World history of mathematics

Mathematics in Africa

Mathematics has emerged in Africa in many varied forms over the millennia. Different civilisations have employed sophisticated arrangements of weights and measures, elaborate counting systems, and have played mathematical games.

Beyond that of the North African Islamic and ancient Egyptian civilisations, the history of mathematics in Africa has been little-studied, and sources are sketchy. Further research will undoubtedly uncover more of the details of African mathematics that have been hidden for centuries.

Early arithmetic?
The Ishango bone is a tool made from a baboon fibula, discovered in 1960 in what is now the Democratic Republic of the Congo. It dates to between 20,000 and 18,000 BCE. Notches carved into the bone have been interpreted as tally marks, evidence of an arithmetical game, a record of lunar cycles, or simply as grooves to aid grip and therefore entirely non-mathematical.

Ancient Egypt

The ancient Egyptian civilisation arose around 3,000 BCE and lasted in various forms for the next 3,000 years. Comprehensive evidence of Egyptian mathematics is rare, since it would have been recorded on papyrus, which has not survived. However, the very few remaining sources allow us to reconstruct the sophisticated arithmetical techniques that Egyptian scribes used in record-keeping. We also have evidence of geometrical understanding: for example, a standard procedure for calculating the area of a circle.

Alexandria in Egypt was also home, in the fourth century CE, to one of the first female mathematicians about whom we have any substantial information: Hypatia, who wrote commentaries on ancient mathematical texts, and is said to have constructed astronomical instruments.

Weights in West Africa

The small hard seeds of the rosary pea were traditionally used as standard weights by various peoples in West Africa, owing to their consistency in size. However, metal weights became the norm following the introduction of metal-working techniques to present-day Ghana, and the expansion of the trans-Saharan gold trade, in the 14th century CE. The Akan people in particular produced brass weights, for weighing gold, that were both elaborately and accurately crafted.

Mathematical Games

Games of chance and strategy have been played throughout Africa over the centuries. One example is morabaraba, a version of the Roman game of nine men's morris, played in South Africa and Botswana. Another game that is particularly widespread, with numerous variants and a host of different names, is mancala – alleged to be one of the oldest games still to be played anywhere. Indeed, mancala tournaments now take place around the world.

One area of mathematics that has seen particular growth in African institutions in recent years is that of machine learning, driven by the foundation of such organisations as Data Science Africa and Deep Learning Indaba.

Founded in 2003, the African Institute for Mathematical Sciences (AIMS) is a network of academic institutions in Cameroon, Ghana, Rwanda, Senegal, South Africa, and Tanzania. Its goal is to promote postgraduate education, research, and public engagement in the mathematical sciences in Africa.
In the past, the various cultures of the continent, particularly of South America, developed a range of number systems, which were employed most extensively in connection with time-keeping.

The Aztecs
The use of base-20 number systems was widespread across South America, both linguistically and notationally. The Aztecs, for example, employed a non-positional base-20 number system with separate signs for 1, 20, 400, and 8000. These numerals may be found in surviving survey documents, which record areas and perimeters of fields – but we do not know how these were measured or calculated.

The Maya
The Maya culture emerged in Mesoamerica around 2,000 BCE, and elements of it survive to the present day. From around the fifth century BCE, the Maya employed an elaborate calendar, consisting of cycles within cycles. In the calendar, a 260-day count was combined with a year of 365 days; the two came back into step every 52 years, known as a Calendar Round. For dates on monuments, the Long Count was used; this was constructed around much larger units of time, some spanning centuries.

Quipu
Rather than recording information in a written form, some Andean civilisations, including the Incas of ancient Peru, developed a system of record-keeping using knots in bundles of wool and cotton cords called quipu (‘talking knots’). The colours of the cords and the spacing of the knots were crucial to the encoding of information on the quipu. Although much of the content of the surviving quipu remains to be deciphered, we know that one of their major uses was the recording of numerical information.

The Americas in the present day
During the past two centuries, the mathematical focus of the Americas has been on Canada and the United States. US-based mathematicians hold the largest number of Fields Medals; so far (2020) only one South American has won the award: Artur Ávila of Brazil. However, the mathematical communities of other countries are now growing. In 1995, the Unión Matemática de América Latina y el Caribe was created, with the initial involvement of Argentina, Brazil, Chile, Colombia, Cuba, Mexico, Uruguay, Venezuela, and Peru; in 2018, Rio de Janeiro became the first city in the southern hemisphere to host an International Congress of Mathematicians. The study of the recent history of mathematics in the Americas is also changing: it has usually concentrated on white male mathematicians, but the contributions of other, previously-overlooked groups are now being recognised.
Mathematics has a long pedigree in China, both as an accomplishment expected of Confucian literati, and as a subject that was key to training as a civil servant.

The Nine Chapters on the Mathematical Art
One of the most influential of the traditional Chinese mathematical texts was the Jiǔzhāng suànshù (The Nine Chapters on the Mathematical Art), probably dating originally from c. 300 BCE, but in use for over 1,000 years as a manual for the training of administrators. The Jiǔzhāng suànshù consists of 246 arithmetical problems and their methods of solution. Its seventh and eighth chapters, for example, deal with the solution of systems of up to five simultaneous equations in five unknowns. The method used, fāngchéng (方程), is equivalent to the process known in the West as Gaussian elimination, and may have been influenced by the use of counting boards in China.

Wang Zhenyi
Although mathematical learning in China was confined largely to men, some women did also engage in scientific studies. Most notable was Wang Zhenyi (王貞儀, 1768–97), who wrote treatises on both astronomy (the procession of the equinoxes) and mathematics (trigonometry).

Because traditional Chinese mathematics focused on methods rather than proofs, it has often been compared unfavourably with the ancient Greek mathematics that has traditionally provided the benchmark for how mathematics ‘should’ be done. Nowadays, however, historians of mathematics strive to treat ancient Chinese mathematics, and other non-Western mathematical traditions, purely on their own terms.

China occupies a prominent position in international mathematics; in August 2002, Beijing became only the second Asian city to host the International Congress of Mathematicians.
India has a mathematical tradition that goes back many thousands of years.

The Indus Valley Civilisation, which emerged around 3,000 BCE, had a centralised system of weights and measures, along with measuring instruments. Although the Indus script has not yet been deciphered, India nevertheless boasts mathematical texts whose originals date from 800–500 BCE (but are now known only from later copies) in the form of the Sulbasutras, which concern geometry and the construction of sacrificial altars.

Classical Period

The so-called ‘classical period’ of Indian mathematics began in the middle of the first millennium CE with the work of Aryabhata I (476–550 CE). His Sanskrit treatise Aryabhatiya (c. 510 CE) deals mostly with plane and spherical trigonometry, but also covers the solution of various types of equations, and contains an approximation of π as 3.1416. Other figures from the classical period are Brahmagupta (c. 598–668 CE), who was the first scholar to set out rules for performing arithmetic with negative numbers and zero, and Bhaskara I (c. 600–680 CE), author of three astronomical treatises.

Kerala School

During the 14th–16th centuries CE, a school of mathematics and astronomy flourished in Kerala in southern India. Often motivated by astronomical problems, the school produced treatises on arithmetic, algebra, geometry, approximation of roots of equations, and magic squares, and also developed power series expansions for trigonometric functions, two centuries before these same ideas appeared in Europe.

Numerals

Indian scholars employed a decimal positional number system from early in the first millennium CE. Initially, empty columns were indicated with a word meaning ‘emptiness’, but eventually a dot, and then a circle, came to be used. This system of numerals was adopted by Middle Eastern scholars in the ninth century, and was transmitted to Europe in the following centuries – both by Italian merchants and via Islamic Iberia – to become the number system that we use today, often termed ‘Hindu-Arabic numerals’.

Ramanujan

India’s most famous mathematician of the twentieth century was Srinivasa Ramanujan (1887–1920). Born in present-day Tamil Nadu, Ramanujan was largely self-taught in mathematics.

In 1913, he began a correspondence with G. H. Hardy in Cambridge, which led to his being invited there the following year. Ramanujan remained in Cambridge until shortly before his early death in 1920, aged just 32; in 1918, he was elected both a Fellow of the Royal Society and of Trinity College, Cambridge.

Indian Women and Mathematics

Like many countries, India has an association devoted to promoting the involvement of women in mathematics: Indian Women and Mathematics, founded in 2009. It holds annual conferences and regional workshops throughout India.
After Japan opened up to the West in the mid-nineteenth century, its government decreed that Western mathematical techniques (洋算 yōsan) should be adopted. The exception to this was that children should still be taught to use the Japanese abacus, the soroban 算盤.

In earlier centuries, Japanese mathematics had been heavily influenced by texts from China, and European missionaries had taught Western mathematics in Japan in the early seventeenth century, but during Japan’s period of isolation from the rest of the world, from the mid-seventeenth century onwards, a Japanese form of mathematics had flourished: wasan 和算.

In the 17th century there arose in Japan a tradition of posing ever harder and more intricate, and often rather artificial, mathematical problems, often taken from Chinese textbooks, whose goal was to challenge the reader rather than to develop a comprehensive understanding of the underlying mathematics. It was out of such account by day) carried out arithmetical experimentations to establish patterns and results.

Using such an approach, Seki was one of the first mathematicians to discover the Bernoulli numbers, and to develop the idea of a determinant. He is also credited with the foundation of the branch of mathematics known as enri 圓理, or circle principles, which sought general methods for calculating areas, volumes, and lengths of curves, and therefore paralleled the development of calculus in Europe.

During the early 20th century, many Japanese students travelled to Europe – Germany, in particular – to study mathematics; in the second half of the century, however, the focus shifted to the USA and to Japanese institutions. In 1990, Kyoto became the first Asian city to host an International Congress of Mathematics.
Some of the world's earliest mathematics was developed in the Middle East by the Sumerian and Babylonian cultures after c. 3,000 BCE, much of which was eventually transmitted to ancient Greece.

A later Middle Eastern mathematical and astronomical school flourished under Islamic influence, particularly during the 9th and 10th centuries CE. The pursuit of knowledge was seen as having religious merit, and certain verses in the Quran were interpreted as encouraging scientific enquiry.

**Babylonian mathematics**
The scribes of ancient Mesopotamia employed a base-60 positional numerical notation, many examples of which appear on surviving clay tablets. Using this notation, the scribes were able to solve a range of arithmetical and geometrical problems, as well as to develop a sophisticated astronomy – their use of base-60 is preserved in our division of an hour into 60 minutes, a minute into 60 seconds, and a circle into 360 degrees.

In his *Compendious Book on Calculation by Completion and Balancing* (كتاب المختصر في حساب الجبر والمقابلة (c. 820 CE)), al-Khwārizmī set out methods for the systematic solution of quadratic equations, which for him fell into six distinct cases because he did not use negative numbers or zero. The ‘al-jabr’ ('balancing') of the text's Arabic title was taken over into European languages as our word 'algebra'. Al-Khwārizmī’s rules for solving equations were presented as step-by-step procedures; a Latinised version of his name gives us the word 'algorithm'.

Two hundred years later, al-Khwārizmī’s methods for solving quadratic equations were extended to the case of cubics by Omar Khayyam (1048–1131), who lived in modern-day Iran. Famous in the West as a poet, Khayyam also studied astronomy and mathematics. As an astronomer, he devised a new solar calendar, while his contributions to mathematics covered both geometry and algebra. In particular, he provided solutions of cubic equations, on a case-by-case basis, as intersections of conic sections.

Maryam Mirzakhani (1977–2017) was born in Iran, but subsequently moved to the United States. In 2014, she became the first woman to win a Fields Medal for "her outstanding contributions to the dynamics and geometry of Riemann surfaces and their moduli spaces."

As well as producing new ideas, Mediaeval Islamic scholars also helped to preserve the knowledge of the ancient world, via their systematic translation of writings whose originals are now lost. Certain texts, such as parts of Diophantus’ *Arithmetica*, a Greek compendium of algebraic problems from the third century CE, are known only through surviving Arabic translations.

Maryam Mirzakhani (1977–2017), the first woman to win a Fields Medal.