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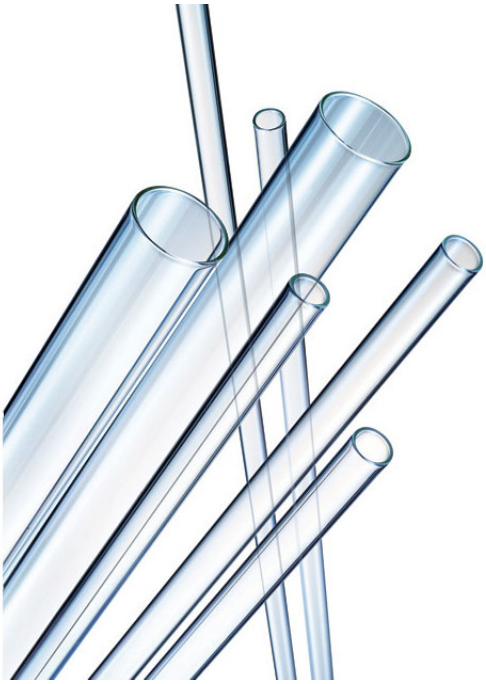

Stability of the Danner Process

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SCHOTT
glass made of ideas

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1 Introduction

Background

Technological advances have led to improvements to glass manufacturing methods.

Glass is made from natural and abundant raw materials that are melted at very high temperatures to form the product. At these high temperatures, glass is a liquid, while at ambient temperature it is a solid. Consequently, hot glass can be poured, blown, pressed, and moulded into almost any shape, which is permanent once the glass is cool.

Glass is an exceptionally versatile material, with almost unlimited potential for new applications. For example, recent developments in the technology sector have driven demand for ultra-thin glass sheets, essential for cutting-edge laptop and phone screens. Inevitably, this increase in demand has necessitated improvements to industrial manufacturing methods for glass sheets which have made it possible to form lighter, stronger, and thinner glass. As a result, glass can now be found everywhere, and has a use in almost all product sectors, from packaging, tableware, and interior design, to electronic appliances, automotives, and fibre optics.

Schott use the Danner process to make glass tubes.

One particularly important use of glass is in the pharmaceutical industry, where glass tubes are used in the form of ampoules, vials, and syringes. Accuracy in the properties of these glass tubes, such as tube wall thickness and tube diameter, is vitally important, since deviations from recommended wall thicknesses can have serious consequences such as the misdosing of patients. Schott AG manufactures these glass tubes using the *Danner process*.

The Danner process

In the Danner process, a slender thread of molten glass flows from a nozzle onto a structure known as the Danner mandrel, as shown in Figure 1. The mandrel rotates at a given angular velocity, which causes the glass to coil around it once contact is made. Since the mandrel is slanted downwards at some known small angle from the horizontal, these glass coils then propagate down the mandrel, accelerated by gravity. The glass eventually falls off the end of the mandrel, where it is pulled away to form a tube.

The glass thread leaves the nozzle vertically and at a prescribed speed. Once the glass makes contact with the mandrel, which is located at a prescribed distance H_{Num} below the nozzle, it moves at the speed at which the mandrel rotates U_M, and in the direction of its motion.

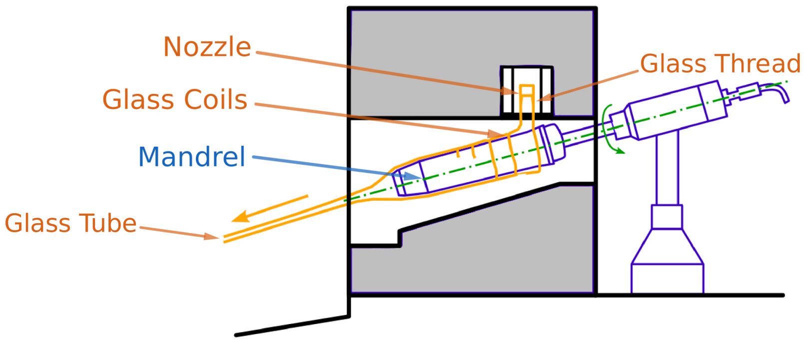

Figure 1 – A schematic of the Danner process.

Sensibly choosing the draw ratio is crucial for achieving high production rate.

The ratio of the glass speed at the nozzle to the glass speed at the mandrel is known as the *draw ratio*. It is known from experience that the choice of a suitable draw ratio is crucial for a high rate of production. Moreover, there are critical draw ratios (upper and lower draw ratio limits) beyond which production becomes impossible.

Our aim is to gain a comprehensive understanding of the flow from the nozzle to the mandrel, and thus to identify draw ratio conditions for the onset of flow instability.

Glossary of terms

- **Draw ratio:** The ratio between the speed of the glass at the mandrel and at the nozzle.
- **Bending stiffness parameter:** The ratio of normal forces acting on the thread to the tension of the thread.
- **Substrate angle:** The angle that the substrate makes with the horizontal.
- **Linear Stability Analysis:** A method for determining the growth of small perturbations from the steady solution in a linear approximation of a non-linear system.

2 Mathematical model for the Danner process

Using the thread's slenderness simplifies our equations.

The stiffness parameter dictates the importance of bending.

We treat the glass thread as a slow-moving, viscous fluid. The general equations for a fluid of this type are called the Stokes equations, which Schott currently solve numerically. To do this accurately is slow, so to save computational expense, we exploit the fact that the glass thread is slender. This allows us to simplify the Stokes equations to a set of thin-film equations known as the Trouton Model. These equations describe conservation of mass and momentum in the thread, which we augment by incorporating terms related to the bending stiffness. The resulting equations can be solved to give the velocity, centreline, and cross-sectional area of the thread. The importance of bending effects is governed by the *bending stiffness parameter*. When forces normal to the thread are more important than tension, the bending parameter is large.

For simplicity, we first consider the paradigm problem of flow falling onto a moving horizontal substrate, rather than the curved mandrel. Later, we extend our model to incorporate non-horizontal substrates, as a proxy for off-centre nozzle positions.

At the nozzle, we prescribe the area, speed, angle, and curvature of the thread. The speed, angle, and curvature are also given at the substrate.

3 Steady state solution

Negligible bending stiffness

Assuming that bending effects are not important, we derive an explicit solution for the velocity, centreline, and cross-sectional area of the thread. We show the shape of the thread in Figure 2 along with results form numerical simulations performed by Schott of the full *Navier-Stokes equations* that describe viscous fluid mechanics. We see that the validity of our assumption of negligible bending depends on the height of the nozzle above the substrate. When the height is small, the tension in the thread dominates the normal force, and so neglecting stiffness in our equations results in a solution which fits Schott's results well. However, as the height increases, the tension in the thread gets smaller, increasing the importance of normal forces. For larger heights, it is no longer valid to neglect the stiffness parameter, and so doing this leads to large differences in the solutions.

Non-negligible bending stiffness

When we consider bending effects, the solutions agree at all heights.

When we consider solutions with non-zero bending stiffness, it is no longer possible to derive a solution explicitly. We therefore solve the equations numerically to find the velocity, centreline, and cross-sectional area of the thread. We show the shape of the thread in Figure 3. We see that, even for the largest feasible Danner process height, our solution agrees extremely well with the numerical simulation carried out by Schott. By including the bending stiffness parameter, we allow our solution to hold true in scenarios where tension is small, or even negative, near the mandrel. Negative tension corresponds to compression, which causes the 'backward heel' shape which we observe in Figure 3.

Our numerical scheme takes less than a second to compute the solution to our simple model. In contrast, it takes almost a day for Schott to compute their full numerical solution.

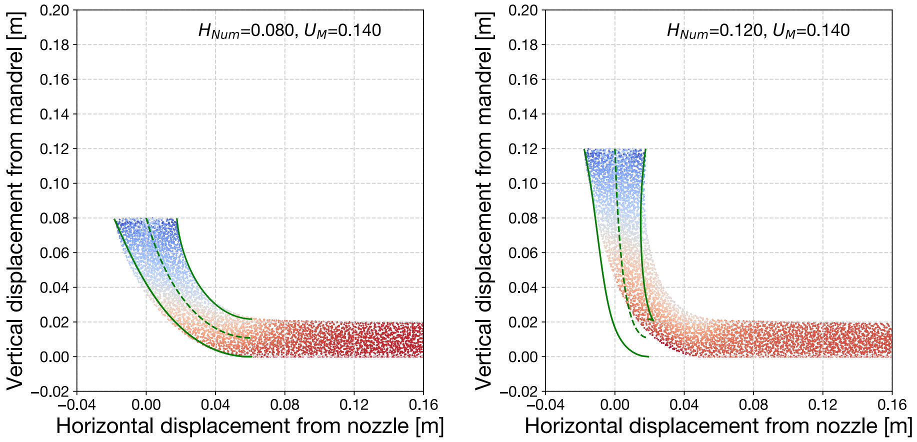

Figure 2 – Graphs showing comparison between the thread shape determined by our simple model (solid lines) and by Schott (coloured dots; colours indicate velocity), for two different heights when bending stiffness is neglected in our model.

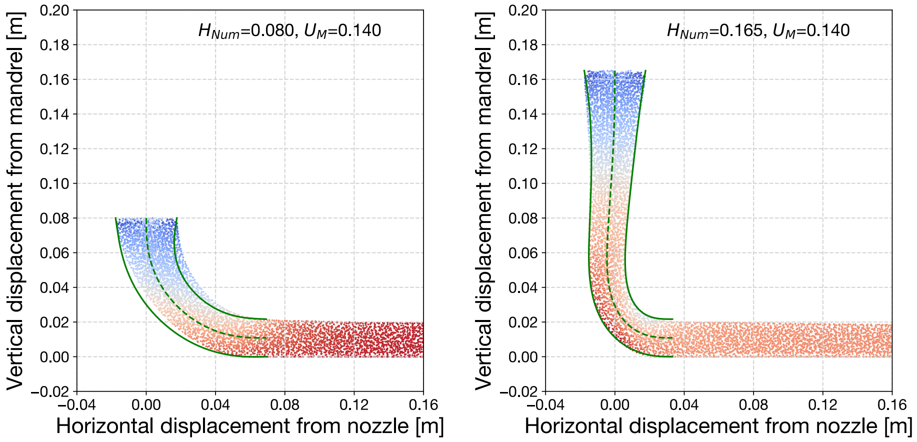

Figure 3 – Graphs showing comparison between the thread shape determined from our simple model (solid lines) and by Schott (coloured dots; colours indicate velocity), for two different heights when bending stiffness is included in our model.

4 Stability of the solution

The critical draw ratio occurs when the growth rate is zero.

Now that we have found the steady state solution, we examine the stability of the system in order to identify the critical draw ratio. To this end, we consider a small perturbation to the steady state solution, and its associated growth rate. The critical draw ratio occurs when the growth rate is zero since for draw ratios exceeding this critical value, any perturbations grow, causing instability. Instability of the system corresponds to undesirable oscillations in the wall thickness of the glass tubes, leading to a decreased product yield.

Substrate angle dependence

The critical draw ratio depends on the angle of the substrate.

We first explore whether the critical draw ratio depends on the *substrate angle*. In Figure 4, we show the growth rate as a function of the draw ratio, for various substrate angles. We see that, as the substrate angle increases, the critical draw ratio decreases. Thus, decreasing the angle increases the stability of our system. In the case where the substrate angle is zero, the critical draw ratio does not exist, and our steady state solution is stable at any draw ratio.

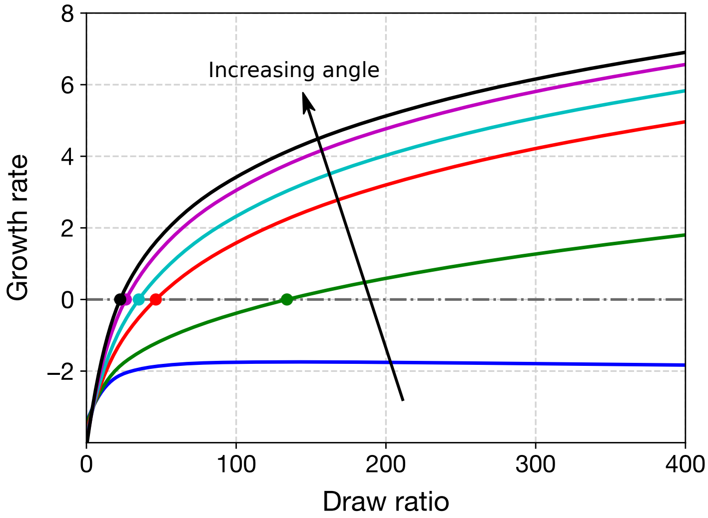

Figure 4 – The growth rate of perturbations as a function of the draw ratio, for various substrate angles. Zero growth rate intersections are highlighted with dots, and indicate the critical draw ratio for a given angle. The substrate angles are 0, 0.01, 0.05, 0.1, 0.3, **and** 1.57**.**

Height dependence

The solution when the substrate is horizontal is always stable.

The critical draw ratio also depends on the height. For non-zero substrate angles, increasing the height decreases the critical draw ratio. As we see in Figure 5, increasing the height in a horizontal substrate scenario drives the negative growth rate closer to zero. However, at each height, the growth rate remains negative regardless of the choice of draw ratio, meaning that the solution remains stable.

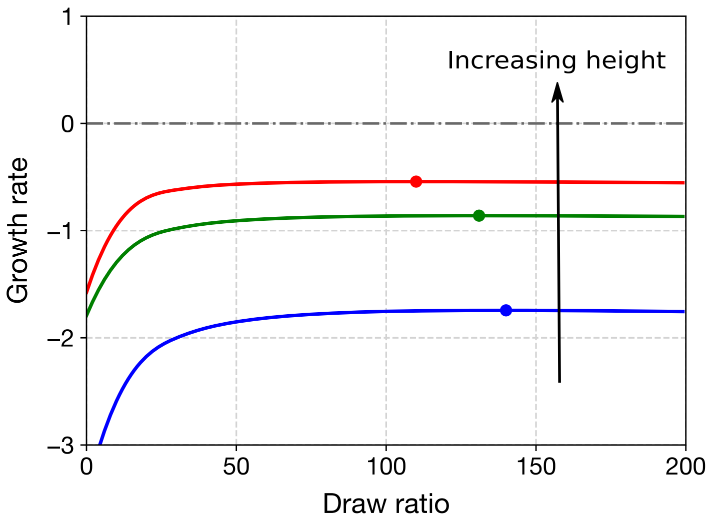

Figure 5 – The growth rate of small steady state perturbations as a function of the draw ratio, for a horizontal substrate and for various thread lengths. The maximum growth rate at each length is highlighted with a dot.

Industrially relevant substrate angle

In the Danner process, the nozzle can be positioned such that the first contact that the thread makes with the mandrel does not occur at the maximum height of the mandrel. Therefore, it is industrially relevant to consider small non-zero substrate angles. In

Figure 6, we show how the critical draw ratio depends on the thread length for a substrate slanted at an angle of 0.1 radians. A critical draw ratio exists for all permitted heights. As the height increases, the critical draw ratio also increases.

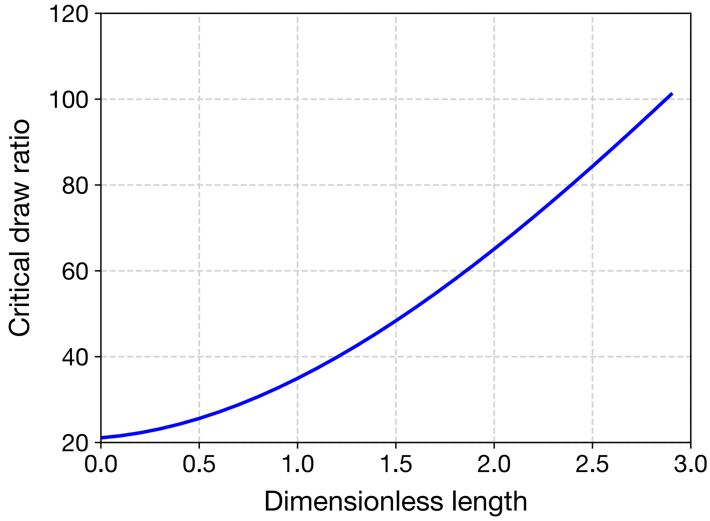

Figure 6 – The critical draw ratio as a function of the dimensionless thread length, for a slightly slanted substrate.

Danner process draw ratios are much smaller than critical draw ratios.

This has industrial implications, since it means that the system gets further from an unstable state as the height is increased. We note that the typical Danner process draw ratios are much smaller than the critical draw ratios that we observe in this slanted substrate case. Therefore, our model suggests that Schott operate the Danner process in a regime which is not vulnerable to instabilities of this type, at least when bending is negligible.

5 Discussion, conclusions, & recommendations

We have derived a mathematical model to describe early stage glass flow in the Danner process for manufacturing glass tubes. Our model utilises the slenderness of the glass thread in order to simplify the fluid mechanics.

In the steady state, and in the absence of bending, we found an explicit solution which compares well with a full numerical solution currently used by Schott, at least for small industrially relevant heights. When we included bending effects, we solved the steady-state system numerically and found excellent agreement for the shape of the thread with Schott's solution, for all industrially relevant heights. Our approach allows the time taken for the computation of the shape, thickness, and velocity of the glass to be reduced from hours to less than a second. We recommend that Schott use our reduced model to rapidly determine the behaviour of the thread when exploring different draw ratio choices.

We also investigated the stability of our model (in the absence of bending effects) by considering small perturbations about our steady state solution. We found stability to be dependent on height, as well as substrate angle. We found that the stability increases as the substrate angle or height are decreased.

A natural extension of our work would be to consider a similar linear stability analysis when bending effects are important, since we have shown that this solution models the Danner process more completely. Another key extension would be a three-dimensional version of our slender model. This would allow for the possibility of out-of-plane oscillations, allowing new instability types. Relevant literature suggests that these instabilities, related to out-of-plane thread buckling, might occur at draw ratios used in the Danner process. These would therefore be particularly relevant to Schott.

6 Potential impact

Our steady state solutions are a very useful output of the project. We have shown that, in some parameter regimes, an explicit solution may be used to find the shape of the early stage Danner process glass flow without the use of numerical methods. Meanwhile, our solver means that Schott can now find a flow solution for any industrially relevant parameter choices in less than a second. This is valuable, since our solution matches Schott's current one, which can take almost a day to find, to a high degree of accuracy.

Our linear stability analysis is another useful output. We have begun to build a picture of how the Danner process reacts to perturbations in mandrel velocity and nozzle position.

Dr Ulrich Lange, Senior Scientist in the Simulation and Data Science team at Schott commented: "*The Danner process is a reliable and robust method for drawing glass tubes, although its basic concept of coiling a thread of molten glass around a rotating mandrel is quite complex from the fluid mechanical point of view. A deeper understanding of the dynamic behaviour of this flow is required to meet the increasing demand for higher precision of the tube geometries and higher yield. The mini-project focused on a small aspect of the dynamics of the coiling flow, but the methods employed are most suitable to get a more comprehensive picture in the near future and enable Schott to identify process conditions which minimize the impact of unavoidable process disturbances (e.g. mechanical vibrations of the mandrel) on dimensional accuracy of the tubes.*"