Examiners' Report: Final Honour School of Mathematics Part B Trinity Term 2017

November 9, 2017

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

  See Table 1.

	Numbers					Percentages %				
	2017	(2016)	(2015)	(2014)	(2013)	2017	(2016)	(2015)	(2014)	(2013)
I	52	(56)	(48)	(49)	(54)	39.39	(39.72)	(32.88)	(31.01)	(34.34)
II.1	64	(58)	(69)	(78)	(78)	48.48	(41.13)	(47.26)	(49.37)	(49.68)
II.2	11	(24)	(25)	(21)	(21)	8.33	(17.02)	(17.12)	(13.29)	(13.38)
III	3	(3)	(3)	(9)	(2)	2.27	(2.13)	(2.05)	(5.7)	(1.27)
P	2	(0)	(1)	(1)	(2)	1.52	(0)	(0.68)	(0.63)	(1.27)
F	0	(0)	(0)	(0)	(0)	0	(0)	(0)	(0)	(0)
Total	132	(141)	(146)	(158)	(157)	100	(100)	(100)	(100)	(100)

Table 1: Numbers and percentages in each class

- **Numbers of vivas and effects of vivas on classes of result.**

  As in previous years there were no vivas conducted for the FHS of Mathematics Part B.

- **Marking of scripts.**

BE Extended Essays, BSP projects, and coursework submitted for the History of Mathematics course, the Mathematics Education course and the Undergraduate Ambassadors Scheme, were double marked.

The remaining scripts were all single marked according to a pre-agreed marking scheme which was strictly adhered to. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**

  See Table 5 on page 13.

B. New examining methods and procedures

This year three changes were made to examining procedures.

Firstly, the length of time allowed for Mathematics unit papers was increased from 1.5 hours to 1.75 hours. Statistics unit papers also increased to 1.75 hours, and Computer Science unit papers to 2 hours. The examination for SB1 Applied and Computational Statistics, a two-unit Statistics course, was increased to 2.5 hours.

Secondly, BEE Extended Essays and BSP Structured Project written reports were marked by the supervisor and one assessor, rather than by two assessors.

Thirdly, candidates taking Part A from 2016 onwards take either 9 or 10 papers in Part A, that is, they must take papers A0, A1, A2, ASO, and five or six out of A3-A11. If a candidate takes 9 papers, paper A2 counts as a double unit and the remaining papers as single units in the Part A USM average. If a candidate takes 10 papers, the two lowest scoring papers from A3-A11 count as half a unit each in the Part A average. In both cases, the classification in Part B depends on the sum of 40% of the Part A average and 60% of the Part B average.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

D. Notice of examination conventions for candidates

The first Notice to Candidates was issued on 15 February 2017 and the second notice on 8 May 2017.

All notices and the examination conventions for 2017 are on-line at http://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments.

Part II

A. General Comments on the Examination

The examiners would like to convey their grateful thanks for their help and cooperation to all those who assisted with this year's examination, either as assessors or in an administrative capacity. The chairman would particularly like to thank Helen Lowe for administering the whole process with efficiency, and also to thank Nia Roderick, Charlotte Turner-Smith, Beth Delaplain and Waldemar Schlackow.

In addition the internal examiners would like to express their gratitude to Professor Higham and Professor Skorobogatov for carrying out their duties as external examiners in a constructive and supportive way during the year, and for their valuable input at the final examiners' meetings.

Standard of performance

The standard of performance was broadly in line with recent years. In setting the USMs, we took note of

- the Examiners' Report on the 2016 Part B examination, and in particular recommendations made by last year's examiners, and the Examiners' Report on the 2016 Part A examination, in which the 2017 Part B cohort were awarded their USMs for Part A;
- a document issued by the Mathematics Teaching Committee giving broad guidelines on the proportion of candidates that might be expected in each class, based on the class percentages over the last five years in Mathematics Part B, Mathematics & Statistics Part B, and across the MPLS Division.

Having said this, as in Table 1 the proportion of first class degrees in Mathematics alone awarded (38.64%) was high, and the proportion of II.2 and below degrees in Mathematics awarded (12.88%) was low, compared to the guidelines. One reason for this is that the examiners consider candidates in Mathematics and in Mathematics and Statistics together when determining USMs, and this year the Mathematics and Statistics candidates performed poorly compared to the Mathematics candidates, so that the averages for the two schools combined (35.2% firsts, and 14.2% II.2 and below) are consistent with the Teaching Committee guidelines.

It seems plausible that the increase in time this year from 1.5 hours to 1.75 hours for Mathematics unit papers may have helped candidates near the II.1/II.2 borderline to perform better, leading to fewer II.2s. The number of candidates was also low (132, compared to an average of 155 over 2008-2016), which may have been in part due to withdrawals by candidates with problems likely to lower their performance, raising the overall standard.

Setting and checking of papers and marks processing

Requests to course lecturers to act as assessors, and to act as checkers of the questions of fellow lecturers, were sent out early in Michaelmas Term, with instructions and guidance on the setting and checking process, including a web link to the Examination Conventions. The questions were initially set by the course lecturer, in almost all cases with the lecturer of another course involved as checkers before the first drafts of the questions were presented to the examiners. Most assessors acted properly, but a few failed to meet the stipulated deadlines (mainly for Michaelmas Term courses) and/or to follow carefully the instructions provided.

The internal examiners met at the beginning of Hilary Term to consider those draft papers on Michaelmas Term courses which had been submitted in time; consideration of the remaining papers had to be deferred. Where necessary, corrections and any proposed changes were agreed with the setters. The revised draft papers were then sent to the external examiners. Feedback from external examiners was given to examiners and to the relevant assessor for response. The internal examiners at their meeting in mid Hilary Term considered the external examiners' comments and the assessor responses, making further changes as necessary before finalising the questions. The process was repeated for the Hilary Term courses, but necessarily with a much tighter schedule.

Camera ready copy of each paper was signed off by the assessor, and then submitted to the Examination Schools.

Except by special arrangement, examination scripts were delivered to the Mathematical Institute by the Examination Schools, and markers collected their scripts from the Mathematical Institute. Marking, marks processing and checking were carried out according to well-established procedures. Assessors had a short time period to return the marks on standardised mark sheets. A check-sum is also carried out to ensure that marks entered into the database are correctly read and transposed from the mark sheets.

All scripts and completed mark sheets were returned, if not by the agreed due dates, then at least in time for the script-checking process.

A team of graduate checkers under the supervision of Helen Lowe sorted all the scripts for each paper for which the Mathematics Part B examiners have sole responsibility, carefully cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way, errors were corrected with each change independently verified and signed off by one of the examiners, who were present throughout the process. A small number of errors were found, but they were mostly very minor and hardly any queries had to be referred to the marker for resolution.

Throughout the examination process, candidates are treated anonymously, identified only by a randomly-assigned candidate number, until after all decisions on USMs, degree classes, Factors Affecting Performance applications, prizes, and so on, have been finalized.

This year, there were a few more corrections to papers announced during the examinations than usual (of 31 papers, 4 papers had one correction, and 4 papers had two separate corrections). There appears to be no pattern on MT/HT or Pure/Applied papers receiving corrections. This may have been a failure of vigilance on the part of the board of examiners, but we also feel that not all of our colleagues put as much effort as they should (and a few, very little effort) into proofreading their draft papers.

Standard and style of papers

At the beginning of the year all setters were asked to aim that a I/II.1 borderline candidate should get about 36 marks out of 50, and that a II.1/II.2 borderline script should get about 25 marks, and emphasising the problems caused by very high marks.

This year one paper (B5.3) turned out to be too easy. This causes problems with determining USMs at the top end.

Setting papers that are significantly too easy (and marking such papers generously) is undesirable from the point of view of fairness. Such papers generate more USMs than usual in the range 80-100 from candidates with close to full marks. An undergraduate who has the good fortune to take an easy paper and score highly will typically receive a rather higher USM than he or she would otherwise have done – perhaps a USM of 100 – and this can easily push an otherwise high II.1 candidate into the first class.

Timetable

Examinations began on Tuesday 23 May and finished on Friday 16 June.

Determination of University Standardised Marks

We followed the Department's established practice in determining the University standardised marks (USMs) reported to candidates. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part A performances of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage). Working with the data for this population, numbers N_1, N_2 and N_3 are first computed for each paper: N_1, N_2 and N_3 are, respectively, the number of candidates taking the paper who achieved in Part A average USMs in the ranges $[69.5, 100]$, $[59.5, 69.5)$ and $[0, 59.5)$, respectively.

The algorithm converts raw marks to USMs for each paper separately. For each paper, the algorithm sets up a map $R \to U$ (R = raw, U = USM) which is piecewise linear. The graph of the map consists of four line segments: by default these join the points $(100, 100)$, $P_1 = (C_1, 72)$, $P_2 = (C_2, 57)$, $P_3 = (C_3, 37)$, and $(0, 0)$. The values of C_1 and C_2 are set by the requirement that the number of I and II.1 candidates in Part A, as given by N_1 and N_2, is the same as the I and II.1 number of USMs achieved on the paper. The value of C_3 is set by the requirement that P_2P_3 continued would intersect the U axis at $U_0 = 10$. Here the default choice of *corners* is given by U-values of 72, 57 and 37 to avoid distorting nonlinearity at the class borderlines.

The results of the algorithm with the default settings of the parameters provide the starting point for the determination of USMs, and the Examiners may then adjust them to take account of consultations with assessors (see above) and their own judgement. The examiners have scope to make changes, either globally by changing certain parameters, or on individual papers usually by adjusting the position of the corner points P_1, P_2, P_3 by hand, so as to alter the map raw $\to$ USM, to remedy any perceived unfairness introduced by the algorithm. They also have the option to introduce additional corners. For a well-set paper taken by a large number of candidates, the algorithm yields a piecewise linear map which is fairly close to linear, usually with somewhat steeper first and last segments. If the paper is too easy or too difficult, or is taken by only a few candidates, then the algorithm can yield anomalous results—very steep first or last sections, for instance, so that a small difference in raw mark can lead to a relatively large difference in USMs. For papers with small numbers of candidates, moderation may be carried out by hand rather than by applying the algorithm.

Following customary practice, a preliminary, non-plenary, meeting of examiners was held ahead of the first plenary examiners' meeting to assess the results produced by the algorithm, to identify problematic papers and to try some experimental changes to the scaling of individual papers. This provided a starting point for the first plenary meeting to obtain a set of USM maps yielding a tentative class list with class percentages roughly in line with historic data.

The first plenary examiners' meeting, jointly with Mathematics & Statistics examiners, began with a brief overview of the methodology and of this year's data. Then we considered the scaling of each paper, making provisional adjustments in some cases. The full session was then adjourned to allow the examiners to look at scripts. This was both to help the external examiners to form a view of overall standards, and to answer questions that had arisen on how best to scale individual papers; for instance, to decide whether a given raw mark should correspond to the I/II.1 or II.1/II.2 borderline, an examiner would read all scripts scoring close to this raw mark, and make a judgement on their standard.

The examiners reconvened and we then carried out a further scrutiny of the scaling of each paper, making small adjustments in some cases before confirming the scaling map (those Mathematics & Statistics examiners who were not Mathematics examiners left the meeting once all papers with significant numbers of Mathematics & Statistics candidates had been considered).

Table 2 on page 10 gives the final positions of the corners of the piecewise linear maps used to determine USMs.

The Mathematics examiners reviewed the positions of all borderlines for their cohort. For candidates very close to the proposed borderlines, marks profiles and particular scripts were reviewed before the class list was finalised.

In accordance with the agreement between the Mathematics Department and the Computer Science Department, the final USM maps were passed to the examiners in Mathematics & Computer Science. USM marks for Mathematics papers of candidates in Mathematics & Philosophy were calculated using the same final maps and passed to the examiners for that School.

Factors affecting performance

A subset of the examiners had a preliminary meeting to consider the submissions for factors affecting performance in Part B. There were twelve Part 13 submissions which the preliminary meeting classified in bands 1, 2, 3 as appropriate. The full board of examiners considered the twelve cases in the final meeting, and the certificates passed on by the examiners in Part A 2016 were also considered. A late application was also considered following the final board meeting. All candidates with certain conditions (such as dyslexia, dyspraxia, etc) were given special consideration in the conditions and/or time allowed for their papers, as agreed by the Proctors. Each such paper was clearly labelled to assist the assessors and examiners in awarding fair marks. Details of cases in which special consideration was required are given in Section E.2.

Table 2: Position of corners of the piecewise linear maps

Paper	P_1	P_2	P_3	Additional Corners	N_1	N_2	N_3
B1.1	(16.09, 37)	(28, 57)	(43, 72)		10	20	7
B1.2	(13.67, 37)	(26.5, 57)	(43, 72)		16	29	9
B2.1	(9.08, 37)	(28, 57)	(36.8, 72)		8	15	0
B2.2	(9.08, 37)	(23, 57)	(36.8, 72)		11	12	0
B3.1	(8.56, 37)	(22, 57)	(40.4, 72)		16	20	1
B3.2	(10, 37)	(28, 57)	(36, 72)		5	2	0
B3.3	(18.1, 37)	(30, 57)	(39, 72)		13	7	0
B3.4	(11.32, 37)	(23, 57)	(36.2, 72)		15	17	1
B3.5	(9.25, 37)	(21, 57)	(35, 72)		15	14	0
B4.1	(10.91, 37)	(21, 57)	(34, 72)		18	18	2
B4.2	(9.25, 37)	(24.5, 57)	(35.6, 72)		19	17	2
B5.1	(11.43, 37)	(19.9, 57)	(30.4, 72)		9	13	7
B5.2	(11, 37)	(22, 57)	(40.5, 72)		13	24	12
B5.3	(18, 37)	(33, 57)	(44, 70)		5	13	6
B5.4	(13.67, 37)	(23.8, 57)	(42, 72)		5	16	7
B5.5	(16.26, 37)	(27, 57)	(40, 72)		8	18	10
B5.6	(15.05, 37)	(26.2, 57)	(35.2, 72)		11	17	5
B6.1	(19.82, 37)	(33, 57)	(41, 72)		8	14	10
B6.2	(15.51, 37)	(31, 57)	(42, 72)		6	9	2
B6.3	(14.82, 37)	(23, 57)	(40, 72)		8	17	5
B7.1	(14.42, 37)	(25.1, 57)	(42.6, 72)		5	13	7
B7.2	(14.94, 37)	(22.5, 57)	(36.5, 72)		3	6	6
B7.3	(16.83, 37)	(25.5, 57)	(39, 72)		4	8	6
B8.1	(13.5, 37)	(27.5, 57)	(41.5, 72)		13	16	3
B8.2	(9.13, 37)	(21, 57)	(41.4, 72)		8	9	2
B8.3	(13.1, 37)	(22.8, 57)	(42.5, 72)		13	31	13
B8.4	(13.96, 37)	(24.3, 57)	(37.8, 72)		6	14	7
B8.5	(10.17, 37)	(17.7, 57)	(34.2, 72)		14	24	5
SB1	(20.05, 37)	(34.9,5 7)	(53, 72)		2	20	7
SB2a	(12.29, 37)	(21.4, 57)	(39.4, 72)		8	26	9
SB2b	(21.19, 37)	(34, 57)	(43, 70)		6	18	5
SB3a	(12.81, 37)	(22.3, 57)	(35.8, 72)		19	38	16
SB3b	(16.6, 37)	(28.9, 57)	(39.4, 72)		10	16	6
SB4a	(16.66, 37)	(28, 57)	(40, 72)		7	24	12
SB4b	(16.26, 37)	(27.5, 57)	(39, 70)		5	22	10

Table 3 gives the rank of candidates and the number and percentage of candidates attaining this or a greater (weighted) average USM.

Table 3: Rank and percentage of candidates with this or greater overall USMs

Av USM	Rank	Candidates with this USM and above	%
93	1	1	0.76
91	2	2	1.52
90	3	3	2.27
85	4	4	3.03
84	5	6	4.55
83	7	7	5.3
81	8	8	6.06
77	9	15	11.36
76	16	17	12.88
75	18	22	16.67
74	23	29	21.97
73	30	33	25
72	34	37	28.03
71	38	44	33.33
70	45	51	38.64
69	52	54	40.91
68	55	62	46.97
67	63	71	53.79
66	72	75	56.82
65	76	82	62.12
64	83	91	68.94
63	92	97	73.48
62	98	102	77.27
61	103	109	82.58
60	110	115	87.12
59	116	118	89.39
58	119	120	90.91
57	121	123	93.18
56	124	124	93.94
53	125	125	94.7
52	126	127	96.21
49	128	128	96.97
45	129	129	97.73
44	130	130	98.48
39	131	131	99.24
36	132	132	100

B. Equality and Diversity issues and breakdown of the results by gender

Table 4: Breakdown of results by gender

Class	Number								
	2017			2016			2015		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	7	45	52	10	46	56	7	41	48
II.1	21	43	64	17	41	58	25	44	69
II.2	5	6	12	10	14	24	8	17	25
III	0	3	3	2	1	3	1	2	3
P	0	2	2	0	0	0	0	1	1
Total	33	99	132	39	102	141	41	107	146
Class	Percentage								
	2017			2016			2015		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	21.21	45.45	39.39	25.64	45.1	39.72	17.07	39.05	32.88
II.1	63.63	43.43	48.48	43.59	40.32	41.13	60.98	41.9	47.26
II.2	15.15	6.06	8.33	25.64	13.73	17.02	19.51	16.19	17.12
III	0	3.03	2.27	5.13	0.98	2.13	2.44	1.9	2.05
P	0	2.02	1.52	0	0	0	0	0.95	0.68
Total	100	100	100	100	100	100	100	100	100

Table 4 shows the performances of candidates broken down by gender. The examiners were concerned to discover, after the class lists were agreed, that the percentage of male candidates awarded first class degrees was over double the percentage of female candidates awarded first class degrees, and that the percentage of female candidates awarded II.2s and below was 2.5 times the percentage of male candidates in the same range.

We would like to bring this year's very significant gender discrepancy to the attention of the department, which we know is already well aware of this issue. We also note that one reason for the increase in time allowed for Mathematics unit papers from 1.5 hours to 1.75 hours introduced this year was that it was believed that some female candidates might be more likely to be adversely affected by time pressure.

C. Detailed numbers on candidates' performance in each part of the examination

The number of candidates taking each paper is shown in Table 5.

Table 5: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
B1.1	38	35.63	8.37	66.21	13.72
B1.2	55	35.73	9.09	66.87	12.81
B2.1	19	33.21	6.63	66.89	12.07
B2.2	22	33.68	7.61	70.5	11.59
B3.1	38	35.39	7.79	69.89	10.47
B3.2	7	35.71	10.32	73.29	16.46
B3.3	20	41.25	5.25	78.8	11.94
B3.4	34	33.71	7.29	70.79	10.88
B3.5	28	33.68	7.15	72.43	10.44
B4.1	39	30.33	6.2	68.15	8.84
B4.2	39	30.28	8.65	65.51	13.12
B5.1	29	25.45	7.16	64.1	11.86
B5.2	50	29.2	10.56	62	14.22
B5.3	26	40.65	7.47	69.12	12.75
B5.4	30	32.7	9.75	65.07	13.86
B5.5	34	33.94	7.73	65.15	12.04
B5.6	34	31.21	6.94	65.35	12.54
B6.1	31	36.55	8.38	65.45	16.43
B6.2	17	36.29	9.27	66	15.94
B6.3	24	35.83	8.67	71.25	13.13
B7.1	24	34.17	9.15	64.96	11.9
B7.2	15	31.4	7.96	67	11.75
B7.3	18	34.78	7.5	68.94	12.1
B8.1	32	38.91	7.78	73.53	14.24
B8.2	19	33.32	9.18	67.37	10.75
B8.3	44	34.43	7.56	66.89	8.88
B8.4	25	31.48	7	65.8	10.23
B8.5	43	29.12	6.99	67.77	8.39
SB1	2	-	-	-	-
SB2a	17	31.41	6.47	65.47	6.63
SB2b	10	41.7	5.31	72.1	12.7
SB3a	55	29.09	6.91	64.11	9.74
SB3b	19	34.37	7.78	65.37	13.5
SB4a	23	34.65	6.83	66.26	10.93
SB4b	19	32.74	6.04	63.32	9.21
CS3a	8	-	-	63	4.24
CS4b	7	-	-	71	8.75
BO1.1	6	-	-	69.5	5.17
BO1.1X	6	-	-	65.33	8.09
BN1.1	6	-	-	69	3.52
BN1.2	5	-	-	-	-
BEE	7	-	-	78.29	7.8
BSP	8	-	-	69.38	4.39
102	2	-	-	-	-
127	1	-	-	-	-
129	1	-	-	-	-

Individual question statistics for Mathematics candidates are shown below for those papers offered by no fewer than six candidates.

Paper B1.1: Logic

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.44	20.83	4.92	35	1
Q2	15.50	15.50	5.22	36	0
Q3	11.00	13.40	3.55	5	3

Paper B1.2: Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.37	16.07	5.44	27	3
Q2	18.90	18.90	5.77	52	0
Q3	16.63	17.68	4.56	31	4

Paper B2.1: Introduction to Representation Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.29	17.29	3.58	17	0
Q2	13.00	13.27	3.67	11	1
Q3	17.08	19.10	6.23	10	2

Paper B2.2: Commutative Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.31	16.31	4.99	16	0
Q2	16.50	16.50	3.63	18	0
Q3	14.85	18.30	7.84	10	3

Paper B3.1: Galois Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.82	20.82	4.16	38	0
Q2	13.97	13.97	5.20	29	1
Q3	14.18	16.56	6.75	9	2

Paper B3.2: Geometry of Surfaces

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.17	21.17	3.31	6	0
Q2	16.33	16.00	7.02	2	1
Q3	15.17	15.17	4.75	6	0

Paper B3.3: Algebraic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.74	19.33	4.04	18	1
Q2	22.11	22.11	3.10	18	0
Q3	19.75	19.75	3.30	4	0

Paper B3.4: Algebraic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.75	17.13	5.83	15	1
Q2	15.71	16.45	4.79	22	2
Q3	17.00	17.00	3.47	31	0

Paper B3.5: Topology and Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.29	16.29	4.34	21	0
Q2	14.30	16.25	6.12	16	4
Q3	17.24	17.95	4.48	19	2

Paper B4.1: Banach Spaces

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.66	14.76	3.77	37	1
Q2	15.31	15.31	3.82	29	0
Q3	16.08	16.08	4.17	12	0

Paper B4.2: Hilbert Spaces

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.18	15.24	4.20	38	1
Q2	12.52	12.58	4.66	24	1
Q3	17.44	18.75	5.90	16	2

Paper B5.1: Stochastic Modelling and Biological Processes

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.69	12.80	3.04	25	1
Q2	12.75	14.17	6.62	18	2
Q3	9.71	10.87	5.14	15	2

Paper B5.2: Applied PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.83	14.18	4.50	40	2
Q2	12.32	13.56	7.37	32	5
Q3	15.29	16.39	6.74	28	3

Paper B5.3: Viscous Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.12	21.12	3.68	26	0
Q2	19.29	19.29	4.49	24	0
Q3	21.00	22.50	2.65	2	1

Paper B5.4: Waves and Compressible Flow

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.09	16.09	3.32	23	0
Q2	13.91	16.71	7.06	17	5
Q3	15.90	16.35	7.39	20	1

Paper B5.5: Mathematical Ecology and Biology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.45	17.80	6.11	10	1
Q2	18.07	18.66	5.35	29	1
Q3	15.00	15.00	4.20	29	0

Paper B5.6: Nonlinear Systems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.34	15.23	4.67	26	3
Q2	12.38	15.18	6.61	11	5
Q3	16.06	16.06	3.96	31	0

Paper B6.1: Numerical Solution of Differential Equations I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.36	14.10	5.18	20	2
Q2	20.62	20.62	4.14	29	0
Q3	17.87	19.46	5.33	13	2

Paper B6.2: Numerical Solution of Differential Equations II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.63	19.60	6.72	15	1
Q2	19.25	19.25	3.70	12	0
Q3	11.10	13.14	5.04	7	3

Paper B6.3: Integer Programming

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.33	20.00	5.03	2	1
Q2	16.73	16.73	5.53	22	0
Q3	18.83	18.83	5.10	24	0

Paper B7.1: Classical Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.30	17.30	4.02	23	0
Q2	18.87	19.71	6.30	14	1
Q3	12.25	13.27	6.17	11	1

Paper B7.2: Electromagnetism

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.86	15.86	4.74	14	0
Q2	16.00	16.00	3.46	7	0
Q3	14.60	15.22	5.38	9	1

Paper B7.2: Further Quantum Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.00	17.00	3.16	14	0
Q2	17.92	19.00	5.28	11	2
Q3	15.67	16.27	6.43	11	1

Paper B8.1: Martingales through Measure Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.63	18.63	4.47	30	0
Q2	20.10	20.29	5.33	28	1
Q3	19.29	19.67	3.09	6	1

Paper B8.2: Continuous Martingales and Stochastic Calculus

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.18	12.18	5.83	11	0
Q2	19.21	19.21	4.73	19	0
Q3	15.78	16.75	5.09	8	1

Paper B8.3: Mathematical Models of Financial Derivatives

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.17	18.17	4.17	35	0
Q2	16.23	16.23	4.43	30	0
Q3	16.12	17.04	5.92	23	2

Paper B8.4: Communication Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.94	14.94	4.04	18	0
Q2	16.27	16.27	4.61	22	0
Q3	14.91	16.00	4.74	10	1

Paper B8.5: Graph Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.05	15.05	3.25	43	0
Q2	14.24	14.24	4.97	41	0
Q3	10.50	10.50	2.12	2	0

Paper SB2a: Foundations of Statistical Inference

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.82	16.63	4.97	16	1
Q2	15.24	15.24	4.10	17	0
Q3	6.50	9.00	3.54	1	1

Paper SB2b: Machine Learning

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.90	21.78	1.60	9	1
Q2	18.50	18.50	5.09	6	0
Q3	19.75	22.00	3.99	5	3

Paper SB3a: Applied Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.76	14.82	3.87	44	1
Q2	16.55	18.20	6.29	20	2
Q3	12.40	12.70	3.72	46	2

Paper SB3b: Statistical Lifetime-Models

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.64	18.62	5.20	13	1
Q2	17.36	17.36	3.26	11	0
Q3	15.71	15.71	4.55	14	0

Paper SB4a: Actuarial Science I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.48	16.48	4.12	21	0
Q2	17.45	17.45	3.61	22	0
Q3	22.33	22.33	1.15	3	0

Paper SB4b: Actuarial Science II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.47	16.47	3.20	15	0
Q2	16.40	17.36	4.67	14	1
Q3	14.50	14.67	3.34	9	1

Assessors' comments on sections and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. The examiners have not included assessors' statements suggesting where possible borderlines might lie; they did take note of this guidance when determining the USM maps. Some statistical data which can be found in Section C above have also been removed.

B1.1: Logic

Question 1: This question was done by most candidates and usually to a very high standard (many 25/25 results). The biggest challenge was to derive the Proof by Contradiction-Rule in the standard calculus.

Question 2: Part (a) and (b) were standard bookwork, though in (b) the base case for the induction (atomic formulas) was often dealt with in a hand waving manner rather than using induction on the complexity of terms. In part (c) it was somewhat surprising how few candidates managed to fully write down all the axioms for a dense linear ordering without endpoints.

Question 3: This question was not very popular (only 13 attempts, all but one gaining rather low marks). Parts (b) and (c) required a good understanding of the proof of the Completeness Theorem.

B1.2: Set Theory

Problem 1. Part (a) was generally well done. Most proofs of (iii) were attempted via transfinite induction This required some care as the statement only holds for ordinals $\omega \leq \alpha$. Often this was not handled correctly. Part (b) was also generally well done, with most realizing the key distinction between parts (ii) and (iii). In part (c), (ii) was much like a homework problem, and was frequently well done, while (iii) was found challenging, not the many people provided a fully satisfactory answer.

Problem 2. Part (a) was generally very well done, with most seeing the straightforward reduction of Rotnac's theorem to Cantor's theorem. Some proved Rotnac's theorem directly using the idea of the proof of Cantor's theorem, which was nice. In (b) (ii), several students applied Foundation to a descending chain of elements. This is OK but some care is required as one really needs a descending sequence of sets (i.e. a function $\omega \to Y$). Much easier is to assert that X, being non-empty, must have an ϵ-minimal element, and by transitivity this element must be the empty set. Part (iii) generated some very circuitous arguments, but many found the key through Foundation. In part (c), parts (i) and (ii) were generally well done, with few problems applying recursion. Part (iii) was also done well by many.

Problem 3. Part (a), all bookwork but (iii) somewhat subtle, was done very well by most who attempted it. Part (b) (ii) and (iii) are similar to problems appearing on several past papers, and were generally well done, (ii) by transfinite induction and (iii) by recursion. In part (c), trying to work with an injective map $\omega \to X$ led to difficulties, which were sometimes overcome; much more direct and appropriate for well-ordered sets was to work with an order-isomorphism, giving an order embedding of ω onto some initial segment, and then the contradiction is clear.

B2.1: Introduction to Representation Theory

Question 1: Popular question. Many candidates guessed correctly that the answer to the last part was no, but then tried to find examples with A semisimple.

Question 2: The last two parts were quite challenging. In (d) very few candidates realized and proved that the image of a representation of a finite abelian group G over C must be diagonalisable.

Question 3: Many good solutions. Many candidates attempted the last part without proving that χ^n is a character of G, which is the key ingredient here.

B2.2: Commutative Algebra

Question 1: Popular question, but last part was quite challenging with only a handful of complete solution. Surprisingly nobody noticed that the nilradical of R is not nilpotent and hence R cannot be Noetherian.

Question 2: Another very popular question with good results. Few candidates realized that part (c) needs an application of Nakayama's Lemma.

Question 3: Many good attempts. Last part (d) was more challenging and in particular was important to prove that in part (c) P_2 cannot be equal to $pZ[t]$ for a prime integer p.

B3.1: Galois Theory

My personal feeling is that the paper went pretty well. Everybody did question 1 (over the rationals). Most of the students also chose question 2, which dealt with function fields in positive characteristic (which I think was a good sign, as this was a quite exotic exercise). They performed very well on question 1, and quite OK on question 2. The students sometimes failed when dealing with basic properties of finite fields, and sometimes wrote contradictory statements when computing the Galois Group of the polynomial in question 2 (some students did not understand the additive structure of $\mathbb{F}_{25}$). They probably should do more exercises on these.

B3.2: Geometry of Surfaces

Candidates did well on question 1 with some perfect or near-perfect answers.

Candidates found the calculus part of 1(c)(iii) the hardest. One near perfect answer to (2), but generally candidates found 2 (c)(i) difficult (perhaps because they did not fully use the hint).

Question 3 received mostly very good answers, but none in the 20+ raw mark range, as candidates tried but failed to tackle 3 (d) in $\mathbb{R}^3$−coordinates (it is much easier if one tackles it in local θ, γ coordinates).

B3.3 Algebraic Curves

Question 1: Mostly well done. Candidates frequently failed to show a, b, c nonzero in (b), or λ, μ, ν distinct in (c). Noone got full marks in (d); most candidates noticed that the issue was in taking square roots in R, but almost everyone wrote $x_1^2 + x_2^2 + x_3^2 = 0$ as a possible local model, although this is empty, and in the mixed signs case, specified λ, μ, ν distinct rather than $\pm\lambda, \pm\mu, \pm\nu$ distinct.

Question 2: The bookwork in (a),(b) was almost always well done. Candidates who could do the calculation in (c) successfully (who were, pleasingly, in the majority) scored highly on this question.

Question 3: The least popular question, with only 4 attempts.

B3.4: Algebraic Number Theory

Question 1: (a) This was straightforward for all who attempted it.

(b) Most people did well on this problem. Some had marks deducted for not proving the correct formula for the minimal polynomial.

(c) This problem was challenging for quite a few people. Those who didn't remember the formula for the discriminant in terms of the derivative of the minimal polynomial easily made calculation errors. Also, the argument for using the Eisenstein polynomial to show that one gets the full ring of integers was missed by many.

(d) I thought this problem would be challenging, but a surprising number of people computed the norms correctly and found a unit. Some people made calculation error while trying to compute the norm directly from the definition.

Question 2:

(a) This problem was easy for most people. Some made the mistake of incorrectly factorising the prime 3.

(b) This was a relatively challenging problem. Because of the slight complication in the form of the ring of integers, small errors appeared in the determination of units. But overall performance on this problem was adequate.

(c) This problem was difficult. The 'only if' direction was solved correctly by many. However, showing that the congruence condition implies that the prime is reducible in the ring of integers proved elusive. Some students gave a somewhat inadequate argument using Legendre symbols.

(d) This problem was challenging for many people. Many noted that the minimal polynomial should factorise into linear terms modulo the prime. However, realising that the multiplicative group of the residue field should contain a 5-th root of 1 was difficult.

Question 3:

(a) This problem was mostly solved correctly. A few students missed the relation between the three primes dividing 2, 3, and 5.

(b) This problem seemed also to be familiar. Some errors occurred in the proof that $(y + \sqrt{-30})$ and $(y + \sqrt{-30})$ are coprime.

(c) This problem was difficult, in part because the formulation of the problem made it substantially more complicated than intended. One should have assumed n odd. Without this assumption two difficulties come up. The first is that $-d$ could be 1 mod 4, in which case the ring of integers of $\mathbb{Q}(\sqrt{-d})$ becomes slightly complicated. The second difficulty is the coprimeness of $(1 + \sqrt{-d})$ and $(1 - \sqrt{-d})$, which is crucial for the argument. In spite of this, one or two people gave solutions that came close to being complete.

B3.5 Topology and Groups

This paper was slightly harder than in previous years, and the spread of marks achieved by the candidates was significantly broader.

Question 1: The bookwork on this question was relatively challenging. It required students to show that homotopy equivalent, path-connected spaces have isomorphic fundamental groups. A significant number of students assumed that the homotopy equivalences and any relevant homotopies preserved basepoints. This was clearly unjustified, as no-one included these properties as part of the definition of a homotopy equivalence in the previous sub-part. As a result, little credit was given for these solutions.

Part (b) (i) - (iii) was new material. Many could do (i) and (ii) but most found (iii) challenging.

Part (c) was more group-theoretic and was generally fairly well done. In this question, the easiest way to establish that a homomorphism was an isomorphism was to exhibit an inverse. Many students got unstuck by trying to show injectivity and surjectivity directly. Partial credit was given here if just surjectivity was established. Many students realised in (c) (iii) that y is not in the image of ϕ_3 and hence ϕ_3 is not an isomorphism. Many arguments were intuitive rather than a rigorous discussion of reduced words, and received partial credit.

Question 2: The bookwork on push-outs and the Seifert-van Kampen theorem was very standard and was well done. Some students attempted to prove that the fundamental group of $S^1 \times D^2$ is $\mathbb{Z}$ by using SVK. This is not the right way to do it, and all attempts along these lines came unstuck. Instead, an obvious homotopy retraction to the circle is the right thing to use.

The application of SVK in (b) was more challenging. Roughly half of all students could make a reasonable attempt at it, although relatively few could give a solution to (b)(iii).

Question 3: The bookwork here was straightforward and a central part of the course. It was well done. But when it came to adapting the bookwork in (b)(i), a minority of students gave an ill-defined homomorphism. Part (b)(ii) on covering spaces was quite easy but was not generally well done. The construction of the covering space $\tilde{X}$ was achieved only by a minority of the students. Many answers attempted to show that $\tilde{X}$ was two tori or two circles attached together in some way. Instead it is a single torus and a single circle glued together at two points.

B4.1: Banach Spaces

Question 1 (Separability; Hahn–Banach Theorem)

Part (a) was answered badly despite appearing in the Part A Metric Spaces course and in a B4.1 problem sheet. Inequalities were handled poorly and often incorrectly and some candidates swapped lim and inf without realising this amounted to assuming the result they were asked to prove.

Part (b) worked well, with a variety of valid examples provided and most candidates demonstrating understanding. Most of those who went astray did so because they forgot that a real normed space has to be closed under addition and scalar multiplication.

Part (c) was a minor variant on a standard HBT application and was answered well. A few candidates omitted to prove or at least to observe that their functionals were well defined. A few others went off course by trying to use part (a), and so specified a functional which was not linear.

Part (d) was intended to be challenging to get fully correct, and so it proved. A number of candidates had all the right ideas, including realising that part (a) was relevant, and lost their way only on the fine detail. At the other end, those who had not appreciated that separability was crucial wrote fallacious answers.

Question 2 (Stone–Weierstrass Theorem, subalgebra form)

Parts (a) and (b) were generally done well. A common mistake was omission of 'subspace' from the (standard) definition of linear sublattice, despite this being clearly a necessary condition for the argument in (b) to work.

Part (c) was new. Subpart (i) involved classical analysis and was done poorly. Most candidates were able to sum the expression for $B_n(f_a)$ but very few seemed to realise that they needed to prove that this converges *uniformly* to f_a on $[0, 1]$ and arguments given for pointwise convergence were unconvincing. Subpart (ii), an application of (b), was handled much better, though it was disappointing that a number of candidates tried to apply it to a subset of $C[0, 1]$ which was not a subspace.

In (d), many did comment that the result in (b) supplied an explicit sequence of approximating polynomials but failed to observe that a somewhat stronger assertion could be made. All missed the point that the proof of (b) involves just polynomial approximation to $t \mapsto \sqrt{t}$ and not the full Weierstrass theorem.

Question 3 (Sequence spaces: completeness; spectral theory)

The bookwork in (a) caused few problems. Part (b) proved a good test of candidates' abilities, with opportunities for slick, economical answers missed by those who failed to see how to relate each subpart to the previous one. No one spotted that M_α maps into c_0 when $\inf \alpha_i = 0$ and so cannot then be surjective. There were good answers to part (c) (and part (d)) from candidates who had not dawdled over (b), with clear understanding of the background spectral theory displayed. It was perhaps too much to expect that justification would be supplied for the values of the limits arising in (c)(ii) and (c)(iii) and very few marks were deducted if reasons were not given.

B4.2: Hilbert Spaces

Everybody has attempted the first question. Bookwork in (*a*)(*i*) and (*b*)(*i*) has been done well with minor exceptions. Problems in (*a*)(*ii*) and (*b*)(*ii*) are not straightforward as the bookwork. In particular, in (*a*)(*ii*), many students realised that the sequence of norms of minimal norm elements is growing and bounded from above. However, how to deduce that the corresponding sequence of minimal norm elements is a Cauchy sequence was not clear for majority of students. In (*b*)(*ii*), the most challenging part is to prove the uniqueness of the norm-preserving extension. The right approach is based on the Riesz Representation Theorem and the Projection Theorem rather than the Hahn-Banach Theorem.

Approximately, a half of students took the second question. Problem (a)(ii) turned out to be the most difficult in the entire exam paper. The bookwork part (*a*)(*i*) shows that one should use the Banach-Steinhaus theorem. Very few students selected a reasonable dense set on which the strong convergence to the identity operator takes place. Further technical step is to analysis the uniform convergence of a particular sequence of functions on a closed interval. Part (*b*) of the second question contains relatively difficult bookwork part related to the Open Mapping Theorem. It has been mostly done well. Problem *b*(*iii*) is in fact an application of the above mentioned bookwork. However, it has been realised by minority of students who attacked this problem. Problem (*b*)(*iv*) is the direct consequence of *b*(*iii*) applied to the identity operator.

Another half of students attempted the third question, in which the bookwork was done well. In problem part (*b*), the first two questions have been answered by the majority of students while the third one, *b*(*iii*), is more complicated. The difficult part is the proof of the completeness of the operator space in question. Finally, in the last problem part (*c*), one should refer to part (*b*)(*i*) and select a basis in an optimal way.

B5.1: Stochastic Modelling and Biological Processes

Question 1. This question was answered by the majority of candidates. Parts (a) and (b), being bookwork and a simple application, were well done. Many candidates struggled to solve the equation in (c), and few candidates made progress in (d).

Question 2. This question was answered by about half the candidates. Part (a) was standard and well done by the majority. In part (b) many candidates struggled to write down the correct equations for the boundary compartments.

Question 3. This question was answered by about half the candidates. In part (a) many candidates derived a system of ordinary differential equations that was not closed.

B5.2: Applied PDEs

Q1 Most students attempted this question. Common mistakes in the first part included missing the distinction between the values of $s^{\pm}$ corresponding to each region, and incorrectly identifying the domain of definition, which was $y < x/2$ for both regions. The domain of definition is almost automatic to see with a simple sketch of the characteristics (straight lines through the origin), but very few candidates actually made use of a sketch. Part c was conceptually the most challenging, and only a small number of candidates correctly identified the dividing points for which characteristics travel *into* the domain.

Q2 Part b was the most challenging part of this question. The conservation of flux across the shock requires showing that there is zero jump of $u(v-V)$ across the shock, where V is the shock velocity and hence $v-V$ is the velocity entering the shock. Showing this is a very simple algebraic manipulation, but very few candidates approached it correctly. Part c was mostly done well, though several candidates stopped at $t = 1/(2s)$, obtained from setting the Jacobian to be zero, without identifying an actual earliest time.

Q3 This question was attempted by the fewest number of candidates, though average scores were the highest for those that did attempt it. Full marks on part c required properly justifying adding three image points with the proper signs, i.e. an argument about how the points take care of the boundary conditions .

B5.3: Viscous Flow

Questions 1 and 2 were the most popular questions. All questions were generally well done.

- **Question 1:** The bookwork parts were well done in general. Some candidates did not use a corollary to Reynolds transport theorem (or apply Reynolds transport theorem component wise) when computing $\frac{d}{dt}\iiint_{V(t)} \rho \mathbf{u} dV$. Part (b) was well done. Minor slips occurred in computing expressions for the flux.

- **Question 2:** This question was well done. Some candidates introduced the incorrect scaling for pressure in (a). Some candidates did not correctly explain why the $\partial p/\partial x$ term does not appear in equation (3). Part (d) was well done. In part (e) there were some problems when finding an expression for the dimensional leading-order component of the shear stress σ_{12}.

- **Question 3:** This was the least popular question. However, those that did this question did it very well.

B5.4: Waves and Compressible Flow

Q1: The first two parts were well done, though many candidates did not explain the use of Bernoulli's equation to derive the dynamic boundary condition in (a) well (e.g. they identified the arbitrary constant with a variable pressure instead of with atmospheric pressure), and a number of candidates were confused about the amplitude of the solutions in (b) being arbitrary. Part (c) was found difficult and no candidates were able to derive the given expression for $\eta(x,t)$. One or two candidates correctly identified that the solution is dominated by $\cos(3\pi x/a)$ for the very last part, but none bothered to sketch it as requested.

Q2: Part (a) was done well, although some candidates gave lengthy regurgitation of bookwork that was unnecessary and in some cases did not actually provide a coherent explanation. Part (b) was done surprisingly poorly, with many students having difficulty solving the transformed problem. A common error was to look for a 'separable solution' $\hat{\phi}(x,\ell,z) = f(x)g(\ell)h(z)$, including the transformed wave-number. Part (c) was on a problem sheet and was done reasonably well. The unseen part (d) was found more difficult, with just one candidate giving a complete solution.

Q3: The bookwork in parts (a) and (b) here was generally done well, though some candidates found very long-winded ways of showing that the flow was homentropic in (a), and some got lost in algebra in (b). There were a number of excellent solutions to part (c), and a number of very confused answers including some who had the wrong picture of the domain and some who attempted to insert shocks into their solutions.

B5.5: Further Mathematical Biology

- **Question 1:** Those candidates who attempted this question typically produced good answers. Parts (a) and (b) contained fairly standard bookwork and were well done, while part (c) was more challenging. While the majority of students were able to derive expressions relating the parameters D_1, D_2, γ and c, few were able to solve them as the algebra was quite involved.

- **Question 2:** Most candidates did well on this question. They were able to derive the stated inequalities in parts (b) and (c), having noted that the stated equations were a variant on the standard PDEs for pattern formation. The final part (d) was algebraically challenging and few candidates were able to sketch their results.

- **Question 3:** This question was completed reasonably well, with most candidates scoring full marks on parts (a) and (b). Parts (c) and (d) were more challenging, with some candidates failing to notice that the nutrient concentration attained its minimum value on the outer boundary ($x = L(t)$) and/or deducing that at steady state the tissue comprised an inner, nutrient-rich region and an outer, region in which cell death was active.

B5.6: Nonlinear Systems

Q1: Most students answered easily Part a, b(ii), c(i) but had difficulties articulating a proof for b(i). Students who drew possible maps were helped in their reasoning. Only a few students realised that question c(ii) could be easily answered using the result of b(i). As a consequence, most students failed to properly answer c(ii).

Q2: Only a few students attempted this problem. Part (a) of the question was relatively straightforward and students mostly answered it correctly. However, Part (b), was more of a challenge for the students despite the fact that it was a direct application of the same algorithm as in part (a). Students who managed to answer demonstrated a full grasp of the method.

Q3: Almost all students tried this question and many of them did well. The question was mostly theoretical and students who learned the material managed to give proper definitions of basic concepts and apply them directly. Part d required the computation of an integral and many students struggled. It seems that students are not required to manipulate integrals routinely and have difficulties with basic integration methods. The last part (e) proved to be too difficult for most students with only a handful showing a good enough understanding of the material to be able to relate the gradient of a first integral to the variational equations around a homoclinic orbit.

B6.1: Numerical Solution of Differential Equations I

Question 1

Most students attempted this question, but many had difficulties. Part a) was generally well done, although some students had trouble identifying the constants. Almost everyone had difficulties with part b), with too much being assumed about the function f, and many people getting into difficulty truncating the Taylor series. Part c) also caused a lot of trouble, with very few students being able to accurately derive an adaptive method that uses the two methods given.

Question 2

This question was done by almost all candidates, and was generally done well. A few students had difficulty reproducing the proof of the result in part b). In part c), some students didnt check for consistency or zero-stability, or failed to accurately state Dahlquists theorem. Part c)ii was generally well done.

Question 3

This was the least popular question, but those that attempted it generally did well. The main source of dropped marks in part a) was forgetting to state the boundary conditions for the recurrence relation. Part b) was generally done well, although some candidates got into difficulty showing that the function is bounded below by -1. Part c) posed difficulties for some candidates who didnt understand the requirement for positivity of the coefficients, which resulted in missing one (or more) of the conditions required for the discrete maximum principle to be satisfied.

B6.2: Numerical Solution of Differential Equations II

Question 1 was addressed by most candidates. All in all, questions of this type have been around in previous papers for several years and this was reflected in the relatively good performance of the candidates, except on the bits that were actually different from previous papers.

Question 2 corresponded to a main theorem from the textbook. Only a few candidates managed to follow all steps of the proof. Some of them actually proved a more general theorem given in the lecture notes, and ended up doing much more work.

The classification of each scheme was done correctly by most of the students attempting question 2, except from the last part, where no one realised that no conflict arises with the theorem regarding high order linear schemes. Also, for scheme iv) many did not realised that the scheme had only one possible value of λ to be TVD.

Question 3 was not very popular. Only the first bit was addressed by a handful of students, but the main steps of parts (b)–(d) were untouched, even if the result was actually left as exercise during the lectures.

B6.3: Integer Programming

Q1 saw the least uptake of the three, presumably because the topic was Lagrangian relaxation, with only 3 serious attempts.

Q2 was attempted by nearly everyone. There was a good spread of marks, and the problem worked well in allowing to distinguish between levels of ability at both ends of the scale.

Q3 was slightly easier than the other two and was attempted by everyone. It was a good discriminator of abilities at the lower and middle range of the scale, but with hindsight it should have contained more of a sting in the tail to better distinguish candidates at the top end of the scale.

Overall the exam worked well and seems to have given the students the chance of showing what theyve learned whilst discriminating well between different levels of mastery of the material. I was impressed by the amount of good mathematical reasoning I saw and felt that the students had worked really hard and achieved a good level of not only knowledge, but also skill and intuition in the subject.

B7.1: Classical Mechanics

Question 1 is on Lagrangian mechanics. Answers to part (a) were generally of a very high standard. The exception was part (a)(ii): one effectively needs to show that $\frac{\mathrm{d}}{\mathrm{d}t}f(\mathbf{q})$ satisfies Lagrange's equations, which follows from an application of the chain rule. A fair number of candidates did this correctly, but it entirely eluded others. Part (b)(i) required candidates to find the Lagrangian for a simple mechanical system; this was very well answered (apart from a few computational errors). Many candidates found the correct quadratic Lagrangian in (b)(ii), but only a handful correctly used this to find the initial angular accelerations.

Question 2 is on rigid body mechanics. Parts (a) and (b) are bookwork, while parts (c) and (d) are applications of Euler's equations that were either covered in lectures, or closely modelled on an example in the printed lecture notes. Candidates who had learned the course material well scored very highly in the question. There were a good number of largely complete solutions, scoring either full marks or close to full marks.

Question 3 is on Hamiltonian mechanics. The Lagrangian and Hamiltonian for a charged particle in a magnetic field, relevant to parts (a) and (b), were covered as an example in the lectures. A few candidates incorrectly identified $m\dot{\mathbf{r}}$ as the momentum canonically conjugate to $\mathbf{r}$. The computations are most easily done using a few vector identities, *e.g.* the scalar triple product identity, and a good number of candidates took this approach. A fair number also got bogged down quite early on in the question.

B7.2: Electromagnetism

The exam turned out to bit a little harder than the setter intended. Having said that, about 40% of the candidates handed in excellent work.

- Q1 Many marks were lost in part (c) and also in part (b) where students made various errors in the computations.
- Q2 There was a typo in part (ii) (where it says "part (a)" it should have said "part (b)i"). This was compensated by generous marking and by not taking points off where a student used the "wrong" field, as long as they were correct conceptually. Not many students were able to get to the last part of the question (part b(iii)).
- Q3 There was also a typo in this question in the second set of equations in part (a). Almost all students who attempted this question got to the right answer. The typo was compensated by generous marking of the question. Part (b) seemed to have been hard and there were many errors in part (ii) and (iii).

B7.3 Further Quantum Theory

The questions were intentionally somewhat easier than previous years and candidates were able to obtain good marks.

The first question was popular on an early part of the course, and was well done except for the middle calculation that often was not properly completed, and the last part that was answered too briefly.

The second question was a relatively routine question on the variational principal, although the calculation did cause some difficulties for some candidates.

The third question was a little nonstandard, but nevertheless attracted a good number of strong attempts.

B8.1: Martingales Through Measure Theory

Overall this is not a particularly hard paper, and most candidates were able to start with parts of the book-work and achieved a good score for each question attempted without much struggle if they understood the material.

Question 1. This is the most popular question, all candidates except one attempted this question. Most candidates were able to state the definitions in (a) and (b) correctly, but a few of them forget to say for (b) the totality measures should be the same for the uniqueness lemma. A few candidates had difficulty to define a proper π-system to argue correctly the independence of two σ-algebras following the hint, by using the uniqueness lemma. Some candidates could not justify properly the computations for the conditional expectation which should be evaluated by a few basic properties about conditional expectations.

Questions 2. This is another question most candidates attempted, and scored quite well. Many candidates lost some marks for part (a)(ii), and could not combine two inequalities together to justify the arguments. A few candidates could not provide a proof for part (b) (ii) to show the convergence almost surely by using Borel-Cantelli lemma, by noticing that an integer valued sequence converges if and only if it is eventually constant.

Question 3. A few candidates attempted this question, and most of them did quite well for parts (a), (b)(i)(ii), but a few of them had difficulty to arrive the estimate in part (b) (iii) by using Doob's L^2-inequality. Most of those attempted were able to find the bracket process in part (c), and to conclude the convergence, but several of them could not present the arguments clearly, and lost a few marks for this part.

B8.2: Continuous Martingales and Stochastic Calculus

Each of Questions 1 and 3 was attempted by roughly half of the students sitting the exam, while question 2 was attempted by nearly everybody. Question 1 proved to be the hardest. The difficulty was caused by the second part where students were asked to compute the quadratic variation of a continuous Gaussian martingale X with independent increments. While

this was an easy computation if one used the fact that $X_t^2 - \langle X \rangle_t$ is a martingale, it was much more involved if one tried to use the definition of the quadratic variation via limits of sums of squares of increments, as many students did. Numerous students got stuck on such computation but did not seek to compute $\langle X \rangle$ in another way. Those who did typically scored very well and the results of this question displayed the largest variance among the three questions.
Question 3 saw a higher average mark and lower variance than Question 1. Most common mistakes involved failure to define the stochastic integrals properly, in particular to explain why the second integral's definition did not depend on T in the first integral, failure to justify the use of Itô's formula or the optional sampling theorem, or errors in computing the derivatives.
Question 2 had the highest standard of answers with mean raw mark of 20 and median of 21. Students often worked hard to justify taking the limits where the use of optional sampling theorem was justified on unbounded horizon and sometimes this led to errors when re-arranging terms. Common mistakes also involved not justifying the application of earlier results in the last part of the problem or identifying a wrong value for θ.

B8.3: Mathematical Models of Financial Derivatives

Question 1: The solution involved finding the price of a European option, with arbitrary payoff $P_o(S_T)$, assuming that the underlying asset evolved according to the process

$$dS_t = \alpha\, S_t\, dt + \sigma\, dW_t.$$

To do this question it was necessary to know how to solve this SDE, how to derive the Black-Scholes equation for an option which depended on this asset and the Feynman-Kăc theorem to price the option.

To the surprise of the examiner, who thought this was the most difficult of the three, candidates on the whole did well on this question.

Question 2: The question involved solving the SDE

$$\frac{dS_t}{S_t} = \mu\, dt + \sigma\, dW_t$$

then using the solution to find the Black-Scholes value of a European option with the payoff $\max(\log(S_T/K, 0)$. Then one had to show that if $V(S, t)$ was a solution of the Black-Scholes equation then so too was $(S/B)^\gamma\, V(B^2/S, t)$, where $\gamma = 1 - 2r/\sigma^2$ and, finally, the students were asked to price the option described above but where a barrier, B with $0 < B < K$, was introduced.

The main problems were in applying the Feynman-Kăc formula to get the price of the option with payoff $\max(\log(S/K), 0)$ and in proving that if $V(S, t)$ is a solution of the Black-Scholes equation then so too is $(S/B)^\gamma V(B^2/S, t)$.

Question 3: This involved firstly proving an arbitrage result about American and European call options (with the same parameters and no dividends) and hence deducing that it was never optimal to exercise an American option before expiry (assuming the interest rate was strictly positive). Secondly it involved solving a perpetual American call option (with positive dividends and interest rate) to find both the price and the optimal exercise boundary, $\hat{S}$. Finally, it was assumed that the holder of the American call option chose an arbitrary exercise price $\bar{S}$ (but greater than the option's strike); in this case you had to show either that

- if $\bar{S} < \hat{S}$ then the option could be made more valuable by increasing the value of $\bar{S}$—by differentiating the solution with respect to $\bar{S}$; or
- if $\bar{S} > \hat{S}$ then the option price went below the payoff for $S \approx \bar{S}$, which represents an arbitrage — in this case you differentiate the solution with respect to S and show that at $\bar{S}$ the slope is greater than the payoff's slope.

The main problem with this question seemed to be a lack of time to complete it.

B8.4: Communication Theory

All three questions were approximately equally popular. Most students managed to gain all points for 1a,2a,3a.

In 1b) most students realized that $a = 1$ has to be treated separately, but curiously not many realized that this is also the case for $a = -1$. In 1c), most students managed to write down the joint information between the input and output of the cascaded channel in terms of entropies but did not get further. A common mistake was to get confused by the chain rule.

In 2a)(i) a common mistake was to state $\leq 1 + H(X)$ as upper bound, which is however not sharp. In 2b), most students managed to calculate the minimum but a recurring mistake was to forget the normalizing factor for the expected length $L^\star$. In 2b)(ii), some students ignored the implicit restriction that ℓ_x has to be integer-valued in order to construct a code. In 2c), many attempts had the right intuition to exploit that $p_1 > \frac{2}{5}$ has consequences for the probability masses of subtrees. However, very few managed to turn this into a rigorous argument.

In 3b) only a few students realized that a simple calculation can be given by conditioning. On the other, most students managed to get far by a direct calculation where sums are spelled out. Part i of 3c) was quite well received, and most students who attempted this question got the right answer. On the other hand, in 3c)(ii) only a few students managed to write down the minimizer.

B8.5: Graph Theory

Every single candidate attempted question 1, and almost all attempted question 2.

Question 1 is not that easy. The results are simple at an intuitive level, but giving proper proofs isn't so easy. Most candidates gave some sort of partly-valid argument. Disappointingly, this was true even for (a), which was written out in full detail in the notes and gone through slowly in lectures.

The bookwork part of question 2 was overall OK; a fair number of good answers, and quite a few partial answers at least showing some of the ideas. Several candidates wasted time proving (a version of) what was given in the hint. Nobody managed the last part of (b) completely, though a few candidates came close. For the construction, several candidates found their own solutions rather than using the (not very helpful, in hindsight) hint.

Only 3 candidates attempted question 3; those that did, did not do it well.

BO1.1: History of Mathematics

Both the extended coursework essays and the exam scripts were blind double-marked. The marks for essays and exam were reconciled separately. The two carry equal weight when determining a candidate's final mark.

The paper consisted of two halves, which carry equal weights. In section A ('extracts'), the candidates were invited to comment upon the context, content and significance of two samples of historical mathematics (from a choice of six). Out of nine candidates, one person answered each of questions 1 and 6; two people answered questions 3 and 4. The most popular questions were 2 and 5, which each attracted answers from six candidates.

The unpopularity of questions 4 and 6 is probably connected with the fact that the relevant topics were touched upon only briefly within the lecture course. The extract presented to candidates in question 1 was almost entirely symbolic in nature and rather tricky to interpret, so candidates may not have felt that there was much here to get hold of. It was disappointing not to see any comment here on the homogeneity of Harriot's equation: the appearance of "000" on the right-hand side. Notational issues (namely Maclaurin's abundance of dots) may also have put people off question 3, even though this extract related to a topic (calculus/infinite series/rigour in analysis) that was at the core of the lecture course. There were some misinterpretations of the language of the extract in question 3: "the law of continuation" does not refer to the convergence of the series; no candidate understood what "z being supposed to flow uniformly" meant (i.e., that $\dot{z}$ is constant).

Despite being a popular choice, the quality of answers to question 2 varied significantly, with several candidates failing to note that the notion of a limit was the key concept here; too many candidates implied that Newton's *Principia* is about calculus. Answers to question 5 were also quite varied, with some candidates being inclined to read more into the presence of the εs and δs on the page than was warranted (they do not correspond to the modern conventions of using these symbols); the popularity of this question probably stemmed from the familiarity of the relevant concept (i.e., that of a Riemann integral) from Prelims.

Some candidates organised their answers to questions from section A under the three headings 'context', 'content', 'significance', with material mostly being distributed correctly. Some candidates were inclined to go beyond the immediate content of the extract in their descriptions thereof, but this was not a major problem. Slightly more serious was the vagueness found in some answers under the heading 'significance': some candidates merely stated that the mathematics at hand "was very significant" without attempting to explain how or why.

In section B of the paper, candidates were invited to write an essay, taken from a choice of three. Questions 7, 8 and 9 attracted answers from two, four, and three candidates, respectively. As for questions 4 and 6 above, the material relevant to question 7 was covered in just one lecture, so this probably explains its unpopularity. On the whole, answers to questions from section B were quite well done, and tended to be better than those for section A, barring a tendency towards vagueness in answers to question 9, and the inclusion of too much material on mathematicians other than Lagrange in answers to question 8. Lagrange's work on mechanics was only rarely mentioned.

For the coursework essays, the candidates were invited to write about some aspects of the context, development, reception, etc., of the Quadratic Reciprocity Theorem. Although it could not be assumed that all candidates taking O1 had already met this theorem in the Part A Number Theory short option, a summary of the relevant mathematics was provided, and this appears to have been assimilated well by all candidates. With regard to the coursework essays, it should be emphasised that the assessors were looking for the use of primary sources (a central feature of O1), and penalised where they did not find it.

BEE, BSP and BOE essays and projects

Mark reconciliation was handled for essays and projects as part of the same exercise. Some assessors/supervisors did not make the deadline for submitting marks so the procedure was handled on a rolling basis once initial suggested marks were received, but overall the process went smoothly.

If the proposed marks were sufficiently close, as set out in the guidelines, then the supervisor and assessor were informed that the automatic reconciliation procedure would be applied unless they indicated that they wished to discuss the mark further. If the proposed marks differed sufficiently from each other, then the supervisor and assessor were asked to confer in order to agree a mark.

BN1.1: Mathematics Education

The assessment of the course is based on:

- Assignment 1 (Annotated account of a mathematical exploration) 35%
- Assignment 2 (Exploring issues in mathematics education) 35%

- Presentation (On an issue arising from the course) 30%

Each component was double-marked, with Dr Jenni Ingram (JI) or Dr Gabriel Stylianides (GS) plus myself (NA) as assessors. Each component was awarded a USM (agreed between assessors for double-marked components), and then an overall USM was allocated according to the weightings above. Where a significant difference between marks awarded by the two assessors arose or marks were across a grade boundary, scripts were discussed in more detail before agreeing a mark.

There were 8 students on the course this year, all but one of whom also went on to study for the BN1.2 (Undergraduate Ambassador Scheme) in Hilary Term. We were pleased to be able to award three Firsts and four very high Upper Seconds, and this years marks have a higher mean and smaller variance in comparison with the previous year.

BN1.2: Undergraduate Ambassadors Scheme

The assessment of the course is based on:

- A Journal of Activities (20%)
- The End of Course Report, Calculus Questionnaire and write-up (35%)
- A Presentation (and associated analysis) (30%)
- A Teacher Report (15%)

The Course Report was double-marked, with Dr Gabriel Stylianides (GS) and myself (NA) as assessors. I was sole assessor for the Presentation and the host school teacher provided grades for the Teacher Report. As recorded in the table below, each component was awarded a USM (agreed between assessors for double-marked components), and then an overall USM was allocated according to the weightings above. Where a significant difference between marks awarded by the two assessors arose (these are underlined in the table), this was discussed in more detail before agreeing a mark.

There were 7 students on the course this year, all of whom had previously studied for the BN1.1 course in Mathematics Education in Michaelmas Term. All students engaged well with the practical aspects of the course leading to the majority receiving first class marks in these areas. Pleasingly, as last year, several candidates were awarded first class marks for their reflective writing too. There was only one candidate who achieved an overall mark of 70 or more, but all other candidates achieved upper second class marks. This is a similar (narrow) distribution to previous years.

Statistics Options

Reports of the following courses may be found in the Mathematics & Statistics Examiners' Report.

SB1 Applied and Computational Statistics

SB2a: Foundations of Statistical Inference

BS2b: Statistical Machine Learning

SB3a: Applied Probability

SB3b: Statistical Lifetime Models

SB4a: Actuarial Science I

SB4b: Actuarial Science II

Computer Science Options

Reports on the following courses may be found in the Mathematics & Computer Science Examiners' Reports.

OCS1: Lambda Calculus & Types

OCS2: Computational Complexity

Philosophy Options

The report on the following courses may be found in the Philosophy Examiners' Report.

102: Knowledge and Reality

127: Philosophical Logic

129: Early Modern Philosophy

E. Comments on performance of identifiable individuals

Removed from public version.

F. Names of members of the Board of Examiners

- **Examiners:**
  Prof Helen Byrne
  Prof Philip Candelas
  Prof Des Higham (External)
  Prof Dominic Joyce (Chair)
  Prof Frances Kirwan
  Dr. Neil Laws
  Prof Irene Moroz
  Prof Alexei Skorobogatov (External)

- **Assessors:**
  Dr Nick Andrews
  Dr Siddharth Arora
  Prof Ruth Baker
  Prof Dmitry Belyaev
  Dr Christian Bick
  Prof Francois Caron
  Prof Coralia Cartis
  Prof Jon Chapman
  Prof Andrew Dancer
  Prof Xenia de la Ossa
  Dr Jamshid Derakhshan
  Prof Jeffrey Dewynne
  Dr Richard Earl
  Prof Artur Ekert
  Dr Ali El Kaafarani
  Prof Karin Erdmann
  Prof Patrick Farrell
  Prof, Victor Flynn
  Prof Andrew Fowler
  Dr Kathryn Gillow
  Prof Christina Goldschmidt
  Prof Alain Goriely
  Prof Ben Green

Dr Stephen Haben
Dr Abdul-Lateef Haji-Ali
Prof Ben Hambly
Prof Ian Hewitt
Dr Christopher Hollings
Dr Jenni Ingram
Prof Minhyong Kim
Prof Jochen Koenigsmann
Prof Yakov Kremnitzer
Prof Marc Lackenby
Prof Alan Lauder
Prof Lionel Mason
Prof Kevin McGerty
Dr Giacomo Micheli
Prof Michael Monoyios
Prof Derek Moulton
Dr Peter Neumann
Prof Nikolay Nikolov
Prof Harald Oberhauser
Prof Jan Obloj
Dr Christophe Petit
Prof Jonathan Pila
Prof Hilary Priestley
Prof Zhongmin Qian
Dr Tyrone Rees
Prof Gesine Reinert
Prof Christoph Reisinger
Prof Alexander Ritter
Dr Ricardo Ruiz Baier
Prof Alexander Scott
Prof Dan Segal
Prof Dino Sejdinovic
Dr Gregory Seregin
Prof James Sparks
Prof Gabriel Stylianides
Prof Balazs Szendrői
Prof Jared Tanner
Dr Philippe Trinh
Dr Robert Van Gorder
Prof Sarah Waters
Dr Catherine Wilkins

Dr Yufei Zhao