EPSRC Centre for Doctoral Training in Industrially Focused Mathematical Modelling

Distribution of signal-to-interference in cellular wireless networks

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1. Introduction

Background

A new way of connecting mobile devices to cellular networks are femtocells (see Figure 1), which are small, low-power cellular base stations, typically distributed in homes and businesses. If deployed, network operators such as BT are likely to incorporate them in WiFi access points. The femtocells will provide additional cellular coverage to users, especially indoors, by filling gaps of coverage areas of the main network and eliminating loss of signal inside buildings. Furthermore, the capacity of the main network is improved as traffic is off-loaded through the user's network (internet) via the femtocells, so fewer users need to connect to the main network, which is done traditionally via cellular towers (called macrocells). Network operators in the UK are considering the deployment of home femtocells, and are therefore investigating the potential performance of such systems.

SIR is a key quantity in cellular networks, as it determines the datarate that a user can receive.

In this report, we consider the problem of finding the distribution of the *signal-to-interference ratio* (SIR) in a femtocell network. The SIR of a particular *user equipment* (UE), such as a mobile phone, is the ratio between the received signal power from the serving cellular base station and the total interference power from all other base stations which transmit on the same frequency band. SIR is a key performance indicator in cellular wireless networks, as it determines, using Shannon's law, the maximal achievable datarate that a user can receive. Our aim is to find the distribution via the complementary cumulative distribution function (ccdf) of SIR in wireless networks which gives us the proportion of users that can achieve a given input SIR on a single frequency band.

Figure 1: Model of a BT Smart Hub. Femtocells, if deployed, will be similar devices.

Simplified mathematical models are useful to quickly test out different transmission scenarios without having to rely on slow, complex simulations.

In this project, we use simplified mathematical models from the literature for general cellular wireless networks and apply them to the femtocell case. When tractable analytical solutions to the simplified mathematical problem exist, they provide quick approximations to realistic scenarios, making it possible to test out different scenarios without having to rely on slow and expensive commercial software to perform complex simulations or conduct field tests which are both expensive and prone to inaccuracy due to noise. In our work, we test how well the mathematical model agrees with more realistic data as well as data from commercial software, and apply the mathematical model to a system with inter-cell interference coordination (ICIC).

2. Mathematical model

The mathematical model for the cellular network that we use consists of a random model for the spatial locations of the femtocells (cells) coupled with a standard power loss propagation model with random channel effects.

Our mathematical model consists of a random model for cell locations and a standard propagation model for the signal.

We assume that the locations of base stations are randomly distributed and we use a homogeneous Poisson Point Process (PPP) to provide our spatial model (see Figure 2 for one realisation of a PPP). Since the exact locations of femtocells will often be decided by the user and not by the network provider, it is reasonable to assume the positions of the femtocells are random. However, the PPP model has two features that likely do not occur in practice: First, there is no minimum distance between points in the PPP model, resulting in clusters of points which typically would not occur in a femtocell deployment. Second, we assume that the density of points in the PPP model is constant in space, which is likely not going to be true in reality due to parks or lakes which cannot contain any femtocells. Despite these two shortcomings, the PPP model is widely used in the wireless communication literature due to its mathematical tractability. Finally, we assume in our model that UEs are distributed uniformly at random, and that each UE connects to the nearest cell.

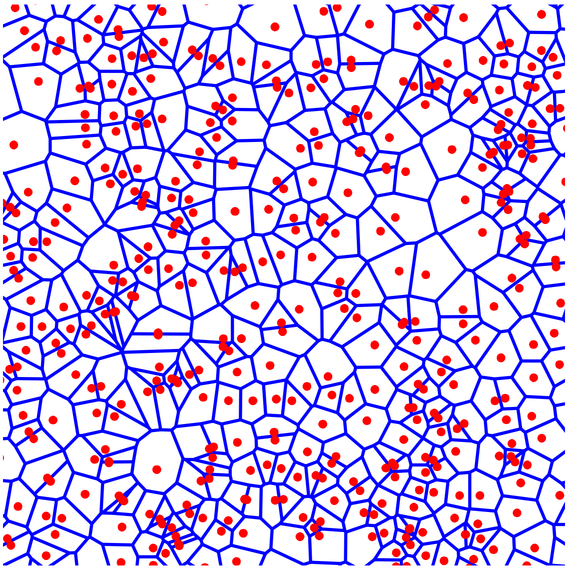

Figure 2: One realisation of a PPP (red dots) with Voronoi cell boundaries (blue curves). The Voronoi cell of a point is the region in which the given point is closer than any other point. Hence all UEs in a Voronoi cell connect to the associated PPP point (red dot).

Our baseline model is the PPP model for cell locations with signals experiencing pathloss and Rayleigh fading.

In a wireless network, attenuation of the radiated signals from cells to the UE can be caused by propagation effects. The main three propagation effects are (i) *pathloss* (signals lose power with distance), (ii) *multipath fading* (reception of multiple copies of the same signal), and (iii) *shadowing* (blocking of the signal due to large obstacles such as buidling). In our propagation model, we include includes pathloss and Rayleigh fading (a specific model of multipath fading), but we omit shadowing for the sake of mathematical tractability. Furthermore, we assume that all cells transmit at the same power.

This mathematical model has been widely studied in the literature. Andrews et al. [1] derived a tractable analytical formula for the ccdf of SIR for the PPP model with pathloss and Rayleigh fading (the baseline model described above). In this report, we will compare this analytical formula with results from simple simulations of more realistic scenarios and data from complex simulations using commercial software. It has been observed in the literature that the ccdf of SIR in other spatial or propagation models is merely a horizontally shifted version of the SIR curve from the PPP model. Ganti and Haenggi [2] derived analytical expressions for estimating this shift (or gap) to the ccdf of the SIR from the baseline model. They derived two approximations, one based on the gap on the left asymptotic end (for low SIR values) and the other one based right asymptotic end (for high SIR values). While the computation of the gap estimation approximations involves simulations as well, it gives an approximation of the entire shape of the ccdf of SIR, including the probability of rare events, whereas direct simulations exhibit much larger errors for estimating the probability of rare events. We use both approximations in our work and call the approximation 'gap estimation'.

Glossary of terms

- UE: User equipment (e.g. mobile phone).
- Cell: Cellular base station (femtocells in our case).
- SIR: Signal-to-interference ratio $= \frac{\text{received signal power}}{\text{total interference power}}$.
- ccdf: Complementary cumulative distribution function. The ccdf of SIR gives the proportion of UEs that can achieve a given SIR, and hence a datarate.
- PPP model: Model for locations of cell location, used because of its mathematical tractability.
- Baseline model: Our baseline model is a PPP model for cell location with pathloss and Rayleigh fading.

Comments

- Both the spatial and the propagation model use many assumptions for the sake of mathematical tractability and these assumptions may not be true in real-world scenarios. However, mathematical models that capture the important qualitative features can give good approximation of the real system.

- The propagation model is widely used in the literature and adapted versions of it are often used in commercial software.

3. Comparison of PPP model with using house locations for femtocell locations

The SIR distribution in urban areas agrees well with what the PPP model predicts. Suburban and rural areas show some discrepancy, but are well approximated using gap estimation.

We aim to validate the predictions of the ccdf of SIR of the PPP model with Rayleigh fading (using the analytical results from [1]) by changing the spatial model and instead using a more realistic dataset for femtocell locations. Note that we do not alter the propagation model. As discussed above, PPP points may not be a realistic representation of femtocell locations, due to the clustering of points occurring in a PPP. Hence we use house locations data from Ordnance Survey OpenData (OS Open Map Local) and assume that there is one femtocell located in each house (100% penetration case). We ran simulations to compute the ccdf of SIR in urban, suburban, and rural areas using the standard propagation model described in Section 2 with pathloss and Rayleigh fading and with UEs distributed randomly. We compare the results with the analytical formula for the ccdf of SIR of the PPP model and also apply the two approximation using gap estimation (see Figure 3). In our simulations, we observed that the ccdf of SIR in urban areas agrees very well with the PPP model (e.g. see Figure 3a). On the other hand, the ccdf of SIR (blue dotted curve in Figure 3b) in suburban and rural areas show significant discrepancies to the ccdf of SIR (red curve in Figure 3b) of PPP model, displaying worse performance than the PPP model would have predicted. However, gap estimation (in particular, the approximation using the left asymptotic gap (see Figure 3b)) is often able to predict the discrepancy fairly well. In our simulations, we found the gap estimation to fail only in areas which contain both highly dense and very sparse areas. In the femtocell application, the urban case is the most relevant, as rural and suburban users would mainly connect to macrocells and it appears that the PPP model is a good approximation for urban areas.

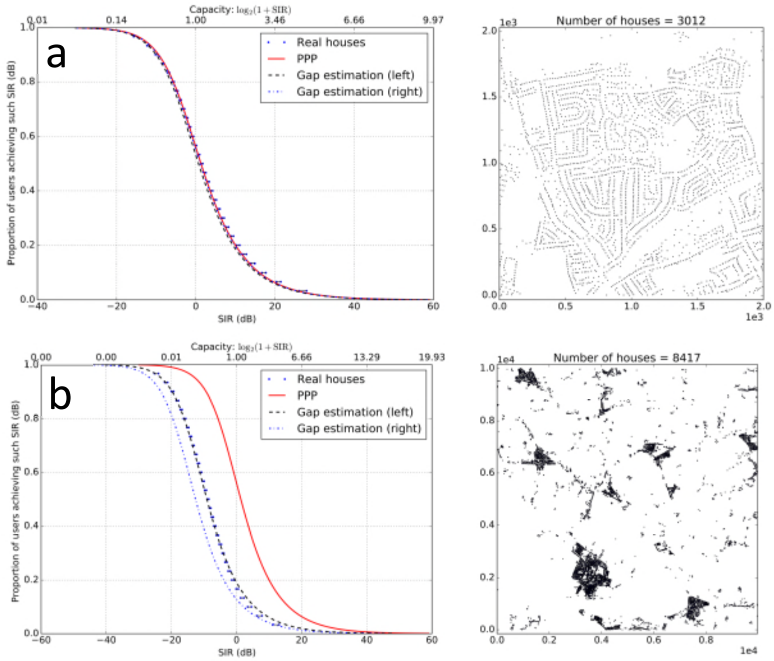

Figure 3: Comparison of the ccdf of SIR from the PPP model (red solid) and from the model using real house locations (blue dots) (a) in an urban area (2km by 2km) and (b) in a suburban/rural area (10km by 10km). The black dashed and blue dot dashed curves show the approximated SIR curve using gap estimation. Results from urban areas agree very well with the analytical formula for the PPP model. In suburban/rural cases, there is a significant discrepancy which is approximated well by gap estimation. The further toward the right the curve is, the better the performance.

4. Comparison with commercial software

The ccdf of SIR from the PPP model agrees well with SIR data from an urban area from simulations produced by commercial software.

So far, we have been using simplified signal propagation models, which may cause inaccuracies. To validate the results with even more realistic data, we obtained data for the SIR distribution of an urban region in London produced using Atoll, a complex commercial software produced by the company Forsk. The software takes into account shadowing due to elevation and buildings, and uses more sophisticated propagation models in order to find the SIR distribution. We observe that the data from the Atoll simulations agrees well with both the PPP model and the model using actual house locations (see Figure 4), with the latter being closer to the Atoll data. There is some discrepancy between the two methods on the low SIR end (left end), which is likely due to Atoll automatically setting SIR values to -6.5 dB, if they fall below this value. This strong agreement suggests that the PPP model is a valid approximation for finding the SIR distribution in urban areas.

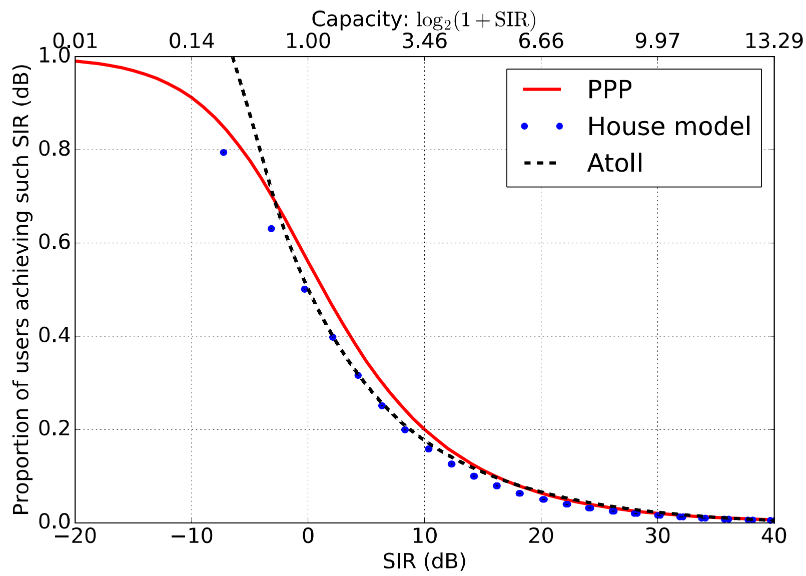

Figure 4: Comparison of the PPP model (PPP, red solid) and model with real house locations (house model, blue dots) with data from commercial software (Atoll, black dashed).

We derived an analytical formula for one implementation of ICIC, in which the interference from nearest interferers is reduced.
This formula can be used predict performance of ICIC for different power levels of the nearest interferers.

5. Inter-cell interference coordination

In Figure 4, we see that, without any additional mechanism to improve the performance of the system, the SIR for more than 40% of UEs is below 0 dB, a regime in which connectivity to the network is typically impossible. For this reason, it is critical for network providers to improve the SIR and with that the performance of the system. One such way to improve the SIR is to use inter-cell interference coordination (ICIC). Under ICIC, cells close together use different frequency spectra or time slots, reducing the interference to UEs that receive signal from either of them. In practice, this reduces the interference from the strongest interferers for UEs that would otherwise achieve very low SIR.

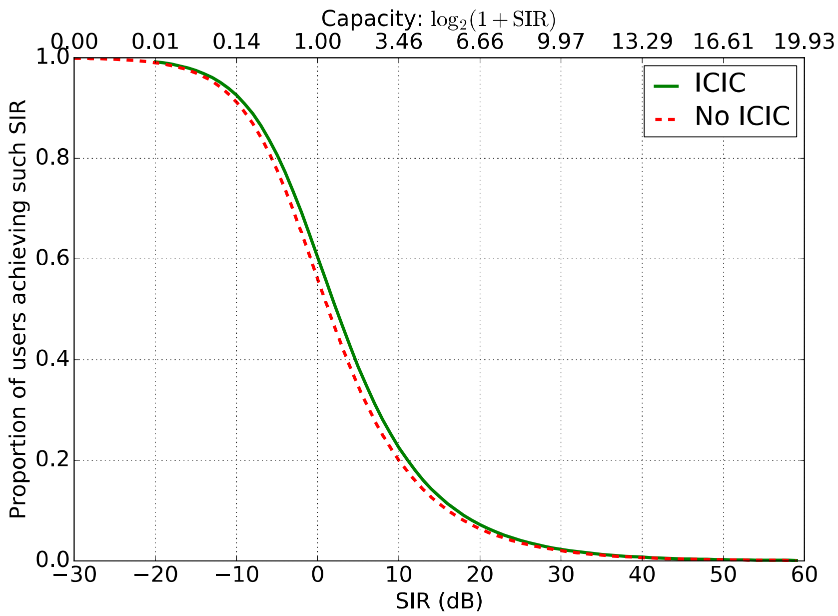

Figure 5: Performance of 2-ICIC compared to no-ICIC, when P_1 = 0.5. We see that the average gain (horizontal shift) for UEs is only about 2 dB. This indicates that 2-ICIC is not likely to improve the performance much.

In the previous sections, we have shown that the PPP model with Rayleigh fading can be used for predicting the SIR distribution of urban areas. We test the performance of ICIC by deriving analytical solutions to the ICIC case in a PPP model with pathloss and Rayleigh fading. We consider the case in which the nearest interfering cell transmits at reduced power P_1, which represent the best-case scenario for a 2-cell coordination (2-ICIC). In Figure 5, we see that the average gain is about 2 dB (as the curve is shifted to the right by 2 dB) in 2-ICIC when the nearest cell transmits at half power and we also compare our analytical solution with the case without ICIC. The formula can be extended to the case when the next nearest interferers are also at reduced powers, and is useful to test out different scenarios of ICIC.

6. Conclusion and recommendations

PPP models are good approximations of real femtocell systems. We can use PPP models to test out different system set-ups such as ICIC.

We have shown that the SIR distribution from PPP models agrees very well with more realistic simulations obtained from both commercial software and by using actual house location data for urban areas. For suburban and rural areas, there is a significant discrepancy between the SIR distribution from the PPP model and these other approaches. However, the gap (or shift) from the PPP model can often be approximated using results derived in [2]. Urban areas are the main application area for femtocells, so our results indicate that PPP models can be used to predict the SIR distribution for femtocell deployment. Furthermore, as an example of how PPP models can be applied, we derived analytical formulas for ICIC in a PPP model, which can be used to predict the performance of a wireless system with ICIC.

Therefore, we would recommend using PPP models to predict the performance of a femtocell deployment in urban areas under different scenarios such as ICIC or other transmission coordination schemes. In a PPP model framework, analytical formulas for the SIR distribution can be found for many different scenarios, such as ICIC.

7. Potential Impact

We have found that PPP models can be used as first-order approximations of a femtocell system in urban areas and hence are useful for finding the potential performance of a femtocell system without needing to rely on complex simulations. The analytical formula for ICIC that we derived is useful to find the performance gain using a given ICIC set-up and extends the existing literature on ICIC.

Dr. Keith Briggs, research mathematician at BT Technology, Strategy and Operations, commented: "*The wireless research team at BT is very interested in ways of predicting the performance of small-cell deployments, and the methods Fabian has developed and tested demonstrate that in many cases simple mathematical models will be sufficiently accurate for our purposes. Furthermore, the ICIC work has potential to be considerably extended.*"

References

1. J. G. Andrews et al (2011) *A Tractable Approach to Coverage and Rate in Cellular Networks* IEEE Transactions on Communications. **59**, no. 11. pp. 3122—3134

2. R. K. Ganti and M. Haenggi (2015) *SIR asymptotics in general cellular network models*, IEEE Transactions on Communications **64** no. 3, pp. 1009—1013