

Complete the Square

1. Here are some 4-digit square numbers, but I've hidden the middle two digits. In each case, find all possible 4-digit square numbers that the number could be.

8 1

4 9

6 1

2. How many four-digit square numbers end in 0?
How many four-digit square numbers end in 1?
How many four-digit square numbers end in 2?
How many four-digit square numbers end in 3?
How many four-digit square numbers end in 4?
How many four-digit square numbers end in 5?
How many four-digit square numbers end in 6?
How many four-digit square numbers end in 7?
How many four-digit square numbers end in 8?
How many four-digit square numbers end in 9?

3. Here's a six-digit square number.

Oops, sorry, I've hidden all the digits, so you can't tell which square number it is. If I revealed the second digit, the fourth digit, and the sixth digit, would you definitely be able to tell what the square number is? Or are there two different square numbers with the same second digit, the same fourth digit, and the same sixth digit?

[Hint: you don't need to list all the six-digit square numbers to prove this. How many possibilities are there for what you'll see when I reveal the second, fourth, and sixth digits?]

4. What if, instead, I revealed the first digit, the third digit, and the fifth digit? Would knowing those three digits always let you identify the square number?

Extension: $2n$ digits, and I'll reveal every other digit starting with the first. Does that always let you identify the square?