\, \bar{\boldsymbol{\sigma}}^0$$

Upshot

Data $\bar{\boldsymbol{\sigma}}^0$ on $\mathscr{I}_{\mathbb{C}}$

$\Leftrightarrow$

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Solutions to good cut equation

$\Leftrightarrow$

Points in $\mathcal{H}$-space

Historical aside

Newman's $\mathcal{H}$-space construction came *before* the NLG!

Initially, shown using direct computation w/ metric on $\mathcal{H}$-space [Newman]

$$\mathrm{d}s^2_{\mathcal{H}} = \left[\frac{1}{16\pi \mathrm{i}} \oint \frac{\mathrm{D}\lambda \wedge \mathrm{D}\hat{\lambda}}{(\mathrm{d}G)^2}\right]^{-1}$$

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Subsequently adapted to the NLG [Hansen-Newman-Penrose-Tod]

Impact today

Linear version of $\mathcal{H}$-space construction:

- Worldsheet descriptions of scattering, soft theorems & Ward idents at $\mathscr{I}$ [TA-Casali-Skinner, Geyer-Lipstein-Mason]
- Celestial interpretation of twistor chiral algebras

  [TA-Mason-Sharma, TA-Bu-Casali-Sharma, Donnay-Friedel-Herfray]
- Link between celestial and Carrollian holography

  [Donnay-Fiorucci-Herfray-Ruzziconi]

Non-linear version:

- Bulk reconstruction [Engelhardt-Horowitz]
- Intrinsic conformal geometry of $\mathscr{I}$ [Herfray, Bekaert-Donnay-Herfray]
- Non-linear extrapolate dictionary for SD fields

  [Friedel-Pranzetti-Raclariu, Kmec-Mason-Ruzziconi-Srikant, Cresto-Friedel,

  Kmec-Mason-Ruzziconi-Sharma]
- Scattering amplitudes & chiral algebras on $\mathcal{H}$-spaces

  [TA-Mason-Sharma, Bogna-Mason, TA-Bu-Zhu]

There is more we can be doing with $\mathcal{H}$-space:

- Properties of celestial/Carrollian amplitudes on $\mathcal{H}$-space
- Realization in twisted holography

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- Properties of celestial/Carrollian amplitudes on $\mathcal{H}$-space
- Realization in twisted holography

Finally, I'd like to discuss a further aspect of Newman's later work that follows on from the $\mathcal{H}$-space story

[Newman]

[Winicour, Lind, Messmer, Kozameh, Silva-Ortigoza, Santiago-Santiago, TA]

Worldlines in $\mathcal{H}$-space

Consider a worldline $x^{\alpha\dot\alpha} = \xi^{\alpha\dot\alpha}(\tau)$ in an $\mathcal{H}$-space

Induces a 1-param. family of good cuts: $u = G(\tau, \lambda, \bar\lambda)$.

Expand in (spin-weighted) spherical harmonics $Y^s_{\ell\, i_1\cdots i_\ell}$:

$$G(\tau,\lambda,\bar\lambda) = \frac{\langle\lambda\,\hat\lambda\rangle}{\sqrt{2}}\,\xi^0 - \frac{1}{2}\,\xi^{\alpha\beta}\,\lambda_\alpha\,\hat\lambda_\beta + \xi^{\alpha\beta\gamma\delta}\,\frac{\lambda_\alpha\,\lambda_\beta\,\hat\lambda_\gamma\,\hat\lambda_\delta}{\langle\lambda\,\hat\lambda\rangle} + \cdots$$

$$= \frac{\xi^0(\tau)}{\sqrt{2}} - \frac{1}{2}\,\xi^i(\tau)\,Y^0_{1\,i} + \sum_{\ell\geq 2}\xi^{i(\ell)}(\tau)\,Y^0_{\ell\,i(\ell)}$$

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Expand in (spin-weighted) spherical harmonics $Y^s_{\ell\, i_1 \cdots i_\ell}$:

$$G(\tau,\lambda,\bar{\lambda}) = \frac{\langle \lambda\, \hat{\lambda}\rangle}{\sqrt{2}}\,\xi^0 - \frac{1}{2}\,\xi^{\alpha\beta}\,\lambda_\alpha\,\hat{\lambda}_\beta + \xi^{\alpha\beta\gamma\delta}\,\frac{\lambda_\alpha\,\lambda_\beta\,\hat{\lambda}_\gamma\,\hat{\lambda}_\delta}{\langle \lambda\,\hat{\lambda}\rangle} + \cdots$$
$$= \frac{\xi^0(\tau)}{\sqrt{2}} - \frac{1}{2}\,\xi^i(\tau)\,Y^0_{1\,i} + \sum_{\ell\geq 2}\xi^{i(\ell)}(\tau)\,Y^0_{\ell\, i(\ell)}$$

Note: higher coefficients $\xi^{i_1\cdots i_\ell}(\tau)$ for $\ell \geq 2$ determined by (ξ^0, ξ^i) and $\bar{\boldsymbol{\sigma}}^0$

In terms of this harmonic expansion

$$\bar{\boldsymbol{\sigma}}^0 = \bar{\eth}^2|_\tau G(\tau, \lambda, \bar{\lambda}) = 24\, \xi^{ij}(\tau)\, Y^2_{2\,ij}(\lambda, \bar{\lambda}) + \cdots$$

for $\bar{\eth}|_\tau \leftrightarrow$ derivative while holding τ fixed

Stereographic angle of asymp. shear free NGC at $\mathscr{I}_{\mathbb{C}}$:

$$L = \bar{\eth}|_\tau G(\tau, \lambda, \bar{\lambda}) = \xi^i(\tau)\, Y^1_{1\,i} - 6\, \xi^{ij}(\tau)\, Y^1_{2\,ij} + \cdots$$

valued in $\mathcal{O}(1, -1)$

Null rotation

Let $\{l^{\alpha\dot{\alpha}}, n^{\alpha\dot{\alpha}}, m^{\alpha\dot{\alpha}}, \bar{m}^{\alpha\dot{\alpha}}\}$ be Bondi tetrad for which the NGCs labeled by $\xi^{\alpha\dot{\alpha}}(\tau)$ have asymp shear $\bar{\sigma}^0$

$$l \cdot n = 1 = -m \cdot \bar{m}$$

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Rotate to frame where l now points along NGC at $\mathscr{I}_{\mathbb{C}}$:

$$\begin{aligned} l^{\alpha\dot{\alpha}} &\to l^{\alpha\dot{\alpha}} - \frac{L}{r} m^{\alpha\dot{\alpha}} - \frac{\bar{L}}{r} \bar{m}^{\alpha\dot{\alpha}} + O(r^{-2}) \\ m^{\alpha\dot{\alpha}} &\to m^{\alpha\dot{\alpha}} - \frac{\bar{L}}{r} n^{\alpha\dot{\alpha}} + O(r^{-2}) \\ n^{\alpha\dot{\alpha}} &\to n^{\alpha\dot{\alpha}} \end{aligned}$$

Asymptotic Weyl curvature

Assuming peeling (not safe in general – talk by Bieri), Weyl curvature

$$\bar{\Psi}_a(u, r, \lambda, \bar{\lambda}) = \frac{\bar{\psi}^0_a(u, \lambda, \bar{\lambda})}{r^{a-5}} + O(r^{-a+6}), \quad a = 0, \ldots, 4$$

for $\bar{\psi}^0_a \in \mathcal{O}(3 - a, -9 + a)$ the *asymptotic Weyl curvatures*

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Rough intuition:

- $\bar{\psi}_4^0 = -\ddot{\bar{\sigma}}^0 \leftrightarrow$ characteristic data
- $\bar{\psi}_2^0 \leftrightarrow$ Coulombic data
- $\bar{\psi}_1^0 \leftrightarrow$ dipole moment, angular momentum

Under the null rotation to align with asymp. shear free NGCs generated by $\xi^{\alpha\dot{\alpha}}(\tau)$:

$$
\begin{aligned}
\bar{\psi}_4^0 &\to \bar{\psi}_4^0 \\
\bar{\psi}_3^0 &\to \bar{\psi}_3^0 - L\,\bar{\psi}_4^0 \\
\bar{\psi}_2^0 &\to \bar{\psi}_2^0 - 2\,L\,\bar{\psi}_3^0 + L^2\,\bar{\psi}_4^0 \\
\bar{\psi}_1^0 &\to \bar{\psi}_1^0 - 3\,L\,\bar{\psi}_2^0 + 3\,L^2\,\bar{\psi}_3^0 - L^3\,\bar{\psi}_4^0 \\
\bar{\psi}_0^0 &\to \bar{\psi}_0^0 - 4\,L\,\bar{\psi}_1^0 + 6\,L^2\,\bar{\psi}_2^0 - 4\,L^3\,\bar{\psi}_3^0 + L^4\,\bar{\psi}_4^0
\end{aligned}
$$

Newman's procedure

1. Rescale τ so that $\xi^0(\tau) = \tau$, assume $v^i(\tau) = \xi^{i\prime}(\tau)$ small
2. Identify $\ell = 1$ harmonic of $\bar{\psi}_1^0$ with *complex dipole moment*:
$$\psi_1^{0\,i} := -\frac{6\sqrt{2}}{c^2}\left(D^i + \frac{\mathrm{i}}{c}J^i\right)$$
3. Single out *unique* 'centre-of-mass' worldline
$$\xi^{\alpha\dot{\alpha}}(\tau) = \xi_{\mathrm{R}}^{\alpha\dot{\alpha}}(\tau) + \mathrm{i}\,\xi_{\mathrm{I}}^{\alpha\dot{\alpha}}(\tau)$$
in $\mathcal{H}$-space by requiring $\psi_1^{0\,i} \to 0$ under null rotation by L
4. Obtain evolution equations (pert. in velocity) from asymp. Bianchi idents

Leads to some remarkable results – expand Bondi mass aspect

$$M_B = -\frac{2\sqrt{2}\,G}{c^2}\,M - \frac{6G}{c^3}\,P^i\,Y^0_{1\,i} + \frac{\sqrt{2}\,G}{24\,c^4}\left(Q^{ij\,\prime\prime}_{\rm mass} - \mathrm{i}\,Q^{ij\,\prime\prime}_{\rm spin}\right)Y^0_{2\,ij} + \cdots$$

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Then Newman's procedure leads to:

$$D^{i\,\prime} = P^i - \frac{12\,G}{5\,c^6}\,\epsilon_{jk}{}^i\left(Q^{lj\,\prime\prime}_{\rm mass}\,Q^{lk\,\prime\prime}_{\rm spin}\right)' + \cdots$$

$$J^{i\,\prime} = -\frac{12\,G}{5\,c^6}\,\epsilon_{jk}{}^i\left(Q^{lj\,\prime\prime}_{\rm mass}\,Q^{lk\,\prime\prime\prime}_{\rm mass} + Q^{lj\,\prime\prime}_{\rm spin}\,Q^{lk\,\prime\prime\prime}_{\rm spin}\right) + \cdots$$

$$M' = -\frac{G}{5\,c^7}\left(Q^{ij\,\prime\prime\prime}_{\rm mass}\,Q^{ij\,\prime\prime\prime}_{\rm mass} + Q^{ij\,\prime\prime\prime}_{\rm spin}\,Q^{ij\,\prime\prime\prime}_{\rm spin}\right) + \cdots$$

$$P^{i\,\prime} = \frac{2\,G}{15\,c^6}\,\epsilon_{jk}{}^i\left(Q^{jl\,\prime\prime\prime}_{\rm mass}\,Q^{kl\,\prime\prime\prime}_{\rm spin} - Q^{jl\,\prime\prime\prime}_{\rm spin}\,Q^{kl\,\prime\prime\prime}_{\rm mass}\right) + \cdots$$

Upshot

- Get EoMs for motion of isolated object with internal structure
- Complex nature of good cuts $\leftrightarrow$ spin/twist effects
- Low-order results reproduce known Newtonian dynamics
- Reproduces many known GR results (Bondi mass loss)
- Makes some new predictions; e.g. momentum-spin coupling in the mass dipole
- Extension to Einstein-Maxwell

The bad news

But there are some problems/mysteries:

- Physical idents based on linear theory – not clear this is unique procedure at non-linear
- Not clear *why* this works: get meaningful (Lorentzian) results from intrinsically *complex* setup
- How to include multiple sources or interactions?
- Connection/comparison with standard PN/EOB results not clear [Kozameh-Ortega-Rojas, Kozameh-Nieva-Quiroga]

Eternal optimist:

In the spirit of Ted, **view all problems as opportunities!**

Bonkers pipe-dream:

Celestial approach to post-Newtonian/Effective-One-Body approximation?

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Thanks!