Undergraduate Summer Project Opportunities

2022-2023

Below you will find a list of possible opportunities for summer projects. In each case, the supervisor listed has indicated that they are happy to be approached by undergraduates interested in their project or in working on a project in that area, but it will be up to the supervisor and student to discuss possibilities. There is no guarantee that a supervisor will end up supervising a particular project or particular student. It is likely to depend in part on finding suitable funding, but your first step should be to find a supervisor who is willing to supervise your project. You are of course also welcome to approach other members of the Mathematical Institute to see whether they will supervise a project for you. The projects are not listed in any particular order.

Flood prediction using machine learning

Dr Yixuan Sun (suny@maths.ox.ac.uk)

Due to Climate change, extreme weathers appear more often. In 2021 alone, we have seen flood in Germany, China and the USA. The extreme weather can cause enormous damage to the society, including life loss, property damage and productivity loss, to name just a few. To be able to predict the extreme weather such a flood becomes very valuable and necessary. Given the frequency increase of the extreme weather, we do get more data about the extreme weather which we don't have access to before. Though more available, the data are still sparse. In this work, we propose a study on current flood models with new data and we also like to build new models based on the current model and new input from the data. We aim to get a better understanding of the cause of the flood in relation to the precipitation, drainage system of the cities (UK cities in particular) and possible measures to prevent or reduce the damage caused by flood. Few available sources of the data are Global Flood Database and Historic Flood Map (from UK government).

Possible avenues of investigation:
1.Literature review of the current study of the flood models and available data set.
2.Literature search in the filtration, oil industry and cardiovascular system for inspiration
3.Literature review for current machine learning tools for such investigation
4.Literature review for image processing and graph theory approach
5.Running simulation with available software package for fluid
6.Running machine learning package for flood model validation and prediction

Pre-requisite knowledge (listed as essential, recommended, useful)
1.Mathematical modelling (Recommended: A10)
2.Basic fluid dynamics and Partial differential equations (Recommended: C5.7 Topics in Fluid Mechanics)

3.Simulations skills in python or C++ (Essential)
4.Optimization (Recommended: B6.2 Optimisation for Data Science)
5.Familiarity with machine learning prior to starting the dissertation (Essential)
Useful reading

C. M. Bishop. Pattern Recognition and Machine Learning
S. Raschka, and V. Mirjalili. Python Machine Learning: Machine Learning and Deep Learning with Python, scikit-learn, and TensorFlow
A. Racca, L. Magri, 2022, Statistical Prediction of Extreme Events from Small Datasets, *Lecture Notes in Computer Science (including Subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, Vol:13352 LNCS, ISSN:0302-9743, Pages:707-713

Further references
I.Goodfellow. Deep Learning (Adaptive Computation and Machine Learning Series)

Group actions on hyperbolic metric spaces

Prof. Harry Petyt (petyt@maths.ox.ac.uk)

This is a project in geometric group theory, which uses tools from geometry to understand finitely generated groups. Hyperbolic spaces are a well-studied family of negatively-curved metric space that makes up an important part of the field. If a group admits a nice "geometric" action on a hyperbolic space, then it turns out that it must have some rather strong properties, but many groups of interest do not admit such actions. However, if we consider weaker kinds of action then we can still gain insights on many "non-positively curved" groups.
The idea for the project will be to look at ways of constructing hyperbolic spaces that different families of groups act on, and to see what consequences this has for the groups in question.

Length of project: Flexible, but expecting 6-8 weeks.
Who might be interested: Students interested in metric geometry and infinite groups.

Ideals in Iwasawa algebras

Prof. Konstantin Ardakov (ardakov@maths.ox.ac.uk)

An Iwasawa algebra is a completed group ring of a compact p-adic Lie group with coefficients in either a p-adic ring or a field of characteristic p. These objects control the p-adic representation theory of the group in question, and therefore are of interest in several areas of algebraic number theory, including Iwasawa theory and the deformation theory of Galois representations. They also happen to be Noetherian rings, which are non-commutative in general. A long-standing project asks for teh classification of the prime ideals in these rings. Although there has been a lot of progress in the case where the coefficient ring has characteristic zero, the question is still open in general in that case, and even more so in the case of mod-p coefficients. One fruitful avenue of attack has been to study the action of the Iwasawa algebra on certain p-adic completions of representations of Lie algebras arising classically, such as Verma modules, highest weight modules more generally, Whittaker modules or the metaplectic representation. In this project the students will look at these particular examples in cases of low rank and aim to establish their faithfulness as modules over the Iwasawa algebra.

Length of Project: flexible.

The evolving risk of local SARS-CoV-2 outbreaks

Prof. William Hart (william.hart@maths.ox.ac.uk)
Dr Robin Thompson (University of Warwick)

While the worst of the COVID-19 pandemic is (hopefully) behind us, there remains a risk of localised SARS-CoV-2 outbreaks in populations including schools, universities, workplaces, and care homes. This risk will change over time because of cycles of increasing and waning immunity generated by infections and booster vaccination campaigns, as well as other factors such as seasonal effects and the continued evolution of the SARS-CoV-2 virus.
In this project, you will develop a mathematical framework for estimating the evolving risk of local SARS-CoV-2 outbreaks, and then implement your modelling approach using a programming language of your choice (likely Matlab or Python). Relevant data, describing how antibody levels change within a person over time since they last received a vaccine dose, will be provided to you. You will use the antibody data to predict how the local SARS-CoV-2 outbreak risk evolves over time following a booster vaccination campaign. Some of the following possible further directions could then be considered:

- Comparing your predictions to simulations of a stochastic epidemic model.
- Comparing your predictions to an approximate approach in which immunity levels are assumed to remain "frozen" following the introduction of the virus into the local population.
- Considering different types of local population and/or considering booster vaccines restricted to certain population sub-groups.
- Considering both infection- and vaccine-derived immunity.
- Extending the framework to incorporate further data describing how viral loads change during a SARS-CoV-2 infection, and considering the effect of other interventions such as regular antigen testing on the outbreak risk.

This project will provide insight into how booster vaccination campaigns (and potentially other interventions) can be designed to mitigate the risk of local SARS-CoV-2 outbreaks, and therefore presents an opportunity to conduct mathematical research with positive real-world impacts.
Length of project:
Flexible (6-8 weeks)
Who might be interested: Students interested in mathematical and computational modelling of infectious disease epidemics.

Computational efficient Deep Neural Networks

Prof. Coralia Cartis (cartis@maths.ox.ac.uk)
Prof. Jared Tanner (tanner@maths.ox.ac.uk)
Dr Jan Fiala (AMD)

The success of DNNs (such as convolution nets for image recognition) is undisputed, however the resource needs (in computation and memory) might be significant and potentially limiting in terms of speed of inference and prohibitive for certain type of applications (e.g., low-memory edge devices deployment). Significant effort is focused on increasing the efficiency of DNN inference while maintaining highly accurate results via various compression techniques, such as pruning. It has been shown that it is possible to drop even 95% of parameters of the net while maintaining similar accuracy, however, achieving further reductions is challenging as well as translating such reductions into actual speed-ups on modern hardware architecture. Another open issue is the memory footprint during inference as the tensors passed in hidden layers might be huge and their compression is the next focus point. This project may be joint with the microchip company AMD, co-supervised by Dr Jan Fiala. Experience with Python is essential.

Length of project: 6-8 weeks, flexible.

Who might be interested: Students wishing to develop skills in deep learning, the main algorithm in modern artificial intelligence and machine learning. Students wishing to become increasingly experience in developing computer software.

Parameter identifiability analysis of spatiotemporal models of cell invasion

Dr Alexander Browning (browning@maths.ox.ac.uk)

Prof. Ruth Baker (ruth.baker@maths.ox.ac.uk)

Mathematical models play an increasingly important role in the interpretation of biological data, and provide a lens through which to better understand critical biological processes ranging from cell behaviour to drug resistance in cancerous tumours. A key question is whether parameters in a mathematical model can be identified from the available experimental data. Over the past few decades, tools have been developed to analyse this question for ordinary differential equation models (that typically neglect space). There are no commonly adopted tools for the spatiotemporal models that are often crucial for understanding dynamics related to cell migration.

The aim of this project is to develop an identifiability analysis framework that can be applied to partial differential equation models of cell migration. Such techniques will have broad applicability in mathematical modelling, and provide essential insights that guide future experimentation and data collection. The project will be motivated by published experimental data of melanoma cell migration, and will involve the development of mathematical models alongside statistical and identifiability analysis techniques.

Length of project: 8 weeks
Who might be interested: Students interested in mathematical biology, statistics, and partial differential equations.

Reactive decontamination of porous media

Dr Ellen Luckins (luckins@maths.ox.ac.uk)

A porous medium that has been contaminated with a hazardous chemical agent (for instance, in a chemical weapons attack) is typically decontaminated by applying a cleanser liquid to the surface of the contaminated material. The cleanser reacts into the porous medium, neutralising the agent in a chemical reaction. It is crucial for both public safety and the environment that the agent is completely removed. Ideally, we'd like to choose an appropriate cleanser and apply it in the best way in order to both (i) complete the decontamination as fast as possible, and (ii) minimise the amount of cleanser that is wasted in the process.

Effective models for the decontamination have recently been derived and studied eg: [1, 2]. These consist of an (advection--) diffusion PDE for the transport of the cleanser chemical, with the decontamination chemical reaction included either at a free boundary of the domain (at the agent-cleanser interface) or within the bulk. The "cleaning boundary condition" describing how cleanser is added at the top of the material has so far been very simple (a Dirichlet boundary condition for the cleanser concentration). However, this simple Dirichlet condition probably isn't correctly capturing what is happening physically.

In this project we would investigate what the correct cleaning boundary condition should be, for some different cleaning scenarios (e.g.: applying the cleanser and leaving it, compared with continually scrubbing the surface). This will require lots of modelling (deciding what are the right equations to write down to describe the physical processes) and asymptotic methods such as boundary-layer analysis. There may additionally be scope for multiple-scales analysis (averaging, or homogenisation) in time.

Please feel free to get in touch via email if you have any questions about this project, or if you would like to discuss an alternative project idea of your own.

[1] MP Dalwadi , D O'Kiely, SJ Thomson, TS Khaleque, CL Hall. Mathematical modeling of chemical agent removal by reaction with an immiscible cleanser. SIAM J. Appl. Math. 77(6), 1937–1961 (2017)

[2] EK Luckins, CJW Breward, IM Griffiths, Z Wilmott**.** Homogenisation problems in reactive decontamination. Eur. J. Appl. Math. 31(5), 782-805 (2019)

Length of the project: Flexible, at least 6 weeks

Who might be interested: Those interested in continuum modelling, applications of PDE models, asymptotic analysis, perhaps some numerical solution of PDEs

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Building combinatorial models of interesting spaces

Dr Davide Spriano (spriano@maths.ox.ac.uk)

This is a project in geometric group theory, namely the branch of mathematics that studies group in relation to their actions on metric spaces.
A stepping stone in modern geometric group theory is the Sageev construction, namely a procedure that allows to build a nice combinatorial object, called a cube complex, very weak hypotheses on a metric space. The main idea is to look at ways to subdivide your metric space into two pieces, this is called a wall, and look at the combinatorial data that arises from how the walls interact with each other.
In this project we want to look at other possible ways to extract combinatorial data from a metric space, and see which information they encode, in particular in presence of a group action.

Length of project: around 8 weeks, flexible.

Who might be interested: Students interested in group theory, metric geometry and/or infinite graphs

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Nonlinear elasticity and Newton's method in infinite dimensions

Dr Charles Parker (parker@maths.ox.ac.uk)
Dr Francis Aznaran (aznaran@maths.ox.ac.uk)

Solving nonlinear partial differential equations describing the deformation of solid structures is a ubiquitous problem in many areas of engineering. This project will study the Newton–Kantorovich iteration, the most widely used method for the numerical solution of nonlinear PDEs, which generalises the Newton–Raphson method (for root finding of scalar functions) to infinite-dimensional normed vector spaces. This will be combined with the finite element method (FEM), which provides approximate solutions to the unknowns in terms of piecewise polynomials, to generate computer simulations of some interesting elasticity problems motivated by concrete real-world applications.

We will study the abstract theory of Newton's method and the FEM, and code numerical experiments for solid mechanics problems to give results with which to compare the theory. Beyond this, the project can either be more theoretical or more computational according to the student's taste, as there are several directions to take:

- alternatives to Newton linearisation (such as the Picard iteration),
- conditions of local or global convergence of these iteration schemes,
- problematic parameter regimes in the physics such as incompressibility of the solid material being modelled,
- experimentation with and study of more complicated elasticity models, or
- experimentation with and study of more sophisticated finite element discretisations.

This project will require familiarity with the python language, or a willingness to learn in time for the start of the project.

Length of project: 5-8 weeks.

Who might be interested: Students interested in applying abstract analysis to concrete problems of interest in mechanics. They will have just finished 3rd or 4th year in summer 2023, and have taken any of the courses B4.1 and B4.2 Functional Analysis I and II, B6.1 Numerical Solution of Partial Differential Equations, C4.3 Functional Analytic Methods for PDEs, or C6.4 Finite Element Methods for PDEs. B4.3 Distribution Theory and C5.2 Elasticity and Plasticity may also be helpful.

Undecidable problems in group theory

Prof. Marco Linton (Marco.Linton@maths.ox.ac.uk)

In 1910, Max Dehn formulated three fundamental decision problems in the theory of infinite groups, the most important of which is known as the word problem. The word problem for a given group asks whether there is an algorithm to decide if any two elements, given as products of generators, are equal. The first examples of finitely presented groups with undecidable word problem first appeared in the work of Novikov and Boone in the 50s. Nevertheless, most natural classes of groups do admit a solution to their word problem. This project will involve exploring the boundary between decidability and undecidability of problems within classes of groups of modern interest such as free groups, one-relator groups, hyperbolic groups or automatic groups.

Length of Project: flexible.

Sumsets and structure in additive combinatorics

Dr Yifan Jing (mudgal@maths.ox.ac.uk)
Dr Akshat Mudgal (jing@maths.ox.ac.uk)

This is a project in additive combinatorics, an area lying at the interface of combinatorics, number theory and harmonic analysis. Topics in this field often concern the structure of sets which behave well under addition or those that exhibit various additive patterns of interest. Another topic of research involves analysing the interplay of addition and multiplication on finite sets of integers/real numbers/finite fields.
In this project, the student would begin by studying some basic materials/papers associated with these subjects. Next aim of the project would be to work on a research problem on related topics. For instance, given some fixed arithmetic pattern X and some ambient set A of integers, one might be interested in finding the largest subset B of A which does not contain an affine copy of X. Another such example is -- given a medium sized subset C of some finite field and bivariate polynomial P, how large can the image P(C,C) be?

Length of the project: Around 8 weeks, but there is flexibility regarding this
Who might be interested: Students interested in additive combinatorics.