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Studying Mathematics at Oxford

University Mathematics

Few people who have not studied a mathematics or science degree will have much idea what modern mathematics involves. Most of the arithmetic and geometry seen in schools today was known to the Ancient Greeks; the ideas of calculus and probability you may have met at A-level were known in the 17th century. And some very neat ideas are to be found there! But mathematicians have not simply been admiring the work of Newton and Fermat for the last three centuries; since then the patterns of mathematics have been found more profoundly and broadly than those early mathematicians could ever have imagined. There is no denying it: mathematics is in a golden age and both within and beyond this university’s “dreaming spires”, mathematicians are more in demand than ever before.

One great revolution in the history of mathematics was the 19th century discovery of strange non-Euclidean geometries where, for example, the angles of a triangle don’t add up to 180°, a discovery defying 2000 years of received wisdom. In 1931 Kurt Gödel shook the very foundations of mathematics, showing that there are true statements which cannot be proved, even about everyday whole numbers. A decade earlier the Polish mathematicians Banach and Tarski showed that any solid ball can be broken into as few as five pieces and then reassembled to form two solid balls of the same size as the original. To this day mathematics has continued to yield a rich array of ideas and surprises, which shows no sign of abating.

Looking through any university’s mathematics prospectus you will see course titles that are familiar (e.g. algebra, mechanics) and some that appear thoroughly alien (e.g. Galois Theory, Martingales, Communication Theory). These titles give an honest impression of university mathematics: some courses are continuations from school mathematics, though usually with a sharp change in style and emphasis, whilst others will be thoroughly new, often treating ideas on which you previously had thought mathematics had nothing to say whatsoever.

The clearest change of emphasis is in the need to prove things, especially in pure mathematics. Much mathematics is too abstract or technical to simply rely on intuition, and so it is important that you can write clear and irrefutable arguments, which make plain to you, and others, the soundness of your claims. But pure mathematics is more than an insistence on rigour, arguably involving the most beautiful ideas and theorems in all of mathematics, and including whole new areas, such as topology, untouched at school. Mathematics, though, would not be the subject it is today if it hadn’t had been for the impact of applied mathematics and statistics. There is much beautiful mathematics to be found here, such as in relativity or in number theory behind the RSA encryption widely used in internet security, or just in the way a wide range of techniques from all reaches of mathematics might be applied to solve a difficult problem. Also with ever faster computers, mathematicians can now model highly complex systems such as the human heart, can explain why spotted animals have striped tails, can treat non-deterministic systems like the stock market or Brownian motion. The high technical demands of these models and the prevalence of computers in everyday life are making mathematicians ever more employable after university.
The Oxford System

Students at Oxford are both members of the University and one of 29 colleges, and mathematics teaching is shared by these two institutions. Oxford's collegiate system makes both study, and the day-to-day routine, a rather different experience from other universities.

Most of the teaching of mathematics in Oxford, especially in the first two years of a degree, is done in tutorials. These are hour long lessons in college between a tutor, who is usually a senior member of the college, and a small group of students (typically a pair). This form of teaching is very flexible and personalized, allowing a tutor time with the specific difficulties of the group and allowing the students opportunities to ask questions. It is particularly helpful for first year mathematicians who naturally begin university from a wide range of backgrounds and syllabi. College tutors follow closely their students' academic progress, guide them in their studies, discuss subject options and recommend textbooks, as well as being able to answer questions about Oxford generally. Colleges are much more than just halls of residence though, each being a society in its own right, and there will be other students studying mathematics (and other subjects) in college who, invariably, will prove a help with study and often friends during university and beyond.

Mathematicians from across all the colleges come together for lectures which are arranged by the University. This is usually how students first meet each new topic of mathematics. A lecture is a 50 minutes talk, usually given in the new Mathematical Institute, with up to 280 other students present. Unsurprisingly there is less (but, by no means, no) chance to ask questions as the lecturer discusses the material, gives examples, provides slides and make notes at the boards. The lecturer will, like your college tutors, be a member of the Faculty, but usually a tutor at a different college to your own. For most students the material of a lecture is presented too intensely to take in at all once, and so it falls to a student to review their lecture notes and other textbooks, determine which elements are still causing difficulty, and try to work through these. To help, the lecturer, or a college tutor, will set exercises on the lecture and these problems will typically form the basis of the next tutorial in college.

By the third and fourth years the subject options become much more specialized and are taught in intercollegiate classes organized by the University. These are given by a class tutor (usually a member of Faculty or senior graduate who has taken and passed teaching qualifications) and a teaching assistant. They range in size – typically there are 8-10 students – and there is again plenty of chance to ask questions and discuss ideas with the tutors.

College tutors mark their students’ tutorial work each week, commenting on progress being made and, at the end of a term, your various tutors will write reports on that term’s work and discuss these with you. Most college tutors also set college exams, called collections, at the start of each term, to check progress and as practice for later university examinations. The results of collections will not count towards the degree classifications, awarded at the end of the third and fourth years.

See http://www.ox.ac.uk/admissions/undergraduate/colleges and https://www.maths.ox.ac.uk/study-here/undergraduate-study/which-college for links to the colleges’ webpages.
The New Institute

The Mathematics Department has now settled in to a brand new £72M Institute on Woodstock Road as part of the new Radcliffe Observatory Quarter. You find more information and pictures at: https://www.maths.ox.ac.uk/study-here/undergraduate-study/why-oxford

The Degree Structure

There are three and four year degrees in Mathematics (BA/MMath) and also in the various joint courses: Mathematics and Statistics (BA/MMath), Mathematics and Computer Science (BA/MMathCompSci) and Mathematics and Philosophy (BA/MMathPhil).

New: There is also now a fourth year stream – Mathematics and Theoretical Physics – whereby students study for an MMathPhys. See page 22 for details.

All of these mathematics degrees have a strong reputation, academically and amongst employers. The joint degrees with Philosophy and with Computer Science contain, roughly speaking, the pure mathematics options. The Mathematics and Statistics degrees have the same first year as the Mathematics degrees, before the emphasis in options increasingly moves towards probability and statistics. Each degree boasts a wide range of options, available from the second year onwards. They will train you to think carefully, critically and creatively about a wide range of mathematical topics, and about arguments generally, with a clear and analytical approach.

The degree structures and the assessment of these degrees have much in common. (See later sections for more on the specifics of each degree.) The first year mathematical content of each degree contains core material, covering ideas and techniques fundamental to the later years. At the end of the academic year, in June, there are five university examinations, known as the Preliminary Examination (or “Prelims”). Students taking the examination are awarded a Distinction, Pass or Fail. The vast majority of students pass Prelims at the first attempt. Those who do not pass Prelims in the first instance in June, may resit one or more of the examinations in September. Successful students may then continue their degree.

The first term of the second year involves the last of the core, compulsory courses (Linear Algebra, Metric Spaces, Complex Analysis, Differential Equations) and some options. From the second term onwards a wide range of options becomes available. Typically a student takes five of nine “long options” in the first two terms and three of seven “short options” in the third term. These vary from pure topics like number theory, algebra and geometry, through to applied areas such as fluid dynamics, special relativity and quantum theory. Other options are also available in the joint degrees which reflect the nature of the speciality. Students can choose mainly pure, mainly applied, or a mixture of topics. There are university examinations at the end of the year; no classification is made at that stage though the marks achieved count towards the classification awarded in the third year.

You will be asked to choose between the three and four year degrees in the second term of their third year. In the third and fourth years there are again a large number of options available, including the chance to write a dissertation and other options which include practical work or projects. Some of these options build on material from earlier courses, whilst others introduce entirely new topics. Some third year courses, and almost all the fourth year courses, bring you close to topics of current research. You may choose a
A varied selection of options or a more specialized grouping reflecting your future academic or career intentions. There are again university examinations at the end of third year, some of which may be replaced with equivalent project work.

You will receive a classification (First, Upper Second, Lower Second, Third, Pass, Fail) based on the assessment of your examinations, practicals and projects from the second year and third years (i.e. not counting your Prelims results), and a further separate classification similarly assessed on your fourth year (if applicable).

Libraries
Students normally buy a certain number of basic textbooks, but typically find that libraries cover other more specialist needs. Each college has its own library from which its undergraduates may borrow books. These libraries usually have copies of all recommended books for core courses and many others. The hugely resourced Radcliffe Science Library (or RSL) www.bodleian.ox.ac.uk/science has both a lending-library and a reference library.

The Invariant Society
The Oxford University Invariant Society www.invariants.org.uk is the undergraduate mathematical society. Its primary aim is to host weekly popular mathematically-related talks by notable speakers, on a wide variety of topics. Past speakers have included Benoit Mandelbrot, Sir Roger Penrose, Marcus du Sautoy and the author Simon Singh. The Invariants also run the Undergraduate Seminars, where current undergraduates give talks on subjects outside the degree’s curriculum and further organize social events.

The Mirzakhani Society
The Mirzakhani Society is a society for women studying maths. Its aim is to support students through providing a space to discuss issues that women may encounter during their degrees. It holds weekly ‘Sip and Solve’ meetings with tea and cake, and other events such as socials and talks. It is open to both undergraduates and postgrads, and has a wide mix of people at its events.

MURC and JCCU
The Mathematical Undergraduate Representative Committee (informally known as MURC) – see www.maths.ox.ac.uk/~murc – is a student body representing the interests of students in mathematics and the joint degrees. It consists of a representative from each college, elected by the undergraduate mathematicians of the college. This committee passes its views on syllabus and examination changes and general matters such as the timing of lectures.

MURC operates a secondhand book scheme whereby all mathematicians are able to buy and sell books. This scheme is particularly useful for ‘freshers’ (first year undergraduates) since they are able to obtain cheaply some of their textbooks as soon as they arrive at Oxford. The Representative Committee appoints eleven junior members (i.e. undergraduate students) to the Joint Consultative Committee with Undergraduates (JCCU); the other six members of the committee are members of the Departments and Division. This committee meets once a term and its discussions concern the syllabus, teaching, library facilities, open days and general aspects of examinations. It is also available for consultation by the departments on any of these matters and is responsible for discussing feedback from lecture surveys.
Admissions and Preparation for the Course

Admissions

The following applies to prospective students for the Mathematics degree, or for any of the joint degrees, who are considering applying in October 2015 for entry in 2016 or 2017. Much like applying for any other UK university, applications to Oxford are made through UCAS, though the deadline is earlier, on October 15th. Your application may include a preference for one college, or may be an “open” application in which case a college is assigned to you.

On November 4th, 2015 (as in 2007-14) the Mathematical Sciences Admissions Test will be sat by candidates in their schools, colleges or at a test centre. The test, which lasts 2½ hours, will be in the same format as in 2007-14 and these past tests, and two further specimen tests, are available with solutions at https://www.maths.ox.ac.uk/study-here/undergraduate-study/maths-admissions-test All applicants attempt the first multiple-choice question, and then four from six longer questions depending on their proposed degree. Instructions are in the test on which questions to complete. No aids, calculators, formula booklets or dictionaries are allowed. A syllabus for the test is available at the above website; it roughly corresponds to material from the A-level modules C1 and C2, though the questions are devised to test for a deeper understanding of, and imagination with, the syllabus’ methods and material.

The distribution of the test will be administered by the Admissions Testing Service and all applicants must separately register with them by the application deadline (15th October), through their school or college or through a test centre. Schools can register with the Admissions Testing Service to become test centres, but please note that this takes a minimum of 24 hours; see http://www.admissiontestingservice.org/our-services/subject-specific/mat/about-mat/

All applicants are expected to take the Admissions Test on the above date and must notify the Admissions Testing Service as soon as possible in the event of any potential difficulties or schedule clashes.

Applicants will be shortlisted for interview in Oxford on the basis of their test marks and UCAS form, with around 3 applicants per place being shortlisted. (Currently there are around 5-6 applicants per place.) During your stay (typically being for 2-3 nights), meals and accommodation are provided by the college you applied to, or were assigned. During this time the college arranges for some current students to be available to answer your questions about university mathematics and the college and to give you an alternative view of Oxford. In the event of a shortlisted overseas applicant being unable to travel to Oxford a Skype, video or telephone interview may be arranged.

Interviews in Oxford take place in mid-December at the college with at least one more interview guaranteed at another college. Typically, interviews last 20-30 minutes with one or two interviewers, and you may have more than one at a particular college. Applicants for the joint degrees with Philosophy and with Computer Science should expect at least one interview on each discipline. In interview, you may be
asked to look at problems of a type that you have never seen before. We want to see how you tackle new ideas and methods and how you respond to helpful prompts, rather than simply find out what you have been taught. Interviews are academic in nature, essentially imitating tutorials, this being how much of Oxford’s teaching is done; feel free to ask questions, do say if unsure of something, and expect hints.

If your application is unsuccessful with your first college, another may make an offer; around 20–25% of offers made are not by the applicant’s first college. Around 20% of all applicants are currently made conditional offers. In Mathematics, Mathematics and Statistics, Mathematics and Philosophy, the standard offers are (i) A*A*A with A* grades in Maths and Further Maths or A*AAa with A*a in A2 Maths and AS Further Maths or A*AA with A* grade in Maths (ii) 39 with 766 at HL including 7 in HL Maths for IB applicants; (iii) AAB/AA for those taking Advanced Highers. In Mathematics and Computer Science, the standard A-level offer is A*AA with A*A between Maths and Further Maths if taken, and otherwise with the A* in Maths. Applicants are informed of their college’s decision by mid-January. Information on typical offers involving international qualifications can be found at www.ox.ac.uk/admissions/undergraduate_courses/international_applicants/international_qualifications

Website Links and Email Addresses
Information about admissions, the University and colleges, is on the University website www.ox.ac.uk/admissions or in the University’s Undergraduate Prospectus – the University prospectus is circulated to schools, can be ordered from this website or by writing to The Undergraduate Admissions Office, University Offices, Wellington Square, Oxford OX1 2JD. Each college has a specific prospectus, obtainable by writing to the college’s Tutor for Admissions, or online from college websites (see https://www.ox.ac.uk/admissions/undergraduate/colleges/college-listing ).

There are two departmental open days (2/5/15, 9/5/15), for which registration is required, and three others (1/7/15, 2/7/15, 18/9/15) when colleges also have open days. At these there will be talks on each of the Mathematics degrees and the joint degrees. There will be plenty of chance to meet current lecturers and students. See https://www.maths.ox.ac.uk/study-here/undergraduate-study/open-days for full details. Colleges also hold open days; see the University prospectus or www.admissions.ox.ac.uk/opendays At the July and September open days, registration is not required for the repeated morning and afternoon sessions in the Mathematical Institute, but you may need to register with a college if you plan to attend their programme of talks, though all colleges will be open to visitors without booking and often tours will be available.

- www.maths.ox.ac.uk – the Mathematical Institute.
- www.stats.ox.ac.uk – the Statistics Department.
- www.cs.ox.ac.uk/ – the Department of Computer Science.
- www.philosophy.ox.ac.uk – the Philosophy Faculty.
- www.ox.ac.uk/admissions – the Admissions Office’s webpage for prospective undergraduates, which includes summaries of all of the colleges.
• www.ox.ac.uk/admissions/undergraduate_courses/student_funding/ – information on student funding and the Oxford Opportunity Bursaries.
• undergraduate.admissions@maths.ox.ac.uk – an email address for any enquiries about admissions relating to Mathematics or its Joint Degrees; copies of this prospectus (for UK addresses) can be requested here.
• undergraduate.admissions@admin.ox.ac.uk – an email address for general enquiries about undergraduate admissions; copies of the University’s prospectus (for UK addresses) can be requested here.
• https://www.maths.ox.ac.uk/study-here/undergraduate-study – the Mathematical Institute’s page for prospective undergraduates; this includes specimen admissions tests.
• www.admissions.ox.ac.uk/opendays – a webpage with the dates and details of college and departmental open days.

The Mathematics Department welcomes applications from disabled students and is committed to making reasonable adjustments so that disabled students can participate fully in our courses. You can find out more about the accessibility of our building at: https://www.admin.ox.ac.uk/access/dandt/mpls/andrewwilesbuilding/ We encourage prospective disabled students to contact the Department’s Administrator (departmental-administrator@maths.ox.ac.uk) at their earliest convenience, to discuss particular needs and the ways in which we could accommodate these needs. See also: www.admin.ox.ac.uk/eop/disab – the University Disability Office’s website which includes FAQs and further information.

Admissions FAQs
For more FAQs see: https://www.maths.ox.ac.uk/study-here/undergraduate-study/frequently-asked-questions

Q. How do I choose a college?
See www.ox.ac.uk/admissions/undergraduate_courses/colleges/
A: There are 29 undergraduate colleges with students taking mathematics, each having 5–10 mathematics students per year. These colleges have tutors and students enough to provide all the support you need. Colleges differ much more in their size, age, location than they do in their teaching of mathematics. Not all colleges, though, take students in the joint degrees. You can find the number of joint degree students and tutors at a college in tables in the university prospectus or at the above website. To help make a choice it’s best to review college prospectuses (usually available to order from college websites) and, if possible, to attend a college open day, at which you will have a chance to meet the college’s mathematics tutors and some students. Alternatively you can make an open application and a college will be assigned to you. Remember your chosen or assigned college is simply the first to consider your application, which will be considered by others. If unsuccessful at the first college, another college may make an offer or you may be made an open offer in which you are guaranteed a place to study at Oxford with your college to be confirmed after your A-level results (or equivalent).

Q. What A-levels do I need?
A: If you are taking A-levels then you need to be taking A-level Mathematics, and Further Mathematics A-level is highly recommended. The standard conditional offer, if
you are taking A2 Further Mathematics, is A*A*A with A* grades in Mathematics and Further Mathematics (except for the joint degree with Computer Science where the offer is A*AA with A*A in some order in Mathematics and Further Mathematics). We encourage students to take what mathematical extension material is available to them (e.g STEP/AEA), but any offer would not depend on these. We strongly recommend Further Mathematics to AS or A2 but recognize that it is not available to many students; single A-level mathematicians successfully study at Oxford, the transition being more difficult, but the tutorial system is especially suited to treating the individual educational needs of students. Philosophy A-level is not required for Mathematics and Philosophy. Recent experience of writing essays, though by no means essential, may be helpful.

Q: How do I prepare for the test?
A: At [https://www.maths.ox.ac.uk/study-here/undergraduate-study/maths-admissions-test](https://www.maths.ox.ac.uk/study-here/undergraduate-study/maths-admissions-test) you’ll find eight past papers and two specimen tests, with solutions, and a syllabus for the test (which roughly corresponds to the C1, C2 modules from A-level Mathematics). We recommend that you attempt some of these tests under timed conditions.

Q: How do I prepare for interview?
A: While styles differ somewhat, in an interview a tutor will typically discuss problems involving new mathematical ideas, building from a familiar or accessible starting point. The tutor will be interested to see how you respond as the problem is adapted and new ideas introduced, and in how well you can express your arguments. Do share what you’re thinking and don’t be afraid to admit that you haven’t yet covered a topic at school – other questions can be tried, or some help given. As practice you might find it helpful to talk to a school teacher about a favourite area of mathematics or go through a longer MAT question with them. Remember that the interview will be academic and mathematical in nature. You will most likely be asked to think about new and unfamiliar mathematics, though no specialized mathematical knowledge will be expected of you beyond what you have met at school or college, but you may well be expected to employ mathematical techniques with which you should be familiar and so it is always a good idea to revise past material you have already met.

Q: Should I do Additional Further Maths/more maths modules? Will this make me more likely to be offered a place?
A: If you have the spare time, and are intending to study maths at university, then doing STEP or AEA papers would be better preparation than doing more modules for the sake of it. However, if you’re particularly interested in S6 or M5 then by all means take the extra modules.

Q: I haven’t studied philosophy, can I still apply to the Maths and Philosophy degree?
A: Yes, absolutely, studying philosophy is not a prerequisite for applying to the Maths and Philosophy degree. For more information see the Maths and Philosophy course page.
Admissions Criteria For The Mathematical Sciences Admissions Group

The following Honour Schools (both three and four year) fall within the aegis of the Group:

- Mathematics
- Mathematics & Philosophy
- Mathematics & Statistics
- Mathematics & Computer Science
- Computer Science
- Computer Science and Philosophy

and the criteria will be measured with full regard to their differing requirements.

Candidates will be invited to take the Admissions Test (on 4th November 2015) and to come for interview in Oxford* if their application gives evidence of the motivation and ability (including an appropriate mathematical background) to undertake what are demanding courses at one of the world's leading universities, sufficient to offer the possibility of final selection given the overall field of applicants. In the case of candidates whose first language is not English, an English language qualification (such as IELTS level 7) will form part of the requirements of any offer. See www.ox.ac.uk/admissions/undergraduate_courses/courses/courses_and_entrance_requirements/english_language.html for details.

(*If unable to travel for interviews in Oxford, colleges may arrange for candidates to be interviewed in their home countries via video-conferencing, Skype or telephone.)

During the selection process, tutors will seek a demonstration of the skills and/or the aptitude necessary for the successful study of the course in question together with the motivation to undertake a demanding programme on that course, and will assess these via:

i. the Admissions Test, and
ii. interviews (when held),

taking into account the level of relevant existing knowledge and experience.

Tutors will, in addition to assessing aptitude and technical skills, seek in successful candidates:

a. a capacity to absorb and use new ideas,
b. the ability to think and work independently, and
c. perseverance and enthusiasm,

in each case to be assessed in respect of the course applied for. Evidence of the extent to which these criteria have been met will be taken from the performance in (i) and (ii) above, together with:

iii. past examination records, and
iv. references and the personal statements contained on the UCAS form.

Candidates will also have the opportunity to present any special factors that they would wish to be considered.

Candidates interviewed in Oxford will be interviewed by at least two colleges. An overall assessment of the strength of each candidate relative to the field of all applicants at this stage will be made on the basis of the criteria detailed above. Ultimate selection is necessarily competitive since the number of places is limited. However, through early identification during the interview process of strong candidates who may not gain places at their first or second choice colleges, the Mathematical Sciences Admissions Group takes active steps to ensure that (whenever possible) such candidates may be offered places at other colleges.

Applicants for a joint degree with Philosophy should also note the Faculty of Philosophy's Admissions Criteria available at http://www.philosophy.ox.ac.uk/admissions/undergraduate/criteria_of_admission_for_philosophy_in_other_joint_degree_courses

Deferred Entry Policy For The Mathematical Sciences Admissions Group

Deferred entry applications in Mathematics, Computer Science, and their Joint Degrees, will be considered from applicants who have planned structured activities in their gap year; activities might include technical employment relevant to Mathematics, Statistics or Computer Science, teaching abroad or a gap year programme. If uncertain, applicants should raise any questions with the tutors at their chosen/allocated college. Tutors may discuss details of the gap year during interviews. After discussion with the candidates, some deferred entry applicants may be offered an immediate place instead. There is no policy for making more demanding offers to candidates seeking a deferred offer. Tutors will typically set successful gap year applicants academic work to be completed during the year or the summer before their first term in Oxford.
Preparation for the Oxford Mathematics Course

Whilst some courses, early in the degree, have a first-principles approach and assume very little mathematical knowledge, other areas would prove rather difficult without certain ideas and techniques being familiar. The following is a list of topics, largely in pure mathematics, most of which we would expect you to have studied before starting the course (but many students will have a few gaps, especially those who have not taken two A Levels in mathematics):

- Polynomials and basic properties of the roots of polynomial equations.
- Partial fractions.
- Simultaneous equations.
- Inequalities and their manipulation.
- Basic properties of triangles and circles.
- Equations of the parabola, ellipse and hyperbola.
- Elementary properties of lines and planes in three dimensions.
- Simple treatment of finite and infinite series including arithmetic and geometric progressions.
- Product, quotient and chain rules of differentiation.
- Solving simple differential equations.
- Integration by parts.
- Recognition of the shape of a plane curve from its equation, maxima and minima, tangents and normals.
- Binomial Theorem, combinations.
- Taylor series, the binomial series for non-integer exponent.
- Matrices and determinants.
- Induction.
- Complex numbers – their algebra and geometry.
- Exponential and trigonometric expansions and Euler’s relation between them.
- Standard integration techniques and spotting substitutions.
- Second-order differential equations with constant coefficients.

As A-level syllabuses contain such varying amounts of applied mathematics, that is topics such as mechanics, probability and statistics, very little prior knowledge is assumed here. You may find the early parts of some courses repeat material from your A-level whilst other topics may be almost completely new to you. Typically though, even the “old” material will be repackaged and presented with a different emphasis to school mathematics.

After A-level (and other) results come out in the summer, tutors usually write to students joining their college in October, enclosing (with their congratulations) preparatory exercises on topics like the ones above, often with a suggested list of helpful text books. These two months are an important chance for you to read up on any gaps in your knowledge of the topics above or to refresh your knowledge of those that have become “rusty”. Similar sheets are available at https://www.maths.ox.ac.uk/study-here/undergraduate-study/practice-problems

On the next page is a selection of mathematical texts; some are technical books aimed at bridging the gap between A-level and university mathematics, which will help you fill in those gaps over the summer; others aim to popularize mathematical ideas, the history of a topic or theorem, or are biographies of great mathematicians, which may give you a flavour for how mathematics is discovered and the variety of topics studied at university. Of course, you aren’t expected to buy or read all, or any, of them, and the list is far from comprehensive, but browsing a selection of these or similar books will
help you make more informed choices about university mathematics.

**Bridging Material**
Alcock, Lara *How to Study for a Mathematics Degree* (2013)
Allenby, Reg *Numbers and Proofs* (1997)
Zawaira, Hitchcock *A Primer For Mathematics Competitions* (2009)

**Popular Mathematical Texts**
Acheson, David *1089 and All That* (2002)
Bellos, Alex *Alex’s Adventures in Numberland* (2010)
Clegg, Brian *A Brief History of Infinity* (2003)
Courant, Robbins and Stewart *What is Mathematics?* (1996)
Devlin, Keith
- *The Unfinished Game* (2008)
Elwes, Richard
- *MATHS 1001* (2010)
- *Maths in 100 Key Breakthroughs* (2013)
Hofstadter, Douglas *Gödel, Escher, Bach: an Eternal Golden Braid* (1979)

Polya, George *How to Solve It* (1945)
Singh, Simon
- *Fermat’s Last Theorem* (1998)
Stewart, Ian

**History and Biography**
Derbyshire, John *Unknown Quantity – A Real and Imaginary History of Algebra* (2006)
Hellman, Hal *Great Feuds in Mathematics* (2006)
Hodges, Andrew *Alan Turing: The Enigma* (1992)
Kline, Morris *Mathematics For The Non-mathematician* (1967)
Pesic, Peter *Abel’s Proof* (2004)

You may also find useful a study guide *How do undergraduates do mathematics?* for incoming Oxford mathematicians
[https://www0.maths.ox.ac.uk/system/files/study-guide/guide.pdf](https://www0.maths.ox.ac.uk/system/files/study-guide/guide.pdf)
and a set of bridging material
[https://www.maths.ox.ac.uk/study-here/undergraduate-study/bridging-gap](https://www.maths.ox.ac.uk/study-here/undergraduate-study/bridging-gap)
The Mathematics Course

Mathematics is the language of science and logic the language of argument. Science students are often surprised, and sometimes daunted, by the prevalence of mathematical ideas and techniques which form the basis for scientific theory. The more abstract ideas of pure mathematics may find fewer everyday applications, but their study instills an appreciation of the need for rigorous, careful argument and an awareness of the limitations of an argument or technique. A mathematics degree teaches the skills to see clearly to the heart of difficult technical problems, and provides a “toolbox” of ideas and methods to tackle them.

The Mathematics degrees can lead to either a BA after three years or an MMath after four years, though you will not be asked to choose between these until your third year. Both courses are highly regarded: the employability of graduates of both degrees is extremely high, and BA graduates can still go on to second degrees, Masters or PhDs. For the BA, a final classification (First, Upper Second, Lower Second, Third, Pass, Fail) is based on second and third year assessment. MMath students receive this classification and also a similar assessment separately on the fourth year.

First Year (Prelims)

The first year of the course ends with the Preliminary Examination in Mathematics (or “Prelims”). The precise syllabus will appear in due course in the Course Handbook, available at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/handbooks-synopses.

On arrival, you will receive a Course Handbook and supplements to this are issued each year which give detailed synopses of all courses and a supporting reading list for each course of lectures.

The first year course consists of lectures on the following topics:

- Introduction to Pure Mathematics
- Linear Algebra (I, II)
- Geometry
- Groups and Group Actions
- Analysis (I, II, III)
- Dynamics
- Probability
- Statistics and Data Analysis
- Introductory Calculus
- Multivariable Calculus
- Fourier Series and Partial Differential Equations
- Constructive Mathematics
- Computational Mathematics

The last course involves practical computing classes using MATLAB – a popular piece of mathematical software. The course involves introductory sessions in the first term, and two projects in the second term, which count towards Prelims.

There are no lectures in the second half of third term, so that you can concentrate on revision. The end of year examination consists of five written papers, each 2½ hours long; no books, tables or calculators may be taken into the examination room. You are examined on your knowledge of the whole syllabus and your results are overall awarded a Distinction, Pass or Fail. The vast majority of students pass all their papers, but anyone failing one or more papers will need to retake some or all of the papers in September in order to continue on to the second year.
Second Year

In the first term of the second year there are four compulsory courses totalling 64 lectures:

- Linear Algebra
- Metric Spaces
- Complex Analysis
- Differential Equations

Second year students must also take five of the following “long” (16 lectures) options from the first two terms:

- Rings and Modules
- Integration
- Topology
- Differential Equations II
- Numerical Analysis
- Waves and Fluids
- Quantum Theory
- Probability
- Statistics

and take three of the following “short” (8 lectures) options from the third term:

- Number Theory
- Group Theory
- Projective Geometry
- Introduction to Manifolds
- Integral Transforms
- Calculus of Variations
- Graph Theory
- Special Relativity

At the end of the year, each student will sit 15 hours of exams with each core topic and long option being examined over 1½ hours and the short options being examined together in a 1½ hours paper.

Third and Fourth Years

In the third and fourth years still more options become available, including non-mathematical material such as the philosophy or the history of mathematics, an extended essay, a structured projects option and the opportunity to assist teachers in local schools. Students choose eight units, with written exams at the end of the year (some of which may be replaced by practicals or projects). The fourth year range of options is still wider with students taking eight units in all, and again exams at the end of the year. A student must achieve at least an overall upper second in their second and third years to progress to the fourth year. A typical list of third year options is below, though this may vary somewhat from year to year.

- Logic
- Set Theory
- Representation Theory
- Commutative Algebra
- Galois Theory
- Algebraic Number Theory
- Geometry of Surfaces
- Algebraic Curves
- Topology and Groups
- Banach Spaces
- Hilbert Spaces
- Dynamics and Energy Minimization
- Techniques of Applied Mathematics
- Applied Partial Differential Equations
- Viscous Flow
- Waves and Compressible Flow
- Classical Mechanics
- Electromagnetism
- Further Quantum Theory
- Introduction to Quantum Information
- Mathematical Ecology and Biology
- Nonlinear Systems
- Martingales Through Measure Theory
- Modelling Financial Derivatives
- Communication Theory
- Graph Theory
- Integer Programming
- Extended Essay
- Structured Projects
- History of Mathematics
- Mathematics Education
- Undergraduate Ambassadors Scheme
- Applied Statistics
- Statistical Inference
- Applied Probability
- Statistical Life-time Models
- Actuarial Science
- Computational Complexity
- Lambda Calculus and Types
- Numerical Solution of PDEs
- History of Philosophy
- Knowledge and Reality
- Philosophy

(Abel 1824, Gauss 1830)

(Kermack, McKendrick 1927)
The Mathematics and Statistics Course

The twentieth century saw Statistics grow into a subject in its own right (rather than just a single branch of mathematics), and the applicability of statistical analysis is all the more important in the current information age. The probabilities and statistics associated with a complex system are not to be lightly calculated, or argued from, and the subjects contain many deep results and counter-intuitive surprises.

The Mathematics and Statistics degrees (a three year BA or a four year MMath) teach the same rigour and analysis, and many of the mathematical ideas, as the Mathematics degrees and further provide the chance to specialize in probability and statistics, including some courses only available to students on the Mathematics and Statistics degrees. For the BA, a final classification (First, Upper Second, Lower Second, Third, Pass, Fail) is based on second and third year assessment. MMath students receive this classification and also a similar assessment separately on the fourth year.

The course has been accredited by the Royal Statistical Society and graduates of the course can gain exemptions from certain professional examinations for the Institute of Actuaries, given satisfactory performances in the relevant papers.

First Year (Prelims)
The first year of the joint degree is identical to the first year of the Mathematics degree and ends with the Preliminary Examination (or “Prelims”), with the joint degree students sitting the same five university examinations at the end of the first year.

The Course Handbook is available at http://www.stats.ox.ac.uk/current_students/bammath

On arrival, you will receive a Course Handbook and supplements to this are issued each year which give detailed synopses of all courses and a supporting reading list for each course of lectures.

The first year course consists of lectures on the following topics:
- Introduction to Pure Mathematics
- Linear Algebra (I, II)
- Geometry
- Groups and Group Actions
- Analysis (I, II, III)
- Dynamics
- Probability
- Statistics and Data Analysis
- Introductory Calculus
- Multivariable Calculus
- Fourier Series and Partial Differential Equations
- Constructive Mathematics
- Computational Mathematics

The last course involves practical computing classes using MATLAB – a popular piece of mathematical software. The course involves introductory sessions in the first term, and two projects in the second term, which count towards Prelims.

There are no lectures in the second half of third term, so that you can concentrate on revision. The end of year examination consists of five written papers, each 2½ hours long; no books, tables or calculators may be taken into the examination room. You are examined on your knowledge of the whole syllabus and your results are overall awarded a Distinction, Pass or Fail. The vast majority of students pass all their papers, but anyone failing one or more papers will need to retake some or all of the papers in September in order to continue on to the second year.
Second Year
There are six compulsory core lecture courses in the second year totaling 96 lectures:

- Linear Algebra
- Metric Spaces
- Complex Analysis
- Differential Equations
- Probability
- Statistics

with students then choosing three of the following “long” (16 lectures) options from the first two terms:

- Rings and Modules
- Integration
- Topology
- Differential Equations II
- Numerical Analysis
- Waves and Fluids
- Quantum Theory
- Statistical Programming & Simulation

and taking three of the following “short” (8 lectures) options from the third term:

- Number Theory
- Group Theory
- Projective Geometry
- Introduction to Manifolds
- Calculus of Variations
- Graph Theory
- Special Relativity

At the end of the year, each student will sit 15 hours of exams with each core topic and long option being examined over 1½ hours and the short options being examined together in a 1½ hours paper.

Third and Fourth Years
In subsequent years there is a wide choice of topics in mathematics and statistics, including mathematical finance, actuarial science and mathematical modelling. There will be examinations at the end of each year, and a compulsory statistics project for those progressing to the fourth year.

At the end of year four, each student must achieve at least an overall upper second in their second and third years to be able to progress to the fourth year.

In the third year there is one mandatory course in

- Applied Statistics

and at least one must be chosen from

- Statistical Inference
- Applied Probability

A typical set of third year optional courses is listed below, though the list may change from year to year.

- Actuarial Science
- Martingales Through Measure Theory
- Modelling Financial Derivatives
- Continuous Martingales & Stochastic Calculus
- Logic
- Set Theory
- Representation Theory
- Commutative Algebra
- Geometry of Surfaces
- Topology and Groups
- Algebraic Curves
- Banach Spaces
- Hilbert Spaces
- Techniques of Applied Mathematics
- Applied Partial Differential Equations
- Dynamics and Energy Minimization
- Classical Mechanics
- Electromagnetism
- Viscous Flow
- Waves and Compressible Flow
- Further Quantum Theory
- Introduction to Quantum Information
- Mathematical Ecology and Biology
- Nonlinear Systems
- Galois Theory
- Algebraic Number Theory
- Communication Theory
- Graph Theory
- Integer Programming
- Structured Projects
- Extended Essay
- History of Mathematics
- Undergraduate Ambassadors Scheme
- Numerical Solution of PDEs

For more information see www.stats.ox.ac.uk/current_students/bammath
The Mathematics and Philosophy Course

This course brings together two of the most fundamental and widely applicable of intellectual skills. Mathematical knowledge, and the ability to use it, is the most important means of tackling quantifiable problems, while philosophical training encourages the crucial abilities to analyze issues, question received assumptions and articulate the results clearly. Logic, and the philosophy of mathematics, provide natural bridges between the two subjects.

The Mathematics and Philosophy degrees (a three year BA or a four year course MMathPhil) teach a mixture of these two disciplines during the first three years, with a first year core syllabus and options becoming widely available from the second year. In the third and fourth years students may choose to specialize entirely in mathematics or philosophy or to retain a mixture. You will not need to choose until the end of your third year whether to continue on to the fourth. For the BA, a final classification (First, Upper Second, Lower Second, Third, Pass, Fail) is based on second and third year assessment. MMathPhil students receive this classification and also a similar assessment separately on the fourth year.

The mathematics in the degree essentially consists of the pure mathematics courses from the Mathematics degrees. Whilst the mathematical content is less in quantity, the level is just as demanding: prospective students are expected to be studying A-level Mathematics, or the equivalent, with A-level Further Mathematics highly recommended, if available at your school. Preferentially, students should also study an A-level which involves some essay writing. Note that Philosophy A-level is not a requirement, though candidates will be expected at interview to show a strong capacity for reasoned argument and a keen interest in the subject.

First Year (Prelims)

The first year of the joint degree ends with the Preliminary Examination (or “Prelims”).

The first year course covers core material and includes lectures on the following topics in mathematics

- Introduction to Pure Mathematics
- Linear Algebra (I, II)
- Groups and Group Actions
- Analysis (I, II, III)
- Probability
- Introductory Calculus

as well as philosophy courses in

- Elements of Deductive Logic
- Introduction to Philosophy

There are no lectures in the second half of third term, so that you can concentrate on revision. The end of year examination consists of five written papers, three in mathematics (two are 2½ hours long, one is 2 hours long) and two are in philosophy (each lasting 3 hours long); no books, tables or calculators may be taken into the examination room. You are examined on your knowledge of the whole syllabus and your results are overall awarded a Distinction, Pass or Fail. The vast majority of students pass all their papers, but anyone failing one or more papers will need to retake some or all of the papers in September in order to continue on to the second year.
Subsequent Years

The second and third years include compulsory courses in each discipline:

- Linear Algebra
- Metric Spaces
- Complex Analysis
- Set Theory and Logic
- Knowledge and Reality
- Philosophy of Mathematics

Also, in the second year, students will typically choose 4 of the following options (some of which are double (D)):

- Rings and Modules (D)
- Integration (D)
- Topology (D)
- Probability (D)
- Number Theory
- Group Theory
- Projective Geometry
- Introduction to Manifolds
- Graph Theory

There are mathematics examinations at the end of the second year, totally 7½ hours. At the end of the third year, there are six three hour papers (or equivalent), with at least two in mathematics and at least three in philosophy.

A student must achieve at least an overall upper second in their second and third years to be able to progress to the fourth year.

The fourth year of the course allows you the opportunity to specialize entirely in mathematics, in philosophy or to retain a mixture. The philosophy units and extended essays are approximately equivalent to three units of mathematics. Specifically the pathways in the fourth year involve taking 3, 2, 1 or 0 philosophy units and 0, 3, 6 or 8 mathematics units respectively. There are examinations at the end of the year with the option of replacing some of these papers with a philosophy thesis or a mathematics dissertation.

Informal descriptions of the philosophy courses can be found at http://www.philosophy.ox.ac.uk/undergraduate/course_descriptions

Recommended Philosophy Reading

Prior study of philosophy is in no way a prerequisite for this degree. It is clearly sensible, though, to find out more about the subject first. Here are some recommendations for philosophy and logic reading, to complement the earlier list of mathematical texts. Selected reading from one or more, or similar texts, will help you get a flavour of the degree.

- Simon Blackburn's *Think* (Oxford)
- One or more of the shorter dialogues of Plato such as *Protagoras*, *Meno* or *Phaedo*. (Each widely available in English translation.)
- Bertrand Russell's *The Problems of Philosophy* (Oxford University Press).
- Martin Hollis's *Invitation to Philosophy* (Blackwell).
- Thomas Nagel's *What Does It All Mean?* (Oxford University Press).
- P.F. Strawson's *Introduction to Logical Theory* (Methuen).
The Mathematics and Computer Science Course

Mathematics is a fundamental intellectual tool in computing, but computing is increasingly also a tool in mathematical problem solving. This course concentrates on areas where mathematics and computing are most relevant to each other, emphasizing the bridges between theory and practice. It offers opportunities for potential computer scientists both to develop a deeper understanding of the mathematical foundations of the subject and to acquire a familiarity with the mathematics of application areas where computers can solve otherwise intractable problems. It also gives mathematicians access to both a practical understanding of the use of computers, and a deeper understanding of the limits to the use of computers in their own subject. This training leads to a greater flexibility of approach and a better handling of new ideas in one of the fastest changing of all degree subjects.

The Mathematics and Computer Science degree can lead to either a BA after three years or an MMathCompSci after four years, though you will not be asked to choose between these until your third year. For the BA, a final classification (First, Upper Second, Lower Second, Third, Pass, Fail) is based on second and third year assessment. MMathCompSci students receive this classification and also a similar assessment separately on the fourth year.

First Year (Prelims)
The first year of the course ends with the Preliminary Examination in Mathematics (or “Prelims”). The current syllabus is contained in the Course Handbook, available at www.cs.ox.ac.uk/teaching/handbooks.html

On arrival, you will receive a Course Handbook and supplements to this are issued each year which give a detailed synopsis of all courses and a supporting reading list for each course of lectures.

The first year consists of lectures on the following core topics:

- Introduction to Pure Mathematics
- Linear Algebra (I, II)
- Groups and Group Actions
- Analysis (I, II, III)
- Continuous Mathematics
- Probability
- Functional Programming
- Design and Analysis of Algorithms
- Imperative Programming (I, II)

Most of the computer science topics have associated practicals which are taken into account in Prelims.

There are no lectures in the second half of third term, so that you can concentrate on revision. The end of year examination consists of five written papers, of between two and three hours in length with three on Mathematics and two on Computer Science; no books, tables or calculators may be taken into the examination room. You are examined on your knowledge of the whole syllabus and your results are overall awarded a Distinction, Pass or Fail. The vast majority of students pass all their papers, but anyone failing one or more papers will need to retake some or all of the papers in September in order to continue on to the second year.
Second Year
At the beginning of the second year students take three of the “core” mathematics courses

- Linear Algebra
- Metric Spaces
- Complex Analysis

Also, in the second year, students will typically choose 4 of the following options (some of which are double (D)):

- Rings and Modules (D)
- Integration (D)
- Topology (D)
- Probability (D)
- Numerical Analysis (D)
- Number Theory
- Group Theory
- Projective Geometry
- Introduction to Manifolds
- Graph Theory

In computing, students are required to take the following four courses

- Object-Oriented Programming
- Concurrency
- Models of Computation
- Logic and Proof

Other second year mathematical options are also available with a tutor’s consent, but may involve a student needing to catch up on some prerequisite material.

These topics are examined at the end of the second year. Most of the computer science courses include practicals which will also be assessed.

Third and Fourth Years
In the third year there is a still wider range of options available – students take eight courses in all, at least two of which must be in Mathematics and at least two of which must be in Computer Science. See the link

www.cs.ox.ac.uk/teaching/mcs/PartB/

for further information on all the options available. They will be examined at the end of the year, and in practicals in some cases.

A student must achieve at least an overall upper second in their second and third years to be able to progress to the fourth year.

In the fourth year students may choose to specialize entirely in mathematics or computer science or to retain a mixture. Every student must take either a Mathematics Dissertation (and six further units) or a Computer Science Project (and five further units); each unit is assessed by a 1½ examination or equivalent.

If you are interested in finding out more about this course, or about the course in Computer Science, details can be obtained from the Academic Administrator, Department of Computer Science, Parks Road, Oxford OX1 3QD.

Or see the website www.cs.ox.ac.uk/

The Department of Computer Science also runs open days on the same days as four of the Mathematics Department open days – in 2015 the dates are Saturday 9th May, Wednesday July 1st, Thursday July 2nd, and Friday September 18th – details of which can be found at

http://www.cs.ox.ac.uk/ugadmissions/open_days/open_days.html
The Mathematical & Theoretical Physics 4th Year

This new course (starting 2015/16) is a 4th year Masters level course, which unites these two classic disciplines. Theoretical physics utilizes many mathematical techniques, and there are many elegant mathematical proofs to be found in string theory, quantum field theory, and other realms of study usually considered to be applied mathematics.

As this is a 4th year course, you cannot apply for it as a prospective undergraduate. Instead students who are in their 3rd year of Mathematics, Physics, or Physics and Philosophy degrees can apply to transfer onto this 4th year. As with our other joint degrees, in this course you may choose to be highly specialised or gain a broad knowledge of the discipline.

Students must choose at least 10 options (with each option being a 16 hour lecture course) from the following list:

- Quantum Field Theory
- Statistical Mechanics
- Introduction to Quantitative Computing
- Nonequilibrium Statistical Physics
- Kinetic Theory
- Viscous Flow
- General Relativity I and II
- Perturbation Methods
- Scientific Computing I and II
- Numerical Solutions to Differential Equations I and II
- Numerical Linear Algebra
- Groups and Representations
- Algebraic Topology
- Algebraic Geometry
- Advanced Fluid Dynamics
- Soft Matter Physics
- Nonlinear Systems
- Advanced Quantum Field Theory
- Waves and Compressible Flow
- String Theory I and II
- Networks
- Plasma Physics
- Supersymmetry and Supergravity
- Galactic and Planetary Supergravity
- Stellar Astrophysics
- Cosmology
- Applied Complex Variables
- Differential Geometry
- Geometric Group Theory
- Conformal Field Theory
- Introduction to Gauge-String Duality
- Topics in Soft and Active Matter Physics
- Complex Systems
- Turbulence
- Advanced Quantum Computing
- Topics in Quantum Computing
- The Standard Model
- Beyond the Standard Model
- Critical Phenomena
- Geophysical Fluid Dynamics
- Advanced Plasma Physics
- Astrophysical Fluid Dynamics
- Nonperturbation Methods in Quantum Field Theory
- High-Energy Astrophysics
- Astroparticle Physics
- Quantum Field Theory in Curved Space
- Dissertation

Students may also choose a maximum of 3 options from Part B and Part C Mathematics courses, or Part C Physics courses.

A student must achieve at least an overall upper second in their second and third years to be eligible to apply for this fourth year.

Applicants for this course will be assessed on the basis of their academic performance and the compatibility of their previous programme of study. Anyone unsuccessful in the application may choose to continue on their current degree. For more information please see the course website at http://mmathphys.physics.ox.ac.uk/
**Careers**

Demand for mathematics graduates has always been strong, but has been growing rapidly with the increased use of highly technical mathematical models and the growing prevalence of computers.

Almost 30% of our graduates continue on a course of further study, ranging from a research degree in mathematics to a postgraduate course in teacher training. Mathematics at Oxford has many very active research groups, ranging from Geometry, Group Theory, Topology, and Number Theory to the applied research groups of the Centre of Mathematical Biology, the Oxford Centre for Industrial and Applied Mathematics, Numerical Analysis and Stochastic Analysis.

There are no clearly defined career routes after a mathematics degree, unlike Law or Medicine. However, a degree in mathematics gives you excellent quantitative skills, which are applicable in a wide range of careers. Our graduates have gone into careers as consultants, analysts and a variety of financial roles. Additionally many of our graduates go into academic-related positions, such as research roles in companies, the intelligence services or the civil service.

Not only are there many career options after graduating, the average starting salary for our mathematics graduates in 2014, six months after finishing their degree, was £27,800 compared with the national average of £23,000.

In order to support your future career, the University runs a Careers Service which offers free advice and services such as internship programmes at companies across the world, advice sessions from alumni, and tailored careers advice. This service is available to you for life, so we can support you whenever you need us.

"Studying maths at Oxford gave me the analytical and reasoning skills I use in my job as a Public Health Intelligence Officer, as well as teaching me a great deal about communicating difficult mathematical/statistical concepts and how to translate public health questions (e.g. "Does this service work?") into questions that can be answered well by data -- and translating the answers back out again."

"Oxford has given me the opportunities to get where I am today through two main areas in my personal development: academia, as the drive and discipline required to complete a degree at Oxford have to come from yourself; and the inter-personal skills developed through sport, student politics, and relaxing in the bar with very bright and interesting people."

*All material and course details are correct at the time of writing.*
Puzzle Number 1

The Lotka-Volterra equations show the interaction of a population of predators (F, for foxes) and a population of prey (R for rabbits). For positive $\alpha, \beta, \gamma, \delta$ the interactions are given by:

\[
\frac{dF}{dt} = \alpha RF - \beta F \quad \frac{dR}{dt} = \gamma R - \delta RF
\]

a. What do you think $\alpha$, $\beta$, $\gamma$, and $\delta$ represent?
b. What is the population equilibrium (i.e. when $\frac{dF}{dt} = \frac{dR}{dt} = 0$)? Is there an interpretation of either of the equilibrium states?
c. If we let $\alpha = 1.5$, $\beta = 1$, $\gamma = 3$, $\delta = 1$, plot how $F$ changes with $R$. What shape is traced out? What does this tell us about the system?

Puzzle Number 2

Imagine a traveller on an integer number line who travels between adjacent integers. At each time step the traveller either moves one to the right (increasing the integer) or move one to the left (decreasing the integer) with equal probability. Let us restrict the traveller to moving between 0 and 5 inclusive, and if the traveller ever reaches 0 or 5 the walk ends.

a. If the traveller starts on 2, what is the probability that the traveller ends up at 0 after 5 steps? What is the probability that the traveller ends up at 5?

b. After 100 steps, what is the mean number of steps to the left? What is the mean number of steps to the right?

c. What does the distribution of possible locations after $N$ steps look like? How does this vary with $N$?

Now consider an infinite number line.

Puzzle Number 3

Imagine a flock of chickens where every chicken pecks every other chicken. Pecking isn’t transitive in general, so just because a chicken $C_1$ pecks a chicken $C_2$ which pecks another chicken $C_3$, it doesn’t necessarily mean that $C_1$ also pecks $C_3$. However, if $C_1 \rightarrow C_2 \rightarrow C_3$ (where arrows represent pecking), then we say that $C_1$ threatens $C_3$. A king chicken is a king, $K$, if it either pecks every other chicken directly or threatens every other chicken.

a. Start with the chicken that pecks the most other chickens. Can you prove that that chicken must always be a king chicken?
b. If you take a chicken, $C$, in a flock can you prove that if that chicken is pecked by other chickens then one of the chickens that pecks $C$ must be a king chicken?
c. Can a flock ever have two king chickens?

Puzzle Number 4

Fibonacci numbers are numbers defined by the recurrence relation $F_n = F_{n-1} + F_{n-2}$, with starting values $F_1 = 1$, $F_2 = 1$. Zeckendorf’s Theorem states that every positive integer can be represented by the sum of non-consecutive Fibonacci numbers $F_n$ where $n \geq 1$.

a. Verify this for the number 400.
b. Can you prove Zeckendorf’s Theorem for any positive integer?
c. The positive integers $a_1 \leq a_2 \leq a_3 \leq \ldots$ have the property that, even when any one of the $a_i$ is removed from the list, every positive integer can be written as a sum of the remaining $a_i$ without repeat. Show that $a_n \leq F_n$ for $n \geq 1$. 


The Equations

The equations in the margins of this prospectus represent a wide range of mathematics, dating from ancient times to the present day. Here is a brief summary of their various meanings and importance. Further details can be found on them in some of the popular reading recommended later or on the internet through search engines.

\( p.2 \) – A famous infinite series of Euler (its summing was known as the Basel problem), more generally he calculated the sum of reciprocals of all even powers. Schrodinger’s equation describing a wave function in quantum theory.

\( p.3 \) – Dal Ferro’s (and later Tartaglia’s) formula for the roots of a cubic equation. \( \pi \) is transcendental, i.e. not the solution of any polynomial with integer coefficients; Lindemann’s result finally showed that a circle cannot be squared.

\( p.4 \) – D’Alembert’s wave equation for small transverse vibrations of a string, an early example of a mathematical model. Euclid’s proof of an infinity of prime numbers (from his Elements) is one of the most aesthetic in mathematics.

\( p.5 \) – Newton’s proof that the planets’ elliptical orbits were a consequence of the inverse square law of gravity was the first significant application of calculus.

\( p.6 \) – The area of a triangle in non-Euclidean (hyperbolic) geometry in terms of its angles. Hales’ recent proof of Kepler’s conjecture on sphere packing – the proof takes up 250 pages in total and 3Gb in programs and data. \( p.7 \) – Fermat’s Little Theorem (\( p \) is prime) – an early theorem from Number Theory. Maxwell’s equations of electromagnetism relating the magnetic field \( B \) and electrical field \( E \).

\( p.8 \) – the sum of the reciprocals of primes diverges. Shannon’s Noiseless Coding Theorem states that the minimum average codeword length for an infinite alphabet is \( \alpha \), read aleph-null.

\( p.9 \) – Lorentz’s formula for the contraction in lengths an observer traveling at speed \( v \) will perceive, here \( c \) is the speed of light.

\( p.10 \) – Euler’s formula relates the number of vertices, edges and faces on a polyhedron (with no holes), it was one of the first topological results in mathematics. \( p.11 \) – a formula for finding the roots of a quadratic equation, and similar ones for cubics (see page two) and quartics; Abel and Galois showed quintics cannot be solved by radicals.

\( p.12 \) – the Cayley–Hamilton Theorem states that a matrix satisfies its characteristic polynomial \( \lambda(A) = \det(A - \lambda I) \).

\( p.13 \) – Wiles’ proof of Fermat’s Last Theorem (\( x^y + y^z = z^r \) are whole numbers) was the biggest headline in 20th century mathematics. Archimedes’ early estimate for \( \pi \) used a polygon with 96 sides inscribed in a circle.

\( p.14 \) – there is a formula for finding the roots of a quadratic equation, and similar ones for cubic (see page two) and quartics; Abel and Galois showed quintics cannot be solved by radicals.

\( p.15 \) – the Central Limit Theorem from probability, here the random variables \( X \) have the same distribution. Euler’s formula relates the number of vertices, edges and faces on a polyhedron (with no holes), it was one of the first topological results in mathematics. \( p.16 \) – the Gauss–Bonnet Theorem shows that the total curvature of a closed surface relates only to its topology. The Navier–Stokes equations describe the behaviour of viscous flow; their solution is the subject of a Millennium prize. \( p.17 \) – the isoperimetric inequality relates the length of a closed curve and the area it bounds. \( p.18 \) – the Prime Number Theorem is an estimate for the number of primes \( \pi(n) \) less than a given number \( n \); it was first conjectured by Gauss in 1792. Verhulst modeled the logistic map with the growth rate \( r \), at time \( t \) with (intrinsic) growth rate \( r \) and population capacity \( K \); May then investigated a discrete version of the model involving the logistic map with \( r \) again denoting growth rate; the population can show widely varying behaviour depending on \( r \) – extinction, stable growth, alternating populations or chaos.

\( p.19 \) – Turing’s result that there is no algorithm to decide if a program will stop. The open Millennium Problem that NP–hard problems (ones where solutions can be checked in polynomial time) are the same as P–hard problems (ones that can be solved in polynomial time).

\( p.20 \) – rearranging Leibniz’s series from page ten can give any answer. Shannon’s Noiseless Coding Theorem states that the minimum average codeword length \( m(S) \) exceeds the entropy \( H(S) \) of the source \( S \). This was one of the first results in Information Theory. \( p.21 \) – the canonical partition function describes the states of a system of fixed composition which is thermal equilibrium with a heat bath. Each state has a varying energy and is given a different probability depending on its total energy. This equation is used in statistical mechanics, and is different from classical thermodynamics in considering uncertainty. Statistical mechanics describes behavior in areas as diverse as magnetism and voting.

The Poincaré Conjecture is the only Millennium Problem to be solved. It claims that a three–dimensional space which is connected, finite, has no boundary, and has the property that each loop in the space can be shrunk to a point, is homeomorphic (identical) to a sphere.