Suggested title of dissertation:

Mathematical modelling of tumour angiogenesis

Dissertation supervisor:

Prof Helen Byrne and Prof Philip Maini

Description of the proposal:

Angiogenesis is the process by which the body regenerates new blood vessels after injury. This naturally occurring process is hijacked by cancer cells, which use it to enhance nutrient delivery to the growing tumour and promote tumour invasion and spread. Many cancer therapies, such as chemotherapy and radiotherapy, are enhanced in well-oxygenated tissues, so it is important to understand how angiogenesis contributes to the spatio-temporal dynamics of oxygen delivery. This multiscale process has been widely modelled but there are still outstanding questions to be answered. Most models are either of coupled partial differential equation type, or hybrid cellular automata - where cells are modelled discretely, coupled to signalling cues modelled by continuum reaction-diffusion type equations.

Possible avenues of investigation:

Model how one goes from a discrete to a continuum description of the so-called "snail-trail model of angiogenesis and extend models from the existing literature to include uptake of signalling molecule.

Pre-requisite knowledge:

B5.5 Further Mathematical Biology (essential) (`https://courses.maths.ox.ac.uk/node/36417`)
B5.2 Applied Partial Differential Equations (essential) (`https://courses.maths.ox.ac.uk/node/36395`)
B6.1 Numerical Solution of Differential Equations I (useful) (`https://courses.maths.ox.ac.uk/node/36448`)
B5.1 Stochastic Modelling of Biological Processes (useful) (`https://courses.maths.ox.ac.uk/node/36382`)

B6.2 Numerical Solution of Differential Equations II (useful) (`https://courses.maths.ox.ac.uk/node/36456`)

Useful reading:

J.D. Murray, Mathematical Biology, II: Spatial Models and Biomedical Applications, Springer, Chapters 1, 5 (2003)

D. Balding, S. McElwain, A mathematical model of tumour-induced capillary growth, J. Theor. Biol., 114, 5373 (1985)

HM Byrne, MAJ Chaplain, Explicit solutions of a simplified model of capillary sprout growth during tumor angiogenesis, Applied Maths Letters, 9, 6974 (1996)

S. Pillay, H.M. Byrne, P.K. Maini, Modeling angiogenesis: a discrete to continuum description, Phys. Rev. E., 95, 012410 (12 pages) (2017)

Further references:

G Vilanova, I Colominas and H Gomez (2013). Capillary networks in tumour angiogenesis: from discrete endothelial cells to phase-field averaged descriptions via isogeometric analysis. Num Methods Biomed Eng 29(10): 1015:1037.