Examiners' Report: Final Honour School of Mathematics Part C Trinity Term 2017

October 26, 2017

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

  See Table 1, page 1.

- **Numbers of vivas and effects of vivas on classes of result.**
  As in previous years there were no vivas conducted for the FHS of Mathematics Part C.

- **Marking of scripts.**
  The dissertations and mini-projects were double marked. The remaining scripts were all single marked according to a pre-agreed marking scheme which was very closely adhered to. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**
  See Table 7 on page 8.

Table 1: Numbers in each class

	Number					Percentages %				
	2017	(2016)	(2015)	(2014)	(2013)	2017	(2016)	(2015)	(2014)	(2013)
I	48	(44)	(45)	(45)	(56)	57.14	(50.57)	(46.39)	(45.92)	(47.46)
II.1	23	(31)	(39)	(42)	(41)	27.38	(35.63)	(40.21)	(42.86)	(34.75)
II.2	12	(9)	(13)	(11)	(15)	14.29	(10.34)	(13.4)	(11.22)	(12.71)
III	1	(3)	(0)	(0)	(4)	1.19	(3.45)	(0)	(0)	(3.39)
F	0	(0)	(0)	(0)	(2)	0	(0)	(0)	(0)	(1.69)
Total	84	(87)	(97)	(98)	(118)	100	(100)	(100)	(100)	(100)

B. New examining methods and procedures

Two changes were made to examining procedures in mathematics. Firstly, the length of time allowed for mathematics unit papers increased from 1.5 hours to 1.75 hours. Secondly, dissertations were marked by the supervisor and one assessor, rather than by two assessors.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

D. Notice of examination conventions for candidates

The first notice to candidates was issued on 15th February 2017 and the second notice on 8th May 2017. These contain details of the examinations and assessments.

All notices and the examination conventions for 2017 examinations are on-line at http://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments.

Part II

A. General Comments on the Examination

The examiners would like to thank in particular Helen Lowe, Waldemar Schlackow and Charlotte Turner-Smith for their commitment and dedication in running the examination systems. We would also like to thank Nia Roderick, and the rest of the Academic Administration Team for all their work during the busy exam period.

We also thank the assessors for their work in setting questions on their own courses, and for their assistance in carefully checking the draft questions of other assessors, and also to the many people who acted as assessors for dissertations. We are particularly grateful to those—this year the great majority—who abided by the specified deadlines and responded promptly to queries. This level of cooperation contributed in a significant way to the smooth running of what is of necessity a complicated process.

The internal examiners would like to thank the external examiners Professor Chris Howls and Professor Simon Blackburn for their prompt and careful reading of the draft papers and for their valuable input during the examiners' meeting.

Timetable

The examinations began on Monday 29th May and finished on Saturday 10th June.

Medical certificates and other special circumstances

The examiners were presented with factors affecting performance applications for seven candidates.

Setting and checking of papers and marks processing

Following established practice, the questions for each paper were initially set by the course lecturer, with the lecturer of a related course involved as checker before the first draft of the questions was presented to the examiners. The course lecturers also acted as assessors, marking the questions on their course(s).

The internal examiners met in early January to consider the questions on Michaelmas Term courses, and changes and corrections were agreed with the lecturers where necessary. The revised questions were then sent to the external examiners. Feedback from external examiners was given to examiners, and to the relevant assessor for each paper for a response. The internal examiners met a second time late in Hilary Term to consider the external examiners' comments and assessor responses (and also Michaelmas Term course papers submitted late). The cycle was repeated for the Hilary Term courses, with two examiners' meetings in the Easter Vacation; the schedule here was much tighter. Following the preparation of the Camera Ready Copy of the papers as finally approved, each assessor signed off their paper in time for submission to Examination Schools in week 1 of Trinity Term.

A team of graduate checkers, under the supervision of Helen Lowe, Jan Boylan and Hannah Harrison, sorted all the marked scripts for each paper of this examination, carefully cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way a number of errors were corrected, each change was signed by one of the examiners who were present throughout the process. A check-sum is also carried out to ensure that marks entered into the database are correctly read and transposed from the marks sheets.

Determination of University Standardised Marks

The Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average in each class over the last four years, together with recent historic data for Part C, the MPLS Divisional averages, and the distribution of classifications achieved by the same group of students at Part B.

The examiners followed established practice in determining the University standardised marks (USMs) reported to candidates. This leads to classifications awarded at Part C broadly reflecting the overall distribution of classifications which had been achieved the previous year by the same students.

We outline the principles of the calibration method.

The Department's algorithm to assign USMs in Part C was used in the same way as last year for each unit assessed by means of a traditional written examination. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part B classification of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage). Working with the data for this population, numbers N_1, N_2 and N_3 are first computed for each paper: N_1, N_2 and N_3 are, respectively, the number of candidates taking the paper who achieved in Part B overall average USMs in the ranges $[70, 100]$, $[60, 69]$ and $[0, 59]$, respectively.

The algorithm converts raw marks to USMs for each paper separately (in each case, the raw marks are initially out of 50, but are scaled to marks out of 100). For each paper, the algorithm sets up a map $R \to U$ (R = raw, U = USM) which is piecewise linear. The graph of the map consists of four line segments: by default these join the points $(100, 100)$, $P_1 = (C_1, 72)$, $P_2 = (C_2, 57)$, $P_3 = (C_3, 37)$, and $(0, 0)$. The values of C_1 and C_2 are set by the requirement that the proportion of I and II.1 candidates in Part B, as given by N_1 and N_2, is the same as the I and II.1 proportion of USMs achieved on the paper. The value of C_3 is set by the requirement that P_2P_3 continued would intersect the U axis at $U_0 = 10$. Here the default choice of *corners* is given by U-values of 72, 57 and 37 to avoid distorting nonlinearity at the class borderlines.

The results of the algorithm with the default settings of the parameters provide the starting point for the determination of USMs. The examiners have scope to make changes, usually by adjusting the position of the corner points P_1, P_2, P_3 by hand, so as to alter the map raw $\to$ USM, to remedy any perceived unfairness introduced by the algorithm, in particular in cases where the number of candidates is small. They also have the option to introduce additional corners.

Table 2 on page 5 gives the final positions of the corners of the piecewise linear maps used to determine USMs from raw marks. For each paper, P_1, P_2, P_3 are the (possibly adjusted) positions of the corners above, which together with the end points $(100, 100)$ and $(0, 0)$ determine the piecewise linear map raw $\to$ USM. The entries N_1, N_2, N_3 give the number of incoming firsts, II.1s, and II.2s and below respectively from Part B for that paper, which are used by the algorithm to determine the positions of P_1, P_2, P_3.

Following customary practice, a preliminary, non-plenary, meeting of examiners was held two days ahead of the plenary examiners' meeting to assess the results produced by the algorithm alongside the reports from assessors. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low. These revised USM maps provided the starting point for a review of the scalings, paper by paper, by the full board of examiners.

Table 2: Position of corners of piecewise linear function

Paper	P_1	P_2	P_3	Additional corners	N_1	N_2	N_3
C1.1	(2.53, 37)	(20, 50)	(42, 70)		2	7	0
C1.2	(8, 37)	(25, 50)	(43, 70)		2	7	0
C1.3	(6.03, 37)	(22, 57)	(36, 72)		8	10	0
C1.4	(10, 37)	(20, 57)	(38, 72)		4	11	0
C2.1	(12.75, 37)	(22.2, 57)	(36, 72)		6	1	0
C2.2	(5.29, 37)	(22, 57)	(33.2, 72)		10	4	0
C2.3	(6.09, 37)	(15, 50)	(32, 72)		2	1	0
C2.4	(8, 37)	(20, 50)	(36, 72)		5	4	0
C2.5	(15.34, 37)	(26.7, 57)	(43.2, 72)		5	2	0
C2.6	(15.8, 37)	(27.5, 57)	(35, 72)		4	1	0
C2.7	(8.62, 37)	(23, 57)	(40, 70)		10	11	0
C3.1	(8.62, 37)	(19, 57)	(32, 72)		8	4	0
C3.2	(7.58, 37)	(27, 57)	(35, 72)		9	8	0
C3.3	(11.66, 37)	(20.3, 57)	(35, 72)		1	2	0
C3.4			(39,72)		9	4	0
C3.5	(16.49, 37)	(28.7, 57)	(37, 72)		5	2	0
C3.6	(12.81, 37)	(28, 57)	(42, 70)		6	6	0
C3.7	(12, 37)	(25, 60)	(35, 72)		8	9	0
C3.8	(8, 37)	(25, 57)	(40, 72)		9	8	0
C4.1		(24, 60)	(33, 72)		9	8	0
C4.2	(10.91, 37)	(23, 57)	(35, 72)		8	6	0
C4.3	(8.16, 37)	(28, 57)	(38.2, 72)		5	4	0
C4.6	(11.72, 37)	(26, 57)	(38.4, 72)		3	3	0
C4.8	(5.06, 37)	(22, 57)	(33, 72)		6	3	0
C5.1	(8.33, 37)	(23, 57)	(36, 70)		8	6	0
C5.2	(8, 37)	(24, 57)	(39, 72)		8	4	0
C5.3	(15, 40)	(30, 60)	(40, 70)		0	1	0
C5.4					8	17	2
C5.5	(10.51, 37)	(24, 57)	(36, 72)		17	11	0
C5.6	(7.53, 37)	(25, 57)	(32, 70)		11	9	0
C5.7	(8, 37)	(29, 57)	(37, 70)		8	4	0
C5.9	(10.51, 37)	(27, 57)	(36, 72)		6	1	0
C5.11	(14.94, 37)	(26, 57)	(42, 72)		9	3	0
C5.12	(14.59, 37)	(25.4, 57)	(38, 66)		10	9	0
C6.1	(13.33, 37)	(30, 60)	(40, 70)		3	7	0
C6.2	(6.84, 37)	(25, 57)	(37.4, 72)		7	12	1
C6.3	(8.62, 37)	(29, 57)	(41, 70)		4	11	1

Paper	P_1	P_2	P_3	Additional corners	N_1	N_2	N_3
C6.4	(8.27, 37)	(30, 57)	(43, 70)		7	6	0
C7.4	(5.11, 37)	(26, 57)	(41, 72)		1	5	0
C7.5	(18.21, 37)	(33, 57)	(41, 72)		0	1	0
C8.1	(16.77, 37)	(29.2, 57)	(38.2, 72)		5	3	1
C8.2	(16.77, 37)	(29.2, 57)	(37, 71)		4	2	1
C8.3	(12.86, 37)	(23, 57)	(42, 72)		6	22	2
C8.4	(5.74, 37)	(23, 60)	(38, 72)		8	24	2
SC1	(15.56, 37)	(27.1, 57)	(42,70)		8	17	3
SC2	(17.64, 37)	(30.7, 57)	(42, 70)		7	20	5
SC4	(6.72, 37)	(22, 57)	(38, 70)		6	15	4
SC5	(12.87, 37)	(26, 57)	(37, 70)		3	6	3
SC6	(14.48, 37)	(25.2, 57)	(43, 70)		4	14	2
SC7	(15.8, 37)	(27.5, 57)	(44, 70)		1	8	1

Table 6 on page 7 gives the rank of candidates and the number and percentage of candidates attaining this or a greater (weighted) average USM.

Table 4: Percentile table for overall USMs

Av USM	Rank	Candidates with this USM or above	%
88	1	1	1.19
87	2	4	4.76
85	5	6	7.14
84	7	7	8.33
83	8	8	9.52
82	9	9	10.71
81	10	10	11.9
80	11	11	13.1
79	12	14	16.67
78	15	16	19.05
77	17	20	23.81
76	21	24	28.57
75	25	30	35.71
74	31	33	39.29
73	34	35	41.67
72	36	40	47.62
71	41	46	54.76
70	47	48	57.14
69	49	50	59.52
68	51	54	64.29
67	55	57	67.86
66	58	60	71.43
64	61	62	73.81
63	63	65	77.38

Av USM	Rank	Candidates with this USM or above	%
62	66	68	80.95
61	69	70	83.33
60	71	71	84.52
59	72	73	86.9
58	74	74	88.1
57	75	76	90.48
56	77	78	92.86
55	79	79	94.05
54	80	80	95.24
53	81	81	96.43
52	82	82	97.62
51	83	83	98.81
49	84	84	100

B. Equality and Diversity issues and breakdown of the results by gender

Table 6: Breakdown of results by gender

Class	Number								
	2017			2016			2015		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	11	37	48	10	34	44	8	37	45
II.1	5	18	23	10	21	31	7	32	39
II.2	2	10	12	4	5	9	3	10	13
III	0	1	1	0	3	3	0	0	0
F	0	0	0	0	0	0	0	0	0
Total	18	66	84	24	63	87	18	79	97
Class	Percentage								
	2017			2016			2015		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	61.11	56.06	57.14	41.67	53.97	50.57	44.44	46.84	46.39
II.1	27.78	27.27	27.38	41.67	33.33	35.63	38.89	40.51	40.21
II.2	11.11	15.15	14.29	16.67	7.94	10.34	16.67	12.66	13.40
III	0	1.52	1.19	0	4.76	3.45	0	0	0
F	0	0	0	0	0	0	0	0	0
Total	100	100	100	100	100	100	100	100	100

C. Detailed numbers on candidates' performance in each part of the exam

Data for papers with fewer than six candidates are not included.

Table 7: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
C1.1	9	33.56	11.17	66.22	16.63
C1.2	8	34.62	9.71	65	9.95
C1.3	18	30.28	6.05	66.28	7.64
C1.4	14	29	9.54	65.36	12.52
C2.1	7	36.29	7.04	74.86	10.27
C2.2	14	33.21	9.09	73.21	13.42
C2.3	3	-	-	-	-
C2.4	9	25.22	12.07	57.78	17.42
C2.5	7	42.14	6.64	77.57	12.43
C2.6	5	-	-	-	-
C2.7	21	36.67	7.96	70.48	9.69
C3.1	12	31.58	9.2	72.92	12.54
C3.2	17	32.71	6.76	68.71	10.71
C3.3	3	-	-	-	-
C3.4	13	44.62	4.86	86.69	11.35
C3.5	7	35.86	7.97	70.86	15.97
C3.6	12	42.17	8.38	80.33	16.68
C3.7	17	28.29	8.91	63.47	13.51
C3.8	16	30.88	12.76	65.56	18.03
C3.9	3	-	-	-	-
C4.1	17	29.12	7.11	66.35	12.04
C4.2	14	33.79	6.66	71.64	10.51
C4.3	9	34	8.51	67.78	13.23
C4.6	6	34.5	7.31	68.33	11.2
C4.8	8	27.5	9.81	65.5	13.56
C5.1	14	33.5	8.17	69.5	11.66
C5.2	12	31.33	8.86	64.17	9.98
C5.3	1	-	-	-	-
C5.4	22	70.36	5.89	70.36	5.89
C5.5	28	32.96	7.02	69.5	10.29
C5.6	20	30.95	7.67	68.25	12.7
C5.7	12	33.08	9.95	66.67	14.23
C5.9	7	33.86	6.54	69.14	10.32
C5.11	12	42	6.58	79.25	13.11
C5.12	18	40.61	5.99	73.94	11.29
C6.1	8	38.5	7.23	72.62	13.06
C6.2	19	30.79	7.7	64.68	9.9
C6.3	16	34.75	8.6	64.38	10.41
C6.4	13	40.92	7.78	73.08	13.49

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
C7.4	6	34.5	9.29	66.5	10.73
C7.5	1	-	-	-	-
C8.1	8	34.62	7.8	66.75	13.34
C8.2	7	37	6.53	72	13.44
C8.3	28	33.36	5.19	65.57	4.8
C8.4	31	29.48	7.48	64.81	7.53
SC1	13	39.85	7.4	73.08	12.26
SC2	16	39.25	6.74	70.44	12.34
SC4	8	28.88	11.51	63.12	13.53
SC6	4	-	-	-	-
CCS1	4	-	-	-	-
CCS2	1	-	-	-	-
CCS4	2	-	-	-	-
CCD	33	-	-	77.91	7.59
COD	2	-	-	-	-

The tables that follow give the question statistics for each paper for Mathematics candidates. Data for papers with fewer than six candidates are not included.

Paper C1.1: Model Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.88	18.88	5.11	8	0
Q2	10.50	10.50	4.20	4	0
Q3	18.17	18.17	8.66	6	0

Paper C1.2: Gödel's Incompleteness Theorems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.00	19.00	5.10	6	0
Q2	9.40	14.00	6.66	3	2
Q3	17.29	17.29	8.01	7	0

Paper C1.3: Analytic Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.33	13.33	2.31	12	0
Q2	17.21	17.21	5.07	14	0
Q3	14.40	14.40	2.84	10	0

Paper C1.4: Axiomatic Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.44	12.25	4.53	8	1
Q2	15.29	15.29	4.73	14	0
Q3	15.67	15.67	7.34	6	0

Paper C2.1: Lie Algebras

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.29	19.29	3.04	7	0
Q2	17.00	17.00	4.80	7	0
Q3	6.00			0	1

Paper C2.2: Homological Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.78	15.78	4.84	9	0
Q2	15.73	15.73	6.17	11	0
Q3	18.75	18.75	3.54	8	0

Paper C2.4: Infinite Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.63	13.00	6.86	7	1
Q2	10.43	12.00	6.16	6	1
Q3	12.80	12.80	9.23	5	0

Paper C2.5: Non-Commutative Rings

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.43	21.43	3.10	7	0
Q2	20.71	20.71	5.31	7	0

Paper C2.7: Category Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.30	20.30	5.45	20	0
Q2	16.39	16.39	4.22	18	0
Q3	15.20	17.25	4.71	4	1

Paper C3.1: Algebraic Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.67	14.50	7.64	2	1
Q2	14.36	14.36	4.37	11	0
Q3	17.45	17.45	5.24	11	0

Paper C3.2: Geometric Group Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.41	16.41	3.62	17	0
Q2	16.25	16.25	4.67	16	0
Q3	17.00	17.00		1	0

Paper C3.4: Algebraic Geometry

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.75	21.75	3.95	4	0
Q2	23.36	23.36	1.50	11	0
Q3	20.92	21.45	3.80	11	1

Paper C3.5: Lie Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.50	16.50	2.17	6	0
Q2	23.00	23.00		1	0
Q3	18.43	18.43	5.16	7	0

Paper C3.6: Modular Forms

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.63	17.63	6.55	8	0
Q2	23.60	23.60	1.58	10	0
Q3	21.50	21.50	4.59	6	0

Paper C3.7: Elliptic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.63	12.63	5.26	8	0
Q2	14.23	14.23	5.31	13	0
Q3	14.43	15.00	5.92	13	1

Paper C3.8: Analytic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.64	12.70	7.74	10	1
Q2	15.08	15.55	7.86	11	1
Q3	15.77	17.82	7.69	11	2

Paper C4.1: Functional Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.00	15.00	4.08	17	0
Q2	14.07	14.07	4.21	14	0
Q3	11.00	14.33	6.68	3	1

Paper C4.2: Linear Operators

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.85	19.85	2.97	13	0
Q2	15.60	16.75	5.03	4	1
Q3	12.50	13.45	5.14	11	1

Paper C4.3: Functional Analytical Methods for PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.33	15.20	6.86	5	1
Q2	17.56	17.56	2.13	9	0
Q3	18.00	18.00	9.45	4	0

Paper C4.6: Fixed Point Methods for Nonlinear PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.17	17.17	4.45	6	0
Q2	17.33	17.33	5.50	6	0

Paper C4.8: Complex Analysis: Conformal Maps and Geometry

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.57	16.57	5.35	7	0
Q2	8.67	8.67	4.18	6	0
Q3	17.33	17.33	0.58	3	0

Paper C5.1: Solid Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.38	17.38	3.75	13	0
Q2	22.00	22.00		1	0
Q3	15.79	15.79	4.87	14	0

Paper C5.2: Elasticity and Plasticity

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.92	14.92	5.88	12	0
Q2	12.67	13.60	3.88	5	1
Q3	18.43	18.43	2.51	7	0

Paper C5.5: Perturbation Methods

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.33	16.33	4.27	9	0
Q2	14.88	15.46	4.64	24	1
Q3	17.61	17.61	4.25	23	0

Paper C5.6: Applied Complex Variables

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.55	17.80	6.17	10	1
Q2	15.31	15.31	5.19	16	0
Q3	14.00	14.00	4.33	14	0

Paper C5.7: Topics in Fluid Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.33	13.33	8.16	6	0
Q2	18.80	18.80	3.74	10	0
Q3	16.13	16.13	4.22	8	0

Paper C5.9: Mathematical Mechanical Biology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.00	18.00	4.20	7	0
Q2	15.50	16.60	3.73	5	1
Q3	11.33	14.00	5.51	2	1

Paper C5.11: Mathematical Geoscience

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	22.60	22.60	3.78	5	0
Q2	18.11	18.11	4.76	9	0
Q3	22.80	22.80	1.32	10	0

Paper C5.12: Mathematical Physiology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.79	18.79	3.93	14	0
Q2	21.63	21.63	2.75	16	0
Q3	20.33	20.33	2.73	6	0

Paper C6.1: Numerical Linear Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.60	15.60	6.73	5	0
Q2	18.40	21.25	6.84	4	1
Q3	20.71	20.71	2.43	7	0

Paper C6.2: Continuous Optimization

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.33	16.33	3.58	18	0
Q2	13.58	13.58	6.47	12	0
Q3	14.44	16.00	5.83	8	1

Paper C6.3: Approximation of Functions

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.88	16.88	4.19	16	0
Q2	14.17	15.20	5.56	5	1
Q3	19.09	19.09	5.50	11	0

Paper C6.4: Finite Element Methods for Partial Differential Equations

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.17	18.40	7.33	5	1
Q2	22.82	22.82	2.09	11	0
Q3	18.58	18.90	4.19	10	2

Paper C7.4: Introduction to Quantum Information

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.00	15.00	7.18	5	0
Q2	17.80	17.80	3.03	5	0
Q3	11.75	21.50	11.27	2	2

Paper C8.1: Stochastic Differential Equations

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.83	20.20	7.17	5	1
Q2	17.38	17.38	1.69	8	0
Q3	12.33	12.33	8.33	3	0

Paper C8.2: Stochastic Analysis and PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.67	19.67	3.06	3	0
Q2	16.17	16.17	4.17	6	0
Q3	20.60	20.60	3.65	5	0

Paper C8.3: Combinatorics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.74	19.74	3.31	27	0
Q2	13.74	13.77	2.81	26	1
Q3	14.33	14.33	5.77	3	0

Paper C8.4: Probabilistic Combinatorics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.48	15.80	4.02	30	1
Q2	13.73	13.73	5.19	22	0
Q3	13.00	13.80	4.11	10	2

Paper SC1: Stochastic Models in Mathematical Genetics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.00	16.00	7.07	4	0
Q2	20.25	20.25	2.83	12	0
Q3	21.10	21.10	3.70	10	0

Paper SC2: Probability and Statistics for Network Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.22	15.40	5.89	5	4
Q2	18.92	19.92	4.44	12	1
Q3	20.80	20.80	3.00	15	0

Paper SC4: Statistical Data Mining and Machine Learning

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.50	15.50	5.55	8	0
Q2	8.33	8.33	4.04	3	0
Q3	16.40	16.40	5.68	5	0

D. Recommendations for Next Year's Examiners and Teaching Committee

None

E. Comments on papers and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. Some data to be found in Section C above have been omitted.

C1.1: Model Theory

Question 1

Almost everyone attempted this question. The first part asked for a proof of the upward Loewenheim-Skolem theorem, which was given correctly with very few exceptions. The same was true for the proof of quantifier-elimination in part 2(i); small mistakes included neglecting to prove the tautological direction, that a witnessed statement implies an existentially quantified one. Part 2(ii) could have been done rapidly using the Tarski-Vaught criterion, but very few chose to invoke it. Some gave a full proof of Tarski-Vaught in this setting, which is of course fine. Many others attempted to use the fact that in models of T, any sentence ϕ is equivalent to a quantifier-free one, ψ. (Some constructed a different quantifier-free equivalent for B and for C, which is problematic later on.) Further, the truth value of ψ in B, A, C is the same; this suffices to show that $Th(B) = Th(C)$, which most people did correctly. However as we do not know that $A \models T$ (this is what we are asked to prove), we do not know if ϕ, ψ are equivalent in A so this approach does not show that $A \models \phi$.

Part (iii), the example, was generally well done with many different examples given; a few neglected to give any indication of proof, or gave an incorrect proof.

Question 2

About half the examinees attempted Question 2. Parts (a,b) are entirely book work, and were largely done well. There was some confusion between the existence of an elementary embedding from A to B, and A actually being a substructure of B; the same in part (c); which was however done well on the whole. Few people got part (d) correctly. All answered correctly that not every theory has a universal model, but few gave a correct example. Perhaps the majority wrote that the universal model is unique up to isomorphism, but in general it is not (the notions of 'universal' and 'saturated' do not coincide.)

Question 3

Part (a) did not pose a problem. Part (b) was to be answered using the Los-Vaught test, via $\aleph_1$-categoricity; the majority did so well. In part (c), a part of the problem is to similarly state that the theory of infinite vector spaces over the 2-element field is complete. Many ignored this and proceeded directly to the description of the finite vector spaces, and their theories. The non-finite axiomatizability (d) was usually handled well.

C1.2: Gödel's Incompleteness Theorems

Question 1 covered the First Incompleteness Theorem and some elements of the Second Incompleteness Theorem. Part (a) was bookwork, proving that if PA is consistent, $PA \nvdash G$. Part (b) required candidates to infer from (a) that $PA \cup \{\sim G\}$ is consistent and hence by the completeness theorem for first-order logic (a prerequisite for this course) $PA \cup \{\sim G\}$ has a countable model, and then to show that G is true, in order to infer, on the assumption that PA is sound, that a countable model of $PA \cup \{\sim G\}$ cannot be isomorphic to the standard model. Part (c) was a demonstration that the non-derivability of the negation of the Gödel sentence of a system cannot be established just from the assumption that the system is consistent. Part (d) was to show that $PA \nvdash \sim G$ can be proved from the necessary and sufficient condition $PA \cup \{Con_{PA}\}$ is consistent. This requires arithmetization of the inference from consistency of PA to $PA \nvdash G$, a key lemma in the proof of the Second Incompleteness Theorem. The instruction in (d) to take $Pr(v_1)$, G, and X as in (c) was interpreted by 10 of the candidates who answered this question as allowing the assumption of what was to be taken as given in (c), on which construal (d) was exceedingly easy. Two candidates interpreted (d) as intended and gave fully correct answers.

Question 2 Part (a) was bookwork, asking candidates to prove that the diagonal lemma for PA is provable *in* PA, given the main lemma for this result. Part (b) called for the application of (a) to prove that for $G(v_1, v_2)$ a formula in the language of PA, with two free variables, there exists a sentence D such that $PA \vdash (D \equiv G(\overline{\ulcorner D \urcorner}, \overline{\ulcorner \sim D \urcorner}))$, given that the function $f(v_1) = \ulcorner \sim E_{\dot{v}_1} \urcorner$ is strongly definable in PA. This application of the Diagonal Lemma was not covered in the course. Part (c) asked candidates to determine the truth or falsity of a sentence X, known to exist by (b), such that $PA \vdash (X \equiv (Pr(\overline{\ulcorner X \urcorner}) \wedge Pr(\overline{\ulcorner \sim X \urcorner}))$, and whether X is provable or refutable in PA. The key to answering (c) was to show that $PA \vdash (Pr(\overline{\ulcorner X \urcorner}) \wedge Pr(\overline{\ulcorner \sim X \urcorner}) \equiv Pr(\overline{\ulcorner (X \wedge \sim X) \urcorner}))$. This equivalence is clearly true by the validity of the natural deduction rules for conjunction, and readily provable by P_1 and P_2. The three candidates who answered (c) did not see this point, and their attempted solutions could only result in partial marks. None of these three candidates attempted (d), which was on a property of the Rosser proof predicate not covered directly in the course.

Question 3 This question covered Löb's Theorem (proved from the properties of a provability predicate), the Second Incompleteness Theorem (proved from Löb's Theorem), the Fixed Point Theorem for the system of provability logic, GL, and the application of the Fixed Point Theorem and the Soundness Theorem for GL to establish an interpretation of the diagonal sentence for $(Pr(v_1) \supset X)$ in PA, which though similar to results proved in the course, was new, and brought out understanding of the significance of the Fixed Point Theorem for GL. This was the last examinable result covered in the course, and it was pleasing that candidates showed mastery of it as much as of earlier results.

C1.3: Analytic Topology

Question 1 The bookwork was done generally very well.

In (b)(i) a number of approaches were tried, but most of them failed to use local finiteness in a meaningful way (namely by seeing that $\bigcup_{j\geq n} \overline{A_j}$ is closed). In (b)(ii), it appeared more difficult than anticipated to deduce from (i) that every locally finite family must be finite (as otherwise it would contain a countable locally finite family).

Part (c)(i) was done mostly well. In part (c)(ii) the majority of candidates tried to show that $\beta\mathbb{N}$ is not countably compact, although a number of them even stated that compact spaces are countably compact.

Question 2 Most candidates could do either the bookwork or large parts of part (b) but only few managed to complete both of these. Various interesting approaches were taken in (b) with every implication proven by someone but most candidates could not prove all of them. A few candidates missed to remark on the existence of compactifications when observing that (3) implies (4).

Those who attempted part (c) could mostly see that the existence of such a function implies normally placed via $g^{-1}(1/n, \infty)$ and most tried to use functional normality of Y for the converse, but only few could complete the proof.

Question 3 Part (a) and parts (b)(i) and (b)(ii) were mostly done well, but then a lot of candidates seem to have run out of time. In (b)(iii) only few even noticed that they need to show that pointwise convergence implies p-convergence. No candidate supplied detailed examples.

C1.4: Axiomatic Set Theory

Generally the standard of answers was pleasingly high. Every candidate attempted Question 2 whilst Questions 1 and 3 were equally popular as a second question.

Question 1: This question gives an idea on how one might prove the independence of **Choice** from **ZF**. Although the required arguments in this question were essentially elementary, especially the later parts proved challenging with only one candidate producing a good attempt at the very last part.

In part (b) a number of candidates tried to directly prove the assertion by induction on α instead of considering $\mathrm{rk}_V(x)$ for $x \in V_A$.

In part (c)(i), a number of ingenious arguments were made instead of noting that by (b) and a very easy inductive argument, $A = \{x \in V_A : (\forall t\ t \notin x)^{V_A}\}$ which is clearly preserved by $\in$-automorphisms.

Question 2 This question explores an alternative construction of van Neumann's universe V and hence an alternative way to prove the consistency of **Foundation**.

In part (a)(ii) and (iii) the main mistake was using $\{x, y\} \in z$ or $\langle x, y\rangle \in f$ without defining this (as for example $\exists v \in f\ v = \{x, y\}$.

In part (b)(ii) a number most candidates used the recursion theorem with $F(x) = \bigcup x$ or $F(x) = x \cup \bigcup x$, but some then defined $TC(a) = \bigcup_{\alpha \in \mathbf{On}} G(\alpha)$ without arguing why this should be a set. The alternative approach to define $TC(a) = \bigcap\{T : a \subseteq T \wedge T \text{ transitive }\}$ suffers from the fact that this is not a well-defined set unless it is shown that there is some transitive set containing a.

In part (c)(iii) a lot of candidates didn't argue that the 'minimal' element of a non-empty $x \in W$ belongs to W by transitivity, essentially 'forgetting' about the relevant relativization.

Similarly in part (c)(iv), candidates were often not careful enough about relativization and absoluteness. A common proof attempt was to show that $TC(x)^A = TC(x)$ and then argue that 'having an $\in$-minimal element' was absolute. However, because $\mathcal{P}(TC(x))$ might be non-absolute (between A and U), this approach does not work.

Finally in part (d), a number of candidates didn't argue why they could assume that if x is well-founded then $x \notin TC(x)$ (and hence $x \notin x$).

Question 3 In part (a)(iv) a number of candidates assumed that an unbounded function can be taken to be weakly increasing without proving it.

In part (b)(i) it is important to note that H_κ is transitive so that if $d \in H_\kappa$ then $x \in d \leftrightarrow x \in d \wedge x \in H_\kappa$.

In part (c), it is necessary to check that if $x \in H_\kappa$ then (by regularity of κ) $\mathcal{P}(x)^{H_\kappa} = \mathcal{P}(x)$.

C2.1: Lie Algebras

All students attempted Question 1. The solutions were generally correct. There were mistakes in the explicit calculation of the Killing form for sl(2), and in (b)(iii), a common mistake was to think that it was sufficient to remark that I is contained in the radical of κ^I. Another mistake was to only compute the diagonal blocks of the matrix of an $ad(x)$ in (c)(i). In (c)(iii), some solutions missed the necessary condition $a^2 + b^2 = 0$, and most solutions did not remark on (or verified) the Jacobi identity.

Almost all students attempted Question 2. This proved more difficult. For (b)(ii), a common mistake was to say that $gl(n)$ is a simple Lie algebra and to apply (b)(i) directly. In b(i), there were mistakes in the last part of the proof (the application of Weyl's theorem). Finally, (c)(ii) posed difficulties.

Only two students attempted Question 3. The difficult parts proved to be finding the Cartan matrix and Dynkin diagram (F_4) in (b)(ii), finding complete arguments in (c).

C2.2: Homological Algebra

I was generally happy by the way the students performed. There were no particular difficulties encountered by the candidates that I could notice.

C2.3: Representation Theory of Semisimple Lie Algebras

Problem 1: The students found part (c) difficult, no complete solution was given. In part a(ii) and part (b), there were a variety of results used, depending on how much of the general theory of Verma modules was employed. The intention was to solve this problem with only sl(2) theory (except for part (c)), but full credit was given to correct solutions as long as the general properties used were stated clearly and correctly. There was a type in part (b)(ii), the word "simple" was missing, but the candidates assumed this, as intended.

Problem 2: Part (a)(iv) was difficult, no complete solution was given. Also no solutions for Part (b)(iii), but this is an easy consequence of the previous parts, particularly (a)(iv).

Problem 3: This was mostly bookwork or similar, but it involved more advanced material. Part (b)(iv) caused difficulties, and there were also mistakes in the computation of the highest weight of the module in (c).

C2.4: Infinite Groups

Each of the questions obtained a few excellent answers and a few good partial answers. The level seemed appropriate.

C2.5: Non-Commutative Rings

Questions 1 and 2 were the only questions attempted by the students. They were mostly well done. Some students did not fully prove the existence of a right inverse in 1.d) and hence lost marks. One student gave the universal property of the localisation instead of the definition and another forgot the definition of the multiplicative subset generated by a subset of a ring.

C2.6 Introduction to Schemes

Question 1 is a special case of Yomeda's lemma, which need to be reproved.

Question 2 In question 2, it is important to note that R_r is a quotient of the R-algebra $R[t]/(r \cdot t - 1)$. In question 2 (b), (i) the fact that X is a scheme is irrelevant.

Question 3 was not attempted.

C2.7 Category Theory

Q1 was attempted by most of the candidates. Part a and b were book work or similar and part c was new.

Q2 was also attempted by most candidates. Part a was bookwork and parts b and c were new. Very few candidates attempted

Q3. Part a was bookwork and parts b and c were new.

C3.1: Algebraic Topology

Question 1. Very few candidates attempted this question even though it was probably the easiest on the exam, perhaps reflecting less comfort with algebraic than topological aspects of the course. There was one strong answer.

Question 2. Most candidates attempted this question, but many struggled with one or more sections of it. There was one strong answer. (b) Most candidates made reasonable attempts in this part, though there were some errors of detail and here and there a lack of clarity about the relative cohomology case. (c) For part (i) a number of candidates did a computation rather than simply applying Lefschetz duality. In part (iii), most but not all attempts to directly compute the simplicial cup product went awry, but there were a number of good answers using excision and identifying the quotient space and its cohomology ring.

Question 3. All candidates attempted this question, and the standard of answers was reasonable overall. There were three nearly perfect answers and a number of other decent ones. (a) A number of candidates either misidentified the boundary map in the Mayer–Vietoris theorem, or simply recapitulated the standard snake procedure for determining the boundary map without actually identifying explicitly the map in this case. (c) (i) Many candidates used a modified form of Mayer–Vietoris for the mapping torus (also obtainable as a long exact sequence of a pair), with two instead of four copies of the homology of K —this substantially reduced the complexity of the calculations. Nevertheless, in many cases despite obtaining the correct exact sequence, candidates made mistakes in computing the kernels and cokernels and piecing together the answer. (ii) There were many correct answers here, though also a few that mistakenly claimed the manifold was of dimension 2.

C3.2 Geometric Group Theory

Q1 This was a basic question about presentations and algorithmic problems. All students attempted this. Part a was done well. Many students had difficulties with part b.i. They did not realize that to produce a list they had to run two procedures 'in parallel' and that it is not possible to give a yes/no answer whether a given map extends to a homomorphism. Similarly in part b.ii some candidates did not realize that to check surjectivity it is enough to check whether the generators are in the image. Several did not explain properly how to check whether generators are in the image. In part b.iii most candidates did not realize that they had to search for appropriate homomorphisms $H' \to G$ rather that $H \to H'$.

Quite a few candidates who did not manage to do part b assumed the results and gave either complete or partial solutions to part c. Only 1 candidate managed to solve all parts of this question.

Q2 This was a question on amalgamated products and actions on trees attempted by most students. Part a was done generally well. In part b many students couldn't show that if a group has an infinite centre then a finite index subgroup has infinite centre too.

Part c.i was generally well done but some candidates failed to give a convincing proof as they tried to define the fixed point rather than simply show its existence.

Several students solved part c.ii using induction.

Few students did part c.iii. Very few realized that part c.ii was relevant and that one could use the action on the subtree of b.ii rather than T.

Q3 This question was attempted by only 2 students. It was on the last part of the course dealing with quasi-isometries and hyperbolic groups.

Both candidates did well in parts a and b gaining most points but they failed to tackle part c.

C3.3: Differentiable Manifolds

Question 1: Part (a) was bookwork, and well done. There were no serious attempts on parts (b),(c).

Question 2: This was the easiest question, and everyone attempted it. Only a minority could do (c)(ii).

Question 3: Not an easy question, but mostly well done by more able candidates.

C3.4: Algebraic Geometry

Most candidates attempted questions 2 & 3. The quality of the scripts was exceptional, with almost flawless answers to every questions, even though the exam was about the same difficulty as in previous years.

The most common difficulty was question 3(d) with candidates not spotting an inverse map or running out of time.

Only one candidate had time to attempt three questions.

In question 2(b), R stands for the coordinate ring, this was announced during the exam, none of the candidates were confused in their answers to 2(b).

C3.5: Lie Groups

Question 1

This question was about basic structure theory for Lie algebras and subalgebras. Although the material was fairly elementary no candidates got all parts fully out. Parts (a),(b),(c) were done fairly well, although some candidates were too sketchy in their proof that $SL(n, R)$ was a manifold. Part (d), on definiteness and nondegeneracy, proved more difficult than expected. Some candidates got tangled up in the algebra here. Part (e), on using the maximal compact subgroup to find the topology of $SL(2, R)$ proved quite challenging, though a couple of candidates made good progress here.

Question 2

This question was on representation theory and characters, with some specific calculations in the case of $SU(2)$ and $SO(3)$. For some reason this question did not prove popular. Both candidates knew the bookwork on characters and complete reducibility very well, and could also do the character calculations for $SO(3)$.

Question 3

This was probably the most sophisticated question, using Schurs lemma and abelian representation theory to show that a certain noncompact Lie group had no faithful finite-dimensional representation and hence was not a matrix Lie group. All candidates attempted the question and some did very well indeed, showing they had a real grasp of the material around Schurs lemma. The bookwork was generally done very well. The last 2 parts were more difficult but some candidates managed the commutator calculation in (d) very well and were then able to put everything together for the final part (e).

C3.6: Modular Forms

Question 1: This question was on the first half of the course, but was probably the most challenging owing to its length: quite a lot of rather long pieces of work seen in the lecture course or problem sheets had to be recalled during the question. One candidate obtained full marks, and several very high marks, but overall the question had a broader spread than any other.

Question 2: This question was the most popular and most straightforward of the three, largely depending on recalling some simple bookwork and formulas from the course and applying the latter, the only part requiring some originality being (b)(iii). Unfortunately though the question was marred by a small error in (b)(ii): a direct application of the valence formula in (b)(i) shows the order of vanishing is 1 not 2. This difference between 1 and 2 is not so very significant in itself, but of course it may have hindered candidates attempting the question in several ways. First, candidates may have doubted that they correctly remembered the valence formula. Second, they might have wasted time trying to find an error in the simple calculation in part (b)(ii). Third, the error might have thrown them "off their stride" for (b)(iii). I took great care during marking to address all of these possibilities, being in particular generous with the marking in (b). Specifically, candidates who modified their statement of the valence formula in (b)(i) so that it gave the (incorrect) answer 2 in (b)(ii) did not lose marks. Second, candidates who left their statement of the valence formula in (b)(i) untouched, but doctored their working in (b)(ii) to produce the wrong answer 2 did not lose marks. Third, of course candidates who defied the incorrect claim in the question and stated that the answer was 1 did not lose marks. Finally, in marking (b)(iii) I insisted that candidates must get the main ideas to obtain full marks (namely, an application of the valence formula forces g to be non-vanishing away from the elliptic point, and then by various possible short routes one reaches it must be a multiple of f^2); however, given that candidates may already have wasted time on the earlier parts, and be uncertain whether they had the valence formula correct, I did not insist on the same level of detail for this argument as I would have done ordinarily.

Overall then I am confident that candidates who attempted this question did not fair less well than those who did not. Indeed, largely because the question was the most straightforward one, all candidates who did this question achieved at least as high a mark on it as they did on the other question they attempted (in fact, they all obtained high marks on this question).

Question 3: This question was on the last part of the course, and was similar in feel to some others on recent exam papers. Almost all candidates who attempted it got high marks, but it was the least popular of the three. No particular difficulties were encountered.

C3.7: Elliptic Curves

In Q1, parts (a),(b) were well answered, but many candidates found part (c) difficult. In Q2(b)(i), many candidates worked through each of the three possibilities at length, without seeing the way to use the first to shorten the work for the second and third parts. Q3(b),(c) had a large range of quality of answers; some candidates worked these through very well and others had trouble with a number of the homogeneous spaces.

C3.8 Analytic Number Theory

Question 1

This question proved a little harder than I expected. Very few candidates managed to deal with the sum of $(-1)^n n^{-s}$ correctly, appealing to gross abuses of the alternating series test. One or two candidates did find a nice way of estimating $n^{-s} - (n+1)^{-s}$ by writing this as an integral. Other parts of (b) were also a little harder than anticipated, especially finding an expression for $D_f(s)$ in terms of $\zeta(s)$, which is really just a matter of manipulation of series. By contrast, part (c) was done a little bit better than I expected, with quite a few candidates obtaining a correct expression for the Dirichlet series $D_f(s)$.

Question 2

I was quite surprised at how well this question was done, especially the bookwork part (b), which was one of the more obscure pieces of bookwork in the course. In fact this was done so well relative to the very straightforward (and still bookwork, but not so obviously so) part (c) that I found it a little depressing. Many candidates found an alternative way to do (d) based on differentiating ζ'/ζ, which I hadn't anticipated. How well this plan was carried out varied considerably, but a number of candidates obtained essentially full marks via this route.

Question 3

Predictably, most candidates with any knowledge of the course could do the bookwork half of the question well. I was quite surprised by the number of candidates who managed correct solutions to the last part which, though certainly not hard, required a proper understanding of the o() notation as well as the idea of performing a truncation in the vertical direction.

C4.1: Functional Analysis

The paper was taken by 18 students, which appears to be the largest number in over ten years for this course. The overall standard was high, and several of the candidates impressed me with their rather original approaches to some of the unseen parts of questions.

Question 1. The bookwork in part (a) was well done. Part (b) separated the candidates even though bits of it had appeared on a problem sheet; in the tricky final part nobody gave a really convincing proof that the space X must indeed be reflexive, and several candidates instead produced bogus counterexamples. Part (c) consisted of two unseen but standard problems which were handled fairly well by most of the candidates who got this far.

Question 2. Part (a) was standard bookwork and caused few difficulties. Part (b) ought to have been familiar from a problem sheet but defeated some of the less well-prepared candidates, or at least cost them a lot of time. Part (c) guided the candidates through a proof of Sobczyk's theorem, usually with moderate success. Only a small number of candidates noticed that bits of this proof, including the final part, were related to ideas explored on a problem sheet.

Question 3. The bookwork in part (a) was well done, as on the whole was part (b)(i) despite the candidates' reluctance to follow the implicit hint in part (a) and look for a subsequence argument. The later bits of part (b) caused more difficulties even though many of the arguments required little beyond elementary linear algebra. Candidates who managed to reach part (c) tended to waste time by proving laboriously that T, an integral operator of rank one, is compact.

C4.2: Linear Operators

Q.1: All but one candidate attempted this question on material which came from the early part of the course, and they scored high marks. The only systematic defect was failure to exhibit clearly why all the limits in (c)(iii) coincide.

Q.2: This question was not very popular, possibly because candidates had not revised the proof of the Hille-Yosida Theorem. It produced reasonably high marks from those who attempted it.

Q.3: This question produced rather low marks. In (b)(i), most candidates did not see that they should show that $B(0, x) \in (X \times \{0\})^\perp$. In (c), candidates who tried to identify the range of $A - i$ failed to allow for a constant of integration. A few candidates tried to argue from knowledge (perhaps from another course) that a different operator is self-adjoint, but they failed to take proper account of the boundary conditions of the operator in this question.

C4.3: Functional Analytic Methods for PDEs

Question 1 was answered satisfactorily by most. The last part of the last question, c.iii surprisingly wasn't addressed by most. It was meant to be relatively straightforward. Some obtained full marks.

Question 2 was answered satisfactorily by most. Part (d) proved elusive, and none of the students obtained full marks on it. It could be done independently of the rest of the problem, in fact. Most student tried to use the previous parts to do it, which turned out to be more difficult than a direct proof.

Question 3 wasn't very popular, however the students who chose it did very well.

C4.6 Fixed Point Methods for Nonlinear PDEs

Question 1: attempted by all the candidates. Parts (a) and (b) of the question were both bookwork and were generally solved very well, with some incomplete answers occurring in (b) as the explanation of which of the two intersection points in the construction of the counterexample had to be chosen was missing. Part (c) of the question was new. Most candidates gave at least a partial answer to (i), where the key point is that the map can be extended by a constant to a continuous map on the whole ball, while the second part was only attempted by a few students. Part (d) was standard and very well solved

Question 2: attempted by all candidates. Part (a) was bookwork and correctly solved by all except one student. Part (b) was a standard application of seen material, where for (i) all that had to be shown was that the map is bounded as it is linear. Very few students realised that the constraint on λ means that one does not even need to use the Poincaré estimate and a surprising number of solutions to this part were either unnecessary complicated or incorrect. The main part (c) was then a standard variation of one of the main methods learned in the course for proving the existence of solutions. There were two possible approaches, either using Schauder's Fixed point theorem or the constructive method of sub and supersolutions and most students chose the former. Surprisingly the students had more difficulties with the very simple parts of the solution, in particular in showing that the function f_λ is increasing for suitable λ, where only one student thought to differentiate the function, than with the more difficult parts. While some very few students clearly struggled with some of the basic concepts of the theory of PDEs and Sobolev spaces, most of the students solved the question well.

Question 3, despite being a combination of bookwork and standard application of seen material was only attempted by one student who submitted a solution to the first few parts but also submitted solutions to both other questions.

C4.8 Complex Analysis: Conformal Maps and Geometry

Q1: This is by far the most popular question attempted by all but one candidate. There were no particularly difficult bits.

Q2: In part (c) many candidates incorrectly applied Laurent expansion and almost all had difficulty showing that the highest power that could appear in the expansion is z. In part c)iv) many candidates did not prove that the coefficients in the Laurent expansion must be real.

Q3: Those who attempted this question got very good marks. No one could find an elementary solution in part c) which required no conformal maps.

C5.1: Solid Mechanics

Question 1
Parts (a) and (b) were solved nearly completely by most candidates with mostly only minor mistakes. In part (b) most candidates recalled the ericksen-Rivlin representation and used this to solve the problem. The interpretation of the result in (b) as an experimentally helpful result posed difficulties to some candidates. Part (c) was challenging to most candidates;

many used the inclusion $SO(3) \subset \mathcal{U}$ to invoke the structure results for isotropic materials, but then had difficulties in arguing that in the case of the larger material symmetry group $\mathcal{U}$ the stored energy function only depends on the determinant. The second part of (c) was only completely solved by very few candidates. Here the main difficulty was that in order to use the blow-up of the stored energy function as $\det(F) \to 0$, it is necessary to consider a sequence of matrices, which have to be related through the action of the group of material symmetries $\mathcal{U}$ to obtain a contradiction.

Question 2
Question 2 was only attempted by a single candidate, who dealt very well with the problem and only made minor mistakes.

Question 3
The bookwork of part (a) was well solved by most candidates. In part (b) a few candidates struggled with the expression for $D_W F$. A number of mistakes were made in passing from the Piola-Kirchhoff stresses to the Cauchy stresses. The last question (c) was attempted by a number of candidates. Here two main difficulties were encountered: A number of candidates struggled with formulating the statement that was to be proved. Only very few candidates then used the symmetry T to conclude the argument.

C5.2: Elasticity and Plasticity

Q1: Generally the first parts were done well: most candidates were able to make a good attempt at showing the conservation of energy (though a few details, such as the importance of the symmetry of τ_{ij}, were sometimes skipped). Similarly, the description of the boundary conditions in part (b) and the solution of the problem to give the desired expressions for the coefficients R and T in part (c) were generally well done. However, only a very few candidates seemed to understand that part (d) concerned total internal reflection covered, with only one candidate correctly deriving the skin depth in part d(iii). No candidates appreciated that the quadratic dependence of the flux $\mathcal{F}$ on w and its derivatives required careful treatment of the real parts and the associated time average in part d(ii).

Q2: This question was generally poorly done. A few candidates did not realise that the statement "you may neglect the mass of the membrane" meant that $p(x)$ vanishes outside the contact region. The calculation of the equilibrium resting depth in (a)(ii) was also poorly done in general. No candidate successfully derived the corresponding (logarithmic) relationship for a sphere in part (b) despite this being very close to one of the contact problems covered in lectures. The attempts that were made on part (c) generally tackled (i) well, though no candidates were able to derive the appropriate expressions for $t_{\max}$ or the contact time in parts (ii) and (iii), respectively.

Q3: The first parts of this question were well done. In part (a)(iv), some candidates simply showed that the given form of w satisfied the relevant boundary conditions without any mention of w needing also to be harmonic. In part (b) candidates who worked exclusively in complex variables generally did not find the term linear in z in part (b)(ii). Part (c) was generally poorly done with no good attempts made at either the geometry in part (c)(i) or the $\epsilon \ll 1$ expansion required in (c)(ii).

C5.3: Statistical Mechanics

All the questions were done competently by the four MMathPhys students.

C5.5: Perturbation Methods

Question 1. This was the least popular question. The bookwork in part (a) was very well done. In part (b) the application of Laplaces method was reasonably well done, though some candidates lost marks for failing to justify the size of the error term during each step of the argument or for failing to verify that the expansions are self consistent. In part (c) nearly all of the candidates identified the correct steepest descent contour, but only a handful dealt successfully with all of the steps required to derive the leading-order term in each regime.

Question 2. This was the second most popular question. In part (a) the bookwork on stating and applying Van Dykes Matching Rule was well done by all but a handful of candidates. In part (b) the application of the principle of dominant balance caused more problems than anticipated: many attempts did not consider all of the different cases in the expansion of $f(\epsilon^{\alpha}X;\epsilon)$ as $\epsilon \to 0^+$, though the minority that did then made efficient use of the hint. In part (c) the application of boundary layer theory caused even more problems, with many candidates failing to seek a boundary layer at both $x = 0$ and at $x = 1$ (the former despite observing in part (a) the non-uniformity in the expansion of $f(x;\epsilon)$ near $x = 0$).

Question 3. This was the most popular question. In part (a) the application of multiple scales theory was very well done on the whole. While many lost marks for algebraic slips in the derivation of the secularity condition or for failing to find the real part of $A(T)$ in the two cases, there were many excellent solutions. In part (b) the application of WKB theory to a third-order ordinary differential equation was well done by a significant minority, the majority receiving only partial credit.

C5.6: Applied Complex Variables

1. This was by far the least popular question but was done well by many who attempted it. The basic conformal mapping was mostly handled competently, although few properly justified the final Möbius map to permute the points on the real axis. Those who could see how to use the hint generally managed the rather unpleasant calculations needed for part (c).

2. This was the most popular question. Few candidates convincingly performed the contour integration for part (b); many failed to use a consistent definition of the multifunction throughout. The bookwork in part (c) was mostly ok, though few candidates gave good explanations for the properties of H, in particular why it should be real on Γ.

3. This question was generally not well done. In part (a), most candidates seemed to understand the method but were let down by very many errors in basic manipulation (integration by parts, partial fractions, etc). Two candidates were misled by the hint inserted by the external examiner into expressing $g(\zeta)$ in terms of an unevaluated

integral, making it harder to determine Γ. In part (b), most candidates were able to derive equation ($\star$), but the Wiener–Hopf decomposition caused many problems, particularly amongst those who were unable to identify the points $k = \pm 1$ in the Argand plane. Very few candidates completed the contour integration required for part (iv).

C5.7: Topics in Fluid Mechanics

The most popular question was Q2, which was perhaps the most predictable being perhaps closest to a classical problem in fluid mechanics (one of the cases where Rayleigh-Benard convection can be treated analyticall), followed by Q3 and then Q1, which overall seemed to be a bit tougher. Only two students picked both Q1 and Q3.

The difficulty in Q1 seemed to be to get the initial similarity solution right (in part a)). Some had the correct form of the self-similar equation and even the solution but got the exponents (slightly) wrong. This allowed them to continue with most of the question. Only very few students got major parts of c) and even fewer got close to using the information correctly for obtaining the required terms in the expansion.

The first parts of Q2 were well done but quite a few students had problems in the last part where they got lost in the algebra or did not realise how to determine the critical Rayleigh-number (but a few did get to the end and did get the Ra_c value right.)

In Q3, nearly everyone got 100% of the available marks in part a). Quite a few students struggled with the algebra of the rescaling or working out the initial conditions. Errors made here made it harder to get a solvable problem for part c), but some students also missed that the question asked for steady state problems. Only very few got right to then end and got the correct ODE for the solution in part c) and got the (essentially) correct answer.

C5.9: Mechanical Mathematical Biology

Overall the student performance on the examination was very pleasing. The questions were not straightforward and yet many students from this small cohort performed well.

Question 1. The most popular question, and most candidates did very well until the final part, where there was a spread in attainment.

Question 2. This was also popular, though many students often did not Taylor expand sufficiently carefully to deduce the results that were given in the intermediate parts of the question. This did not preclude further progress and otherwise good answers were generally presented until the final part, which the students tended to find difficult, though there were some solid attempts.

Question 3. This was unpopular and attempted only by a very small minority and as a third question attempt; these students found the final part was very difficult.

C5.11: Mathematical Geoscience

Q1: This question had only 6 attempts but these were all very good. The most difficulty was had with the bookwork in part (a). The unseen parts (b)-(d) were quite straightforward and were generally done well with occasional slips.

Q2: This question was found to be the hardest. All candidates managed the bookwork in parts (a) and (b) well. Part (c) was similar to a question on the problem sheets. There were one or two excellent solutions, but most candidates struggled to keep track of all the different characteristics and how they continued beyond $t = \epsilon$, leading to some very confused answers.

Q3: This was a straightforward question that was generally done well. Many candidates gave expressions for α in part (a) that were not dimensionless and therefore clearly wrong. The unseen part (c) was done well.

C5.12: Mathematical Physiology

- **Question 1** This question was the second most popular, and was well done in general. In part (c) some candidates were confused when considering the $(w - I^*, v)$ phase plane, and plotted the nullclines incorrectly. Depending on what was plotted there were also errors when indicating w_* and w^* on the phase plane. In part (d) some candidates failed to spot that c was positive, which lead to errors when plotting the nullclines in part (d)(iii). Additionally, some candidates marked the direction of the trajectory joining A to B incorrectly.

- **Question 2** This was the most popular question, and was well done. Some candidates were unable to give a brief justification for (a)(iii). In part (b)(ii), some candidates did unnecessary work by analysing the nature of complex roots of $(\star)$.

- **Question 3** This was the least popular question. Those that did it did very well. Some candidates made mistakes in section (c)(ii).

C6.1: Numerical Linear Algebra

This seems to have been a reasonably successful exam with a range of marks including many high marks for the small number of undergraduate candidates.

C6.2: Continuous Optimisation

The paper was more accessible than previous years and the students seemed a lot more comfortable with each of the questions.

C6.3 Approximation of Functions

Problem 1. Every student without exception did this problem, and there was a good spread of results. On part (d), quite a few students assumed that $x_0, \ldots x_n$ were Chebyshev points, even though in the very first line they are defined merely as “distinct points in $[-1, 1]$”. This was unfortunate.

Problem 2. In part (e), there was an error in the solution sheet ($K \geq 9$ should have been $K \geq 7$). The question remains fine, however, and of course those students who got this far were marked against the correct solution.

Problem 3. It seems to have been the easiest of the problems, with only parts (c) and (e) requiring much thought and understanding.

C6.4: Finite Element Methods for Partial Differential Equations

While the course lecturer changed this year, the examination largely followed the pattern of previous years. We expect the questions in future examinations to feature more novel material.

Q1: This question contained some new material that was not covered in previous years. This had two consequences: first, many students avoided this question for the (more familiar) questions 2 and 3; second, this question tended to reveal a spread among the students that was not revealed by the other questions. In Q1 (a), several students neglected to define the Riesz map. Several students had difficulty with Q1 (b) (iii)-(iv); this was surprising, as it forms the central idea in the proof of one of the keystone theorems of the course (the Lax–Milgram Theorem). The material in Q1 (c) had been emphasised heavily in the lecture notes and lectures, but several students could not explain the central concept. The answer to Q1 (e) relied on knowledge of a basic fact from Part A Numerical Analysis.

Q2: This question was quite formulaic and similar to those of previous years. As a result, it did not yield a significant spread of marks, with every student attempting the question achieving at least 20/25 marks. The only significant common difficulty was supplying a correct definition of the function space $H^1(\Omega)$ in Q2 (a).

Q3: This question was also similar to those of previous years, but achieved a better spread of marks. In Q3 (a), some students did not explicitly demand that the boundedness constant be finite. The solution of Q3 (e) relied on knowledge of the Sobolev embedding theorem, which had not been emphasised in previous years.

C7.4: Introduction to Quantum Information

Question 1

This was the most popular question on the paper and generally well done. Parts (a) and (b) were bookwork and the solutions were mostly flawless. In part (c) some students did not notice that the measurement outcome $x = 0$ is inconclusive as there can be errors in both qubits. Most marks were lost in part (e) as many candidates struggled to describe the recovery procedure. Many good attempts at part (f).

Question 2

Fairly well done question. Very few students showed that $P_{\pm}$ satisfy conditions of orthogonal projectors. Many struggled with part (e). There were various attempts at part (f) but most students got the right idea.

Question 3

The bookwork in part (a) and the calculation of the Bloch vector in parts (c) and (d) caused no problems. However surprisingly many students struggled to prove positivity of a CPTP map in part (b). Common sloppiness and algebraic mistakes lead to mark losses in part (e).

C7.5: General Relativity I

Question 1

This exercise dealt with tensors in GR. While most students who attempted this exercise solved the first $\frac{2}{3}$, the unfamiliar nature of the last part caused problems to all of the students.

Question 2

Most students did well in this exercise, few were able to find $k(y)$ in (d). Happily, most had the idea for the correct parametrization in (c).

Question 3

This was attempted by the fewest students. While nearly everyone solved the introductory parts, the final part seemed too difficult for most.

C8.1: Stochastic Differential Equations

Question 1 was the most popular, Question 3 the least popular. Most students managed to gain all points part a) of the questions they attempted.

1b(i) was successfully solved by most students; however surprisingly many students failed to work out $\int_0^1 \mathbb{E}[|B_t|]dt$ by using the Gaussianity of B_t. 1c) was successfully solved by nearly all students who attempted it.

Most students realized that 2b) was a simple variation of a theorem discussed in the lecture. However, few managed to give a complete argument for 2b(ii). Here, a common mistake was not to justify uniform integrability. Similarly, the first part of 2c) was solved by most students by making use of 2b), some students gave a direct proof using Ito. For 2c), most attempts realized that the symmetry of Brownian motion plays a role but a common mistake was to use optimal stopping to early and ignore that N is not bounded.

Most solutions of 3b(i) and 3b(ii) were correct. Only one student found the right answer for 3b(iii) which follows immediately if one realizes that local time only increases if Brownian motion crosses 0. In 3c)(i) a common mistake was a calculation error in the derivation of the cumulative distribution function.

C8.2: Stochastic Analysis and PDEs

The exam went well in that all candidates were able to display their knowledge of the course and there were very good attempts at all questions.

Question 1 This began with standard bookwork which was generally well done. The later questions on the resolvent were new but generally good attempts were made.

Question 2 The bookwork was well done by most candidates. Even though only a standard analysis of convergence of an integral was required, only one person could correctly determine the final condition for the process not to hit 0.

Question 3 This was well done with most able to score well on the bookwork. The final new example was solved completely by one person.

C8.3: Combinatorics

Questions 1 and 2 were the most popular. Question 1 was generally done well. The bookwork in Question 2 was generally answered correctly, but many candidates had difficulty with the unseen parts, missing the idea of working with subspaces of $\mathbb{F}_2^n$. Few candidates attempted the third question, although the unseen parts were in fact quite straightforward.

C8.4 Probabilistic Combinatorics

The average marks on the questions were rather similar, suggesting they were well matched in difficulty, with Question 1 slightly easier.

Almost all candidates attempted Question 1. (a) was mostly well done (sometimes with a lot more detail than required, e.g., proving Markov's and Chebyshev's inequalities). (b)(i) was not well done. Counting the number of possible copies of H exactly is slightly fiddly (because H might not be connected and so might not have a unique bipartition); most candidates tried this but gave incorrect formulae. It's also not necessary for a Theta(.) result; it's simpler to give upper and lower bounds separately. (b)(ii) is a very minor variation of bookwork and was generally well done. Only a few candidates managed (c) - many didn't spot that it's not a question about probabilities.

The bookwork in Question 2 was mostly well done; the next two parts much less so. (b) should be very easy, but many candidates tried to compare Delta with μ^2 instead of with mu (to use part (a)), and most missed that one can start by saying $WLOGp = Cn^{-2/5}$. The answers to (c) were rather poor (for example considering the size of the largest clique in the complement, rather than the number of cliques needed to cover the complement). The basic strategy is as used for $G(n, p), p$ constant, in the lectures, but with much simpler calculations.

Question 3 was the least popular though, in contrast to recent years, attempted by a reasonable number of candidates. The bookwork (which is almost all of the question) was mostly well done, though a few candidates just analyzed the random walk without making the connection to the graph via an exploration process (which is where most of the work is). The last part was intended to be straightforward but most candidates did not get very far with it.

Statistics Units

Reports on the following courses may be found in the Mathematics and Statistics examiners' report.

SC1: Stochastic Models in Mathematical Genetics

SC2: Probability and Statistics for Network Analysis

SC4: Data Mining and Machine Learning

SC5: Advanced Simulation Methods

SC6: Graphical Models

SC7: Bayes Methods

Computer Science

Reports on the following courses may be found in the Mathematics and Computer Science examiners' report.

Quantum Computer Science

Categories, Proofs and Processes

Computer Animation

F. Comments on performance of identifiable individuals

Removed from the public version of the report.

G. Names of members of the Board of Examiners

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  Prof. C Douglas
  Prof. A Goriely (Chair)
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