Examiners' Report: Final Honour School of Mathematics Part B Trinity Term 2014

October 28, 2014

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

  See Table 1.

	Numbers					Percentages %				
	2014	(2013)	(2012)	(2011)	(2010)	2014	(2013)	(2012)	(2011)	(2010)
I	49	(54)	(57)	(54)	(55)	31.01	(34.34)	(34.34)	(36.24)	(35.71)
II.1	78	(78)	(79)	(67)	(61)	49.37	(49.68)	(47.59)	(44.97)	(39.61)
II.2	21	(21)	(21)	(19)	(28)	13.29	(13.38)	(12.65)	(12.75)	(18.18)
III	9	(2)	(5)	(7)	(9)	5.7	(1.27)	(3.01)	(4.70)	(5.84)
P	1	(2)	(3)	(2)	(0)	0.63	(1.27)	(1.81)	(1.34)	(0)
F	0	(0)	(0)	(0)	(1)	0	(0)	(0)	(0)	(0.65)
Honours (unclassified)	0	(0)	(1)	(0)	(0)	0	(0)	(0.6)	(0)	(0)
Total	158	(157)	(166)	(149)	(154)	100	(100)	(100)	(100)	(100)

Table 1: Numbers and percentages in each class

- **Numbers of vivas and effects of vivas on classes of result.**

  As in previous years there were no vivas conducted for the FHS of Mathematics Part B.

- **Marking of scripts.**

  The following were double marked: whole unit BE Extended Essays, BSP projects, and coursework submitted for the History of Mathematics course, and the Undergraduate Ambassadors Scheme.

The remaining scripts were all single marked according to a pre-agreed marking scheme which was strictly adhered to. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**

  See Table 5 on page 18.

B. New examining methods and procedures

There was a small procedural change relating to BSP projects.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

There have been syllabus changes and structural changes in the first two years for the students who will be taking Part B next year. This will not cause substantial changes in the methods and procedures in Part B, but there will be changes to the syllabus of some courses, and to fine details of the examining procedures. We anticipate that there will be some need to set some papers for the old syllabus and this will require the setting of some extra questions.

In May we received guidance from the MPLS Division on certain matters arising from the 2013 examinations. The Mathematical Institute's procedures already met most of the guidance, but there are two minor exceptions. One of these concerns principles by which doubly-marked assignments are reconciled, and we discuss that in the context of BE Extended Essays and BSP projects in Section II.A. The other point relates to the handling of extraneous material in scripts (i.e., material which is crossed out or marked as rough work). The Division asks each department to make their policy clear and recommends that the policy should be uniform for Prelims and Finals. The Mathematical Institute's guidance to candidates says that they should cross out anything which they do not want to be marked, but the guidance to markers is that they must leave some trace that each page has been marked. This is usually interpreted as putting some sign on each page that has anything written on it, even if it is crossed out or marked as rough work. It should also be noted that the number of answers that will be marked is limited in Prelims but not in Finals.

To remove uncertainty and to comply with the Division's position, we **RECOMMEND** that the Mathematical Institute should clarify its policy concerning extraneous material for the benefit of candidates, assessors and examiners.

D. Notice of examination conventions for candidates

The first Notice to Candidates was issued on 21 February 2014 and the second notice on 5 May 2014. These notices can be found at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions, and contain details of the examinations and assessments.

The Examination Conventions for 2014 examinations are on-line at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. General Comments on the Examination

The examiners would like to convey their grateful thanks for their help and cooperation to all those who assisted with this year's examination, either as assessors or in an administrative capacity. However we, and the Chairman in particular, do wish to single out for special mention Helen Lowe for providing excellent administrative support throughout and Charlotte Turner-Smith for her help and support whenever this was needed. We are extremely grateful to Waldemar Schlackow for the excellent work he has done in maintaining and running the database, assisting the examiners in the operation of the scaling algorithm, and in generating output data as requested by the examiners. We are also grateful to Nia Roderick and Jessica Sheard for assistance during the logging in and checking of scripts.

In addition the internal examiners would like to express their gratitude to Professor Lister and Professor Thomas for carrying out their duties as external examiners in a constructive and supportive way during the year, and for their valuable input at the final examiners' meeting.

Standard of performance

The proportion of Mathematics candidates awarded a First was lower than usual and lower than the MPLS target. Indeed, we expect that it will be lower than the University average (not yet available) for the first time in many decades (excluding the Part B classifications between 1997 and 2007 when only those candidates not proceeding to Part C were classified in Part B). If Mathematics and Mathematics & Statistics are aggregated (as they are for a large part of the classification process), the proportions become somewhat higher but still a little lower than recent years.

In reaching this outcome, we took note of

- the Examiners' Report on the 2013 Part B examination, and in particular recommendations made by last year's examiners, and the Examiners' Report on the 2013 Part A examination, in which the 2014 Part B cohort were awarded their USMs for Part A;
- a document issued by the Mathematics Teaching Committee giving broad guidelines on the proportion of candidates that might be expected in each class, based on the class percentages over the last five years in Mathematics Part B, Mathematics & Statistics Part B, and across the MPLS Division.

The 2013 Part A Examiners noted that unusually few of their candidates received average marks of 70 or more, and unusually many had average marks below 60, reflecting their view of the field of candidates. We were conscious that this potentially has a double effect on Part B classifications, firstly because Part A carries 40% of the weight, and secondly because the scaling algorithm proposes a very similar distribution of marks to the Part A marks unless the Part B examiners act to change that. When we first ran the scaling algorithm we found that the proportion of Firsts remained almost exactly the same as in Part A, but the proportion of Lower Seconds and below fell to a level which was in line with recent years. The overall effect of our adjustments to the scaling functions was more or less neutral, and our reading of borderline scripts led us to conclude that the proportions were appropriate for this field.

There was a small group of candidates who performed very poorly in Part B, and descended into the Third Class. Consequently the size of this class increased compared with Part A 2013.

Setting and checking of papers and marks processing

Requests to course lecturers to act as assessors, and to act as checkers of the questions of fellow lecturers, were sent out early in Michaelmas Term, with instructions and guidance on the setting and checking process, including a web link to the Examination Conventions. The questions were initially set by the course lecturer, in almost all cases with the lecturer of another course involved as checkers before the first drafts of the questions were presented to the examiners. Most assessors acted properly, but a few failed to meet the stipulated deadlines (mainly for Michaelmas term courses) and/or to follow carefully the instructions provided.

The internal examiners met at the beginning of Hilary Term to consider those draft papers on Michaelmas Term courses which had been submitted in time; consideration of the remaining papers had to be deferred. Where necessary, corrections and any proposed changes were agreed with the setters. The revised draft papers were then sent to the external examiners. Feedback from external examiners was given to examiners and to the relevant assessor for response. The internal examiners at their meeting in mid Hilary Term considered the external examiners' comments and the assessor responses, making further changes as necessary before finalising the questions. The process was repeated for the Hilary Term courses, but necessarily with a much tighter schedule.

Camera ready copy of each paper was signed off by the assessor, and then submitted to the Examination Schools.

Except by special arrangement, examination scripts were delivered to the Mathematical Institute by the Examination Schools, and markers collected their scripts from the Mathematical Institute. Marking, marks processing and checking were carried out according to well-established procedures. Assessors had a short time period to return the marks on standardised mark sheets. A check-sum is also carried out to ensure that marks entered into the database are correctly read and transposed from the mark sheets.

All scripts and completed mark sheets were returned, if not by the agreed due dates, then at least in time for the script-checking process. A team of graduate checkers under the supervision of Helen Lowe sorted all the scripts for each paper for which the Mathematics Part B examiners have sole responsibility, carefully cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way, errors were corrected with each change independently verified and signed off by one of the examiners, who were present throughout the process. A small number of errors were found, but they were mostly very minor and there were very few queries which had to be referred to the marker for resolution.

Standard and style of papers

As in past years several papers turned out to be too easy. There were two extremely severe cases (where the algorithm proposed mapping 48 or 49 to 70), one severe case (45 to 70), and two moderately severe cases (40 or 43 to 70). The (extremely) severe cases are very difficult to handle: one has to compromise between (extremely) severe downward scaling at the top and a sharp corner on the one hand, and widespread high scaled marks on the other hand. Since easy papers usually attract some marks of 50, and many marks of 45 or more, it is impossible to avoid this problem.

More unusually, we had some papers which turned out to be too difficult. There were two extremely severe cases, and two moderate cases. However these papers were much less difficult to scale fairly. The reason is that even in the extremely severe cases there were hardly any candidates with less than 10 marks (out of 50), and the severe upward scaling can be confined to that region, so the steep upward scaling and sharp corner may affect no, or very few, candidates.

In addition we again felt that some papers had excessively lengthy questions, and/or excessive amounts of standard material, with little testing of problem-solving skills. This was discussed in detail in the report of the 2013 Examiners. We had slightly adjusted the guidance sent out to assessors about setting questions and mark-schemes, in particular to discourage lengthy bookwork, repetition of questions from problem sheets, and non-linear mark-schemes, and to encourage more unseen problems, including easy problems. The Teaching Committee had approved such a shift in October 2013, but we were constrained in implementing it because candidates' expectations had been built up in Mods and Part A, and recent Part B papers, and because some guidance had already been made available to candidates in the form of appendices to the Examination Conventions. The observed effect was small, partly for these reasons, but also because some assessors appeared not to read our guidance even where it was highlighted as being different from the last few years.

We shall pass to next year's Examiners lists of the papers in the three categories above and we **RECOMMEND** that they meet the setters and checkers of each of those papers to discuss how to set more suitable papers.

Papers shared with Part C

Following several anomalies in the 2013 examination, the 2013 Part B examiners recommended the termination of the sharing of examination papers between Parts B and C. It was too late to introduce this in the 2014 examination, but only one paper was shared this time. The process for that paper was made more rigorous, and the scaling function chosen by the Part C Examiners worked fairly for the Part B candidates.

We had understood that the new process was only an interim measure by the Teaching Committee, in the expectation that papers would not be shared in future. We were alarmed to find that the schedule of lectures for 2014/15 shows that there are to be 4 shared courses, and (as we understand it) each will share an examination paper with Part C. Apart from the risk of examining mishaps, this will increase the difficulty of scheduling the examinations (see the Section on Timetable).

We **RECOMMEND** again the termination of sharing of examination papers between Parts B and C. We stress that this does not necessarily imply that the lecture courses should not be shared, nor that the examination papers must be disjoint (although overlap will hamper scheduling).

We remark here that two of the current Part B courses (Energy Minimisation, and Stochastic Calculus) appear to be at, or very close to, the level of Part C courses; and they would probably benefit from being so classified. If the Teaching Committee really believes that shared courses are more beneficial than problematic, we suggest that the Committee should consider giving these courses dual status.

Timetable

Examinations began on Monday 2 June and finished on Friday 20 June.

The Examination Schools originally proposed a timetable in which three papers would have been taken on Saturday 21 June (Week 8), including a very popular paper that afternoon. This would have made the timetable for our meetings extremely tight, it would have required almost all candidates to stay over at least over the Saturday night, and it would have disrupted timetables for the traditional Schools Dinners which involve also Part C candidates whose exams had finished much earlier. We asked if the timetable could be rearranged to finish on Friday, and the Schools were able to achieve this, but it was apparent from the discussions that there was very little slack.

Next year there will be more papers in both Parts B and C, with more shared papers which will be particularly difficult to timetable, and we foresee a danger that it will not be possible to fit all the papers into the 17 days (excluding Sundays) between Monday of Week 6 and Friday of Week 8. Since the Part C and Part B timetables are interrelated, with Part C given priority over Part B for Weeks 6 and 7 in order that their examiners meetings can take place in Week 9, it is not appropriate for either set of Examiners to negotiate individually with the Schools. We therefore **RECOMMEND** that the Mathematical Institute discusses the timetables with the Examination Schools early in the academic year 2014/15 in order to establish whether the exams for Parts B and/or C might have to stretch over a longer period, and if so how best to mitigate the effects.

Reconciliation of marks for projects

The Mathematical Institute has complicated procedures for reconciliation of the marks proposed by the assessors of each BE extended essay and BSP project. In addition to the proposed mark from each assessor, the assessors and the supervisor each propose a range of marks in which they feel the final mark should lie.

In certain circumstances, the Institute's complicated rules allow the Examiners to take the average of the two preferred marks without further consultation. On the other hand, the MPLS Division requires there to be further consultation if the two preferred marks differ by more than 10. Since assessors may give a range much wider than 10 marks, it is possible that the Institute's rules allow averaging without consultation but the Division's rule prohibits it, although this did not happen this year. Indeed a corollary of the Division's rule is that averaging without consultation cannot occur unless the average mark is within 5 of both assessors' marks. Since assessors rarely give a range less than 5 marks either side, the assessors' ranges are almost irrelevant to the question whether the marks can be averaged without consultation.

In cases where further consultation is required, the Institute's rules for BE (but not those for BSP) require the assessors to confer with the supervisor without revealing their proposed mark ranges. We feel that this is unrealistic.

The majority of the essays and projects were ineligible for averaging without further consultation. Nearly all the failures were due to the average of the assessors' marks being outside the supervisor's range. In some cases it took a long time to obtain an agreed mark, and in all cases the agreed mark was close to the average of the assessors' mark.

Each BSP project has three components, and each component is assessed by two assessors, so each candidate is assessed by 6 different people, and the supervisor is often a seventh person. The Institute's procedure do not seem to take account that there is no assessor who has a complete overview of a given candidate, although the assessors of the written work clearly have the most influence on the final mark. In practice, the two proposed marks for the written work are reconciled by following the Institute's procedures and the Division's 10-mark rule, the marks for the oral presentation are reconciled by the two assessors before being passed to the Examiners, and the marks for the peer review are reconciled by the Examiners. The marks are then scaled appropriately by the Examiners to reach an overall mark.

In 2013 the relative weightings of the three components were: written project 70%, oral presentation 20%, peer review 10%, but this was changed to 75:14:10 this year. In addition, a detailed marksheet for the oral presentations was produced. This had the desired effect on the distribution of marks for the oral presentation. However the proposed marks for the peer review were again almost universally very high.

We **RECOMMEND** that the Institute consider revising their rules generally and/or with specific regard to the following:

(i) To incorporate the Division's 10-mark rule into their rules.

(ii) To cease to ask assessors to specify a range of marks.

(iii) To consider the role of the supervisor's range generally and/or specifically with regard to the question whether or not the supervisor should know the assessors' proposed marks when discussions take place.

(iv) Produce a detailed marksheet for the assessment of BSP peer review.

(v) Revise the BSP rules to take account that 6 different assessors consider each candidate.

Consultation with assessors on written papers

This year we had more interaction with assessors about scaling of marks than in the past. We had indicated this at the start of the academic year, and it met some resistance from parts of the Mathematical Institute, but we consider it to have been a marked improvement, and we strongly RECOMMEND it to next year's Examiners.

In recent years the Examiners had asked assessors to propose ranges for raw marks that might map to 70 and 60. Only about half the assessors had responded, and many of the proposals were very inappropriate. In almost all cases no further consultation took place. This year we explained how we would use the proposals, telling assessors that we would consult them after receiving their marks. A large majority of assessors offered their own proposals. Within a few days of receiving the mark-sheets we calculated the marks that the standard algorithm would propose to map to 70 and 60, and compared them with the assessor's suggestions (if offered), and then we reported to the assessor. There were many papers where we felt that the algorithm's suggestions were appropriate but they did not agree with the assessor's suggestions, so we asked the assessors whether they would be content with the algorithm (most replied that they would be). There were several papers where neither the Examiners nor the assessor thought that the algorithm worked appropriately, and in these cases the consultations with the assessors were very useful indeed. There were 4 papers where we felt the algorithm worked appropriately, but the assessor was not content. These were resolved by the Examiners collectively. In three cases we adopted scaling which was intermediate between the algorithm and the assessor, and in one case (where the gap was small) we followed the algorithm.

In addition to the general benefit to the Examiners in consulting assessors to seek advice, we believe that there was an educational benefit to assessors. Most assessors were in fairly close agreement with the algorithm about the mark mapping to 70. However the great majority put the mark mapping to 60 considerably higher than the algorithm suggested. Typically the assessors suggested 30 or above, and the algorithm suggested a mark in the 20s. This has been the pattern since the present algorithm was introduced in 2006, and it can be seen in tables in the subsequent Examiners reports, but Faculty members do not seem to be aware of this. We believe that this consultation, if repeated, will greatly improve awareness of this.

The consultation with assessors was moderately time-consuming, but it was confined to one Examiner (no administrative staff). The time spent on it may well have been recovered by shortening the Examiners meetings involving 10 people. Instead of repeatedly asking the Institute to run the database, we calculated the algorithm's suggestions by hand, using the assessors' mark-sheets and advance knowledge of the values of N_1 and N_2 (as shown in Table 2). This took only a few minutes for each paper, and the actual consultations generally took longer than that. Nevertheless there would be a useful saving of time if the mark-sheets were to show the total mark of each candidate. The assessors have to calculate this in order to enter the checksum, so it would not be an additional burden for them (some assessors included such a column themselves). A column for the total mark could replace the redundant column for the number of empty cover sheets. Ideally for us, the total mark entered would be the sum of the two best marks, whereas the checksum uses the sum of all three marks if the candidate has answered all three questions. However these cases are rare and they could easily be adjusted by hand.

We therefore **REQUEST** that the mark-sheets should include a column showing each candidate's total mark.

Determination of University Standardised Marks

In our opinion and the opinion of many others, the 70/60/50 convention for class borderlines is highly inappropriate for mathematics. Ideally classification would be achieved by simple linear operations on the raw marks, but there is a requirement that classification is determined solely by the standardised marks on each paper. We are expected to award Upper Seconds to about 50% of our candidates, and to achieve that we should have close to 50% of marks between 60 and 69, so for minimal recalibration of a paper with a field of average quality 50% of the marks should be in a 5-mark range (out of 50). Assuming a normal distribution that would indicate a standard deviation of less than 4 marks, and around 98% of marks in the range 25-39. In practice, Part B papers typically have average means in the low 30s, but the standard deviation is usually about 8 marks. We regard such a distribution as much more robust than one with a concentration of marks into a very narrow band. The most important feature of an examination is that the candidates come out in a fair order; marks can then be assigned appropriately by a scaling process (now often called standard setting). When the marks are very concentrated, minor accidents can lead to large distortions in the ordering, and hence to unfair penalties.

In order to resolve this conflict, some non-linearity has to be used somewhere in the process. Some recent examiners asked assessors to aim for minimal scaling of raw marks at the bottom end, and to achieve this by using non-linear marks schemes, so the first few lines of a solution would earn many more marks than the last (problem-solving) part. This approach had some success in reducing scaling at the bottom end (where marks are scarce) but it introduced non-linearity into the mark-schemes, it increased non-linearity of the scaling functions at the upper end (where marks are more common) and it exacerbated the problem of papers with too many high marks. The Mathematical Institute has continued to issue guidance that papers should be set in such a way as to minimise scaling of raw marks, but this has never been achieved, and we do not think it should be an aspiration. We note that neither Computer Science nor Statistics include such guidance.

In our opinion, on a well-set mathematics paper a 1/2.1 borderline candidate should typically score 70-75% of the marks (say, 36 out of 50), and a 2.1/2.2 borderline candidate should score about 50% of the marks (say, 25). This corresponds to a mean about 33 and a standard deviation about 8, and these figures fall in the middle of the range observed in the Part B papers in any year since the switch to short exams. Such papers are easy to scale with fairly gentle corners. The Mathematical Institute (and Department of Statistics, but not Computer Science) has issued guidance that 2.1/2.2 borderline candidates should be "able to clock up" 15 marks out of 25 on each question. We are not sure exactly what this means, and if target marks are to be set we would recommend that 2.1/2.2 borderline candidates should typically get about 25 marks out of 50 (over 2 questions).

We **RECOMMEND** that the Mathematical Institute's *Information for Examiners*, particularly Section D.1, is reviewed in consultation with the Examiners, at the start of the academic year 2014/15 and before being released to candidates as an appendix to the Examination Conventions.

Apart from the additional consultation with assessors, the examiners followed the Department's established practice in determining the University standardised marks (USMs) reported to candidates. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part A performances of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage). Working with the data for this population, numbers N_1, N_2 and N_3 are first computed for each paper: N_1, N_2 and N_3 are, respectively, the number of candidates taking the paper who achieved in Part A average USMs in the ranges $[70, 100]$, $[60, 69]$ and $[0, 59]$, respectively.

The algorithm converts raw marks to USMs for each paper separately. For each paper, the algorithm sets up a map $R \to U$ (R = raw, U = USM) which is piecewise linear. The graph of the map consists of four line segments: by default these join the points $(100, 100)$, $P_1 = (C_1, 72)$, $P_2 = (C_2, 57)$, $P_3 = (C_3, 37)$, and $(0, 0)$. The values of C_1 and C_2 are set by the requirement that the number of I and II.1 candidates in Part A, as given by N_1 and N_2, is the same as the I and II.1 number of USMs achieved on the paper. The value of

C_3 is set by the requirement that P2P3 continued would intersect the U axis at $U_0 = 10$. Here the default choice of *corners* is given by U-values of 72, 57 and 37 to avoid distorting nonlinearity at the class borderlines.

The results of the algorithm with the default settings of the parameters provide the starting point for the determination of USMs, and the Examiners may then adjust them to take account of consultations with assessors (see above) and their own judgement. The examiners have scope to make changes, usually by adjusting the position of the corner points P_1, P_2, P_3 by hand, so as to alter the map raw $\rightarrow$ USM, to remedy any perceived unfairness introduced by the algorithm. They also have the option to introduce additional corners. For a well-set paper taken by a large number of candidates, the algorithm yields a piecewise linear map which is fairly close to linear, usually with somewhat steeper first and last segments. If the paper is too easy or too difficult, or is taken by only a few candidates, then the algorithm can yield anomalous results—very steep first or last sections, for instance, so that a small difference in raw mark can lead to a relatively large difference in USMs. For papers with small numbers of candidates, moderation may be carried out by hand rather than by applying the algorithm.

Following customary practice, a preliminary, non-plenary, meeting of examiners was held two days ahead of the first plenary examiners' meeting to assess the results produced by the algorithm, to identify problematic papers and to try some experimental changes to the scaling in general and of individual papers. This provided a starting point for the first plenary meeting to obtain a set of USM maps yielding a tentative class list with class percentages roughly in line with historic data.

The first plenary examiners' meeting, jointly with Mathematics & Statistics examiners, began with a brief overview of the methodology and of this year's data. Those papers which had been identified as problematic were scrutinised, and provisional adjustments of the scaling were made. The full session was then adjourned to allow the external examiners to look at scripts.

The examiners reconvened and carried out a further scrutiny of the scaling of each paper, making small adjustments in some cases before confirming the scaling map (those Mathematics & Statistics examiners who were not Mathematics examiners left the meeting once all papers with significant numbers of Mathematics & Statistics candidates had been considered).

Table 2 on page 15 gives the final positions of the corners of the piecewise linear maps used to determine USMs.

At their final meeting on the following morning, the Mathematics examiners reviewed the positions of all borderlines for their cohort. For candidates very close to the proposed borderlines, marks profiles and particular scripts were reviewed before the class list was finalised.

In accordance with the agreement between the Mathematics Department and the Computer Science Department, the final USM maps were passed to the examiners in Mathematics & Computer Science. USM marks for Mathematics papers of candidates in Mathematics & Philosophy were calculated using the same final maps and passed to the examiners for that School.

Medical certificates and other special circumstances

The examiners considered medical certificates relating to the Part B examination and also certificates passed on by the examiners in Part A 2013. All candidates with certain conditions (such as dyslexia, dyspraxia, etc) were given special consideration in the conditions and/or time allowed for their papers, as agreed by the Proctors. Each such paper was clearly labelled to assist the assessors and examiners in awarding fair marks. Details of cases in which special consideration was required are given in Sections E.1 and E.2.

Prizes

The value of the Gibbs prizes available in Part B had been slightly reduced this year, as Gibbs prizes for Part A were being awarded this year for the first time. The criteria for the Part B prizes remained the performance in Parts A and B combined. This was natural for this year, since the Part B candidates had not been eligible for any prize on Part A. When we investigated whether next years Part B prizes would be awarded solely for Part B performance, we found that this is not planned. The intention is that the Part B examiners should award prizes for Parts A and B combined. This means that the best students in Part A not only get a prize for that but also get a head start for the following year's prizes. If any of this year's candidates had already been awarded a Gibbs prize for Part A, we would not have wished to award such a candidate a prize unless it was merited on the basis of Part B performance alone. Noting that the prizes are awarded by the Examiners, we **RECOMMEND** that next year's Examiners adopt this principle.

In the absence of a Gibbs prize specifically for Mathematics & Statistics, the Gibbs prizes were awarded to the two most deserving candidates in the combined field for Mathematics and Mathematics & Statistics. The Junior Mathematical Prizes, and the Statistics departmental prize, were awarded to the most deserving candidates in the separate fields for Mathematics and Mathematics & Statistics, respectively, excluding those awarded a Gibbs prize.

Table 2: Position of corners of the piecewise linear maps

Paper	P_1	P_2	P_3	Additional Corners	N_1	N_2	N_3
B1a	(14.36, 37)	(25, 57)	(40, 72)		13	17	8
B1b	(19.65, 37)	(34.2, 57)	(42, 70)		14	23	12
B2a	(11.2, 37)	(19.5, 57)	(30, 72)		10	12	7
B2b	(13.96, 37)	(24.3, 57)	(37.8, 72)		9	9	3
B3a	(9.77, 37)	(20, 57)	(35, 72)		8	10	1
B3b	(11.95, 37)	(20.8, 57)	(41.8, 72)		5	6	1
B3.1a	(10.23, 37)	(19, 57)	(38.8, 72)		10	17	3
B4a	(9, 50)	(12, 57)	(31, 72)		17	13	4
B4b	(13.16, 37)	(22.9, 57)	(35, 72)		15	7	4
B5a	(11, 40)	(23, 57)	(36.2, 72)		13	38	20
B5b	(14.36, 37)	(25, 57)	(40, 72)		13	37	16
B5.1a					2	0	0
B6a	(15.51, 37)	(27, 57)	(42, 72)		13	27	11
B6b	(13.44, 37)	(23.4, 57)	(41.4, 72)		11	23	8
B7.1a	(10.34, 37)	(21, 60)	(33, 72)		11	24	9
B7.2b	(13.04, 37)	(22.7, 57)	(39.2, 72)		6	16	6
B8a	(13.61, 37)	(23.7, 57)	(40.2, 72)		15	43	19
B8b	(15.45, 37)	(26.9, 57)	(37.4, 72)		7	23	12
B9a	(14.65, 37)	(22, 50)	(30, 60)	(42, 70)	14	18	7
B9b	(10.57, 37)	(18.4, 57)	(36.4, 72)		13	16	6
B10a	(6.43, 37)	(20, 57)	(34, 72)		23	18	4
B10b	(8, 50)		(24, 72)		11	8	1
B10.1b	(10.91, 37)	(21, 57)	(34, 72)		27	27	14
B11a	(13.33, 37)	(23.2, 57)	(32.2, 72)		14	14	12
B11b	(12.81, 37)	(22.3, 57)	(35.8, 72)		21	35	18
B21a	(19.93, 37)	(34.7, 57)	(45, 70)	(48, 80)	2	16	10
B21b	(19.36, 37)	(33.7, 57)	(45, 70)	(48, 80)	2	11	7
B22a	(14.82, 37)	(23, 57)	(32.8, 72)		5	3	10
C7.1b	(12.93, 37)	(27, 57)	(37, 72)		3	11	1
OBS1	(10.06, 37)	(21.9, 57)	(35, 72)		15	14	8
OBS2a	(18.15, 37)	(31.6, 57)	(42, 70)		17	15	7
OBS3a	(10.91, 37)	(19, 57)	(34, 72)		34	38	17
OBS3b	(13.21, 37)	(23, 57)	(38, 72)		14	12	8
OBS4a	(17.18, 37)	(29.9, 57)	(40.4, 72)		19	20	14
OBS4b	(15.45, 37)	(26.9, 57)	(37.4, 72)		16	16	12

Table 3 gives the rank of candidates and the number and percentage of candidates attaining this or a greater (weighted) average USM.

Table 3: Rank and percentage of candidates with this or greater overall USMs

Av USM	Rank	Candidates with this USM and above	%
87	1	1	0.63
86	2	2	1.27
83	3	4	2.53
81	5	7	4.43
80	8	9	5.7
79	10	11	6.96
78	12	13	8.23
77	14	16	10.13
76	17	20	12.66
75	21	26	16.46
74	27	29	18.35
73	30	33	20.89
72	34	37	23.42
71	38	43	27.22
70	44	50	31.65
70	44	50	31.65
69	51	54	34.18
68	55	57	36.08
67	58	64	40.51
66	65	74	46.84
65	75	88	55.7
64	89	97	61.39
63	98	105	66.46
62	106	114	72.15
61	115	119	75.32
60	120	127	80.38
59	128	132	83.54
58	133	135	85.44
57	136	139	87.97
56	140	143	90.51
55	144	144	91.14
54	145	145	91.77
53	146	146	92.41
50	147	147	93.04
49	148	150	94.94
49	148	150	94.94
48	151	152	96.2
45	153	153	96.84
43	154	154	97.47
41	155	156	98.73
40	157	157	99.37
39	158	158	100

Table 4: Breakdown of results by gender

Class	Total		Male		Female	
	Number	%	Number	%	Number	%
I	49	31.01	42	36.52	7	16.28
II.1	78	49.37	57	49.57	21	48.84
II.2	21	13.29	9	7.83	12	27.91
III	9	5.7	6	5.22	3	6.98
P	1	0.63	1	0.87	0	0
Total	158	100	115	100	43	100

B. Equal opportunities issues and breakdown of the results by gender

Table 4 shows the performances of candidates broken down by gender.

C. Detailed numbers on candidates' performance in each part of the examination

The number of candidates taking each paper is shown in Table 5.

Table 5: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
B1a	39	32.23	7.72	63.51	10.59
B1b	51	38.1	6.76	65.35	12.56
B2a	31	24.71	7.11	64	10.82
B2b	23	33.39	9.28	67.83	15.07
B3a	19	28.74	6.23	65.74	7.72
B3b	12	35	6.52	67.67	5.71
B3.1a	31	31.42	7.54	67.29	7.74
B4a	34	24.85	9.76	68.09	10.05
B4b	26	33.96	8.93	72.73	14.05
B5a	70	27.01	7.75	61.63	10.38
B5b	67	32.01	8.8	63.97	14.19
B5.1a	2	-	-	-	-
B6a	54	33.81	7.51	64.35	10.7
B6b	43	31.77	10.29	65.48	13.85
B7.1a	45	26.4	6.29	65.07	8.13
B7.2b	29	29.07	9.15	61.03	13.96
B8a	78	30.1	7.73	62.26	10.58
B8b	42	31	5.71	62.67	9.35
B9a	41	38.07	8.86	71.07	15.01
B9b	38	28.34	8.88	65.08	11.8
B10a	41	31.27	8.49	70.49	11.49
B10b	18	20.5	7.76	66.61	9.68
B10.1b	50	30.08	9.09	67.52	14.8
B11a	32	29.12	7.87	66.31	13.69
B11b	73	28.34	7.8	62.48	12.43
B21a	29	37.72	9.55	62.59	16.34
B21b	21	36.71	10.96	61.05	17.24
B22a	15	25.6	8.71	59	16.44
C7.1b	15	31.73	9.21	64.87	15.49
BS1	4	-	-	-	-
BS2a	10	42.1	5.11	75.5	14.41
BS3a	56	30.27	10.13	68.62	16.02
BS3b	11	35.18	7.93	72	13.12
BS4a	26	34.5	8.41	64.58	14.56
BS4b	19	32.16	6.53	64.58	10.57
BE	4	-	-	-	-
BSP	16	-	-	68	7.29
O1	6	-	-	70.67	4.722
CS3a	2	-	-	-	-
CS4b	3	-	-	-	-
N1b	7	-	-	67.142	1.07
102	1	-	-	-	-
122	1	-	-	-	-

Individual question statistics for Mathematics candidates are shown below for those papers offered by no fewer than six candidates.

Paper B1a: Logic

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.2	14.2	3.95	35	0
Q2	18.04	19.42	5.64	24	2
Q3	15.47	15.47	5.26	19	0

Paper B1b: Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.23	21.23	3.78	48	0
Q2	17.5	17.5	3.3	42	0
Q3	15.23	15.75	4.85	12	1

Paper B2a: Introduction to Representation Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14	14	3.37	31	0
Q2	10.21	10.54	3.91	26	2
Q3	9	11.6	7.14	5	2

Paper B2b: Group Theory and an Introduction to Character Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.65	16.65	5.85	20	0
Q2	15.18	15.18	4.79	17	0
Q3	19.67	19.67	4.18	9	0

Paper B3a: Geometry of Surfaces

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.95	18.89	5.83	18	1
Q2	11.17	11.82	5.25	11	1
Q3	8.36	9.5	4.3	8	3

Paper B3b: Algebraic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.8	15.33	7.45	9	1
Q2	19.33	19.33	3.87	12	0
Q3	16.67	16.67	4.51	3	0

Paper B3.1a: Topology and Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.83	15.83	4.47	30	0
Q2	15.63	15.63	4.95	27	0
Q3	15.4	15.4	5.77	5	0

Paper B4a: Banach Spaces

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.69	11.64	6.65	14	2
Q2	12	12.28	3.82	29	1
Q3	13.04	13.04	6.93	25	0

Paper B4b: Hilbert Spaces

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.3	17.3	5.66	23	0
Q2	15.65	16.42	5.06	19	1
Q3	17.3	17.3	4.97	10	0

Paper B5a: Techniques of Applied Mathematics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.36	12.66	4.06	62	5
Q2	13.29	13.62	5.01	60	2
Q3	12.96	16.06	6.79	18	7

Paper B5b: Applied PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.79	14.60	4.91	53	5
Q2	15.81	16.85	6.18	39	4
Q3	16.09	17.00	6.87	42	3

Paper B6a: Viscous Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.69	17.69	3.58	39	0
Q2	16.14	16.46	5.65	41	1
Q3	16.46	16.46	4.41	28	0

Paper B6b: Waves and Compressible Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.65	15	6.5	30	1
Q2	15.41	15.88	6.27	33	1
Q3	16.32	17.04	6.24	23	2

Paper B7.1a: Quantum Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.75	14.95	4.63	43	1
Q2	9.88	11.67	4.83	12	4
Q3	11.05	11.57	3.89	35	3

Paper B7.2b: Special Relativity and Electromagnetism

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12	12	6.45	24	0
Q2	17.11	17.7	5.18	27	1
Q3	11	11	4	7	0

Paper B8a: Mathematical Ecology and Biology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.16	16.16	4.77	77	0
Q2	9.06	10.4	4.11	15	3
Q3	14.81	14.81	4.63	64	0

Paper B8b: Nonlinear Systems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.57	15	3.17	29	1
Q2	16.45	16.45	5.32	33	0
Q3	13.32	14.73	5.1	22	3

Paper B9a: Galois Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20	20	5.92	31	0
Q2	19.68	19.68	4.41	40	0
Q3	11.75	14	4.74	11	5

Paper B9b: Algebraic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.23	13.53	6.07	19	3
Q2	14.31	14.6	5.7	25	1
Q3	14.06	14.22	4.83	32	1

Paper B10a: Martingales through Measure Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.63	16.63	4.95	40	0
Q2	13.66	14.14	5.3	28	1
Q3	12.56	15.79	7.07	14	4

Paper B10b: Continuous Martingales and Stochastic Calculus

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.18	10.18	3.63	17	0
Q2	8.29	8.29	4.07	7	0
Q3	10.21	11.5	5.41	12	2

Paper B10.1b: Mathematical Models of Financial Derivatives

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.71	17.71	5.35	49	0
Q2	12.73	13.02	5.3	44	1
Q3	8.13	9	5.62	7	1

Paper B11a: Communication Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	6	10	4.69	2	2
Q2	13.13	13.43	3.35	30	1
Q3	15.91	15.91	5.72	32	0

Paper B11b: Graph Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.89	15.03	4.41	65	1
Q2	13.44	13.44	4.71	72	0
Q3	13.7	13.78	3.43	9	1

Paper B21a: Numerical Solution of Differential Equations I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.75	17.75	6.95	28	0
Q2	20.07	20.07	3.26	29	0
Q3	11	15	5.66	1	1

Paper B21b: Numerical Solution of Differential Equations II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.71	19.71	4.64	21	0
Q2	14.64	14.64	6.12	11	0
Q3	18.27	19.6	8.03	10	1

Paper B22a: Integer Programming

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	9.47	9.86	4.72	14	1
Q2	15.43	15.43	5.63	14	0
Q3	9	15	9.02	2	2

Paper C7.1b: Quantum Theory and Quantum Computers

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.54	16.58	4.39	12	1
Q2	16.29	16.29	4.7	14	0
Q3	16.33	16.33	7.02	3	0

Paper OBS2a: Foundations of Statistical Inference

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	23.60	23.60	2.07	10	0
Q2	17.78	17.78	4.06	9	0
Q3	25.00	25.00	-	1	0

Paper B12a/OBS3a: Applied Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.87	12.91	4.95	45	1
Q2	16.12	16.12	5.98	51	0
Q3	16.94	18.25	5.59	16	2

Paper OBS3b: Statistical Lifetime-Models

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.89	18.89	4.31	9	0
Q2	15.4	15.4	3.91	5	0
Q3	17.5	17.5	4.6	8	0

Paper OBS4a: Actuarial Science I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.64	16.3	5.36	23	2
Q2	18.35	18.35	5.17	26	0
Q3	11	15	7.11	3	2

Paper OBS4b: Actuarial Science II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.21	13.78	3.72	18	1
Q2	15	15	-	1	0
Q3	18.32	18.32	5.1	19	0

Assessors' comments on sections and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. The examiners have not included assessors' statements suggesting where possible borderlines might lie; they did take note of this guidance when determining the USM maps. Some data to be found in Section C above have also been removed.

B1a: Logic

Most candidates attempted question 1, but (despite the hint) only very few managed to get the derivation in part (a) right and there were only two rigorous solutions to part (d)(ii) and (iii).

There were many fully satisfactory solutions to part (a) and (b)(i) of question 2.

The main difficulty in question 3 was to see the (simple) mathematical argument for part (e)(iii) and how to turn it into a formal derivation.

B1b: Set Theory

Question 1.

This question was generally well done. The most difficult aspects seemed to be around AC: stating it correctly and the bookwork proof that WO implies AC.

Question 2.

Parts a and b were generally well done, though the examples of non-commutativity were frequently given incorrectly. Part c was not done well, despite similar problems appearing on recent papers. Few thought to construct μ in part c(ii) by recursion, and there was quite a bit of incorrect ordinal arithmetic. Very few were in a position to approach c(iii), whose difficulty depended a lot on the choice of construction of mu. Part d was well done.

Question 3.

Fewer students attempted problem 3. Part a, especially the definition of 'class function' was not well done, given that it is straight bookwork. Part b was well done. Students struggled with part c, again most did not provide a recursive definition of U(X). Many nevertheless managed 3(iii). Part d was generally well done.

B2a: Introduction to Representation Theory

Q1. This was the most popular question. Parts (a) and (b) were answered correctly by most candidates but there were few correct answers in (c) to $\mathrm{rad}(A) \subseteq J$. In (d) most candidates quoted Maschke's theorem but many failed to see how it immediately gives $\mathrm{rad}(\mathbb{F}_5 C_6) = 0$.

Q2. Another very popular question. There were very few correct answers to (b), most candidates tried to use (a) which doesn't apply here. Few candidates realised (d) amounts to factorising $x^7 - 1$ over $\mathbb{R}, \mathbb{Q}$ and $\mathbb{F}_7$.

Q3. (a) and (b) were standard. A common answer to (c) was to say that $s = l$ follows immediately from $(R/pR)^s = (R/pR)^l$ but some justification is required to argue with dimension over R/pR.

B2b: Group Theory and an Introduction to Character Theory

B3a: Geometry of Surfaces

1. Candidates who saw the last part geometrically ("pyramid on a polygon", "subdivide a triangle") were ok. Those who tried to write down a planar model often didn't have a triangulation.

2. The bookwork for (a ii) was not well done. Most who tried part (b) made mistakes in calculations.

3. Many candidates wrote down the formula for a triangle in part (b) and then came to grief in (civ). Nobody got it out completely.

B3b Algebraic Curves

Section B
The performance of the students was quite good. Out of the 17 students, 12 got above 30 marks, and 5 got above 40 marks. In fact, one student managed to get the hardest question almost right (with minor errors). Their solutions of the unseen parts was sometimes innovative too. Overall, I was happy with their work.

B3.1a Topology and Groups

The pattern of marks was quite uniform across the three solutions. Only the best candidates scored well on the final part of each question, with 17 marks above 20/25 from 82 attempts (41 candidates).

Question 1. 38 of 41 candidates attempted this question. The accounts of bookwork were generally good but weaker in a(iii) and c(iii). In C(iv), many candidates said something vague concerning simplicial approximation, few related maps $x \to y$ to maps $s^2 \to s^3$ well.

Question 2. 32 of 41 candidates attempted this question. Concise solutions to a(iii) were rare; many failed to focus on the uniqueness aspect of the universal property. (b) and c(i), c(ii) were generally well done. In c(iii) most failed to say/see that $E \to \mathbb{Z}$ is trivial.

Question 3. Only 12 attempts from 41 candidates, but marks spread as for other questions. Many failed to state uniqueness in HLT (part (b)) and marks were lost throughout (b) for failing to specify basepoints of lifts. Many reasonable attempts were made at (c), with most getting good marks on the first part and fewer finding degree -3 cover. Only the very best candidates used the fact that H was not normal.

B4a: Banach Spaces

In general questions turned out to be a bit more difficult than expected.

Question 1: This was the least popular question: only 17 candidates out of 39 attempted it. Many candidates successfully completed book-work in part (a). In part (b) many candidates tried wrong approach to use Weierstarass theorem to approximate the function f by polynomials.

Question 2: Most of candidates successfully answered the bookwork in part (a). There is a mistake in part (b), the norm should be $\|f\| = \|x_0\|/\text{dist}(x_0, Y)$. There were quite a few candidates who nevertheless presented correct solutions. Part (c) turned out to be rather challenging, many candidates used extensions with different norms.

Question 3: This question was the simplest one with many candidates scoring over 20 marks and very few scoring low marks. Most of the candidates successfully attempted the bookwork parts (a) and (b) . The last part of (c) turned out to be challenging as intended. Not so many were able to show that the norm is not achieved, and even fewer showed that this implies that the space is not reflexive. Many candidates tried to show that this implies that the space is not Banach.

B4b: Hilbert Spaces

Question 1 [self-adjoint and unitary operators; invertibility] The bookwork in (a) and (b) caused little trouble. The first part of (c) was essentially B4a material; a number of candidates called on the Inverse Mapping Theorem to show that T^{-1} is bounded when this could be deduced in one line from the facts given; they lost a mark. A number went into bookwork mode for the first part of (d) and proved in general that a self-adjoint operator has real spectrum, not appreciating that invertibility of $\pm iI + T$ is a special case which is simpler to prove. They penalised themselves and did not lose marks. Many got through the end part successfully, if inelegantly.

Question 2 [BCT, UBT and projections on Banach spaces] Virtually all those who attempted this question could reproduce a proof of UBT but rather a large number failed to impose the correct conditions when stating BCT, OMT and UBT. Given the hint included, almost all were able to establish continuity of the projection onto a complemented closed subspace of a Banach space (though a number omitted to specify any norm on $Y \times Z$). The last part was poorly done. Almost everyone realised they needed to use UBT to prove P continuous but few were able to argue correctly to identify its kernel and image and attempts often ended in incoherent rambling. Part of the problem here was a lack of facility with Part A linear algebra, and several candidates tried to use inapplicable facts about orthogonal projections on a Hilbert space.

Question 3 [Complete orthonormal bases; Bergman space] The bookwork part caused some technical problems at the level of Mods convergence. The Bergman space orthonormal basis had appeared on a problem sheet and almost all candidates realised how to proceed to establish completeness. The verification regrettably gave rise to some muddled thinking, with area integrals treated as though they were path integrals. The (new) final part caused no calculational problems, but the required deduction was not often argued properly. The very last bit discriminated well: those who did not get it out either offered no attempt or came to the wrong conclusion.

B5a: Techniques of Applied Mathematics

Question 1: This problem was attempted by nearly all candidates, though the average score was the lowest of all three problems. Part b caused the most difficulties, with few candidates certain how to deal with the delta function in a boundary value problem. A few candidates incorrectly treated the delta function as a constant, only a few saw that the problem should be solved in two intervals just as in the approach with Green's function. Nobody reached the correct expression for the eigenvalues in the final part.

Question 2: Parts a and b were done reasonably well. The problem was largely conceptual, requiring very little computation. Some candidates struggled to make the connection between Fredholm Alternative and Green's function, and thus approached the problem with far more computational effort than was needed. Very few candidates made a reasonable attempt at part c, which was also primarily a conceptual problem, requiring use of the solvability condition in Fredhom Alternative.

Question 3: Very few candidates attempted this problem, though in fact it was probably the easiest of the three, and had the highest average score. In the first part, some candidates failed to realise that constructing a solution about an ordinary point does not require the full Frobenius machinery. Full marks required dealing appropriately with the n even and n odd cases. A variety of successful approaches were constructed in part b, with most candidates recognising that only the leading order behaviour of the trig functions is needed to get the indicial equation.

B5b: Applied PDEs

Overall the students performed well on this paper, with most making good progress with the questions. In more detail:

Question 1. Most candidates completed parts (a), (b) and (c), although few were able to derive the correct expression for $v(x,t)$ and $c(x,t)$. As a result, few were able to complete part (d).

Question 2.
•Parts (a), (b) were standard and completed well by most students.
•Part (c), while standard, caused some challenges, with candidates failing to derive suitable boundary conditions for the Green's function and/or struggling to apply Green's Theorem.
•Part (d) was challenging, with few candidates realising that the method of images was needed and/or how to sum the images to satisfy both boundary conditions.

Question 3. Candidates made a reasonable attempt at part (a) but many failed to apply these results to part (b). Others did not realise how to parametrise the initial data and/or to use to determine p and q.

B5.1a: Dynamics and Energy Minimization

Q1. Two attempts, one fairly good and one very good, but neither doing (d)(i) correctly, the solution to which though short, was perhaps not as easy to see as might have been predicted.

Q2. One fairly good attempt, in which neither (c)(i) nor (c)(ii) was done correctly, even though they were quite easy.

Q3. One good attempt in which (c) would have been almost perfect but for a computational error.

B6a: Viscous Flow

Question 1:
Part (a): The statement of the Transport Theorem and Cauchy's Stress Theorem were in general well done. When deducing the incompressibility condition, some candidates argued that $\partial\rho/\partial t = 0$ rather than $D\rho/Dt = 0$ to incorrectly derive the result. The derivation of the Navier-Stokes equations was excellent. The argument that the stress tensor is symmetric was not well done in general. Some candidates struggled to apply the divergence theorem to the RHS of equation (1) correctly; some were unable to correctly apply the Reynolds transport theorem to the LHS of (1). Part b(i): A number of candidates failed to use the incompressibility condition to show that w is independent of z, instead assuming the result. Otherwise this section, and also section (ii) were generally well done (with algebraic slips occurring in the solution for w). Part (iii) was poorly done and very few students were able to determine the drag. Some students were able to determine the relevant component of the stress tensor, but then failed to integrate over the circumference of the pipe walls. Only a couple of students were able to comment that the total drag is proportional to the product of the pressure gradient and the cross-sectional area.

Question 2:

Part (a): The bookwork part of the question was very well done. Part b(i): some algebraic slips lead to candidates not correctly determining α and β. Some candidates did not correctly argue that for a similarity solution to exist the derived equation must be an ode, which can only be the case if α and β are constant (independent of x). Some candidates lost straightforward marks by not stating the boundary conditions on f. Part b(ii) presented some difficulties. The majority candidates were able to get to $g^2 U_s$ = constant, but some did not use the fact that $\alpha = (U_s g)'g$ = constant (or the β expression) to eliminate U_s (or g) and derive an equation for g (or U_s). Part b(iii): many candidates failed to state that if $l > 0$ then $U_0 > 0$ and hence flow is from left to right. The statement that for the approximation to be valid it is necessary that the pressure gradient is negative was often missing.

Question 3: Part (a): The bookwork to derive the leading order lubrication equations was well done. The physical significance of the boundary conditions at $\hat{z} = 0$ caused some confusion. Part (b): The majority of candidates were able to determine the governing equation for the pressure, and to solve it, although there were some algebraic slips when integrating. Part (c) caused some problems. Many candidates did not realise that to compute the force exerted by the fluid on the upper plate it was necessary to integrate the relevant component of the stress vector along the plate. Some errors were made in obtaining the appropriate dimensions for the force (in particular, the factor of L introduced when integrating along the upper plate with respect to x). Very few candidates equated the computed force with the weight of the plate. While some candidates were able to determine an expression for h/L, very few realised that the prediction holds provided that $h/L \ll 1$, which places a constraint on the relative sizes of W and M.

B6b: Waves and Compressible Flow

The take-up of the 3 questions was fairly even, as was the spread of marks on each one.

Q1: A surprising number of candidates had difficulty with separating variables in part (a). Most managed to find the natural frequencies, although several candidates applied no-flow conditions at the open ends of the cylinder. In parts(b) and (c), many candidates repeated lengthy algebraic procedures to arrive at the Bessel function solution, which is unnecessary given the earlier work. Part (c) was almost identical to a problem sheet question, invoking the radiation condition, but was successfully completed by very few candidates.

Q2: For part (a), a number of candidates drew a diagram and stated that the limit $t \to \infty$ positive and negative contributions would not cancel out at the stationary values of $\phi(k)$, rather than that the cancellation would simply be slower. A number of candidates failed to relate the curvature to the surface displacement η in part (b) and were then forced to take some optimistic leaps in arriving at the dispersion relation in part (c). Very few were successful with the algebraic details to arrive at the final answer in part (d); in particular most candidates were not careful enough with the signs when dealing with square roots.

Q3: This question was generally well done. A common lapse in part (b) was to start with the characteristics being $\dot{x} = 1 \pm 2\sqrt{g}$ before having reasoned that u and c should be constant. Most candidates realised they were looking at an expansion fan, and a pleasing number found the expression for both u and c within it. Part (c) was straightforward and was well done. The sketch of the solution in part (d) was rather poorly done, with only one candidate attaining all 3 marks.

B7.1a: Quantum Mechanics and Electromagnetism

Q1: This question was attempted by 45 students (almost all of them) and was the question with higher average mark (15/25). Most students got parts a and b correctly, but some misunderstood what was being asked in part b. Parts c and d were more challenging with a good spread among the candidates.

Q2: This question was attempted by only 17 students, with an average mark of 10/25. While part a was standard and done by most students, some students had problems understanding part b, which was also long, and I think this part put off many students from trying the question. While this part could be answered by methods learnt in the course (and indeed many students did so) the setting was a little unusual and maybe this should have been asked at a later stage in the question. Still, most students attempting this question made progress, and the low average mark is in part due to the fact that some students have attempted only part a of this question, having focused in the other two questions.

Q3: This question was attempted by 39 students with an average mark of 11/25. While part a was solved by most students, the first half of part b took quite longer than anticipated (this was straightforward but tedious). In retrospective, this part could have been shorter. The other parts went as expected.

Summary: The average this year was 26.4 (with a standard deviation of 6.6), which is around the mark on previous years, but lower than previous year. A posteriori, probably Q2 part b could have been introduced at a later stage, so that students would choose more evenly among the three questions. Furthermore, part b of Q3 could have been shorter. Besides these small changes, I think the level of the questions was adequate. Furthermore, during the examination everything went fine and no problems arose.

C7.1b: Quantum Theory and Quantum Computers

Question 1 was attempted by almost everyone and seemed to have worked well, with a good spread of solutions.

Question 2. There were many good attempts at this question, with only (d), the most novel part, presenting serious difficulties. A number of candidates, however, claimed in (b) that the function had n different values that could be tested, not 2^n.

Question 3. Very few attempts - one excellent, understanding connection between (a iii) and (c). Others less convincing although (b) was well done. In (a), candidates typically neglected to mention the self-adjoint condition.

B7.2b: Special Relativity and Electromagnetism

Q 1: a question on kinematics and word lines, parts (a)-(c) are book work while (d) is essentially new though simple. This wasn't very well done, and few gave the definition of proper tiem in (d)(i).

Q2: The most popular and best done question, on 4-momentum. Quite a few had trouble with kinetic energy, although it is defined in the question.

Q3: The EM question, as usual the least popular. Almost nobody showed that $E + e\phi$ was conserved.

B8a: Mathematical Ecology and Biology

Question 1
(a) and (b) reasonably well done. However, in (b)(ii) a number of candidates did not realise that $u = 1 - \frac{1}{r}$ is a root of the cubic for the period 2 orbit calculation. Some students could not sketch $1 - \frac{1}{r}$ as a function of r. (c) was meant to have students focus on $3 < r < 4$ as (b) focussed on $0 < r > 3$ but the wording of the question was not clear - some candidates thought r was varying, while others focused on $0 < r < 3$. With hindsight it should have read "show that for r fixed, $3 < r < 4$, ...". There were 3 marks for this part and I was generous [but students were confused by this.] The part $r > 4$ was not done properly by anyone - max of u_t is $\frac{r}{4}$ $\therefore$ if$r > 4, u_{t+1} < 0$.

Maths candidates: 78 attempts, 16.1 (average mark).

Question 2
Typo last line of (a)(iii): u and v should be u and w — obvious (no problems).

Most candidates found this challenging as they struggled with phase plane.

Maths candidates: 16 attempts, 10.4 (average mark).

Question 3
(a)(i) most candidates struggled to explain the term.

(a)(iii) Many students wrote $A + 2B \to B$ when it should have been $A + 2B \to 3B$.

(a)(v) virtually no student stated that the concentrations have to be non negative (they focused on real).

(a)(vi) having calculated the steady states in (v), one simply had to simplify the expression for this case, yet many students essentially did (v) over again.

(b) not many candidates got this part.

Maths candidates: 64 attempts, 14.8 (average mark).

B8b: Nonlinear Systems

Question 1: Parts (a) and (b) were well answered, as was (c) (i). Part (c) (ii) was badly answered. Few candidates classified the equilibria properly.

Question 2: Most popular and well answered.

Question 3: Few candidates classified the fixed points as required in (c). Other parts were well answered.

B9a: Galois Theory

Total of Scripts returned = 58
Paper average = 38.5/50

Q1: 46/58 attempts, average = 19.8/25
Comments: Students found this question easy. There were several excellent solutions.

Q2: 57/58 attempts, average = 19.8/25
Comments: Students found this question easy. There were several excellent solutions.

Q3: 19/58 attempts with mark = 12.1/25
Comments: the setter is surprised by the lack of attempts to this question, which was not harder than the first two. Part (b) of the question seemed to be particularly hard.

B9b: Algebraic Number Theory

All 3 questions were answered to a similar standard on average, but they varied substantially in the number of candidates attempting each question: Q1 was the least answered, Q2 in between, and Q3 was answered by easily the largest number of candidates (40 out of 48). For Q2, only a few candidates were able to complete part (d). For Q3, a surprisingly large number of candidates did not estimate the Minkowski bound sufficiently accurately and so wasted time considering extra prime ideals which need not have been included.

B10a: Martingales Through Measure Theory

Question 1 was by far the most popular, attempted by all but one candidate. (a) was generally well done. With hindsight, in part (b) the question should have stated clearly whether the random variables take values in the reals (intended) or extended reals. Almost all candidates assumed the real case. When marking, either interpretation was allowed if followed consistently: a number of candidates gave answers that assumed reals for (i) and extended reals for (iii).

A surprising number of candidates confused the limsup of a sequence of random variables with that of a sequence of events and so wrote nonsense for (b) (i) and (ii).

The last part was meant to illustrate examples of Kolmogorov's 0/1-law, rather than be an application. Most candidates missed that a sum of non-negative terms converges in the extended reals, and that the Monotone Convergence Theorem gives the expectation of the sum; when this is finite then the sum is finite a.s.

Question 2: part (a) was mostly well done, though a few circular answers were given for the last part. Part (b) turned out to be rather tricky. Most candidates gave at least an intuitive answer for independence. A fair number thought of expressing X as a sum and applying the previous parts. None gave a proper justification for the very last part.

Question 3 was least popular, perhaps because it has the least explicit bookwork, though a standard bookwork proof gets you most of the way in part (b). Many candidates didn't prove the result in the hint (even though the hint says 'show'). Unfortunately, quite a few candidates found a different way of applying the hint from what was intended, leading off down a blind alley.

B10b: Continuous Martingales and Stochastic Calculus

B10b Continuous Martingales and Stochastic Calculus is a new course introduced this year to Part B syllabus. It is a challenging course and a large share of its material used to be covered in Part C lectures. While students had problem sheets and a set of"fake exam questions" to prepare for the exam, there was no history of past exam papers often available for other courses. In consequence many students found the paper challenging. Question 1 was answered relatively well however few students were able to answer part (d) of the question and many did not argue part a) fully. The standard of answers to question 2 varied. Students did not remember that M^* was of finite variation could not make any significant progress. Others were able to answer parts a) and b) well but found aspects of part c) very difficult. Question 3 was answered relatively well. Common errors included mistaking the Riemann sum approximation theorem for stochastic integrals with their construction and, often silly, mistakes in computing the covariance function which hindered attempts to answer part d).

B10.1b: Mathematical Models of Financial Derivatives

Question 1 This was the most popular question. All candidates attempted it. It was mostly done very well. In part (b), many candidates forgot to mention that $p^{\mathbb{Q}}$ and $q^{\mathbb{Q}}$ both lay in $(0, 1)$, so defined a probability measure. A few candidates made a poor error in part (c) in describing the valuation algorithm for an American option, comparing the pay-off with a discounted expectation over multiple time periods (from maturity until current time), when in fact one must do the comparison on a one-period basis. Some also forgot to point out that the algorithm starts with $V_n = h(S_n)$. In part (d), some forgot to discount. Hardly any candidates pointed out the exercise policy in part (d) in completely precise language: at one node, the holder would be indifferent as to whether to exercise the option or not, and the holder would exercise in all states at maturity if the option had not been exercised before then.

Question 2 The second most popular question, with nearly all candidates attempting it. There were fewer excellent answers than for Question 1, but there were many reasonable attempts. The most serious misunderstanding was that some candidates failed to see that asset P is not a directly tradeable asset, so in part (a) they got its $\mathbb{Q}$-drift wrong. Some candidates went as far as to invoke trading strategies involving P in part (b), even though the question directly instructed candidates to trade S and Y. Other common errors were to forget the cross-variation terms in using the Itô product rule, and to misunderstand what is meant by a volatility. Nearly all students failed to see that defining a BM $B := \rho W + \sqrt{1 - \rho^2} W^{\perp}$, correlated with W, would give slightly cleaner calculations. In part (c), some candidates chose $z = xy$ rather than $z = \sqrt{xy}$, but this was acceptable. Many candidates failed to quote directly the formula for the expectation of a GBM in (d), though this came up many times in the course.

Question 3 The least popular question, done by only a handful of candidates. There were hardly any excellent attempts. Most got the definitions of quadratic and cross-variation correct, and did a reasonable job of computing $[M]$ and $[M, S]$. There were a fair number errors in part (b) in the replication argument. Some candidates took the maximum of the stock to be a traded asset, which was a serious error. Others forgot to show that the variation with respect to M in the replication argument necessitated the boundary condition $v_y(t, y, y) = 0$.

B11a: Communication Theory

Q1 The least popular question, with only four attempts that counted. This is possibly because it has an unfamiliar appearance; although it is in fact standard, and probably the easiest of the three questions. I guess that this is an unlooked-for corollary of 90 minute examinations, where it makes sense to go for a maximum of previously seen material ; with 3 hours in hand the candidate has time to assess unfamiliar problems. But the shorter exam makes it too risky to gamble with the unfamiliar.

Q2 The first part being routine, this was attempted by almost everyone with reasonable success; though the last part proved to be unexpectedly tricky –despite the hint– so there were not very many alphas.

Q3 Again, the first part is routine, so this also was attempted by almost everyone; with much greater success on the final unseen part here, so that there were a lot of good alpha solutions.

B11b: Graph Theory

Questions 1 and 2 were much more popular then question 3, with all but one candidate selecting question 2. This may have been because of the larger number of marks for bookwork, but it is quite tricky bookwork. Performance on all three questions was similar (slightly lower on question 2), so it doesn't seem to be the case that Q3 was harder.

Q1: Almost all candidates completed (a) successfully, though many followed a route from lectures rather than seeing that it is very easy given the allowed assumption (plus the handshaking lemma). The bookwork in part (b) was well done, though with surprisingly many logic errors (e.g., confusing a function being injective with its being well defined, i.e., each tree having only one code). A reasonable number of candidates managed the first part of (c) (use handshaking), not many thought of using Prüfer codes to then complete the question.

Q2: I don't think this was an easy question. Part (a) was mostly fine. (b) was rather variable. It is entirely bookwork, though leaving out some steps taken in notes. A few candidates blindly followed the notes leading to absurdities (which did not lose marks) such as using (a) to prove that if a graph has all degrees even then it can be decomposed into cycles, and using this only to show that the graph in question contains a cycle, which (a) already gives.

Some attempts at (b) were not rigorous enough - roughly the level of rigour in lectures/notes was expected.

(c) turned out to be difficult. Only one or two candidates managed the proof part, though quite a few came close. It suffices to consider faces with 3 edges and those with more separately, and to note that there are at most m/3 of the former since they are disjoint. Quite a few candidates had the correct intuition that in the extreme case every edge should have a 3-cycle on one side and a 4-cycle on the other. With this it is not too hard to find an example.

Q3: The first two parts were ok. For (c) many missed that in the case of n even all vertices have the same degree. No candidate realised that there is a quick way to deduce the second part of (c) from the first.

The attempts at (d) were mostly not very good, even though it is quite a minor variation of the proof of Turan's theorem, using (c) as an ingredient.

B21a: Numerical Solution of Differential Equations I

Question 1: The question was overall well done by the majority of candidates. Several candidates struggled with expanding a function of two variables into a Taylor series and had difficulties with parts (c) and (d).

Question 2: The question was very well done by most of the candidates.

Question 3: The question concerned the discrete maximum principle for the heat equation. It was only attempted by two candidates, bot of whom progressed very little beyond stating the finite difference approximations in parts a) and b) of the question.

B21b: Numerical Solution of Differential Equations II

Question 1: high level of mastery for this topic with many nearing full marks.

Question 2: good results for part b) maximum principle but less precise results for a) and c).

Question 3: fewer number of attempts, but typically near full marks for those attempting the question.

B22a: Integer Programming

Almost all 26 candidates seemed to have solved the questions in the order given on the exam paper, and as a result, gave answers to Q1 and Q2 only.

Q1: The candidates found this problem harder, including the bookwork parts, resulting in lower than expected marks, although with a reasonably looking dispersion of the distribution.

Q2: This problem was well solved, resulting in a typical distribution of marks, including several that obtained full marks.

Q3: This problem was only solved by 7 candidates, of whom 3 obtained full or nearly full marks, and 4 (all of whom had answered all three questions) obtained low marks. The dispersion of the distribution of marks was larger than for other questions, but the number of sample points is too small to draw strong conclusions, and the 4 candidates who obtained low marks seemed to have used this question as a backup solved in extra time.

BSP: Structured Projects

Assessment for this course is in three parts: a project completed at the end of HT (75%), a peer review completed over the Easter vacation (10%) and a presentation given at the start of TT (15%).

This year students were offered a choice of six topics: mathematical finance (chosen by 5), thermohaline circulation (5), subduction zones (1), graveyards (1), diffusion limited aggregation (1) and Turing boundary conditions (2).

Written projects and peer reviews were double-marked by two assessors, a different pair for each topic. Oral assessments were also double-marked, by a different pair of assessors. The standard of the oral presentations was high and often excellent.

O1: History of Mathematics

Both the extended coursework essays and the exam scripts were blind double-marked. The marks for essays and exam were reconciled separately, and then averaged (with equal weights) to obtain the final mark.

In the first part of the paper ('extracts'), five candidates (out of a total of nine) attempted question 1, and four attempted question 3. These were done reasonably well the material required for these questions was covered in some considerable detail in the lecture course. Thomas Hobbes' criticism of symbolic algebra ("... a scab of symbols ...") proved a popular quotation for inclusion in answers to question 1. Only one candidate tried question 2; likewise, question 6. Although question 2 relates to material that enjoyed a prominent position in one lecture, it lies slightly to the side of the main thrust of that part of the course (the development of calculus), knowledge of which would have perhaps left the candidate better placed to answer questions 1 or 3 instead. The content of question 6 was dealt with in reasonable detail in one lecture, but the mathematics involved might perhaps be characterised as more abstruse than that for the other questions. No candidates attempted question 5 this is not entirely surprising, as question 5 was a 'wildcard' question, the relevant material for which had appeared only briefly in a single lecture. By far the most popular question in this part of the paper was question 4, which seven candidates attempted this may have been because, as well as featuring in the MT lecture course, materials relevant to this question had also been used in the HT reading course. On the whole, the question was quite well done.

One point that was raised by the 'extracts' questions was that the candidates needed to distinguish a little more carefully between the 'context', 'content' and 'significance' of the piece of text under consideration many of the answers to these questions were structured around these terms as subheadings, but the contents of these subsections were muddled in places, with, for example, material that properly belonged under 'significance' appearing instead under 'context'. It should be emphasised that the use of the term 'context' here refers to the context in which a text was written (in which sense, 'context' was used throughout the course), not the context in which it is being read.

Turning to the second part of the paper ('essays'), three candidates attempted question 9, whilst six tackled question 8; no candidates tried question 7. This was a little surprising, given that the relevant subject matter for question 7 featured very prominently in the lecture course. The popularity of question 8 may be explained by its being a more 'general' question than the others: it did not require the candidates to focus on a single topic, but instead gave them an opportunity to draw upon a range of possible material, thus enabling them to play to their strengths. However, question 9 was, overall, done better than question 8.

Regarding the extended coursework essays, these were, on the whole, strikingly different from each other, although they did have a number of points in common. The essays differed similarly in their organisation: some were better structured and more clearly sign-posted than others. Sadly, only about half of the candidates had proof-read properly.

The range of sources used in the construction of the essays was quite wide. Some candidates used primary sources, and used them well, although the majority relied mainly on secondary sources of varied reliability. Two candidates had quotations from primary sources, but, peculiarly, cited secondary sources for them.

Five of the essays showed originality and boldness of assertions put forward one in particular was very good. The others, however, were a little less convincing.

N1b: Undergraduate Ambassadors Scheme

The assessment of the course is based on:

- Journal of Activities (20%)
- End of Course Report, Calculus Questionnaire and write-up (35%)
- Presentation (and associated analysis) (30%)
- Teachers Report (15%)

The Journal and Report were double-marked, with Nick Andrews (NA) and myself (RAE) as assessors. Nick was sole assessor for the Presentation. Each part was awarded a USM, and then an overall USM has been allocated according to the weightings above.

There were 12 students on the course this year, which is a welcome increase over recent years. The quality of the field was rather clear throughout with all students gaining high 2.1 marks overall. Quite a few first class marks were given for individual assessments and not a single 2.2 mark given.

Statistics Options

Reports of the following courses may be found in the Mathematics & Statistics Examiners' Report.

OBS1 Applied Statistics

OBS2a: Foundations of Statistical Inference

OBS3a: Applied Probability

OBS3b: Statistical Lifetime Models

OBS4a: Actuarial Science I

OBS4b: Actuarial Science II

Computer Science Options

Reports on the following courses may be found in the Mathematics & Computer Science Examiners' Reports.

CS3a: Lambda Calculus & Types

CS4b: Computational Complexity

Philosophy Options

The report on the following courses may be found in the Philosophy Examiners' Report.

102: Knowledge and Reality

122: Philosophy of Mathematics

E. Comments on performance of identifiable individuals and other material which would usually be treated as reserved business

Removed from the public version of the report.

F. Names of members of the Board of Examiners

- **Examiners:**
  Prof. C Batty (Chairman)
  Prof. R Hauser
  Prof. N Hitchin
  Prof. J Lister (External)
  Prof. J Marchini
  Prof. I Moroz
  Prof. G Seregin
  Prof. R Thomas (External)

- **Assessors:**
  Dr A Abate
  Prof. F Alday
  Prof. Sir J Ball
  Prof. R Baker
  Prof. D Belyaev
  Prof. M Bridson
  Prof. H Byrne
  Dr M Dallaston
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  Prof. K Erdmann
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Dr R Knight
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Prof. K McGerty
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Prof. I Moroz
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Dr P Neumann
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