

# Examiners' Report: Final Honour School of Mathematics Part C Trinity Term 2015

October 27, 2015

## Part I

### A. STATISTICS

- **Numbers and percentages in each class.**

See Table 1, page 1.

- **Numbers of vivas and effects of vivas on classes of result.**

As in previous years there were no vivas conducted for the FHS of Mathematics Part C.

- **Marking of scripts.**

The dissertations were double marked. The remaining scripts were all single marked according to a pre-agreed marking scheme which was very closely adhered to. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**

See Table 7 on page 8.

Table 1: Numbers in each class

	Number					Percentages %				
	2015	(2014)	(2013)	(2012)	(2011)	2015	(2014)	(2013)	(2012)	(2011)
I	45	(45)	(56)	(45)	(47)	46.39	(45.92)	(47.46)	(45.45)	(46.53)
II.1	39	(42)	(41)	(36)	(37)	40.21	(42.86)	(34.75)	(36.36)	(36.63)
II.2	13	(11)	(15)	(15)	(14)	13.40	(11.22)	(12.71)	(15.15)	(13.86)
III	0	(0)	(4)	(3)	(1)	0	(0)	(3.39)	(3.03)	(0.99)
F	0	(0)	(2)	(0)	(2)	0	(0)	(1.69)	(0)	(1.98)
Total	97	(98)	(118)	(99)	(101)	100	(100)	(100)	(100)	(100)

## **B. New examining methods and procedures**

The University had introduced new procedures for considering factors affecting performance of individual candidates which the examiners followed.

## **C. Changes in examining methods and procedures currently under discussion or contemplated for the future**

The Teaching Committee is considering increasing the time allowed for mathematics papers from 1.5 hours to 2 hours.

## **D. Notice of examination conventions for candidates**

The first notice to candidates was issued on 19th February 2015 and the second notice on 23rd April 2015. These contain details of the examinations and assessments.

All notices and the examination conventions for 2015 examinations are on-line at <http://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments>.

# **Part II**

## **A. General Comments on the Examination**

The examiners would like to thank in particular Helen Lowe, Waldemar Schlackow and Charlotte Turner-Smith for their commitment and dedication in running the examination systems. We would also like to thank Nia Roderick, and the rest of the Academic Administration Team for all their work during the busy exam period.

We also thank the assessors for their work in setting questions on their own courses, and for their assistance in carefully checking the draft questions of other assessors, and also to the many people who acted as assessors for dissertations. We are particularly grateful to those—this year the great majority—who abided by the specified deadlines and responded promptly to queries. This level of cooperation contributed in a significant way to the smooth running of what is of necessity a complicated process.

The internal examiners would like to thank the external examiners Professor Jack Carr and Professor Alexei Skorobogatov for their prompt and careful reading of the draft papers and for their valuable input during the examiners' meeting.

## **Timetable**

The examinations began on Monday 1st June and finished on Monday 15th June.

## **Medical certificates and other special circumstances**

The examiners were presented with factors affecting performance applications for five candidates. They took into account a report of noise disturbance during one examination.

## Setting and checking of papers and marks processing

Following established practice, the questions for each paper were initially set by the course lecturer, with the lecturer of a related unit involved as checker before the first draft of the questions was presented to the examiners. The course lecturers also acted as assessors, marking the questions on their course(s).

The internal examiners met in early January to consider the questions on Michaelmas Term courses, and changes and corrections were agreed with the lecturers where necessary. The revised questions were then sent to the external examiners. Feedback from external examiners was given to examiners, and to the relevant assessor for each paper for a response. The internal examiners met a second time late in Hilary Term to consider the external examiners' comments and assessor responses (and also Michaelmas Term course papers submitted late). The cycle was repeated for the Hilary Term courses, with two examiners' meetings in the Easter Vacation; the schedule here was much tighter. Following the preparation of the Camera Ready Copy of the papers as finally approved, each assessor signed off their paper in time for submission to Examination Schools in week 1 of Trinity Term.

A team of graduate checkers, under the supervision of Helen Lowe, sorted all the marked scripts for each paper of this examination, carefully cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way a number of errors were corrected, each change was signed by one of the examiners who were present throughout the process. A check-sum is also carried out to ensure that marks entered into the database are correctly read and transposed from the marks sheets.

## Determination of University Standardised Marks

The Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average in each class over the last four years, together with recent historic data for Part C, the MPLS Divisional averages, and the distribution of classifications achieved by the same group of students at Part B.

The examiners followed established practice in determining the University standardised marks (USMs) reported to candidates. This leads to classifications awarded at Part C broadly reflecting the overall distribution of classifications which had been achieved the previous year by the same students. We outline the principles of the calibration method.

The Department's algorithm to assign USMs in Part C was used in the same way as last year for each unit assessed by means of a traditional written examination. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part B classification of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage). Working with the data for this population, numbers  $N_1$ ,  $N_2$  and  $N_3$  are first computed for each paper:  $N_1$ ,  $N_2$  and  $N_3$  are, respectively, the number of candidates taking the paper who achieved in Part B overall average USMs in the ranges  $[70, 100]$ ,  $[60, 69]$  and  $[0, 59]$ , respectively.

The algorithm converts raw marks to USMs for each paper separately (in each case, the raw marks are initially out of 50, but are scaled to marks out of 100). For each paper, the algorithm sets up a map  $R \rightarrow U$  ( $R = \text{raw}$ ,  $U = \text{USM}$ ) which is piecewise linear. The graph of the map consists of four line segments: by default these join the points  $(100, 100)$ ,  $P_1 = (C_1, 72)$ ,  $P_2 = (C_2, 57)$ ,  $P_3 = (C_3, 37)$ , and  $(0, 0)$ . The values of  $C_1$  and  $C_2$  are set by the requirement that the proportion of I and II.1 candidates in Part B, as given by  $N_1$  and  $N_2$ , is the same as the I and II.1 proportion of USMs achieved on the paper. The value of  $C_3$  is set by the requirement that  $P_2P_3$  continued would intersect the  $U$  axis at  $U_0 = 20$ . Here the default choice of *corners* is given by  $U$ -values of 72, 57 and 37 to avoid distorting nonlinearity at the class borderlines.

The results of the algorithm with the default settings of the parameters provide the starting point for the determination of USMs. The examiners have scope to make changes, usually by adjusting the position of the corner points  $P_1, P_2, P_3$  by hand, so as to alter the map  $\text{raw} \rightarrow \text{USM}$ , to remedy any perceived unfairness introduced by the algorithm, in particular in cases where the number of candidates is small. They also have the option to introduce additional corners.

Table 2 on page 5 gives the final positions of the corners of the piecewise linear maps used to determine USMs from raw marks. For each paper,  $P_1, P_2, P_3$  are the (possibly adjusted) positions of the corners above, which together with the end points  $(100, 100)$  and  $(0, 0)$  determine the piecewise linear map  $\text{raw} \rightarrow \text{USM}$ . The entries  $N_1, N_2, N_3$  give the number of incoming firsts, II.1s, and II.2s and below respectively from Part B for that paper, which are used by the algorithm to determine the positions of  $P_1, P_2, P_3$ .

Following customary practice, a preliminary, non-plenary, meeting of examiners was held two days ahead of the plenary examiners' meeting to assess the results produced by the algorithm alongside the reports from assessors. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low. These revised USM maps provided the starting point for a review of the scalings, paper by paper, by the full board of examiners.

Table 2: Position of corners of piecewise linear function

Paper	$P_1$	$P_2$	$P_3$	Additional corners	$N_1$	$N_2$	$N_3$
C1.1	(9,37)	(18,57)	(38,72)		4	5	0
C1.2	(8,37)	(18,57)	(41,70)		1	4	0
C1.3	(8,37)	(14,57)	(32.2,72)		7	6	1
C1.4	(8,47)	(15,57)	(30,72)		6	9	0
C2.1	(2.39,37)	(14,57)	(38,72)		3	5	0
C2.2	(8,37)	(16,57)	(31.2,72)		4	5	0
C2.4	(7.81,37)	(17,57)	(32,72)		4	5	1
C2.5	(7.12,37)	(22,57)	(37,72)		3	5	1
C2.6	(6,37)	(18,57)	(26,72)		9	12	1
C3.1	(6,37)	(17,57)	(26.8,72)		5	9	0
C3.2	(7.12,37)	(20,57)	(37,72)		5	8	0
C3.3	(4.82,37)	(18,57)	(33,72)		2	3	0
C3.4	(9.60,37)	(26,57)	(36,72)		3	6	0
C3.5	(7.76,37)	(22,57)	(36,72)		4	6	0
C3.6	(12.31,37)	(28,57)	(36,72)		3	4	0
C3.7	(4.41,37)	(26,57)	(36.6,72)		5	6	0
C3.8	(3.8,37)	(26,57)	(41,72)		4	12	1
C4.1	(12.5,37)	(25,57)	(35,72)		7	2	3
C4.2	(12,37)	(20,57)	(34,72)		6	2	3
C4.3	(11.6,37)	(25.2,57)	(34.2,72)		4	2	2
C4.4	(10.01,37)	(20,57)	(27.8,72)		2	1	1
C4.5	(5,37)	(20,57)	(32,72)		6	3	0
C5.1	(10.66,37)	(23.2,57)	(39,72)		9	18	3
C5.2	(7.12,37)	(15.5,57)	(38,72)		9	14	2
C5.3	(6.48,37)	(19,57)	(38,72)		3	6	2
C5.4	(4.92,37)	(16,57)	(27.2,72)		12	15	3
C5.5	(9,37)	(16,57)	(32,72)		16	24	3
C5.6	(9.56,37)	(24,57)	(39,72)		9	15	1
C5.7	(10,50)	(15,57)	(32,72)		10	14	0
C5.8	(10,46)	(17,57)	(26.4,72)		7	14	2
C5.9	(7.21,37)	(25,57)	(36,72)		5	6	1
C5.11	(8.73,37)	(19,57)	(34,72)		4	8	0
C5.12	(6.11,37)	(19,57)	(40,72)		8	19	1
C6.1	(9.51,37)	(24,57)	(37.2,72)		5	7	2
C6.2	(9,37)	(19,57)	(34.6,72)		4	13	2
C6.3	(9.51,27)	(24,57)	(34.5,72)		5	10	1

Paper	$P_1$	$P_2$	$P_3$	Additional corners	$N_1$	$N_2$	$N_3$
C6.4	(10.34,37)	(27,57)	(42,72)		5	12	2
C7.1	(18,37)	(45,57)	(72,72)		1	2	1
C7.2	(8,37)	(21,57)	(32,72)		2	3	0
C7.3	(9,37)	(24,57)	(36,72)		1	1	1
C7.4	(21.00,37)	(33,57)	(41,72)		1	2	1
C7.5	(12.91,37)	(28.1,57)	(36,72)		1	2	1
C7.6	(13.55,37)	(28,57)	(36,72)		1	2	1
C8.1	(6.57,37)	(26,57)	(38,72)		6	3	1
C8.2	(7.49,37)	(27,57)	(42,72)		6	2	1
C8.3	(9,37)	(22,57)	(32,72)		13	18	1
C8.4	(9,37)	(28,57)	(38,72)		11	16	2
SC1	(9.01,37)	(29,57)	(40,72)		13	16	1
SC2	(6.34,37)	(25,57)	(40,70)		6	14	1
SC3	(9,37)	(23,57)	(35,72)		3	4	1
SC4	(7,37)	(17,57)	(32,72)		6	11	1
SC5	(7.58,37)	(26,57)	(38,72)		5	7	1

Table 4 on page 6 gives the rank of candidates and the number and percentage of candidates attaining this or a greater (weighted) average USM.

Table 4: Percentile table for overall USMs

Av USM	Rank	Candidates with this USM or above	%
91	1	1	1.03
88	2	3	3.09
86	4	4	4.12
85	5	7	7.22
84	8	8	8.25
83	9	10	10.31
81	11	12	12.37
80	13	13	13.40
79	14	16	16.49
78	17	19	19.59
77	20	22	22.68
75	23	27	27.84
74	28	30	30.93
73	31	34	35.05
72	35	36	37.11
71	37	39	40.21
70	40	45	46.39
69	46	53	54.64
68	54	55	56.70
67	56	61	62.89
66	62	64	65.98
65	65	69	71.13

Av USM	Rank	Candidates with this USM or above	%
64	70	72	74.23
63	73	76	78.35
62	77	77	79.38
61	78	80	82.47
60	81	84	86.60
59	85	87	89.69
58	88	88	90.72
57	89	90	92.78
54	91	94	96.91
53	95	97	100

## B. Breakdown of the results by gender

Table 6, on page 7 shows the performances of candidates broken down by gender.

Table 6: Breakdown of results by gender

Class	Total		Male		Female	
	Number	%	Number	%	Number	%
I	45	46.39	37	46.84	8	44.44
II.1	39	40.21	32	40.51	7	38.89
II.2	13	13.40	10	12.66	3	16.67
III	0	0	0	0	0	0
F	0	0	0	0	0	0
Total	97	100	79	100	18	100

### C. Detailed numbers on candidates' performance in each part of the exam

Data for papers with fewer than six candidates are not included.

Table 7: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
C1.1	9	32.56	7.99	68.78	8.30
C1.2	5	-	-	-	-
C1.3	14	25.86	7.55	66.43	8.33
C1.4	15	25.00	7.95	67.33	9.26
C2.1	8	34.50	10.64	73.00	11.38
C2.2	9	27.11	9.08	68.33	11.59
C2.4	10	28.00	7.57	68.50	8.95
C2.5	9	27.78	7.97	62.33	8.76
C2.6	22	19.05	9.33	58.77	14.77
C3.1	14	19.14	7.25	59.07	13.49
C3.2	13	31.69	6.01	67.77	6.11
C3.3	5	-	-	-	-
C3.4	9	37.44	7.11	76.11	12.60
C3.5	10	33.20	9.75	71.80	14.67
C3.6	7	39.43	7.30	79.00	14.41
C3.7	11	30.64	7.89	64.82	10.76
C3.8	17	37.88	8.91	72.29	13.10
C4.1	12	33.25	8.04	70.25	13.21
C4.2	11	30.55	8.52	68.55	11.89
C4.3	8	32.88	9.43	70.38	16.04
C4.4	4	-	-	-	-
C4.5	9	28.89	11.59	69.00	15.95
C5.1	30	38.30	8.15	76.00	12.91
C5.2	25	31.00	8.02	68.72	8.66
C5.3	11	31.27	11.45	69.45	14.68
C5.4	26	24.23	8.83	67.46	11.62
C5.5	41	25.20	9.10	65.37	11.84
C5.6	24	35.38	6.03	70.00	8.49
C5.7	24	29.75	8.63	71.29	10.11
C5.8	21	19.71	6.77	61.05	10.15
C5.9	12	36.83	10.54	76.25	16.35
C5.11	12	29.67	4.42	68.00	5.26
C5.12	28	33.57	7.37	68.71	7.85
C6.1	14	31.64	6.45	66.50	8.79
C6.2	16	23.75	9.98	60.56	13.98
C6.3	16	32.12	5.70	69.12	9.04
C6.4	19	37.00	7.23	68.37	9.62
C7.1	4	-	-	-	-
C7.2	5	-	-	-	-
C7.3	3	-	-	-	-

Paper	Number of Candidates	Avg	StDev	Avg	StDev
		RAW	RAW	USM	USM
C7.4	4	-	-	-	-
C7.5	4	-	-	-	-
C7.6	4	-	-	-	-
C8.1	9	38.11	8.88	76.11	13.91
C8.2	8	40.38	10.60	77.38	17.15
C8.3	30	30.97	6.40	70.67	9.76
C8.4	27	35.30	8.26	70.22	13.49
SC1	15	41.53	6.17	78.73	12.87
SC2	9	37.22	6.38	70.78	11.87
SC4	6	30.67	8.89	72.67	11.962
SC5	3	-	-	-	-
CCS1	1	-	-	-	-
CCS2	2	-	-	-	-
CCS3	2	-	-	-	-
Double Unit CCD Dissertation	41	-	-	73.32	7.69
Double Unit COD Dissertation	2	-	-	-	-

The tables that follow give the question statistics for each paper for Mathematics candidates. Data for papers with fewer than six candidates are not included.

#### Paper C1.1: Model Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.50	19.50	4.32	6	0
Q2	14.63	16.00	5.45	7	1
Q3	12.80	12.80	5.54	5	0

#### Paper C1.3: Analytic Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.5	14.6	5.16	11	1
Q2	11.0	11.0	5.05	9	0
Q3	12.8	12.8	3.69	8	0

#### Paper C1.4: Axiomatic Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.5	13.5	4.58	15	0
Q2	9.5	11.0	2.12	1	1
Q3	11.6	11.6	4.82	14	0

**Paper C2.1: Lie Algebras**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.2	17.2	4.31	6	0
Q2	21.0	21.0	2.12	5	0
Q3	13.0	13.6	7.13	5	1

**Paper C2.2: Homological Algebra**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.3	14.75	6.04	8	1
Q2	14.75	14.75	6.99	4	0
Q3	11.17	11.17	5.199	6	0

**Paper C2.4: Infinite Groups**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.4	14.4	4.01	10	0
Q2	15.7	15.7	4.41	6	0
Q3	9.2	10.5	5.07	4	1

**Paper C2.5: Non-Commutative Rings**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	9.6	10.8	5.18	4	1
Q2	14.7	14.7	4.13	6	0
Q3	13.6	14.9	5.36	8	1

**Paper C2.6: Commutative Algebra**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	9.3	9.3	6.21	22	0
Q2	9.3	9.3	3.13	20	0
Q3	9.0	14.5	10.42	2	2

**Paper C3.1: Algebraic Topology**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	7.9	7.9	2.14	13	0
Q2	12.2	12.2	5.31	10	0
Q3	8.5	10.8	4.59	4	2

**Paper C3.2: Geometric Group Theory**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.0	19.0	3.24	13	0
Q2	13.1	13.1	3.56	11	0
Q3	10.5	10.5	3.54	2	0

**Paper C3.4: Algebraic Geometry**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	22.0	22.0	3.35	6	0
Q2	13.7	13.7	3.06	3	0
Q3	18.2	18.2	3.23	9	0

**Paper C3.5: Lie Groups**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.5	15.5	7.47	10	0
Q2	21.0	21.0	3.16	10	0
Q3	14.4	14.4	5.32	5	0

**Paper C3.6: Modular Forms**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.3	17.3	4.08	6	0
Q2	21.1	21.1	3.39	7	0
Q3	24.0	24.0		1	0

**Paper C3.7: Elliptic Curves**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.8	14.9	6.48	7	1
Q2	15.7	15.7	4.06	10	0
Q3	15.2	15.2	3.77	5	0

**Paper C3.8: Analytic Number Theory**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.3	19.5	7.05	15	1
Q2	16.8	16.8	5.23	13	0
Q3	22.2	22.2	2.71	6	0

**Paper C4.1: Functional Analysis**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.5	18.5	4.72	10	0
Q2	13.0	15.0	6.98	6	1
Q3	15.5	15.5	5.18	8	0

**Paper C4.2: Linear Operators**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.9	17.2	4.14	6	1
Q2	15.0	15.0	4.15	11	0
Q3	13.6	13.6	6.54	5	0

**Paper C4.3: Functional Analytical Methods for PDEs**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.6	16.6	5.04	8	0
Q3	16.3	16.3	5.62	8	0

**Paper C4.5: Ergodic Theory**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	9.86	9.86	2.91	7	0
Q2	14.0	18.3	9.27	3	1
Q3	17.0	17.0	7.52	8	0

**Paper C5.1: Solid Mechanics**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.3	20.3	4.68	30	0
Q2	17.3	17.8	5.37	27	2
Q3	19.7	19.7	2.89	3	0

**Paper C5.2: Elasticity and Plasticity**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.4	16.4	3.10	17	0
Q2	14.7	15.3	5.82	12	1
Q3	14.0	15.0	6.36	21	2

**Paper C5.3: Statistical Mechanics**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.6	17.1	9.01	9	1
Q2	10.5	10.5	3.70	4	0
Q3	16.4	16.4	4.64	9	0

**Paper C5.4: Networks**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	9.14	9.5	4.00	20	1
Q2	13.0	14.3	6.31	19	3
Q3	12.4	13.0	4.93	13	1

**Paper C5.5: Perturbation Methods**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.9	12.9	3.91	34	0
Q2	11.9	11.9	6.07	25	0
Q3	11.7	13.0	6.76	23	3

**Paper C5.6: Applied Complex Variables**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.1	17.1	3.57	10	0
Q2	18.2	18.2	2.85	10	0
Q3	17.1	17.5	3.84	18	1

**Paper C5.7: Topics in Fluid Mechanics**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.8	13.8	4.60	24	0
Q2	7.3	11.8	5.18	4	4
Q3	16.9	16.9	4.92	20	0

**Paper C5.8: Stochastic Modelling of Biological Processes**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.5	10.5	3.87	21	0
Q2	8.6	8.9	4.72	19	1
Q3	9.3	12.5	4.79	2	2

**Paper C5.9: Mathematical Mechanical Biology**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	8.5	12.0	5.43	3	3
Q2	21.3	21.3	4.06	10	0
Q3	17.5	17.5	6.53	11	0

**Paper C5.11: Mathematical Geoscience**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.5	15.5	2.81	6	0
Q2	14.0	14.0	2.89	12	0
Q3	15.8	15.8	2.86	6	0

**Paper C5.12: Mathematical Physiology**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.3	15.3	2.19	15	0
Q2	15.9	16.8	5.94	18	1
Q3	17.7	17.7	5.53	23	0

**Paper C6.1: Numerical Linear Algebra**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.7	15.7	4.14	14	0
Q2	15.9	15.9	3.99	14	0

**Paper C6.2: Continuous Optimization**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.8	11.8	5.99	16	0
Q2	11.6	12.5	5.93	11	1
Q3	10.8	10.8	3.56	5	0

**Paper C6.3: Approximation of Functions**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.5	14.5	5.50	12	1
Q2	18.1	18.1	2.29	16	0
Q3	12.8	12.8	3.19	4	1

**Paper C6.4: Finite Element Methods for Partial Differential Equations**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.1	20.1	3.40	17	0
Q2	17.6	17.6	3.54	15	0
Q3	15.1	16.3	6.67	6	1

**Paper C8.1: Stochastic Differential Equations**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.8	17.8	6.45	4	0
Q2	20.0	20.0	3.07	8	0
Q3	18.7	18.7	5.43	6	0

**Paper C8.2: Stochastic Analysis and PDEs**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.8	21.4	8.86	7	1
Q2	20.4	20.4	4.04	5	0
Q3	17.8	17.8	9.50	4	0

**Paper C8.3: Combinatorics**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.4	17.4	2.73	28	0
Q2	11.5	11.5	4.26	22	0
Q3	16.3	18.8	6.85	10	2

**Paper C8.4: Probabilistic Combinatorics**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.2	17.2	4.74	26	0
Q2	12.8	12.8	4.97	5	0
Q3	18.5	19.2	4.25	23	1

**Paper SC1: Stochastic Models in Mathematical Genetics**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.7	20.7	5.28	11	1
Q2	21.5	21.5	2.36	15	0
Q3	18.0	18.0	4.97	4	0

## Paper SC2: Probability and Statistics for Network Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.4	20.4	2.99	7	0
Q2	17.7	17.7	5.05	6	0
Q3	17.2	17.2	1.30	5	0

## SC3: Statistical Data Mining and Machine Learning

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.0	15.8	6.32	4	1
Q2	13.3	13.3	4.04	3	0
Q3	16.2	16.2	5.54	5	0

## D. Recommendations for Next Year's Examiners and Teaching Committee

### Similarity of questions to those of previous years

There was one case this year of an assessor setting a question which was almost identical to a question from last year's paper. Fortunately this was identified by the examiners, and the question was eventually substantially altered. However, this similarity should have been identified by the checker. Perhaps teaching committee could consider making explicit in the advice to setters and checkers that questions should be substantially different from those of recent years.

### Presence of assessors during exams

Some confusion arose this year from a typo in a paper which was not identified during the first half hour of the examination for which the assessor was present. Although the assessor was contacted by phone, the matter was not satisfactorily resolved, and a small number of candidates were adversely affected. The examiners took some time to determine how to treat fairly these candidates in their final meeting. Had the assessor been present for the whole exam the typo would have been dealt with swiftly and no confusion would have arisen. We would ask teaching committee to consider whether assessors should be present for the duration of an exam, rather than just the first thirty minutes.

### Hand-written sample solutions

Both external examiners commented that typed model solutions made their job of checking the questions much easier, and that some of the hand-written answers were impossible to comprehend. We suggest that all assessors be encouraged to provide typed model solutions.

## Dissertations

The examiners wondered whether it was possible to attain very high marks in a dissertation, and were concerned that this may dissuade very able candidates from choosing to do a dissertation. Teaching committee may like to consider this issue.

## E. Comments on papers and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. Some data to be found in Section C above have been omitted.

### C1.1: Model Theory

The exam questions this year contained slightly stronger “unseen” components, compared to the recent years. This especially effected the marks for questions 2 and 3. Although the unseen parts of the questions could have been answered in a quite short way, none of the candidates gave full enough answers to these and the overall scores for the two questions are all below 21. Question 1 proved to be easier: 6 out of 10 candidates who attempted the question scored 22 and above. Overall the answers were quite satisfactory.

### C1.2: Gödel’s Incompleteness Theorems

Question 1 tested the provability of the diagonal lemma within a given theory, via a sequence of independently important results, a key theorem needed for major results in the second half of the course. This was essentially bookwork. Five of the nine students did very well with it.

Question 2 tested Löbs Theorem and the Second Incompleteness Theorem, and included two previously unseen applications of Löbs Theorem, and one section that tested material from the beginning of the course. Five of the nine students did very well with it.

Question 3 was on provability logic, from the last part of the course.

### C1.3: Analytic Topology

The questions were roughly equally popular.

Overall, bookwork was done mostly well and promising and insightful attempts were made for the other parts. However, too much ‘wishful mathematics’ and incorrect assumptions meant that only few of these promising attempts were made into full and rigorous solutions. In particular Q2(b) seemed difficult.

Also notable was that candidates seemed unwilling to proceed to later parts of a question without fully completing the earlier parts.

Question 1. Although the question explicitly said to not carry out any inductive construction in detail, a lot of candidates still did and thus lost time without gaining marks.

In (b) a lot of candidates asserted that  $[0, 1]$  was homeomorphic to  $\mathbb{R}$ , or if they embedded  $\mathbb{R}$  as  $(0, 1)$  into  $[0, 1]$  did not adjust the extension given by Tietze’s Theorem to ensure that

the values 0 and 1 were not taken.

Question 2. Instead of giving the Stone-Ćech property (continuous maps into any compact Hausdorff space can be extended) a number of candidates only considered compactifications.

In (b) a lots of candidates wrongly assumed (implicitly or explicitly) that projections of closed sets are closed. For the forward implication only very few candidates considered the obvious application of the Embedding Lemma with the family of all continuous real-valued functions.

Question 3. In the answers the relativization of “open” and “closed” were not always clear. In (b), a lot of candidates looked at  $\bigcup \mathcal{C}_0$  instead of its complement. Also, despite proving that  $x \in \overline{U} \cap \overline{V}$ , so that  $x \notin \text{int}(U \cup \{x\})$ , a lot of candidates claimed that  $U \cup \{x\}$  and/or  $V \cup \{x\}$  are in  $\mathcal{C}_0$ .

### C1.4: Axiomatic Set Theory

Most answers dealt with the bookwork reasonably well, but often lacked some precision. For the non-bookwork parts, there were a lot of promising ideas, but again, a lack of precision and scepticism.

Only very few candidates attempted question 2.

Question 1. In (a) a lot of candidates were not careful about obtaining a weakly increasing function into  $\alpha$ .

In (b) for minimality, candidates claimed that  $\alpha \mapsto \kappa_\alpha$  would be unbounded, although  $\kappa \rightarrow \kappa; \alpha \mapsto 1$  and  $\kappa = \sum_{\alpha \in \kappa} 1$  shows that this doesn't quite work. The idea needs a little bit more of an argument to be shown to work.

Parts (c) and (d) saw a wide variety of attempts; the most common mistake was to claim  $\kappa^\mu \leq \kappa$  for  $\mu < \kappa$  (in the part without GCH).

Question 2. There were too few attempts to make informed comments about common difficulties.

Question 3. In (a) the injectivity of the Mostowski collapse was often done in a very handwaving fashion (e.g. ‘by  $R$ -induction’ or ‘by minimality of  $x, y$  such that  $\text{mos}(x) = \text{mos}(y)$ ’) which made the last part of (a) impossible: candidates needed to get from the class of counterexamples to injectivity to a set of counterexamples to apply well-foundedness.

For (b), a lot of attempts used the lexicographic order, although this is clearly not set-like. Also, almost all candidates failed to read the question properly for the deduce part. They mostly showed that  $On^A = On^B \rightarrow (On \times On)^A = (On \times On)^B$  whilst what was asked was (informally)  $\mathcal{P}(On)^A = \mathcal{P}(On)^B \rightarrow \mathcal{P}(On \times On)^A = \mathcal{P}(On \times On)^B$ .

Good attempts were made for (c), but none were carried out entirely correctly.

### C2.1: Lie Algebras

Question 1. Part (b) is bookwork, but there were difficulties in remembering the proof. A common mistake was to use that if  $I$  is an ideal in a semisimple algebra  $\mathfrak{g}$ , then  $\mathfrak{g} = I \oplus I^\perp$ .

This result does not apply directly, since here  $I$  is semisimple and  $\mathfrak{g}$  is not.

Part (c)(ii) was the most difficult part of the question. Most students assumed implicitly that if  $k.x$  is the complement of a codimension one ideal in  $\mathfrak{n}$ , then  $x$  centralises the ideal. This isn't necessarily the case. Instead, one should investigate which conditions a derivation must satisfy in order to vanish on the codimension one ideal.

Question 2. The only difficulty was with part (c)(ii). For this one should compare  $\dim \mathfrak{g}$  and  $\dim V$ , after using (b) to argue that  $C_V$  acts by a scalar.

Question 3. This problem, particularly (a), was found quite long by most students. In part (c), the easiest approach is to use the Cartan decomposition and the easy facts about the commutator of root spaces, e.g.  $[\mathfrak{g}_\alpha, \mathfrak{g}_\beta]C^{\mathfrak{g}_{\alpha+\beta}}$ , to see that with respect to the Cartan decomposition, the killing form consists of five  $2 \times 2$  blocks, each nondegenerate.

In part (b), the easiest solution is to consider the formula of the killing form on the Cartan in terms of evaluation of roots.

## C2.2: Homological Algebra

Question 1. Parts (a) and (b) of the question were bookwork. Students who knew the bookwork well did very well on this question.

Question 2. Part (a) was bookwork. Parts (b) and (c) were new but are very standard material on group cohomology appearing in any textbook. Not many students attempted this question. Most of the students who attempted it did quite well.

Question 3. Parts (a) and (b) were bookwork. Students who knew the bookwork did well on this question.

## C2.4: Infinite Groups

Question 1. This was the most popular question. Parts (a) & (b) were bookwork, surprisingly many errors in part (c) which was close to a question from problem sheets. Only one complete solution to part (d).

Question 2. Mostly standard. The last part was solved by only a few candidates.

Question 3. Seems to be the hardest question despite part (c) being very similar to a question from a problem sheet. Only one complete solution to part (d).

## C2.5: Non-Commutative Rings

Question 1. This was the least popular question, attracting fewer good answers than question 2 and question 3. Several candidates did not appreciate the difference between the definition of prime ideals for non-commutative rings and the corresponding definition in the commutative case! Only one candidate gave a correct answer to part (e), and only two to part (d).

Question 2. Parts (a),(b), (c) were done reasonably well by nearly all candidates who attempted this question. Part (d) proved the most challenging, although two people nearly cracked it.

Question 3. The most popular and most straightforward question. Everything but part (e) was essentially bookwork, but part (e) stumped everyone! This was unfortunate, since the idea behind the solution had already been seen in Exercise 5.5 (c).

## C2.6 Commutative Algebra

Q.1 (a) and (b) bookwork, mostly well done. (b) and (c) quite easy but not many got the idea.

Q.2 (a) bookwork, mostly well done. (b) first step done by many, only a few managed the rider.. (c) very few got the idea. (d) seemed to cause confusion.

Q.3 (a) bookwork: very few recalled the argument. (b) ok. (c) too hard to grasp, really. A few did parts of it but no one got it all.

## C3.1: Algebraic Topology

Question 1. (a) Most candidates incorrectly identified either a transverse or meridian curve of the Mobius strip (rather than the boundary curve of the Mobius strip as asked) with the curve  $RP^2$ . Note that this exact same complex appeared as the sole focus of question 1 of last year's exam, and so should not have caused difficulty. Indeed, parts (a) and (b) of this year's exam are practically the same as question 1 of last year's exam, which also appeared as question 5 of problem sheet 3 this year. Students are highly advised to carefully study past papers and the problem sheets. (b) with one exception, no candidates correctly computed the cohomology of  $D$ , despite the computation being entirely analogous to (or indeed an immediate consequence of, using UCT) question 1(b) from last year's exam, or question 5 of problem sheet 3.

Question 2. (b) Many candidates applied UCT and Poincare Duality effectively in part (i). Note that this part was very close to question 3(b) from last year's exam. A number of candidates supplied erroneous 'proofs' of the statement in part (iii), rather than noting that any nonzero torsion class provides a counterexample. (c) A number of candidates correctly observed the non-obvious fact that case iv occurs as the cohomology of a quotient of the 3-sphere.

Question 3. (a) Numerous candidates failed to accurately specify the attaching maps for the complex, as the identification of antipodal points. (b) In some scripts, attention was paid to computing the homology or cohomology groups, with little or no mention of the cup product. (c) A few candidates saw the general arc a proof might follow for this more conceptual part of the question, but no complete solutions were given.

## C3.2 Geometric Group Theory

Question 1. This was a basic question about presentations and algorithmic problems. All students attempted this and did generally well. In part (a) some candidates did not rule out the case  $G$  cyclic when showing that  $G$  is not free. In part (b) in the algorithm for simple groups some candidates did not explain how to check that the normal closure of an element is the whole group and only got partial credit for their solution. Quite a few candidates had

some difficulty with the characterisation of finite groups and only one candidate managed to answer correctly the last part.

Question 2. This was a question on amalgamated products and actions in trees attempted by most students. Part (a) was done generally well, except for the last question where some students instead of using a homomorphism to an abelian group tried to work directly with generators and did not get very far. In part (b) few students realised that they should use normal forms and many tried to tackle this using the definition of an amalgam instead. Most of them incorrectly stated that the index of  $H$  is 2. In part (c) most candidates stated correctly Kuroch's theorem. No one gave a fully justified answer of the last part but some got partial credit by applying the right method considering the action of the subgroup on the tree of the amalgam.

Question 3. This question was attempted by only 2 students. It was on the last part of the course dealing with hyperbolic groups. In part (a) a candidate confused the definitions of thin and slim triangles. Part (b) was generally done well. Part (c) was not done well - most of it was bookwork- but a rather challenging part of bookwork.

### **C3.3: Differentiable Manifolds**

In general the questions were too hard, at least for this cohort. Each one required a degree of carefulness which was absent, although the basic task was reasonably elementary.

Question 1: Candidates did not understand the coordinate change for the tangent bundle.

Question 2: The main problem was not understanding the basic properties of the Lie bracket of two vector fields. One good solution.

Question 3: They understood what they had to do but fell apart with trying to find the inverse images of the function  $F$ .

### **C3.4: Algebraic Geometry**

The general standard of bookwork and its direct applications was high, but candidates had difficulties solving the last part of the questions.

Question 1. Bookwork in parts (a)-(d) caused no problems. Most students attempted the computational part (e) and achieved reasonable results both in finding the equations of the blow up and the number of iterations needed.

Question 2. This was the least popular question with no complete solution to part (d). Only one candidate attempted to calculate the Hilbert polynomial.

Question 3. The most popular question: all candidates attempted this. High standard of answers although most candidates struggled to give a complete description of the orbits in part (e).

### **C3.5: Lie Groups**

Everyone attempted question 1, candidates then evenly chose either question 2 or question 3.

There was one perfect score.

Candidates often struggled to solve question 1 (b)(ii) fully because they did not use the Taylor series for the exponential of a matrix. Very few did question 1 (b)(iv).

Only one candidate did question 3 (b)(iv), (v). Many tried to solve question 3 (b)(ii) without using the hint or using a different  $f$ . There was some confusion about what the action of  $G$  was on  $\text{Hom}(v, v)$  in question 3 (a)(iii) which solutions tried to camouflage, but which eventually became problematic in 3(b)(i) and (ii) where an explicit formula for the action is needed.

### **C3.6: Modular Forms**

Question 1. This was attempted by all but one candidate. Part (a) was seen material and generally done well. Part (b) was original, but included some fairly routine computations - many of the candidates got the rough idea for (b)(iii), with some giving complete solutions.

Question 2. This was based largely upon material from the lecture notes and problem sheets and attempted by all the candidates. All subparts were completed by some majority among the candidates, no specific difficulties being encountered.

Question 3. This question was based upon the final sections of the course, and only taken by one candidate (who gave a very good solution). Perhaps candidates were discouraged from this question because it involved later material from the course and was not divided so neatly into subparts.

### **C3.7: Elliptic Curves**

Question 1. Part (a): standard bookwork. Most students who attempted this did fine. Part (b): this question proved to be quite difficult. Many were unfamiliar with the valuation argument in part (ii). Essentially no-one did part (iii) adequately. Among the challenges was the factorisation of 459. Part (c): this question also proved quite difficult. Correct application of Hasse's bound was rare. Even essentially correct answers had small flaws. It seems question 1 was quite difficult overall.

Question 2. Part (a): This was mostly fine. Some students failed to identify the group after finding points. Part (b): Some common errors were (i) incorrect attempts to apply the Nagell-Lutz theorem, e.g., wrong computation of discriminant (2) wrong computation of points mod  $p$ . (3). Failure to identify some obvious points. Part (c): this question could have been challenging, but many students found the answer: the point  $(2, 4)$  is non-torsion because it has denominators when doubled.

Question 3. This problem is a standard one, and was mostly handled well. In part (c), some students forgot to identify the Torsion subgroup. In part (b), sometimes, the condition for rank 1 was not clearly identified, leading to confusion in part (c) which no-one did correctly.

### **C3.8 Analytic Number Theory**

Question 1. A popular question with a good spread of marks.

Question 2. The second most popular, again with a good spread of marks. A misprint in part (b) caused problems, and those affected were marked sympathetically. Disappointingly few candidates realised one should use  $f(n) = \Lambda(n) - 1$  in part (c).

Question 3. Only 6 attempts but generally well done, though several candidates took  $c = 1 + 1/\log x$ , instead of using  $c = 1/\log x$ .

#### **C4.1: Functional Analysis**

#### **C4.2: Linear Operators**

The questions appeared to work as intended. The straightforward parts were well answered by most candidates, with marks being lost mostly for omission or error of details. There were a few parts which required rather deep understanding, so very high marks were difficult to achieve.

Q.1. Solutions to part (b) which reduced to the case of multiplication operators were more successful than other attempts. In (b)(iv) few candidates thought of using the decomposition of the space associated with  $E(J)$ .

Q.2. There was a small error in (c) which was corrected during the exam, and its effects were slight. Few candidates saw the connection between the projection  $P$  of rank 2, (b)(iv), and the 2 functionals given in the description of the range of  $A$ .

Q.3. Marks were lost in various places.

#### **C4.3: Functional Analytic Methods for PDEs**

Question 1: attempted by all the candidates. Part (a) of the question contains both bookwork and some variation of bookwork and has been done well. However, in several papers, certain difficulties occur to prove that the corresponding distributions is not regular. In part (b), the first question has been done by the majority of candidates while the second question has not. Here, the key point is to find a partial sum and its structure shows immediately what to do further.

Question 2: attempted by only one candidate. Here, continuity follows from the regularity for distributional solutions to the Poisson equation and the embedding theorems. In the second part, one needs to use density of smooth compactly supported functions in  $H_0^1$ .

Question 3: although not the most difficult question from the technical point of view, it has been attempted by many candidates. Part (a) is purely bookwork although not every candidate gave a full proof. In part (b), the main point is to get a constant in the right hand side that depends on  $k$  linearly. Many candidates realized the point but some of them struggled with an application of Hölder inequality. Finally, the last part (c) can be treated with help of the Taylor series and the majority of candidates do so. Here, typical mistakes are related to splitting the sum in two parts and a correct choice of a parameter  $\lambda$ .

## C4.4 Hyperbolic Equations

Question 1. Part (a), most students got it right. Part (b) was based on the equation derived in (a) and asked students to just solve the ODE. One student did it this way directly. The other students solved it by using an argument that was more complicated than necessary. Part (c) was similar to problem sheets. Almost all did it well except the minor mistakes in calculation. Part (d)(i), two students misread the equation. Other students reduced the problem to an ODE and solved it. Part (d)(ii) was based on part (i). One student did well by using finite speed propagation.

Question 2. This question was rather straightforward compared to question 3. I am surprised that students choose the harder problem during examination.

Question 3. Part (a), bookwork. Part (b)(i), calculus problem. One student did it right. Part (b)(ii), one student did it well. Parts (c) and (d) can be regarded as examples of bookwork. Most students did some reasonable work. The solution is a standard example for Riemann problem in the notes. Unfortunately, when this question was raised in (d), students forgot to treat it as a standard Riemann problem.

## C4.5 Ergodic Theory

Question 1: The attempts on this question were rather disappointing. The first ten marks were available for bookwork, but most candidates were unable to show that all invariant  $f$  are constant in an ergodic system with a completely correct argument, and all candidates overlooked the need (explicitly referred to in the mark scheme) to produce the easy argument for why  $T^{-n}E = E$  up to measure zero if  $E$  is invariant. The last 15 marks were available for saying whether three specific systems (toral automorphisms) were ergodic or weak-mixing. I knew that part (iii) would be found hard (and indeed no candidate got close to it) but parts (i) and (ii) should have been fairly routine given what was in lectures and on the examples sheets. Whilst one or two candidates did answer these parts more or less satisfactorily, most did not. All candidates merely stated that  $r\sqrt{2} + s\sqrt{3}$  is not zero when  $r, s$  are integers, not both zero. The short argument for why this was so was included in the markscheme (are the candidates appealing to a general theorem here that they can state accurately?) so I deducted a mark across the board.

Question 2: This was not a very popular question, though it was done fairly well by those candidates who attempted it. The bookwork part of this question was gone over quite carefully (with an additional handout provided) in my consolidation classes, but two of the four attempts still made little progress with it. It is therefore encouraging that the unseen parts of the question were done comparatively well.

Question 3: This was found to be the easiest question, perhaps unsurprisingly. The first four marks were definitely a gift, gladly accepted by all candidates. Quite a few candidates got the point of (b), which was to use the maximal ergodic theorem to prove that the set of functions satisfying the pointwise ergodic theorem is closed in  $L^1$ , then to observe that trigonometric polynomials satisfy the pointwise ergodic theorem. A few candidates tried to simply repeat the proof of the pointwise ergodic theorem shown in lectures, but this required them quoting a result about decomposition into cocycles which was part of the proof of the  $L^2$  ergodic theorem. I gave a maximum of 8/13 for such attempts. Perhaps I might have made it clearer what candidates were allowed to assume and what they weren't, but most

candidates made the right judgement and further more the whole of part (b) was a result from lectures, and clearly I didn't mean for them to simply quote it! Part (c) was done quite well, though a number of candidates fell in to the trap of not noticing that  $f(x) = [1/x]$  does not lie in  $L^1$ , and so the pointwise ergodic theorem does not immediately apply. Around half the candidates did appreciate this subtly and either suggested or rigorously found a way around it.

### C5.1: Solid Mechanics

Question 1. All students tried this question and most of them did very well. The question was a series of statements to be shown. Aided by the results, students found a way to prove the statements. They showed a good understanding of the basics of nonlinear elasticity and were able to manipulate satisfactorily all computations.

Question 2. This question was answered well and probably a better test of the students' ability. Most students could do the main basic steps but only a few students really understood the last steps of the problem and were able to prove the main results.

Question 3. Only a handful of students attempted this question, probably due to the fact that it was perceived as a word problem and required some understanding of concepts rather than formulaic application of the methods. Once set in equations, the computations were straightforward (much shorter than the other two questions) and the students who attempted this problem did very well.

### C5.2: Elasticity and Plasticity

Question 1: This question was generally well done. However, there was a lack of efficiency in the solution of the fourth order ODE that arose in part a(iv): students preferred to write down the general quartic and then apply the higher order bcs at  $x = L$  right at the end, rather than implementing them as they went. This meant that a number of algebraic errors slipped in. Some of these should have been spotted (e.g. if the vertical displacement of the end of the beam ended up being *positive* or when the shear force at the clamp,  $x = 0$ , vanished). In part (b) very few students obtained the correct result for the bending moment  $M(0)$  but many (correctly) guessed that tapering makes the branch less likely to snap.

Question 2: The first portions of this question were almost universally well done. Common errors in the Love wave portion of the question included using continuity of  $\partial w/\partial y$  across the interfaces (rather than continuity of shear stress,  $\mu_i \partial w_i/\partial y$ ) and seeking a wave solution travelling in the  $y$ -direction, as well as the  $x$ -direction as directed. In part (c), candidates often neglected to calculate the minimum phase speed of each of the even and odd modes; very few candidates were able to obtain the desired result for small, but finite, ratios of the shear moduli.

Question 3: This question was very popular but was more difficult than expected. The first part on the calculation of the maximum shear stress was generally very well done, though some answers failed to realise that the shear stress is  $\mathbf{t} \cdot \mathcal{T} \cdot \mathbf{n}$  and instead tried to maximize the modulus of  $\mathcal{T} \cdot \mathbf{n}$ . For the main parts of the question, the mix of stress and displacement boundary conditions at the inner and outer edges of the annulus was generally handled well, though it did cause some confusion in a few answers. The general form taken by the

stress profile in this annular geometry was also well-recognized though some careless algebra and the failure to realize that if  $\tau_{rr} = A - B/r^2$  then  $\tau_{\theta\theta} = A + B/r^2$  meant a number of solutions went awry at this point. Solutions to the portion of the question on plasticity were generally good but there was some confusion regarding the sign of  $\tau_{rr} - \tau_{\theta\theta}$  within the plastic region.

### C5.3: Statistical Mechanics

Question 1. Question done bimodally, either well or poorly. It is difficult to make this material hard (thermodynamics).

Question 2 (Boltzmann). This question was mostly avoided, two average attempts.

Question 3. This question attracted a decent spread, but again is mostly done by rote - only the last twist caused a problem (and no-one did it, though some came close).

### C5.4: Networks

Question 1. The jump from bookwork to harder material was too steep in problem 1. Part C confounded people who tried this problem, and in retrospect I would like to have started off that part of the problem more gently. The students didn't seem to be thinking about centralities as I had been hoping, and nobody seemed to discern that this amounted to a generalization of eigenvector-like centralities. A couple of students did notice graph Laplacian structure coming into play.

Question 2. This was a well-constructed problem with which I am very pleased. Students submitted some very interesting examples for part d(ii)! Part d(v) was challenging for students, as was intended.

Question 3. This was a well-constructed problem, though it turned out to be slightly harder than problem 2. Parts (a) and (b) were fine for just about all students, as was intended. Part (c) was also fine for most students, though a fair number had trouble with it even though it was directly out of an example discussed in lecture and reinforced in homework. No student got full marks on part (d), which was intended to be challenging. Part (e) also had some challenges, but students did a decent job on it even though it was more difficult than prior parts of the problem.

### C5.5: Perturbation Methods

Question 1. The application of integration by parts in part (a) and of Laplace's method in part (b) were reasonably well done. In part (b) marks were lost for failing to perform correctly an appropriate change of variables, to justify the size of the error term or to verify that the expansions are self-consistent. Only a handful of candidates identified the correct approach to part (c).

Question 2. While the bookwork in part (a) was well done, the dominant balance argument was done poorly or inefficiently, with many attempts failing to consider all of the possibilities. Those successful at part (a) made good attempts at part (b), though a worrying number of attempts failed due to mistakes with the elementary calculus. Only a handful of

candidates identified the correct approach to part (c).

Question 3. The bookwork in part (a) was well done on the whole, with marks largely being lost due to algebraic mistakes. In part (b), there were as many high quality answers as there were low quality ones. While only a handful of candidates gave a convincing argument that  $u_0$  is independent of  $X$ , many made good progress in deriving the relevant expression for  $u_1$ . There were no successful derivations of the ODE for  $u_0(x)$  due to the compound effect of inefficient or inaccurate manipulations, though a handful of candidates were extremely close.

### C5.6: Applied Complex Variables

Question 1. This was the least popular question. In part (a), students lost marks by not giving adequate explanation for their sketches, and several erroneously had the sector angle in the hodograph plane equal to  $\alpha$  rather than  $\pi - \alpha$ . The conformal maps in part (b) were generally handled well. In part (c), no-one gave a convincing statement of the far-field conditions. Only the few best students made it to the end of part (d).

Question 2. This was a popular question on which many students achieved high marks. The bookwork in parts (a) and (b) was generally done well. In part (c), many candidates just tried to reproduce the lecture notes instead of using the multifunction  $\sqrt{z^2 - c^2}$  as suggested. There were very few convincing explanations for the appearance of the holomorphic function  $H(z)$ . In part (d), many students seemed not to appreciate the importance of the singularities in  $H(z)$  being *isolated*. Only a few obtained the required solvability condition in part (d)(ii).

Question 3. This was a popular question. The standard computations and manipulations in parts (a)–(c) were generally done well, although a few candidates were evidently still confused about the concept of analytic continuation. Students received partial credit for clearly stating the residue calculation required for part (d) even if the fiddly algebra defeated them.

### C5.7: Topics in Fluid Mechanics

Question 1. Done by most students, and mostly well done except for the last part which was more difficult.

Question 2. Rarely done. Among those who attempted it, few got to the end; (c), (d) in particular seemed hard.

Question 3. Done by most students. Probably the question got through more smoothly than the other ones. Some students lost marks by being sketchy in (b) or not finishing (c).

### C5.8: Stochastic Modelling of Biological Processes

Overall the performance of the candidates on this exam was disappointing.

Q.1: All candidates attempted this question. This question had a large component of bookwork. Parts (a) and (b) in particular should have been an easy 11 marks, but a surprising number of students could not reproduce the necessary bookwork. Part (c) was

also only a small perturbation from the lecture notes. There was only one serious attempt at part (d). Many students did not recognise the difference between saying the degradation of B is reversible and simply adding production of B in the first compartment. Only a few candidates realised that it was crucial to introduce the degraded B as a new species, in order to keep track of the number of degraded molecules. Perhaps if the term “inactivated” rather than “degraded” had been used more candidates would have realised what was going on.

Q.2: All but one candidate attempted this question. There was a small typo (a  $\lambda$  in part (b) which should have been  $\lambda_0$ ) which was announced at the beginning of the exam and did not cause any candidate any problem. Most candidates managed the bookwork part (a) without difficulty. However, the limit in part (b), although similar to results in the lecture notes, caused more than a little difficulty. Only one candidate managed to derive the correct equation, though there were some near misses. Many forgot to replace  $X(t)$  in  $\lambda^\pm$  with  $x$  in the differential equations for  $p_i^\pm$ . The failure to complete part (b) had a knock on effect, since the remaining parts of the question relied on the equation derived in part (b).

Q.3: There were only four attempt at this question. Part (a) caused some difficulty. Part (b) was completed by all candidates. No candidate managed part (c). None could even write down the correct equation to solve when a radical drift is introduced: all introduced a reaction term in the equation rather than a drift term. Part (d) caused no difficulty for those that attempted it.

### **C5.9: Mechanical Mathematical Biology**

Question 1: Very few students attempted this question. Those who tried had issues with the formulation of the Kirchoff equations in local coordinates. This mistake prevented them from completing the question. One student managed to go through the question almost completely.

Question 2: This question was well answered. The question was a simplification of the general case done in class (the case in the notes was for a function  $h = h(x, y)$ , whereas the question was for  $h = h(x)$ ). Most candidates decided to ignore the simplification and repeated verbatim the lecture notes. At the cost of lengthy computations, most of them obtained the correct response for the form of the shape equation. The last part consisted of solving a linear BVP with constant coefficient of 4th order. While most of the students obtained the correct general solutions, very few obtained the correct particular solution, despite its simplicity. Since this last part only counted for 5 marks, it did not effect the success of the students on this question.

Question 3. This question was simple on a computational level but required some conceptual understanding of the material. The students who understood the theory well managed to answer the question completely.

### **C5.11: Mathematics of Geoscience**

Question 1 and 3. Both questions done similarly. One first class answer and the rest of the responses ordinary.

Question 2. The style of questions for 2(a)-(c) were of a similar structure and style to previous years, with 2(d) being completely new. Part(a) done quite well as straightforward mass conservation. Part (b) not done well at all. Candidates did not read question carefully as  $\epsilon = 0$  was only required for sketching the hydrograph, not for deriving the equation for  $A$ . Part (c) generally well done, though numerous algebraic mistakes were made by most candidates. Stability conditions often stated in reverse. Part (d) only attempted by two candidates in any serious manner, and then, only got halfway through the question.

### C5.12: Mathematical Physiology

This paper was in general well done.

Question 1. In general this question was well done.

- A common error was missing out the factor  $\epsilon$  when computing the Jacobian in the linear stability calculation in (a)(ii) to determine the stability of the equilibrium point  $(0, 0)$ .
- In (b)(ii) some justification was required for why  $v \rightarrow 1$  as  $Y \rightarrow \infty$  (for example, by appealing to the phase plane sketch, or the outer dynamics).
- No candidate was able to complete (b)(iii). While some candidates constructed a piecewise solution, and stated continuity of  $v$  where it takes the value  $a$  (at  $Y = Y_1$ , say), no candidate imposed continuity of the derivative of  $v$  with respect to  $Y$ , and thus no one obtained the required solution.

Question 2. The bookwork components were in general well done, as was the discussion of curvature blocking. Specific problems encountered by candidates were as follows:

- In (a) (iii) some candidates were unable to give an accurate physical interpretation of all the terms in equation (4).
- (b)(i) Some candidates were unable to compute the curvature.
- (b)(iii) Some candidates were unable to linearise the problem (by appealing to the fact that the  $O(1)$  terms balance, and neglecting higher order terms.) Some candidates also did not seek solutions in the form

$$\tilde{r} = \sum_{n=0}^{\infty} a_n(t) \exp(in\theta),$$

or, if they did, could not then interpret the  $n = 1$  and  $n \geq 1$  results.

Question 3.

- (a) Very few candidates gave an accurate explanation of what *all* the terms represent: the most commonly neglected feature was a discussion of  $\tau$ .
- (b) A number of candidates did not explain why in dimensionless form  $p_\tau$  becomes  $p_1$ .

- (d)(ii) A number of candidates failed to justify why  $\omega^* \approx \pi/2$  via a sketch of  $\tan \omega^*$  versus  $-\omega^*/A$ . If this was obtained, many candidates failed to relate this to the period of oscillation, getting confused between dimensionless and dimensional considerations.
- (d)(iii) was poorly done.

### **C6.1: Numerical Linear Algebra**

Question 1. Generally (a) solved well with (b) most easily solved using a block version of Gershgorin disc theorem. Part (c) had few compute the flop count correctly or properly state the algorithm for Cholesky.

Question 2. Parts (a) and (b) solved well. Part (c) few thought of Jacobi and instead considered QR iteration on Simultaneous Iteration

Question 3. Not attempted by any student.

### **C6.2: Continuous Optimisation**

There were a handful of excellent answers. Majority of students found the questions challenging.

### **C6.3 Approximation of Functions**

Question 1. Parts (a)–(d) which were largely bookwork were answered well, parts (e), (f) much less well with (f) having no good solutions.

Question 2. Answered by all candidates. Bookwork (a), (b) uniformly well done. Part (c) mostly well done although only a few justified the representations using Weierstrass Function. Part (d) most wanted to retain abstract representation rather than just calculate explicit coefficients for the function given and consequently could not show the final result although a few appealed to other theorems to justify the form given.

Question 3. Small number of attempts, no real pattern. Those who did the question did not appreciate that the key to (c), (d) uses that  $T_m \cdot T_n \neq T_{m+n}$  but a slightly different formula [this was unseen].

### **C6.4: Finite Element Methods for Partial Differential Equations**

Question 1 was attempted by all but one candidate and there were a number of high marks.

Question 2 was attempted by the majority of candidates. Few were able to directly apply the Aubin-Nitsche argument for the Poisson equation as asked.

Question 3 was the least popular question, but the only one on which a candidate scored 25/25.

## C7.2: Electromagnetism

Question 1: This question was attempted by 19 students (almost all of them) and was the question with higher average mark. I think the level was adequate.

Question 2: This question was attempted by 14 students and was the question with the lowest average mark (9/25). All students but one approached part c in a way that was not the most convenient one. Maybe a hint could have been given in part c, as to which method to use.

Question 3: This question was attempted by 17 students and I think the level was adequate.

Summary: The average this year was 24.9 which seems slightly lower compared to similar subjects, but reasonable. A posteriori maybe a hint in Q2, part c could have been added. Besides this small changes, I think the level of the questions was adequate.

## C7.3: Further Quantum Theory

Questions 1 was not popular but nevertheless attracted some reasonable attempts although the addition of angular momentum part was not well answered. Question 2 was modelled on the calculation for corrections to the ground state energy of the Helium atom in the notes, but the integration defeated most candidates. The third question was generally well done.

## C7.4: Introduction to Quantum Information

Question 1: With only one exception, everyone attempted question 1 and most students were very well prepared for it. Some students apparently studied Simon's algorithm using external resources and learnt the material by heart. In some cases this resulted in answers that correctly described the algorithm, but that were not the exact information the question asked for. Some students were able to provide quite detailed arguments for the last part of the question dealing with the performance comparison between the quantum and the classical algorithm.

Question 2: Less than half of all students attempted this question. However, most students who attempted it did very well, except those who worked already on questions 1 and 3 and just had a go at question 2 in addition. Many students had problems with part (d). A frequent mistake was an incorrect evaluation of the trace of a product of matrices. In the last part (e) of the question, most students had the correct idea for calculating the maximal probability with which ones can distinguish the two qubits. However, only few found the correct value of the trace distance and hence the correct final answer.

Question 3: Most students attempted question 3 and the quality of answers was generally high. From a conceptual point of view, the most difficult part was to calculate the probability for the qubit to be found in state  $|0\rangle$  at the output in the presence of decoherence. Only few students were able to tackle the last part of the question completely, which required an analysis of Deutsch's algorithm in the presence of decoherence.

### **C7.5: General Relativity I**

The performance of the students for each question was mostly in line with the expectations from the B/S/N marking scheme. The exception was question 2, but only one student attempted to solve it. There were few students so it is hard to draw conclusions.

Question 1. Part (a) all who attempted this were successful. Part (b) all got the first part, but only one got the harder part. Part (c) only one got it. Although technically easy, it required a more flexible understanding. Part (d) no one got full marks because the second part was conceptually challenging. Only one showed a serious attempt to relate the concepts involved.

Question 2. Only one student tried question 2 and the result was poor (while the student did much better in a different question). Even though there was a related problem in the example sheets, it apparently proved hard to find the great simplifications which arise for the family of metrics in question.

Question 3. Part (a) all who attempted this were successful. Part (b) all who attempted this were successful. Part (c) only one student got very close. The last part was conceptually challenging.

### **C7.6: Relativity II**

Question 1: The setter was surprised by the low marks for this question. The intent of the question was for the students to apply and generalise what they learned about Killing vectors to conformal killing vectors.

Question 2: This was a question about charged black holes. There were very good attempts to this question.

Question 3: No comments.

### **C8.1: Stochastic Differential Equations**

The exam was well done by many candidates.

Question 1: This was the least popular question and the solutions varied widely in quality. Part (c)'s aim was to establish a more general version of the Dubins-Schwarz Theorem but quite a few candidates found the proof of (i) difficult and only a few used Levy's characterization of Brownian motion in (ii).

Question 2: This was the most popular question. It required repeated use of the Itô formula and some candidates struggled with the calculus! The most difficult part was (b) where only one candidate could correctly deduce that  $Y$  satisfied Kazamaki's criterion. Those that understood the Girsanov Theorem found the tail straightforward.

Question 3: This question was on new material that had been added to the course after the introduction of the course B8.2. The first parts were book work and were generally well done. The final part of the tail, part (d), proved challenging with only a few able to recognise how to use part (i) and no one was able to choose exactly the correct  $H$  and  $G$  to establish the result.

## C8.2: Stochastic Analysis and PDEs

Question 1. This question was popular and, on the whole, rather well done, although some found the last part challenging.

Question 2. Again rather well done. Part (b) was the most challenging for most candidates who struggled to quite pin down the argument for  $M_t^2 - \int_0^t a(x_s) ds$  to be a local martingale.

Question 3. Less popular than the others, but some extremely good attempts (including some novel arguments).

## C8.3: Combinatorics

Question 1: The bookwork parts of this question were mostly answered well. Answers to the unseen parts were more patchy. Surprisingly few candidates spotted that 1(c) can be deduced from 1(b).

Question 2: Again, bookwork parts of the question were mostly answered well. Candidates had more trouble with the unseen parts (and perhaps 2(d) was too difficult).

Question 3: This was generally answered well.

## C8.4 Probabilistic Combinatorics

Question 1. (Lovasz local lemma and application to graph colouring)

(a) Well done, mostly quite easy marks.

(b) All works well with ‘bad’ events  $A(u, v; c)$  that  $u$  and  $v$  both have colour  $c$ , where  $u \sim v$  and  $c \in S(u) \cap S(v)$ ; but many tried less ‘fine’ events and then had problems.

Question 2. (Cliques in random graphs)

This was not popular, and was less well done than the other two questions.

(a) There was a good selection of acceptable answers: the hope was for ‘if  $\mu \rightarrow \infty$  and  $\Delta = o(\mu^2)$  then  $P(X = 0) \rightarrow 0$ ’.

(b) Part (i) was ok; in part (ii) many were not comfortable with the asymptotics.

(c) Few good answers.

Question 3. (Down-sets, Janson inequality, application to number of  $K_4$ s in a random graph)

(a) Straightforward and well done.

(b) (i) Several students got in a tangle, with inequalities back-to-front. (ii) Bookwork, rather easy marks. (There was a typo, that did not seem to cause any problems:  $\sum_{j < i: A_i \cap A_j \neq \emptyset}$  should have been  $\sum_{j < i: E_i \cap E_j \neq \emptyset}$ .)

(c) Hard-earned marks, but many decent attempts. Many got to  $e^{(1+o(1))\mu}$  and claimed the result followed, again showing a lack of comfort with asymptotics.

## **Statistics Units**

Reports on the following courses may be found in the Mathematics and Statistics examiners' report.

SC1: Stochastic Models in Mathematical Genetics

SC2: Probability and Statistics for Network Analysis

SC4: Statistical Data Mining and Machine Learning

SC5: Advanced Simulation Methods

## **Computer Science**

Reports on the following courses may be found in the Mathematics and Computer Science examiners' report.

Quantum Computer Science

Categories, Proofs and Processes

Automata, Logic and Games

## F. Comments on performance of identifiable individuals

*Removed from public version of the report.*

## G. Names of members of the Board of Examiners

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