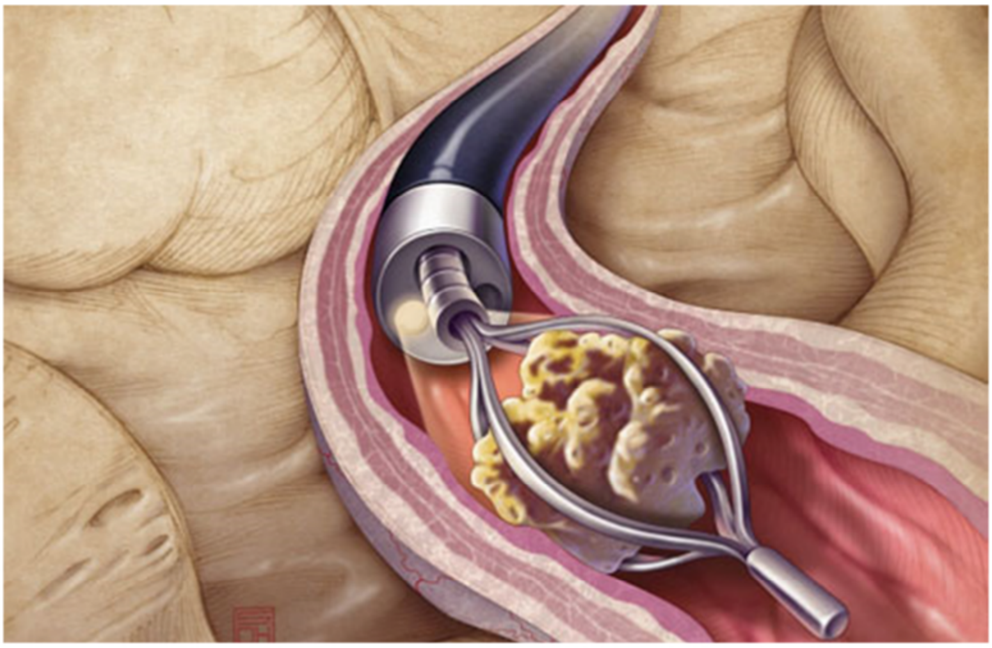
EPSRC Centre for Doctoral Training in Industrially Focused Mathematical Modelling



Mathematical Modelling of Ureteroscope Irrigation

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1. Introduction

Motivation

The cost of kidney stones in the United States is predicted to be 5 billon USD by the year 2030.

The prevalence of kidney stones has increased over time, and in 2010 it was reported that stone disease affects one in eleven people in the United States [3]. Not only does this constitute a significant healthcare burden, but the financial cost of stone disease in the USA by the year 2030 is estimated at over five billion dollars annually [1]. In addition, kidney stones are most common among working age individuals, and indirect implications of this condition include absence from work and lost productivity [2]. Thus, it is of the utmost importance to develop a treatment method that is successful, inexpensive, and requires minimal recovery time.

Background

A common procedure for removing kidney stones is ureteroscopy, or retrograde intrarenal surgery. This involves inserting a medical instrument, a ureteroscope, through the urethra; the urethra provides passage through the bladder to the urinary system, see Figure 1. Often, before the ureteroscope is inserted, an access sheath is placed within the urethra, functioning as a hollow channel to allow easier admittance of the scope. Flexible ureteroscopes give freedom to deflect the distal portion of the scope, allowing access to stones in all parts of the urinary system. Ureteroscopy for kidney stone removal necessitates the use of working tools, such as laser fibres and stone baskets, to break up or remove fragments of stones, respectively. These tools resemble long wires which are passed through the scope to reach the stone. Boston Scientific is a worldwide manufacturer and developer of medical instruments, including ureteroscopes, access sheaths, and working tools.

Irrigation is essential for visualisation of the urinary system during kidney removal surgery, a procedure involving ureteroscopes.

In order for this procedure to be possible, visualisation of the renal system is required, and modern ureteroscopes are fitted with a light and a camera at the tip of the scope, so that the urological environment can be viewed on a screen; urologists use this image to guide their way through the urinary tract to locate stones. This visualisation requires constant irrigation, both to clear the field of view of debris, and to open up the ureter to provide access for the scope. The irrigation is provided by a weak saline solution connected to the scope via irrigation tubing. This irrigation fluid then flows through the working channel of the scope, out the distal end, directly into the urinary system. Our focus will be on modelling the flow of irrigation fluid through the ureteroscope.

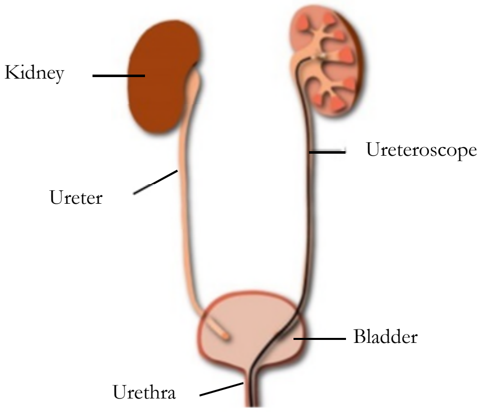

Figure 1: A diagram depicting the urinary system. The ureteroscope is inserted through the urethra, the bladder, the ureter, and into the kidney. (Image reproduced from www.kidneystoners.org).

2. Our Challenge

We construct a mathematical model to optimise irrigation flow.

Optimising the flow of irrigation fluid through the ureteroscope to maintain good visualisation during surgery is a universal urological challenge. Ideal irrigation flow would aid visualisation without causing detrimentally high intrarenal pressures or sufficient force to propel the stone away from the scope. Determining the optimal flow requires a good understanding of the fluid mechanics of ureteroscopes, and how the irrigation system behaves under operating room conditions. In particular, we aim to understand the effects of scope deflection, working tools, and access sheaths on the flow through an isolated ureteroscope. To accomplish this, we construct a mathematical description of the system, relating flow rate to fluid and scope parameters as well as operating conditions. To test our model predictions we also conduct experiments in a controlled laboratory setup, shown in Figure 2.

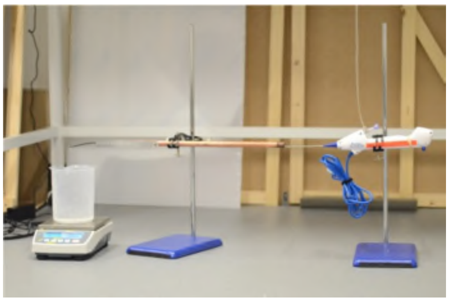

Figure 2: Experimental setup to measure the flow rate of irrigation fluid through the ureteroscope.

Glossary of terms and symbols

- Hydrostatic Pressure Head: Pressure exerted by a stationary column of fluid – depends only upon the height, gravity, and the density of the fluid.
- Volumetric Flow Rate: The volume of fluid that flows, per unit time.

Mathematical model

We consider the effects of scope deflection, working tools, and access sheaths.

We model the flow of irrigation through the ureteroscope outside the urinary system, considering the influence of deflection of the scope as well as the effect of the presence of the working tools. The equations, derived through conservation of mass and a balance of forces, relate the volumetric flow rate of irrigation fluid to properties of the scope and the geometry of the irrigation setup.

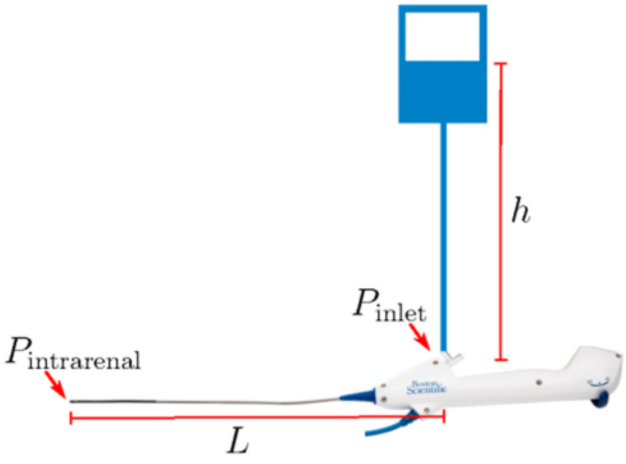

Figure 3: A schematic depicting the ureteroscope system, indicating the height of the hydrostatic head, the inlet and intrarenal pressures, and the length of the scope.

3. Results

We present preliminary mathematical and experimental results for flow through four configurations: a straight scope, a deflected scope, a straight scope with a working tool, and a straight scope through an access sheath.

Straight Scope

Flow rate is dependent on inlet pressure, scope size, and properties of the irrigation fluid.

The volumetric flow rate, Q, of irrigation fluid through the ureteroscope is dependent upon the inlet pressure, $P_{inlet} = \rho g h$, (where ρ is density, g is gravitational acceleration, and h is the hydrostatic head height), the internal scope radius, a, the length of the scope, L, and the viscosity of the irrigation fluid, μ. Assuming the flow remains laminar, the relation is:

$$Q = \frac{a^4 \pi \rho g h}{8 \mu L}$$

Hence, from the model, we anticipate a linear relationship between volumetric flow rate and pressure head height.

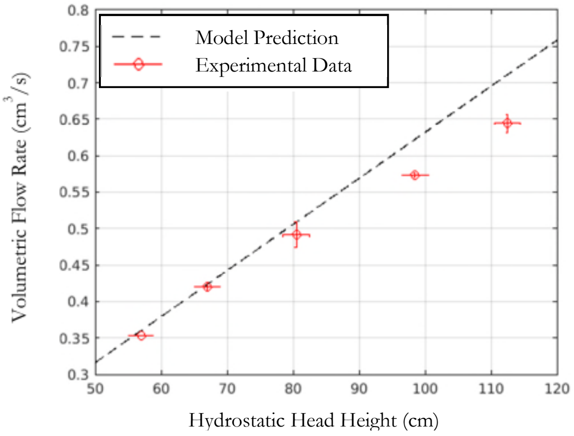

Figure 4: A comparison between theory and experiments for irrigation flow through a straight ureteroscope.

Bends in the irrigation tubing or discrepancy in the radius of the scope are two possible sources of error.

We can see from Figure 4 that this prediction agrees with the laboratory experiments for lower pressure heads, but as the inlet pressure increases, the prediction deviates from the data. One possible cause for this discrepancy is a loss of pressure as the fluid navigates around bends in the experimental set-up; for example, where the fluid enters the scope via the irrigation tubing. This would have a stronger effect at higher flow rates, thus resulting in a non-linear relationship between head height and inlet pressure, shown in Figure 5a. We note that this resembles the shape of the data in Figure 4.

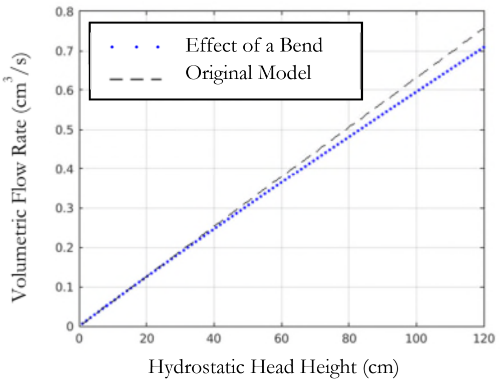

Figure 5a: The effect of a bend on the volumetric flow rate.

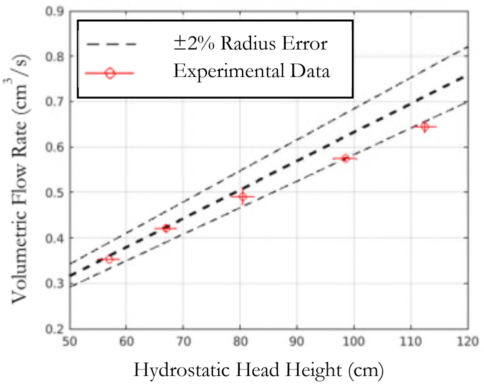

Figure 5b: The effect of working channel radius error on the volumetric flow rate.

Another possible cause for disagreement is a deviation in scope radius. We note that the flow rate scales with the radius to the fourth power, demonstrating a strong sensitivity to internal scope radius. We see from Figure 5b that even a 2% error in the measurement of the scope radius has a significant effect on the predictions of the mathematical model.

Deflected Scope

We now consider the effect of ureteroscope deflection on the volumetric flow rate. In Figure 6, which displays Boston Scientific's LithoVue ureteroscope as it varies from minimal to maximal deflection, we notice that only the tip of a constant length is able to curve, while the majority of the scope remains straight.

Figure 6: The LithoVue ureteroscope at various stages of deflection.

We adapt the previous flow model to describe flow through a scope with a deflected tip; this involves considering flow through a pipe with constant curvature. From curved pipe theory, we deduce that flow rate is dependent upon the Dean number, a dimensionless parameter that relates centripetal, inertial, and viscous forces, and that different flow rate relations can be implemented depending upon the magnitude of this value. From these results, we conclude that the flow through a curved scope depends upon the curvature of the tip, along with the previous relevant parameters. In Figure 7, we plot the volumetric flow rate as a function of curvature. The two dashed lines in Figure 7 show the minimal and maximal theoretical predictions, with regards to the error in measuring the hydrostatic head height. The decrease in flow rate becomes more pronounced at higher head heights, as shown in Figure 5a.

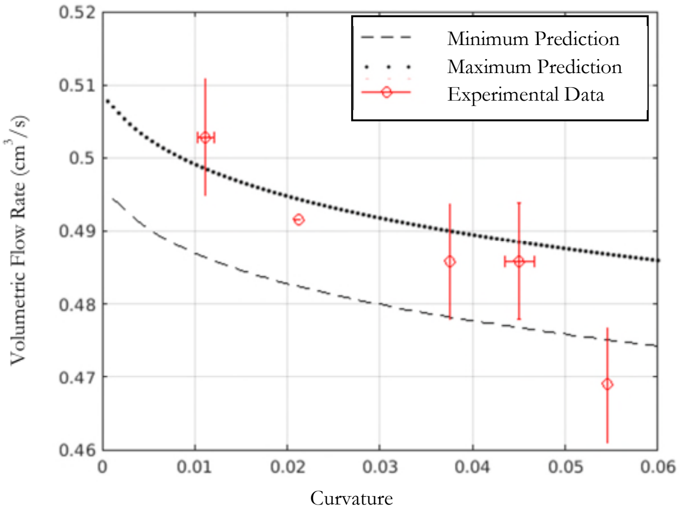

Figure 7: A comparison between theory and experiments for irrigation flow through a deflected scope at a pressure head height of 78.5 ± 1 cm.

Our model also predicts that the flow rate will decrease linearly with the arc length of the curved portion of the scope, although this result is not shown. However, in any case, the decrease in volumetric flow rate is relatively minimal, e.g., the highest curvature in Figure 7, relating to the image on the far right of Figure 6, corresponds to only a 5% decrease in flow rate from that of a straight scope.

Flow decreases with scope deflection.

Working Tools and Access Sheaths

We next consider the effect of including access sheaths around the scope or working tools through the scope on irrigation flow. In Figure 8, we plot the results of experiments measuring the flow rate through a ureteroscope with various Boston Scientific working tools, compared with the predictions of our mathematical model.

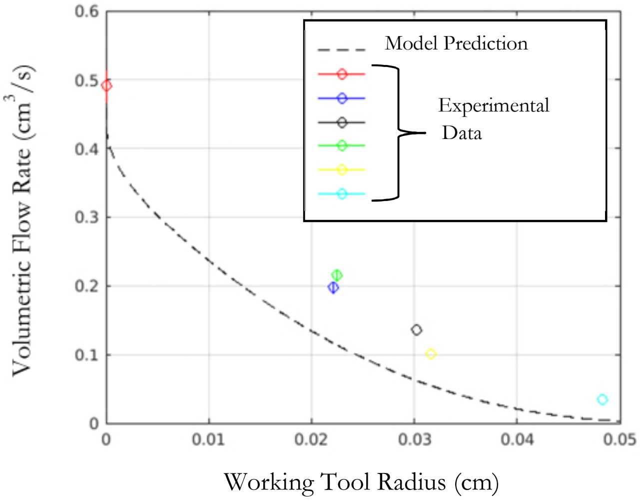

Figure 8: **A comparison between theory and experiments for irrigation flow through a straight scope with a working tool at a pressure head height of 78.5 cm.**

We see that the presence of a working tool reduces the cross-sectional area available for fluid and thus reduces the flow. The degree of reduction increases with the radius of the working tool; a larger tool hinders irrigation flow more. We attribute the discrepancy between the experimental and theoretical results seen in Figure 8 to ignoring the effect of the position of the working tool within the working channel. The theory is built upon the assumption that the working tool is concentric within the working channel, when in actuality, it is likely that it lies along the bottom, leading to an increased flow when compared with the model predictions. In Figure 9, we show a comparison between theory and experiments for flow through the scope when an access sheath is used.

The disparity between theory and experiments for these models can be explained by the positioning of the instruments.

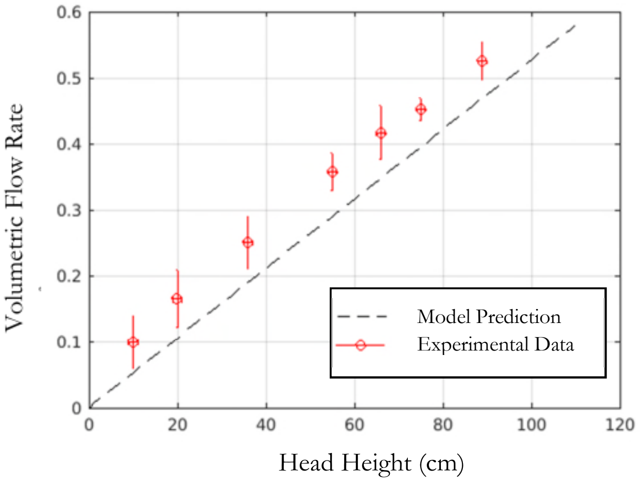

Figure 9: **A comparison between theory and experiments for the irrigation flow when the scope is placed through an access sheath; the fluid flows into a rigid chamber before flowing back through the sheath.**

The access sheath model also under-predicts the volumetric flow rate; this behaviour may also be caused by assuming that the scope is concentric within the access sheath.

4. Discussion, Conclusions & Recommendations

We have presented experimental and theoretical results on the irrigation flow through an isolated uereteroscope. In particular, we have focused on four particular scenarios. In the case of a straight scope with no impediments, the experimental data deviated from the model predictions at high pressure heights, suggesting that care must be taken to consider all aspects of the scope design, such as connections and manufacturing tolerances. With a deflected tip, we observed only a minimal decrease in flow rate, which was in good agreement with experiment. The presence of a working tool, on the other hand, can significantly decrease the flow, and in a nonlinear fashion. Here the underestimate of our model suggests that the tool does not sit concentrically in the scope, a point we will return to in future work; an obvious extension to consider is incorporating the effect of offsetting the inner instrument and to determine whether this increases the accuracy of the models.

Once we have validated models for all potential configurations of the ureteroscope outside the urinary system, we can start including physiological effects, such as considering the deformable nature of the bladder, ureters, and kidneys. Importantly, we will also model the dynamics of kidney stones within the irrigation flow field, so we can use this information to minimise retropulsion of the stone during surgery. Additionally, in further studies, we aim to more explicitly model the inlet pressure, since this is the main controlling parameter used in operating conditions to modify the flow.

5. Potential Impact

The flow of irrigation through ureteroscopes has been examined in previous experimental studies, but we are the first to apply a mathematical modelling approach to this system. Our work serves as preliminary modelling steps towards developing a generic framework for describing flow in ureteroscopes. Such a tool, adaptable to different physiological conditions and scope parameters, would have immense potential value, both in advising scope design and as an optimization guide for physicians in scope procedures.

Tim Harrah, a manager at Boston Scientific, commented: *"After a decade of market leadership making tools used through endoscopes, Boston Scientific has recently entered the market as a producer of visualization systems. For surgeries done using an endoscope, good visualization is synonymous with good fluid management. Unfortunately the literature for fluid delivery in endoscopic surgery is sparse. Work on this project has already contributed meaningfully to our understanding and we are excited to extend the underlying modeling framework to address other opportunities to make endoscopic surgeries more efficient and effective."*

References

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