INDUCTION EXERCISES 2.

1. Show that n lines in the plane, no two of which are parallel and no three meeting in a point, divide the plane into

$$\frac{n^2+n+2}{2}$$

regions.

2. Prove for every positive integer n, that

$$3^{3n-2}+2^{3n+1}$$

is divisible by 19.

3. (a) Show that if $u^2-2v^2=1$ then

$$(3u+4v)^2-2(2u+3v)^2=1.$$

(b) Beginning with $u_0=3, v_0=2$, show that the recursion

$$u_{n+1}=3u_n+4v_n \quad \text{and} \quad v_{n+1}=2u_n+3v_n$$

generates infinitely many integer pairs (u,v) which satisfy $u^2-2v^2=1$.

(c) How can this process be used to produce better and better rational approximations to $\sqrt{2}$? How many times need this process be repeated to produce a rational approximation accurate to 6 decimal places?

4. The Fibonacci numbers F_n are defined by the recurrence relation

$$F_n=F_{n-1}+F_{n-2}, \quad \text{for} \quad n\geq 2$$

and $F_0=0$ and $F_1=1$. Prove for every integer $n\geq 0$, that

$$F_n=\frac{\alpha^n-\beta^n}{\sqrt{5}}$$

where

$$\alpha=\frac{1+\sqrt{5}}{2} \quad \text{and} \quad \beta=\frac{1-\sqrt{5}}{2}.$$

[Hint: you may find it helpful to show first that the two roots of the equation $x^2=x+1$ are α and β.]

5. The sequence of numbers $x_0, x_1, x_2, \ldots$ begins with $x_0=1$ and $x_1=1$ and is then recursively determined by the equations

$$x_{n+2}=4x_{n+1}-3x_n+3^n \quad \text{for} \quad n\geq 0.$$

(a) Find the values of x_2, x_3, x_4 and x_5.

(b) Can you find a solution of the form

$$x_n=A+B\times 3^n+C\times n3^n$$

which agrees with the values of $x_0,\ldots,x_5$ that you have found?

(c) Use induction to prove that this is the correct formula for x_n for all $n\geq 0$.