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Modelling the Thermal Cracking of Composite Electrodes

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1. Introduction

Background

Elkem is a metallurgical company that produces silicon, among other materials. The production of silicon requires extremely high temperatures, and these are achieved within a silicon furnace by adding large amounts of electrical energy, generated using electrodes which are used to conduct the required electricity. These electrodes are required to be continually manufactured within the furnace, as the tip, which is embedded within the raw materials (quartz and carbon, often referred to as the charged material) is degraded. Ideally an electrode would be comprised of pure graphite, due to its high thermal and electrical conductivities, but as this is an expensive material, alternate compositions have been designed. A particular electrode configuration, known as the Persson or composite electrode, is comprised of a graphite core, with a surrounding layer of paste. The paste is made from a mixture of anthracite and coal tar-pitch binder; once the paste has undergone a baking, or hardening, process, the paste behaves similarly to graphite.

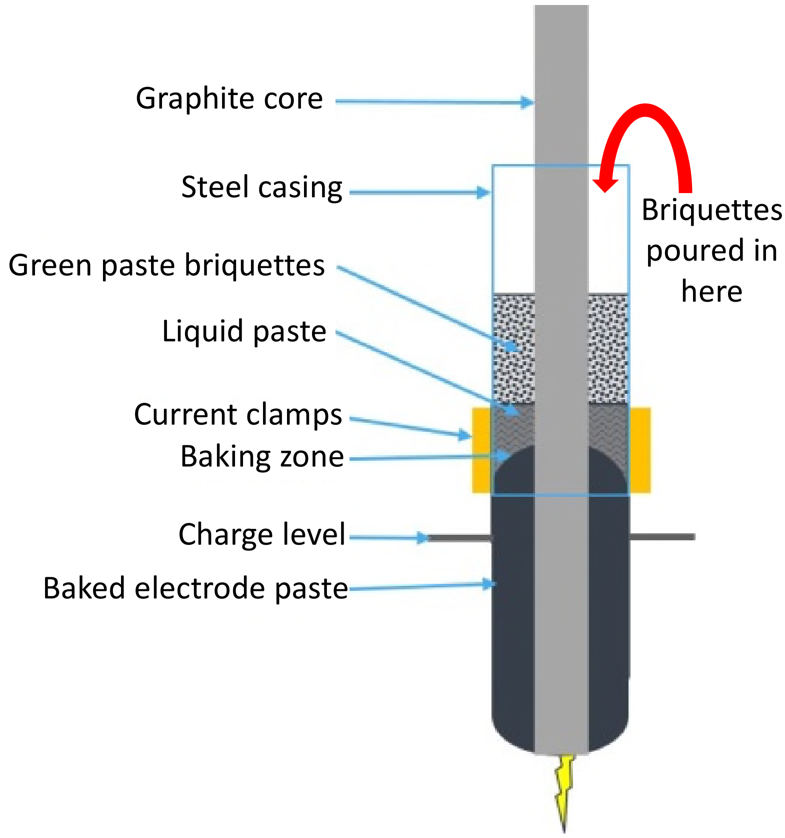

Figure 1: *Schematic of the composite electrode, reproduced from Rolf Birkeland.*

In Figure 1, we see a diagram of the composite electrode while it is forming within the furnace. The paste briquettes are inserted at the top of the system, and as they move through the furnace, they liquidise, bake, and harden. The temperature within the furnace varies, with the highest temperature at the bottom tip of the electrode, and the coolest at the top. To control the conduction of electricity through the electrode, as well as to maintain its height during degradation, the composite electrode is repeatedly moved downwards.

2. Electrode Cracking

Although it is difficult to directly observe the composite electrode as it forms due to the extremely high temperatures within the furnace, cracking of the electrode has been observed, see Figure 2. This picture was obtained after raising the electrode out of the charged material.

Figure 2: *Photo of a cracked electrode, after it has been lifted from the charged material.*

Glossary of terms

- Tensile Stress: Stress leading to expansion.
- Young's Modulus: A material property, measuring the stiffness of a solid material.
- Poisson Ratio: A material property, measuring the expansion of a solid material, normal to the direction of compression.
- Hoop Stress: The force exerted circumferentially (perpendicular to both the axis and the radius of the object).

Mathematical model

A one-dimensional elastic model will allow us to calculate the stresses within the electrode as they vary across the radial coordinate.

We develop a model to describe the stresses within the electrode below the baking zone (Figure 1), when the briquettes have melted and re-hardened to form a continuous, solid paste layer. We model the composite electrode as a cylindrical linearly elastic solid, comprised of two different isotropic materials; the graphite core, and the outer paste layer. We also make the following simplifying assumptions:

- Properties of the system are constant in time.
- The electrode is cylindrically symmetric.
- Axial expansion is negligible when compared with radial expansion.
- We assume there are no external forces acting on the system.

Comments

- Our model is formulated as a single differential equation, which we solve in both the graphite core and the outer paste layer, paying particular attention to the different thermal expansions in these regions.
- Inputs to the model are the temperature-dependent thermal expansion of the graphite and the paste, mechanical properties of the system, and a given radial temperature distribution. We use the model to calculate the radially-dependent

displacement, stresses, and strains over a two-dimensional, axisymmetric cross-section of the electrode.

3. Results

Using prescribed temperature distributions, as well as data describing the temperature-dependent thermal expansion and mechanical properties of the graphite core and electrode paste, we solved the simple mathematical model to predict the stresses within the electrode.

Stress Distribution

In Figure 3, we see that the predicted stresses peak at the interface between the graphite core and the electrode paste, as well as at the outer edge of the electrode. A positive hoop stress indicates a tensile stress, which is most significant in predicting crack propagation. The higher the tensile stress in a particular region, the smaller the defects required for crack propagation. Thus, the results in Figure 3 indicate that small cracks present on the surface, or on the interface between the core and the paste, may be most likely to propagate.

Stresses peak at the surface of the electrode and at the interface between the core and the paste.

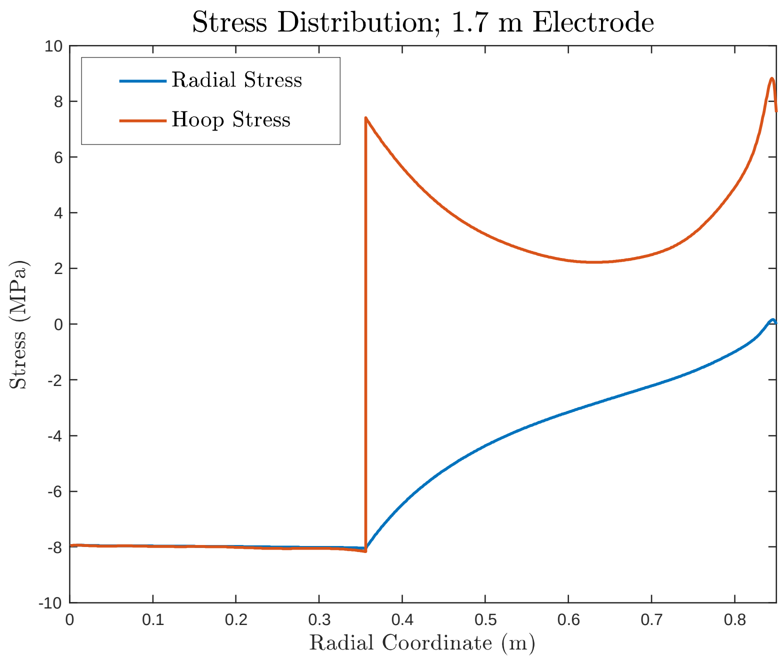

Figure 3: *The predicted stress distribution at the base of the holder. The surface of the electrode is at 0.85 m, while the core has radius 0.356 m.*

The peak in tensile stress on the interface between the core and the paste is due to the differential expansion of these two materials. The build-up of stress near the surface of the electrode is caused by the cooler temperatures here, which is likely to lead to paste shrinkage in a thin surface layer.

Electrode Size

In Figure 4, we show how the maximum hoop stress, calculated using our model, varies with the diameter of the electrode, for a fixed graphite core diameter. We see that the general trend is that stresses increase with the width of the electrode paste. This suggests that a thinner layer of paste on the electrode may be beneficial in mitigating crack propagation.

Stresses increase with the diameter of the electrode (with respect to the diameter of the graphite core).

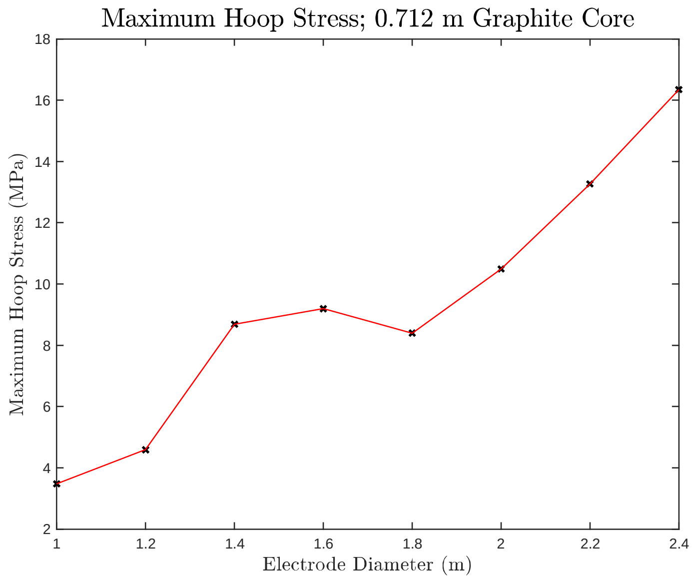

Figure 4: *The relationship between electrode diameter and maximum tensile stress, for a fixed graphite-core diameter.*

However, our model predicts some curious behaviour, due to the shift in global maximum stress between the two maxima seen in Figure 3.

Material Parameters

The stresses within the electrode are dependent upon the material parameters of the electrode paste. In Figure 5, we see the relationship between the maximum tensile stress, and both Young's Modulus and the Poisson Ratio of the paste.

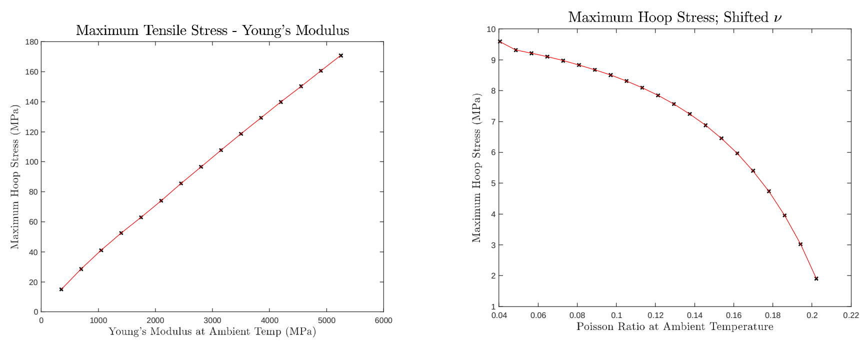

Figure 5: *The relationship between maximum hoop stress, for a fixed graphite-core diameter.*

We see that the hoop stress increases with increasing Young's Modulus; this translates intuitively to larger pressures resulting within stiffer materials. The opposite effect is seen between the Poisson Ratio and the hoop stress.

Electrode Movement

The electrode moves both up and down within the furnace in order to compensate for the degradation of the electrode tip, and to control the passing current. We model this effect on a cross-section of the electrode as an outer layer that has been either heated or cooled after the change in electrode position. The width of this layer, δ, is assumed to be given by a balance between axial movement and radial conduction; thus δ is given by

$$\delta = \sqrt{\frac{\kappa L}{v}},$$

where κ is the thermal diffusivity of the electrode paste, v is the velocity of electrode movement, and L is the vertical distance moved. Incorporating this effect within our model, we see in Figure 6 the resulting stress distribution after lifting a cross-section of the tip of the electrode from the charged material.

Electrode movement leads to a build-up of stress on the surface of the electrode.

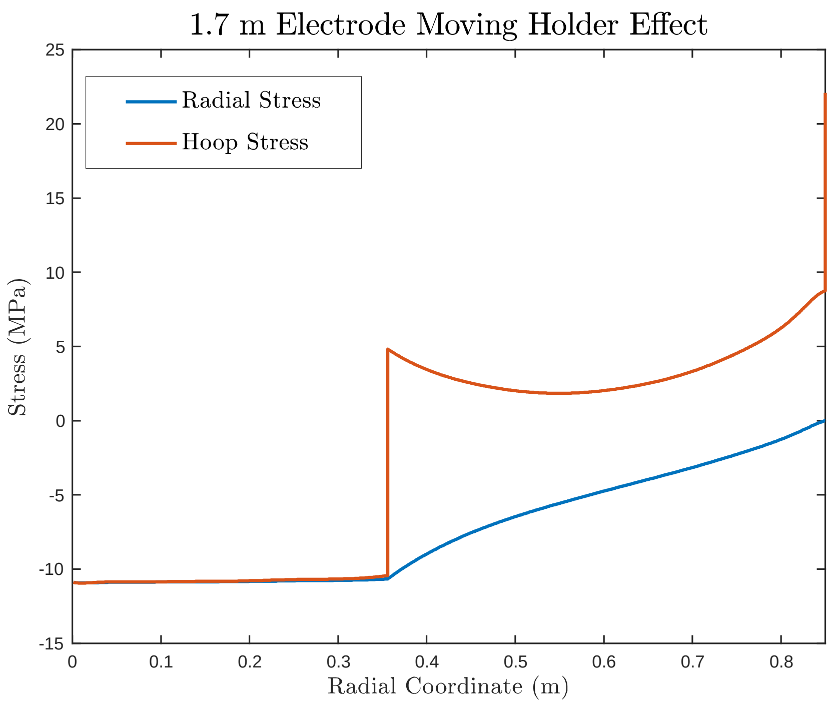

Figure 6: *The stress distribution after moving the tip of the electrode from the charged material.*

We see from the results in Figure 6 that there is a sharp increase in tensile stress in a thin layer on the outer edge of the electrode; this effect may lead to crack-propagation of surface defects.

Expanding the model

- A first improvement to the model would be to replace the constant material properties with those that vary with temperature, where the temperature is a given distribution.

- An additional extension would be to solve for the temperature distribution within the system, by considering a thermal model of the electrode.

4. Conclusions and Recommendations

We have modelled the stresses within the composite electrode, which result from thermal expansion or shrinkage as the electrode forms. We have shown that differing properties within the paste and the graphite lead to heightened tensile stress along the interface between the core and the paste, as well as a build-up of stress at the outside of the electrode. These high stresses could result in propagation of material defects, leading to the observed cracking. We found that the magnitude of this tensile stress increased for larger electrodes (with respect to the size of the graphite core); thus, decreasing this ratio may result in lower forces within the material. Additionally, electrode paste with a high Young's Modulus or low Poisson Ratio resulted in higher stresses. Finally, we considered the effect of electrode movement and found that subjecting a hot section of electrode embedded within the charged material to the cooler ambient gases may result in a heightened peak in tensile stress within an outer layer at the surface of the electrode. To further investigate this effect, it may be useful to obtain a simulated temperature distribution for rapid electrode movement, which could then be analysed using the model presented here.

5. Potential Impact

Rolf Birkeland from Elkem commented on the potential impact of this work: "*The results from the work of Jessica Williams will first and foremost be used by Elkem to improve the electrode operation. Elkem will also use the results in designing the next-generation of composite electrode system and improvement of the electrode paste. The total potential impact on electrode operation and furnace productivity is large. It has been a pleasure to work with Jessica throughout this mini-project.*"