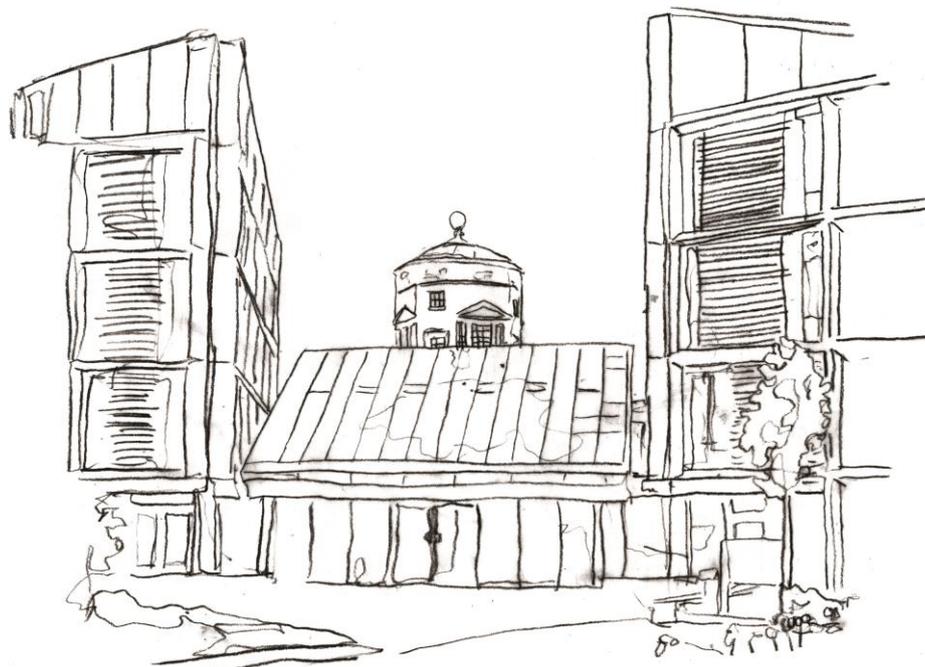


Mathematical Sciences

University of Oxford



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Studying Mathematics at Oxford

University Mathematics

Few people who have not studied a mathematics or science degree will have much idea what modern mathematics involves. Most of the arithmetic and geometry seen in schools today was known to the Ancient Greeks; the ideas of calculus and probability you may have met at A-level were known in the 17th century. And some very neat ideas are to be found there! But mathematicians have not simply been admiring the work of Newton and Fermat for the last three centuries; since then the patterns of mathematics have been found more profoundly and broadly than those early mathematicians could ever have imagined. There is no denying it: mathematics is in a golden age and both within and beyond this university's "dreaming spires", mathematicians are more in demand than ever before.

One great revolution in the history of mathematics was the 19th century discovery of strange non-Euclidean geometries where, for example, the angles of a triangle don't add up to 180° , a discovery defying 2000 years of received wisdom. In 1931 Kurt Gödel shook the very foundations of mathematics, showing that there are true statements which cannot be proved, even about everyday whole numbers. A decade earlier the Polish mathematicians Banach and Tarski showed that any solid ball can be broken into as few as five pieces and then reassembled to form two solid balls of the same size as the original. To this day mathematics has continued to yield a rich array of ideas and surprises, which shows no sign of abating.

Looking through any university's mathematics prospectus you will see course titles that are familiar (e.g. algebra, mechanics) and some that appear thoroughly alien (e.g. Galois Theory, Martingales, Communication Theory). These titles give an honest impression of university mathematics: some courses are continuations from school mathematics, though usually with a sharp change in style and emphasis, whilst others will be thoroughly new, often treating ideas on which you previously had thought mathematics had nothing to say whatsoever.

The clearest change of emphasis is in the need to prove things, especially in *pure mathematics*. Much mathematics is too abstract or technical to simply rely on intuition, and so it is important that you can write clear and irrefutable arguments, which make plain to you, and others, the soundness of your claims. But pure mathematics is more than an insistence on rigour, arguably involving the most beautiful ideas and theorems in all of mathematics, and including whole new areas, such as topology, untouched at school. Mathematics, though, would not be the subject it is today if it hadn't had been for the impact of *applied mathematics* and *statistics*. There is much beautiful mathematics to be found here, such as in relativity or in number theory behind the RSA encryption widely used in internet security, or just in the way a wide range of techniques from all reaches of mathematics might be applied to solve a difficult problem. Also with ever faster computers, mathematicians can now model highly complex systems such as the human heart, can explain why spotted animals have striped tails, and can examine non-deterministic systems like the stock market or Brownian motion. The high technical demands of these models and the prevalence of computers in everyday life are making mathematicians ever more employable after university (see Careers on page 22 for more information).

$$\frac{\pi^2}{6} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots \quad (\text{Euler 1735})$$

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi \quad (\text{Schrödinger 1926})$$



The Oxford System

Students at Oxford are both members of the University and one of 29 colleges, and mathematics teaching is shared by these two institutions. Oxford's collegiate system makes both study, and the day-to-day routine, a rather different experience from other universities.

Most of the teaching of mathematics in Oxford, especially in the first two years of a degree, is done in *tutorials*. These are hour long lessons in college between a tutor, who is usually a senior member of the college, and a small group of students (typically a pair). This form of teaching is very flexible and personalized, allowing a tutor time with the specific difficulties of the group and allowing the students opportunities to ask questions. It is particularly helpful for first year mathematicians who naturally begin university from a wide range of backgrounds and syllabi. College tutors follow closely their students' academic progress, guide them in their studies, discuss subject options and recommend textbooks, as well as being able to answer questions about Oxford generally. Colleges are much more than just halls of residence though, each being a society in its own right, and there will be other students studying mathematics (and other subjects) in college who, invariably, will prove a help with study and often friends during university and beyond.

Mathematicians from across all the colleges come together for lectures which are arranged by the University. This is usually how students first meet each new topic of mathematics. A *lecture* is a 50 minutes talk, usually given in the new Mathematical Institute, with up to 280 other students present. Unsurprisingly there is less (but, by no means, no) chance to ask questions as the lecturer discusses the material, gives

examples, provides slides and make notes at the boards. The lecturer will, like your college tutors, be a member of the Faculty, but usually a tutor at a different college to your own. For most students the material of a lecture is presented too intensely to take in all at once, and so it falls to a student to review their lecture notes and other textbooks, determine which elements are still causing difficulty, and try to work through these. To help, the lecturer, or a college tutor, will set exercises on the lecture and these problems will typically form the basis of the next tutorial in college.

By the third and fourth years the subject options become much more specialized and are taught in intercollegiate classes organized by the University. These are given by a class tutor (usually a member of Faculty or senior graduate who has taken and passed teaching qualifications) and a teaching assistant. They range in size – typically there are 8-10 students – and there is again plenty of chance to ask questions and discuss ideas with the tutors.

College tutors mark their students' tutorial work each week, commenting on progress being made and, at the end of a term, your various tutors will write reports on that term's work and discuss these with you. Most college tutors also set college exams, called *collections*, at the start of each term, to check progress and as practice for later university examinations. The results of collections will not count towards the degree classifications, awarded at the end of the third and fourth years.

See www.ox.ac.uk/ugcolls and www.maths.ox.ac.uk/study-here/undergraduate-study/which-college for links to the colleges' webpages.

$$x^3 + px = q \implies x = \sqrt[3]{\frac{q}{4} + \frac{px^3}{27}} + \frac{x}{2} - \sqrt[3]{\frac{q}{4} + \frac{px^3}{27}} - \frac{x}{2} \quad (\text{dal Ferro c.1500})$$

$$\neg(\exists p(x) \in \mathbb{Q}[x] : p(\pi) = 0) \quad (\text{Lindemann 1882})$$



The Andrew Wiles Building

The Mathematics Department has now settled in to the Andrew Wiles Building on Woodstock Road as part of the new Radcliffe Observatory Quarter. You can find more information and pictures at: www.maths.ox.ac.uk/study-here/undergraduate-study/why-oxford

The Degree Structure

There are three and four year degrees in Mathematics (BA/MMath) and also in the various joint courses: Mathematics and Statistics (BA/MMath), Mathematics and Computer Science (BA/MMathCompSci) and Mathematics and Philosophy (BA/ MMathPhil). There is also now a fourth year stream – Mathematics and Theoretical Physics – whereby students study for an MMathPhys. See page 21 for details.

All of these mathematics degrees have a strong reputation, academically and amongst employers. The joint degrees with Philosophy and with Computer Science contain, roughly speaking, the pure mathematics options. The Mathematics and Statistics degrees have the same first year as the Mathematics degrees, before the emphasis in options increasingly moves towards probability and statistics. Each degree boasts a wide range of options, available from the second year onwards. They will train you to think carefully, critically and creatively about a wide range of mathematical topics, and about arguments generally, with a clear and analytical approach.

The degree structures and the assessment of these degrees have much in common. (See later sections for more on the specifics of each degree.) The first year mathematical content of each degree contains core material, covering ideas and techniques fundamental to the later years. At the end of the academic year, in June, there are five university

examinations, known as the Preliminary Examination (or “Prelims”). Students taking the examination are awarded a Distinction, Pass or Fail. The vast majority of students pass Prelims at the first attempt. Those who do not pass Prelims in the first instance in June may resit one or more of the examinations in September. Successful students may then continue their degree.

The first term of the second year involves the last of the core, compulsory courses (Linear Algebra, Metric Spaces, Complex Analysis, Differential Equations) and some options. From the second term onwards a wide range of options becomes available. Typically a student takes five or six of nine “long options” in the first two terms and three of nine “short options” in the third term. These vary from pure topics like number theory, algebra and geometry, through to applied areas such as fluid dynamics, special relativity and quantum theory. Other options are also available in the joint degrees which reflect the nature of the speciality. Students can choose mainly pure, mainly applied, or a mixture of topics. There are university examinations at the end of the year; no classification is made at that stage though the marks achieved count towards the classification awarded in the third year.

Decisions regarding continuation to the fourth year do not have to be made until the third year. In the third and fourth years there are again a large number of options available, including the chance to write a dissertation and other options which include practical work or projects. Some of these options build on material from earlier courses, whilst others introduce entirely new topics. Some third year courses, and almost all the fourth year courses, bring you close to topics of current research. You may choose a varied selection of options or a more specialized grouping reflecting

$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ (D'Alembert 1746)

there are infinitely many primes (Euclid c.300 BC)



A planet will orbit in an ellipse according to the inverse square law of gravity (Newton 1687)

your future academic or career intentions. There are again university examinations at the end of third year, some of which may be replaced with equivalent project work.

You will receive a classification (First, Upper Second, Lower Second, Third, Pass, Fail) based on the assessment of your examinations, practicals and projects from the second year and third years (i.e. not counting your Prelims results), and a further separate classification similarly assessed on your fourth year (if applicable).

Libraries

Students normally buy a certain number of basic textbooks, but typically find that libraries cover other more specialist needs. Each college has its own library from which its undergraduates may borrow books. These libraries usually have copies of all recommended books for core courses and many others. The hugely resourced Radcliffe Science Library www.bodleian.ox.ac.uk/science has both a lending-library and a reference library.

IT

If you're buying a computer for university, please do consider a laptop over a desktop, so that you can take the laptop to classes. If you don't have your own, the department has several spare laptops that you are welcome to use.

The Invariant Society

The Oxford University Invariant Society www.invariants.org.uk is the undergraduate mathematical society. Its primary aim is to host weekly popular mathematically-related talks by notable speakers, on a wide variety of topics. Past speakers have included Benoit Mandelbrot, Sir Roger Penrose, Marcus du Sautoy and the author Simon Singh.

The Invariants also publishes a termly magazine and runs a puzzle competition.

The Mirzakhani Society

The Mirzakhani Society is a society for women studying maths. Its aim is to support students through providing a space to discuss issues that women may encounter during their degrees. It holds weekly 'Sip and Solve' meetings with tea and cake, and other events such as socials and talks. It is open to both undergraduates and postgrads, and has a wide mix of people at its events.

MURC and JCCU

The Mathematics Undergraduate Representative Committee (informally known as MURC) – is a student body representing the interests of students in mathematics and the joint degrees. It consists of a representative from each college, elected by the undergraduate mathematicians of the college. This committee passes its views on syllabus and examination changes and general matters such as the timing of lectures.

MURC operates a second-hand book scheme whereby all mathematicians are able to buy and sell books. This scheme is particularly useful for 'freshers' (first year undergraduates) since they are able to obtain cheaply some of their textbooks as soon as they arrive at Oxford. The Representative Committee appoints eleven junior members (i.e. undergraduate students) to the Joint Consultative Committee with Undergraduates (JCCU); the other six members of the committee are members of the Departments and Division. This committee meets once a term and its discussions concern the syllabus, teaching, library facilities, open days and general aspects of examinations. It is also available for consultation by the departments on any of these matters and is responsible for discussing feedback from lecture surveys.

$\oint_{\gamma} f(z) dz = 0$ (Cauchy 1825)



Admissions and Preparation for the Course

Admissions

The following applies to prospective students for the Mathematics degree, or for any of the joint degrees, who are considering applying in October 2017 for entry in 2018 or 2019. Much like applying for any other UK university, applications to Oxford are made through UCAS, though the deadline is earlier, on October 15th. Your application may include a preference for one college, or may be an “open” application in which case a college is assigned to you.

The Mathematical Sciences Admissions Test (MAT) is sat by candidates in their schools, colleges or at a test centre. The test, which lasts 2½ hours, will be in the same format as in 2007-16 and these past tests, and two further specimen tests, are available with solutions at www.maths.ox.ac.uk/study-here/undergraduate-study/maths-admissions-test. All applicants attempt the first ten multiple-choice questions, and then four from six longer questions depending on their proposed degree. Instructions are in the test on which questions to complete. No aids, calculators, formula booklets or dictionaries are allowed. A syllabus for the test is available at the above website; it roughly corresponds to material from the A-level modules C1 and C2, though the questions are devised to test for a deeper understanding of, and imagination with, the syllabus’ methods and material.

The distribution of the test will be administered by the Admissions Testing Service and **all applicants must separately register with them by the application deadline (15th October), through their school or college or through a test centre. Schools can**

register with the Admissions Testing Service to become test centres, but please note that this takes a minimum of 24 hours; see www.matoxford.org

All applicants are expected to take the Admissions Test on the above date and must notify the Admissions Testing Service as soon as possible in the event of any potential difficulties or schedule clashes.

Details of the date of the test can be found at: www.matoxford.org.uk

Applicants will be shortlisted for interview in Oxford on the basis of their test marks and UCAS form, with around 3 applicants per place being shortlisted. (Currently there are around 7 applicants per place.) During your stay (typically being for 2-3 nights), meals and accommodation are provided by the college you applied to, or were assigned if you made an open application. During this time the college arranges for some current students to be available to answer your questions about university mathematics and the college and to give you an alternative view of Oxford. In the event of a shortlisted overseas applicant being unable to travel to Oxford a Skype, video or telephone interview may be arranged.

Interviews in Oxford take place in mid-December at the college with at least one more interview guaranteed at another college. Typically, interviews last 20-30 minutes with one or two interviewers, and you may have more than one at a particular college. Applicants for the joint degrees with Philosophy and with Computer Science should expect at least one interview on each discipline. In interview, you may be

A = π - α - β - γ (Lobachevsky 1829, Bolyai 1832)

cubic and hexagonal close packing are the densest possible (Hales 1998)



asked to look at problems of a type that you have never seen before. We want to see how you tackle new ideas and methods and how you respond to helpful prompts, rather than simply find out what you have been taught. Interviews are academic in nature, essentially imitating tutorials, this being how much of Oxford's teaching is done; feel free to ask questions, do say if unsure of something, and expect hints.

If your application is unsuccessful with your first college, another may make you an offer; around 25–30% of offers made are not by the applicant's first college. Around 15% of all applicants are currently made offers, with the majority being conditional offers. In Mathematics,

Mathematics and Statistics, Mathematics and Philosophy, the standard conditional offers are (i) A*A*A with A* grades in Maths and Further Maths *or* A*AAa with A*a in A2 Maths and AS Further Maths *or* A*AA with A* grade in Maths (ii) 39 with 766 at HL including 7 in HL Maths for IB applicants; (iii) AAB/AA for those taking Advanced Highers. In Mathematics and Computer Science, the standard A-level offer is A*AA with A*A between Maths and Further Maths if taken, and otherwise with the A* in Maths. Applicants are informed of their college's decision by mid-January. Information on typical offers involving international qualifications can be found at www.ox.ac.uk/intquals

Website Links and Email Addresses

Information about admissions, the University and colleges, is on the University website www.ox.ac.uk/admissions or in the University's Undergraduate Prospectus – the University prospectus is circulated to schools, can be ordered from this website or by writing to The Undergraduate Admissions Office, University Offices, Wellington Square, Oxford OX1 2JD. Each college has a specific prospectus, obtainable by writing to the college's Tutor for Admissions, or online from college websites (see www.ox.ac.uk/admissions/undergraduate_courses/colleges/).

There are two departmental open days (22/4/17, 29/4/17), *for which registration is required*, and three others (28/6/17, 29/6/17, 15/9/17) when colleges also have open days. At these there will be talks on each of the Mathematics degrees and the joint degrees. There will be plenty of chance to meet current lecturers and students. See www.maths.ox.ac.uk/open-days for full details. Colleges also hold open days; see www.ox.ac.uk/opendays or the University prospectus. At the June and September open days, registration is not required for the repeated morning and afternoon sessions in the Mathematical Institute, but you may need to register with a college if you plan to attend their programme of talks, though all colleges will be open to visitors without booking and often tours will be available.

- www.maths.ox.ac.uk – the Mathematical Institute.
- www.stats.ox.ac.uk – the Statistics Department.
- www.cs.ox.ac.uk/ – the Department of Computer Science.
- www.philosophy.ox.ac.uk – the Philosophy Faculty.
- www.ox.ac.uk/admissions – the Admissions Office's webpage for prospective undergraduates, which includes summaries of all of the colleges.
- www.ox.ac.uk/feesandfunding – information on student funding and the Oxford Opportunity Bursaries.



- undergraduate.admissions@maths.ox.ac.uk – an email address for any enquiries about admissions relating to Mathematics or its Joint Degrees; copies of this prospectus (for UK addresses) can be requested here.
- undergraduate.admissions@admin.ox.ac.uk – an email address for general enquiries about undergraduate admissions; copies of the University’s prospectus (for UK addresses) can be requested here.
- www.maths.ox.ac.uk/study-here/undergraduate-study – the Mathematical Institute’s page for prospective undergraduates; this includes past admissions tests, information about the courses, lecture notes, and extension material.
- www.ox.ac.uk/opendays – a webpage with the dates and details of college and departmental open days.

The Mathematics Department welcomes applications from disabled students and is committed to making reasonable adjustments so that disabled students can participate fully in our courses. You can find out more about the accessibility of our building at: www.admin.ox.ac.uk/access/dandt/mpls/andrewwilesbuilding/. We encourage prospective disabled students to contact the Department’s Administrator (departmental-administrator@maths.ox.ac.uk) at their earliest convenience, to discuss particular needs and the ways in which we could accommodate these needs. See also: www.admin.ox.ac.uk/eop/disab – the University Disability Office’s website which includes FAQs and further information.

Admissions FAQs

For more FAQs see:

www.maths.ox.ac.uk/study-here/undergraduate-study/frequently-asked-questions and www.ox.ac.uk/ask

Q: How do I choose a college?

See www.ox.ac.uk/ugcolls

A: There are 29 undergraduate colleges with students taking mathematics, each having 5–10 mathematics students per year. These colleges have tutors and students enough to provide all the support you need. Colleges differ much more in their size, age, location than they do in their teaching of mathematics. Not all colleges, though, take students in the joint degrees. You can find the number of joint degree students and tutors at a college in tables in the university prospectus or at the above website. To help make a choice it’s best to review college prospectuses (usually available to order from college websites) and, if possible, to attend a college open day, at which you will have a chance to meet the college’s mathematics tutors and some students. Alternatively you can make an *open application* and a college will be assigned to you. Remember your chosen or assigned college is simply the first to consider your application, which will be considered by others. If unsuccessful at the first college, another college may make an offer or you may be made an *open offer* in which you are guaranteed a place to study at Oxford with your college to be confirmed after your A-level results (or equivalent).

Q: What A-levels do I need?

A: If you are taking A-levels then you need to be taking A-level Mathematics, and Further Mathematics A-level is highly recommended. The standard conditional offer, if you are taking A2 Further Mathematics, is A*A*A with A* grades in Mathematics and Further Mathematics (except for the joint degree with Computer Science where the offer is A*AA with A*A in some order in Mathematics and Further Mathematics). We



encourage students to take what mathematical extension material is available to them (e.g. STEP/AEA), but any offer would not depend on these. We strongly recommend Further Mathematics to AS or A2 but recognize that it is not available to many students; single A-level mathematicians successfully study at Oxford, the transition being more difficult, but the tutorial system is especially suited to treating the individual educational needs of students. Philosophy A-level is *not* required for Mathematics and Philosophy. Recent experience of writing essays, though by no means essential, may be helpful.

Q: How do I prepare for the test?

A: You'll find past papers since 2007 and two specimen tests, with solutions, and a syllabus for the test (which roughly corresponds to the C1 and C2 modules from A-level Mathematics) at:

www.maths.ox.ac.uk/study-here/undergraduate-study/maths-admissions-test. We recommend that you attempt these tests under timed conditions.

Q: How do I prepare for interview?

A: While styles differ somewhat, in an interview a tutor will typically discuss problems involving new mathematical ideas, building from a familiar or accessible starting point. The tutor will be interested to see how you respond as the problem is adapted and new ideas introduced, and in how well you can express your arguments. Do share what you're thinking and don't be afraid to admit that you haven't yet covered a topic at school – other questions can be tried, or some help given. As practice you might find it helpful to talk to a school teacher about a favourite area of mathematics or go through a longer MAT question with them. Remember that the interview will be academic and mathematical in nature. You will most likely be asked to think about new and unfamiliar mathematics, though *no specialized mathematical knowledge* will be expected of you beyond what you have met at school or college, but you may well be expected to employ mathematical techniques with which you should be familiar and so it is always a good idea to revise past material you have already met.

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \quad (\text{Lorentz 1904})$$

Q: Should I do Additional Further Maths/more maths modules? Will this make me more likely to be offered a place?

A: If you have the spare time, and are intending to study maths at university, then doing STEP papers would be better preparation than doing more modules for the sake of it. However, if you're particularly interested in the topics covered in later modules then by all means take the extra modules.

Q: I haven't studied philosophy; can I still apply to the Maths and Philosophy degree?

A: Yes, absolutely, studying philosophy is not a prerequisite for applying to the Maths and Philosophy degree though you should be able to demonstrate an interest in philosophy (e.g. through further reading, suggested on page 18).

Q: Are GCSE grades important? / What are the minimum GCSE requirements?

A: We have no minimum GCSE requirements. However, we do look at the proportion of A* you've achieved at GCSE (where applicable) as part of your overall academic achievement. This is contextualised by the GCSE results of the cohort at your school.

Q: What are your admissions criteria? How do you select students?

A: You can find our formal admissions criteria at www.maths.ox.ac.uk/study-here/prospective-undergraduates/how-apply/admissions-criteria



Information for International Students

We welcome applications from international students – currently around 25% of our undergraduate students are from outside the EU and around 8% are from within the EU (but from outside the UK). Colleges and the department provide you with many opportunities to socialise with a wide range of people. If you're feeling homesick, there are over 150 University registered clubs and societies dedicated to a wide range of activities and cultures – for example the German Society (which publishes its own guide for new students), the Chinese Society, and the Italian Society all host a variety of events, including film viewings, talks, and food tasting. You can find a complete list at: www.ox.ac.uk/students/life/clubs/list?wssl=1 . Many colleges offer storage for international students over the short vacations, and some may also offer accommodation throughout the year.

The application process for international students is the same as for UK students – applicants need to apply via UCAS by the 15th October and also, separately, register to sit the Mathematics Admissions Test (MAT). International schools and colleges can become test centres, so you can sit the MAT at your school – however centres should register ideally a month in advance of the UCAS deadline. Alternatively you can sit the MAT at a local open centre – www.admissionstestingservice.org/find-a-centre/

A UCAS application requires you to write a short personal statement focusing on why you are academically suitable for the course you are applying to – this could include mention of extra reading, extra-curricular activities (e.g. Olympiads), or extra classes you have taken. It also requires your school (or a teacher at your school) to write you an academic reference. For more information on applying see www.ox.ac.uk/ucasps Please note that we do not accept additional transcripts, certificates, or references.

Students within the EU will be expected to come in person for interviews in mid-December if shortlisted. Students outside the EU will not be expected to come in person for interview, and Skype interviews will standardly be arranged for these students. Candidates who are Skype interviewed typically have one interview at their first choice college, although some candidates may find themselves being Skype interviewed more than once (often by their second college).

If you are made an offer it may be conditional on you achieving particular grades in your qualifications. We accept a wide range of international qualifications – you can find a complete list at www.ox.ac.uk/intquals . If you have not been taught in English for the last three years your offer may also be conditional on you satisfying English Language requirements. English Language requirements must be met by the 31st July in the year after you apply.

There are a number of scholarships available to international undergraduates – most have closing dates in mid-February after you have received your offer. You can search for scholarships at www.ox.ac.uk/feesandfunding/search

You can find more information about being an international student at Oxford at www.ox.ac.uk/int



Preparation for the Oxford Mathematics Course

Whilst some courses, early in the degree, have a first-principles approach and assume very little mathematical knowledge, other areas would prove rather difficult without certain ideas and techniques being familiar. The following is a list of topics, largely in pure mathematics, most of which we would expect you to have studied before starting the course (but many students will have a few gaps, especially those who have not taken two A Levels in mathematics):

- ✗ Polynomials and basic properties of the roots of polynomial equations.
- ✗ Partial fractions.
- ✗ Simultaneous equations.
- ✗ Inequalities and their manipulation.
- ✗ Basic properties of triangles and circles.
- ✗ Equations of the parabola, ellipse and hyperbola.
- ✗ Elementary properties of lines and planes in three dimensions.
- ✗ Simple treatment of finite and infinite series including arithmetic and geometric progressions.
- ✗ Product, quotient and chain rules of differentiation.
- ✗ Solving simple differential equations.
- ✗ Integration by parts.
- ✗ Recognition of the shape of a plane curve from its equation, maxima and minima, tangents and normals.
- ✗ Binomial Theorem, combinations.
- ✗ Taylor series, the binomial series for non-integer exponent.
- ✗ Matrices and determinants.
- ✗ Induction.
- ✗ Complex numbers – their algebra and geometry.
- ✗ Exponential and trigonometric expansions and Euler’s relation between them.
- ✗ Standard integration techniques and spotting substitutions.
- ✗ Second-order differential equations with constant coefficients.

As A-level syllabuses contain such varying amounts of applied mathematics, that is topics such as mechanics, probability and statistics, very little prior knowledge is assumed here. You may find the early parts of some courses repeat material from your A-level whilst other topics may be almost completely new to you. Typically though, even the “old” material will be repackaged and presented with a different emphasis to school mathematics.

After A-level (and other) results come out in the summer, tutors usually write to students joining their college in October, enclosing (with their congratulations) preparatory exercises on topics like the ones above, often with a suggested list of helpful text books. These two months are an important chance for you to read up on any gaps in your knowledge of the topics above or to refresh your knowledge of those that have become “rusty”. Similar sheets are available at

www.maths.ox.ac.uk/study-here/undergraduate-study/practice-problems

On the next page is a selection of mathematical texts; some are technical books aimed at bridging the gap between A-level and university mathematics, which will help you fill in those gaps over the summer; others aim to popularize mathematical ideas, the history of a topic or theorem, or are biographies of great mathematicians, which may give you a flavour for how mathematics is discovered and the variety of topics studied at university. Of course, you aren’t expected to buy or read all, or any, of them, and the list is far from comprehensive, but browsing a selection of these or similar books will

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

(Leibniz 1674)

$$\Delta_{\psi}(F)\Delta_{\psi}(X) \geq \hbar$$

(Heisenberg 1927)



help you make more informed choices about university mathematics.

Bridging Material

Alcock, Lara *How to Study for a Mathematics Degree* (2012)

Allenby, Reg *Numbers and Proofs* (1997)

Earl, Richard *Towards Higher Mathematics: A Companion* (2017)

Liebeck, Martin *A Concise Introduction to Pure Mathematics* (2000)

Houston, Kevin *How to Think Like a Mathematician* (2009)

Popular Mathematical Texts

Acheson, David *1089 and All That* (2002)

Bellos, Alex *Alex's Adventures in Numberland* (2010)

Clegg, Brian *A Brief History of Infinity* (2003)

Courant, Robbins and Stewart *What is Mathematics?* (1996)

Devlin, Keith
- *Mathematics: The New Golden Age* (1998)
- *The Millennium Problems* (2004)
- *The Unfinished Game* (2008)

Dudley, Underwood *Is Mathematics Inevitable? A Miscellany* (2008)

Elwes, Richard
- *MATHS 1001* (2010)
- *Maths in 100 Key Breakthroughs* (2013)

Gardiner, Martin *The Colossal Book of Mathematics* (2001)

Gowers, Tim *Mathematics: A Very Short Introduction* (2002)

Hofstadter, Douglas *Gödel, Escher, Bach: an Eternal Golden Braid* (1979)

Körner, T. W. *The Pleasures of Counting* (1996)

Neale, Vicky *Closing the Gap: the quest to understand prime numbers* (2017)

Odifreddi, Piergiorgio *The Mathematical Century: The 30 Greatest Problems of the Last 100 Years* (2004)

Piper, Fred & Murphy, Sean *Cryptography: A Very Short Introduction* (2002)

Polya, George *How to Solve It* (1945)

Sewell, Michael (ed.) *Mathematics Masterclasses: Stretching the Imagination* (1997)

Singh, Simon
- *The Code Book* (2000)
- *Fermat's Last Theorem* (1998)

Stewart, Ian
- *Letters to a Young Mathematician* (2006)
- *17 Equations That Changed The World* (2012)

History and Biography

Burton, David *The History of Mathematics* (2007)

Derbyshire, John *Unknown Quantity – A Real and Imaginary History of Algebra* (2006)

Goldstein, Rebecca *Incompleteness – The Proof and Paradox of Kurt Gödel* (2005)

Gray, Jeremy *Hilbert's Challenge* (2000)

Hellman, Hal *Great Feuds in Mathematics* (2006)

Hodgkin, Luke *A History of Mathematics – From Mesopotamia to Modernity* (2005)

Hodges, Andrew *Alan Turing: The Enigma* (1992)

Stedall, Jacqueline *The History of Mathematics: A Very Short Introduction* (2012)

Pesic, Peter *Abel's Proof* (2004)

Reid, Constance *Julia: A Life in Mathematics* (1996)

Stillwell, John *Mathematics and Its History* (2002)

You may also find useful a study guide *How do undergraduates do mathematics?* for incoming Oxford mathematicians

www.maths.ox.ac.uk/system/files/attachments/study_public_0.pdf

and a set of bridging material

www.maths.ox.ac.uk/study-here/undergraduate-study/bridging-gap

$x_A(A) = 0$ (Cayley 1858)

$\forall p(x) \in \mathbb{C}[x] \exists z \in \mathbb{C} : p(z) = 0$ (Gauss 1799)



The Mathematics Course

Mathematics is the language of science and logic the language of argument. Science students are often surprised, and sometimes daunted, by the prevalence of mathematical ideas and techniques which form the basis for scientific theory. The more abstract ideas of pure mathematics may find fewer everyday applications, but their study instils an appreciation of the need for rigorous, careful argument and an awareness of the limitations of an argument or technique. A mathematics degree teaches the skills to see clearly to the heart of difficult technical problems, and provides a “toolbox” of ideas and methods to tackle them.

The Mathematics degrees can lead to either a BA after three years or an MMath after four years, though you will not be asked to choose between these until your third year. Both courses are highly regarded: the employability of graduates of both degrees is extremely high, and BA graduates can still go on to second degrees, Masters or PhDs. For the BA, a final classification (First, Upper Second, Lower Second, Third, Pass, Fail) is based on second and third year assessment. MMath students receive this classification and also a similar assessment separately on the fourth year.

First Year (Prelims)

The first year of the course ends with the Preliminary Examination in Mathematics (or “Prelims”). The precise syllabus will appear in due course in the Course Handbook, available at www.maths.ox.ac.uk/members/student-s/undergraduate-courses/teaching-and-learning/handbooks-synopses

On arrival, you will receive a Course Handbook and supplements to this are

issued each year which give detailed synopses of all courses and a supporting reading list for each course of lectures.

The first year course consists of lectures on the following topics:

- Introduction to Pure Mathematics
- Linear Algebra (I, II)
- Groups and Group Actions
- Analysis (I, II, III)
- Introductory Calculus
- Probability
- Statistics and Data Analysis
- Geometry
- Dynamics
- Constructive Mathematics
- Multivariable Calculus
- Fourier Series and Partial Differential Equations
- Computational Mathematics

The last course involves practical computing classes using *MATLAB* – a popular piece of mathematical software. The course involves introductory sessions in the first term, and two projects in the second term, which count towards Prelims. If you’re buying a computer for university, please do consider a laptop over a desktop, so that you can take the laptop to these classes.

There are no lectures in the second half of third term, so that you can concentrate on revision. The end of year examination consists of five written papers, each between 2–3 hours long; no books, tables or calculators may be taken into the examination room. You are examined on your knowledge of the whole syllabus and your results are overall awarded a Distinction, Pass or Fail. The vast majority of students pass all their papers, but anyone failing one or more papers will need to retake some or all of the papers in September in order to continue on to the second year.

$x^n + y^n = z^n, n > 2 \Rightarrow xyz = 0$ (Wiles 1994)

$3\frac{10}{71} < \pi < 3\frac{1}{7}$ (Archimedes c. 250 BC)



Second Year

In the first term of the second year there are three compulsory courses totalling 64 lectures:

- Linear Algebra
- Metric Spaces and Complex Analysis
- Differential Equations I

Second year students must also take five or six of the following “long” (16 lectures) options from the first two terms:

- Rings and Modules
- Integration
- Topology
- Differential Equations II
- Numerical Analysis
- Waves and Fluids
- Quantum Theory
- Probability
- Statistics

and take three of the following “short” (8 lectures) options from the third term:

- Number Theory
- Group Theory
- Projective Geometry
- Introduction to Manifolds
- Integral Transforms
- Calculus of Variations
- Graph Theory
- Special Relativity
- Mathematical Modelling in Biology

At the end of the year, each student will sit three core papers and six or seven optional papers.

Third and Fourth Years

In the third and fourth years still more options become available, including non-mathematical material such as the philosophy or the history of mathematics, an extended essay, a structured projects option and the opportunity to assist teachers in local schools. Students choose eight units, with written exams at the end of the year (some of which may be replaced by practicals or projects). The fourth year range of options is still wider with students taking eight units in all, and exams at the end of the year.

Currently a student must achieve at least an overall upper second in their second and third years to progress to the fourth year. A typical list of third year options is below, for a current list see www.maths.ox.ac.uk

- Logic
- Set Theory
- Representation Theory
- Commutative Algebra
- Galois Theory
- Algebraic Number Theory
- Geometry of Surfaces
- Algebraic Curves
- Algebraic Number Theory
- Topology and Groups
- Banach Spaces
- Hilbert Spaces
- Stochastic Modelling of Biological Processes
- Dynamics and Energy Minimization
- Applied Partial Differential Equations
- Viscous Flow
- Waves and Compressible Flow
- Further Mathematical Biology
- Nonlinear Systems
- Numerical Solution of Differential Equations I and II
- Integer Programming
- Classical Mechanics
- Electromagnetism
- Further Quantum Theory
- Martingales Through Measure Theory
- Modelling Financial Derivatives
- Communication Theory
- Graph Theory
- Extended Essay
- Structured Projects
- History of Mathematics
- Mathematics Education
- Undergraduate Ambassadors Scheme
- Applied and Computational Statistics
- Statistical Inference
- Statistical Machine Learning
- Applied Probability
- Statistical Life-time Models
- Actuarial Science
- Computational Complexity
- Lambda Calculus and Types
- Early Modern Philosophy
- Knowledge and Reality
- Philosophy of Mathematics

$$\frac{dS}{dt} = -rSI, \quad \frac{dI}{dt} = rSI - bI, \quad \frac{dR}{dt} = bI, \quad (\text{Kermack, McKendrick 1927})$$



The Mathematics and Statistics Course

The twentieth century saw Statistics grow into a subject in its own right (rather than just a single branch of mathematics), and the applicability of statistical analysis is all the more important in the current information age. The probabilities and statistics associated with a complex system are not to be lightly calculated, or argued from, and the subjects contain many deep results and counter-intuitive surprises.

The Mathematics and Statistics degrees (a three year BA or a four year MMath) teach the same rigour and analysis, and many of the mathematical ideas, as the Mathematics degrees and further provide the chance to specialize in probability and statistics, including some courses only available to students on the Mathematics and Statistics degrees. For the BA, a final classification (First, Upper Second, Lower Second, Third, Pass, Fail) is based on second and third year assessment. MMath students receive this classification and also a similar assessment separately on the fourth year.

The course has been accredited by the Royal Statistical Society.

First Year (Prelims)

The first year of the joint degree is identical to the first year of the Mathematics degree and ends with the Preliminary Examination (or "Prelims"), with the joint degree students sitting the same five university examinations at the end of the first year.

The Course Handbook is available at www.stats.ox.ac.uk/current_students/bammath

On arrival, you will receive a Course Handbook and supplements to this are issued each year which give detailed

synopses of all courses and a supporting reading list for each course of lectures.

The first year course consists of lectures on the following topics:

- Introduction to Pure Mathematics
- Linear Algebra (I, II)
- Groups and Group Actions
- Analysis (I, II, III)
- Introductory Calculus
- Probability
- Statistics and Data Analysis
- Geometry
- Dynamics
- Constructive Mathematics
- Multivariable Calculus
- Fourier Series and Partial Differential Equations
- Computational Mathematics

The last course involves practical computing classes using *MATLAB* – a popular piece of mathematical software. The course involves introductory sessions in the first term, and two projects in the second term, which count towards Prelims. If you're buying a computer for university, please do consider a laptop over a desktop, so that you can take the laptop to these classes.

There are no lectures in the second half of third term, so that you can concentrate on revision. The end of year examination consists of five written papers, each 2-3 hours long; no books, tables or calculators may be taken into the examination room. You are examined on your knowledge of the whole syllabus and your results are overall awarded a Distinction, Pass or Fail. The vast majority of students pass all their papers, but anyone failing one or more papers will need to retake some or all of the papers in September in order to continue on to the second year.

$$P\left(\frac{X_1 + X_2 + \dots + X_n - n\mu}{\sigma\sqrt{n}} \leq x\right) \sim \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{1}{2}t^2} dt \quad (\text{Lyapunov 1901})$$

$$V - E + E = 2 \quad (\text{Euler 1750})$$



Second Year

There are five compulsory core lecture courses in the second year totalling 96 lectures:

- Linear Algebra
- Metric Spaces and Complex Analysis
- Differential Equations I
- Probability
- Statistics

with students then choosing three or four of the following “long” (16 lectures) options from the first two terms:

- Rings and Modules
- Integration
- Topology
- Differential Equations II
- Numerical Analysis
- Waves and Fluids
- Quantum Theory
- Simulation & Statistical Programming

and taking three of the following “short” (8 lectures) options from the third term:

- Number Theory
- Group Theory
- Projective Geometry
- Introduction to Manifolds
- Integral Transforms
- Calculus of Variations
- Graph Theory
- Special Relativity
- Modelling in Mathematical Biology

At the end of the year, each student will sit five core papers and four or five optional papers.

Third and Fourth Years

In subsequent years there is a wide choice of topics in mathematics and statistics, including mathematical finance, actuarial science and mathematical modelling. There will be examinations at the end of each year, and a compulsory statistics project for those progressing to the fourth year.

Currently a student must achieve at least an overall upper second in their second and third years to be able to progress to the fourth year.

In the third year there is one mandatory course in

- Applied and Computational Statistics and at least two must be chosen from
- Statistical Inference
- Statistical Machine Learning
- Applied Probability
- Statistical Lifetime-Models

A typical list of third year options is below, for a current list see www.stats.ox.ac.uk/current_students/bammath

- Actuarial Science
- Martingales Through Measure Theory
- Modelling Financial Derivatives
- Continuous Martingales & Stochastic Calculus
- Logic
- Set Theory
- Representation Theory
- Commutative Algebra
- Geometry of Surfaces
- Topology and Groups
- Algebraic Curves
- Banach Spaces
- Hilbert Spaces
- Applied Partial Differential Equations
- Dynamics and Energy Minimization
- Classical Mechanics
- Electromagnetism
- Viscous Flow
- Waves and Compressible Flow
- Further Quantum Theory
- Introduction to Quantum Information
- Further Mathematical Biology
- Nonlinear Systems
- Galois Theory
- Algebraic Number Theory
- Communication Theory
- Graph Theory
- Integer Programming
- Structured Projects
- Extended Essay
- History of Mathematics
- Undergraduate Ambassadors Scheme
- Numerical Solution of DEs I and II

$$\iint_S K \, dA = 2\pi\chi(S) \quad (\text{Bonnet 1848})$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{g}, \quad \nabla \cdot \mathbf{u} = 0, \quad (\text{Navier 1822, Stokes 1842})$$



The Mathematics and Philosophy Course

This course brings together two of the most fundamental and widely applicable of intellectual skills. Mathematical knowledge, and the ability to use it, is the most important means of tackling quantifiable problems, while philosophical training encourages the crucial abilities to analyse issues, question received assumptions and articulate the results clearly. Logic, and the philosophy of mathematics, provide natural bridges between the two subjects.

The Mathematics and Philosophy degrees (a three year BA or a four year course MMathPhil) teach a mixture of these two disciplines during the first three years, with a first year core syllabus and options becoming widely available from the second year. In the third and fourth years students may choose to specialize entirely in mathematics or philosophy or to retain a mixture. You will not need to choose until the end of your third year whether to continue on to the fourth. For the BA, a final classification (First, Upper Second, Lower Second, Third, Pass, Fail) is based on second and third year assessment. MMathPhil students receive this classification and also a similar assessment separately on the fourth year.

The mathematics in the degree essentially consists of the pure mathematics courses from the Mathematics degrees. Whilst the mathematical content is less in quantity, the level is just as demanding: prospective students are expected to be studying A-level Mathematics, or the equivalent, with A-level Further Mathematics highly recommended, if available at your school. Students may find it helpful to study an A-level which involves some essay writing. Note that

Philosophy A-level is *not* a requirement, though candidates will be expected at interview to show a strong capacity for reasoned argument and a keen interest in the subject.

First Year (Prelims)

The first year of the joint degree ends with the Preliminary Examination (or "Prelims").

The first year course covers core material and includes lectures on the following topics in mathematics

- Introduction to Pure Mathematics
- Linear Algebra (I, II)
- Groups and Group Actions
- Analysis (I, II, III)
- Probability
- Introductory Calculus

as well as philosophy courses in

- Elements of Deductive Logic
- Introduction to Philosophy

There are no lectures in the second half of third term, so that you can concentrate on revision. The end of year examination consists of five written papers, three in mathematics (two are 2½ hours long, one is 2 hours long) and two are in philosophy (each lasting 3 hours long); no books, tables or calculators may be taken into the examination room. You are examined on your knowledge of the whole syllabus and your results are overall awarded a Distinction, Pass or Fail. The vast majority of students pass all their papers, but anyone failing one or more papers will need to retake some or all of the papers in September in order to continue on to the second year.

$\mathbb{R}^2 \geq 4\pi A$ (Weierstrass 1870)

$2^{\aleph_0} = c$ (Cantor 1883)



Subsequent Years

The second and third years include compulsory courses in each discipline:

- Linear Algebra
- Metric Spaces and Complex Analysis
- Set Theory and Logic
- Knowledge and Reality
- Philosophy of Mathematics

Also, in the second year, students will typically choose 4 of the following options (some of which are double (D)):

- Rings and Modules (D)
- Integration (D)
- Topology (D)
- Probability (D)
- Number Theory
- Group Theory
- Projective Geometry
- Introduction to Manifolds
- Graph Theory
- Integral Transforms
- Calculus of Variations
- Special Relativity
- Modelling in Mathematical Biology

There are *mathematics* examinations at the end of the second year, totally 7½ hours. At the end of the third year, there are six three hour papers (or equivalent), with at least two in mathematics and at least three in philosophy.

Currently a student must achieve at least an overall upper second in their second and third years to be able to progress to the fourth year.

The fourth year of the course allows you the opportunity to specialize entirely in mathematics, in philosophy or to retain a mixture. The philosophy units and extended essays are approximately equivalent to three units of

mathematics. Specifically the pathways in the fourth year involve taking 3, 2, 1 or 0 philosophy units and 0, 3, 6 or 8 mathematics units respectively. There are examinations at the end of the year with the option of replacing some of these papers with a philosophy thesis or a mathematics dissertation. For a current list of philosophy options see: www.philosophy.ox.ac.uk/undergraduate

Informal descriptions of the philosophy courses can be found at www.philosophy.ox.ac.uk/undergraduate/course_descriptions

Recommended Philosophy Reading

Prior study of philosophy is in no way a prerequisite for this degree. It is clearly sensible, though, to find out more about the subject first. Here are some recommendations for philosophy and logic reading, to complement the earlier list of mathematical texts. Selected reading from one or more, or similar texts, will help you get a flavour of the degree.

- Simon Blackburn's *Think* (Oxford)
- One or more of the shorter dialogues of Plato such as *Protagoras*, *Meno* or *Phaedo*. (Each widely available in English translation.)
- Bertrand Russell's *The Problems of Philosophy* (Oxford University Press).
- Jonathan Glover's *Causing Death and Saving Lives* (Penguin).
- A.J. Ayer's *The Central Questions in Philosophy* (Penguin).
- Martin Hollis's *Invitation to Philosophy* (Blackwell).
- A.W. Moore's *The Infinite* (Routledge).
- Thomas Nagel's *What Does It All Mean?* (Oxford University Press).
- P.F. Strawson's *Introduction to Logical Theory* (Methuen).

(Hadamard & de la Vallee Poussin 1896)

$$\pi(n) \sim \frac{n}{\log n}$$

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K} \right) \quad x_{n+1} = rx_n(1 - x_n) \quad (\text{May 1976}) \quad (\text{Verhulst 1838})$$



The Mathematics and Computer Science Course

Mathematics is a fundamental intellectual tool in computing, but computing is increasingly also a tool in mathematical problem solving. This course concentrates on areas where mathematics and computing are most relevant to each other, emphasizing the bridges between theory and practice. It offers opportunities for potential computer scientists both to develop a deeper understanding of the mathematical foundations of the subject and to acquire a familiarity with the mathematics of application areas where computers can solve otherwise intractable problems. It also gives mathematicians access to both a practical understanding of the use of computers, and a deeper understanding of the limits to the use of computers in their own subject. This training leads to a greater flexibility of approach and a better handling of new ideas in one of the fastest changing of all degree subjects.

The Mathematics and Computer Science degree can lead to either a BA after three years or an MMathCompSci after four years, though you will not be asked to choose between these until your third year. For the BA, a final classification (First, Upper Second, Lower Second, Third, Pass, Fail) is based on second and third year assessment. MMathCompSci students receive this classification and also a similar assessment separately on the fourth year.

First Year (Prelims)

The first year of the course ends with the Preliminary Examination in Mathematics (or "Prelims"). The current syllabus is contained in the Course Handbook, available at

www.cs.ox.ac.uk/teaching/handbooks.html

On arrival, you will receive a Course Handbook and supplements to this are issued each year which give a detailed synopsis of all courses and a supporting reading list for each course of lectures.

The first year consists of lectures on the following core topics:

- Introduction to Pure Mathematics
- Linear Algebra (I, II)
- Groups and Group Actions
- Analysis (I, II, III)
- Continuous Mathematics
- Probability
- Functional Programming
- Design and Analysis of Algorithms
- Imperative Programming (I, II)

Most of the computer science topics have associated practicals which must be passed in order to progress.

There are no lectures in the second half of third term, so that you can concentrate on revision. The end of year examination consists of five written papers, of between two and three hours in length with three on Mathematics and two on Computer Science; no books, tables or calculators may be taken into the examination room. You are examined on your knowledge of the whole syllabus and your results are overall awarded a Distinction, Pass or Fail. The vast majority of students pass all their papers, but anyone failing one or more papers will need to retake some or all of the papers in September in order to continue on to the second year.

the halting problem is undecidable (Turing 1936)

$P = NP$ (Open Millennium Problem)



The terms $1, -\frac{1}{3}, \frac{1}{5}, -\frac{1}{7}, \frac{1}{9}, \dots$, taken in the right order, can converge to any limit (Riemann 1854)

Second Year

At the beginning of the second year students take two of the “core” mathematics courses:

- Linear Algebra
- Metric Spaces and Complex Analysis

Also, in the second year, students will typically choose 4 of the following options (some of which are double (D)):

- Rings and Modules (D)
- Integration (D)
- Topology (D)
- Probability (D)
- Numerical Analysis (D)
- Statistics
- Fluids and Waves
- Quantum Theory
- Number Theory
- Group Theory
- Projective Geometry
- Introduction to Manifolds
- Graph Theory

In computing, students are required to take the following two “core” courses

- Models of Computation
- Algorithms
- Practical course and group practical

Students also take additional Computer Science options in either their second or third years, and a full and current listing of these can be found at www.cs.ox.ac.uk/undergradcourses

Other second year mathematical options are also available with a tutor’s consent, but may involve a student needing to catch up on some prerequisite material.

Students will sit exams at the end of the second year, and some of the computer science courses include practicals which must be passed in order to progress.

Third and Fourth Years

In the third year there is a still wider range of options available – students take ten courses in all, at least two of which must be in Mathematics and at least two of which must be in Computer Science. See the link

www.cs.ox.ac.uk/teaching/mcs/PartB/

for further information on all the options available. They will be examined at the end of the year, and in practicals in some cases.

Currently a student must achieve at least an overall upper second in their second and third years to be able to progress to the fourth year.

In the fourth year students may choose to specialize entirely in mathematics or computer science or to retain a mixture. Every student must take either a Mathematics Dissertation (and six further units) or a Computer Science Project (and five further units); each unit is assessed by examination or equivalent.

If you are interested in finding out more about this course, or about the course in Computer Science, details can be obtained from the Academic Administrator, Department of Computer Science, Parks Road, Oxford OX1 3QD.

Or see the website www.cs.ox.ac.uk/

The Department of Computer Science also runs open days on the same days as four of the Mathematics Department open days – in 2017 the dates are Saturday 22nd April, Wednesday June 28th, Thursday June 29th, and Friday September 15th – details of which can be found at www.cs.ox.ac.uk/opendays

$H(S) \leq m(S) \leq H(S) + 1$ (Shannon 1948)



The Mathematical & Theoretical Physics 4th Year

This new course (which started in 2015/16) is a 4th year Masters level course, which unites these two classic disciplines. Theoretical physics utilizes many mathematical techniques, and there are many elegant mathematical proofs to be found in string theory, quantum field theory, and other realms of study usually considered to be applied mathematics.

As this is a 4th year course, you cannot apply for it as a prospective undergraduate. Instead students who are in their 3rd year of Mathematics, Physics, or Physics and Philosophy degrees can apply to transfer onto this 4th year. As with our other joint degrees, in this course you may choose to be highly specialised or gain a broad knowledge of the discipline.

Students must choose at least 10 options (with each option being a 16 hour lecture course) from the following list:

- Quantum Field Theory
- Statistical Mechanics
- Introduction to Quantitative Computing
- Nonequilibrium Statistical Physics
- Kinetic Theory
- General Relativity I and II
- Perturbation Methods
- Numerical Linear Algebra
- Groups and Representations
- Algebraic Topology
- Algebraic Geometry
- Advanced Fluid Dynamics
- Soft Matter Physics
- Advanced Quantum Field Theory
- String Theory I and II
- Networks
- Plasma Physics
- Supersymmetry and Supergravity
- Galactic and Planetary Dynamics
- Cosmology
- Applied Complex Variables
- Differential Geometry
- Geometric Group Theory

- Conformal Field Theory
- Introduction to Gauge-String Duality
- Topics in Soft and Active Matter Physics
- Advanced Quantum Theory
- Quantum Matter
- Turbulence
- Advanced Quantum Computing
- Topics in Quantum Computing
- The Standard Model
- Beyond the Standard Model
- Critical Phenomena
- Geophysical Fluid Dynamics
- Advanced Plasma Physics
- Astrophysical Fluid Dynamics
- Astrophysical Gas Dynamics
- Nonperturbation Methods in Quantum Field Theory
- Astroparticle Physics
- Radiative Processes and High Energy Astrophysics
- Quantum Field Theory in Curved Space
- Dissertation

Students may also choose a maximum of 3 options from Part B and Part C Mathematics courses, or Part C Physics courses.

A student must achieve at least an overall upper second in their second and third years to be eligible to apply for this fourth year.

Applicants for this course will be assessed on the basis of their academic performance and the compatibility of their previous programme of study. Anyone unsuccessful in the application may choose to continue on their current degree. For more information please see the course website at <http://mmathphys.physics.ox.ac.uk/>

$$P = \sum_k \exp \frac{-E_k}{k_B T} \quad (\text{Boltzmann 1884})$$

Every simply connected, closed 3-manifold is homeomorphic to the 3-sphere.
(Poincaré conjectured 1904, solved by Perelman 2003)



Careers

Demand for mathematics graduates has always been strong, but has been growing rapidly with the increased use of highly technical mathematical models and the growing prevalence of computers.

Over 30% of our graduates continue on a course of further study, ranging from a research degree in mathematics to a postgraduate course in teacher training. Mathematics at Oxford has many very active research groups, ranging from Geometry, Group Theory, Topology, and Number Theory to the applied research groups of the Centre of Mathematical Biology, the Oxford Centre for Industrial and Applied Mathematics, Numerical Analysis and Stochastic Analysis. You can find out more at: www.maths.ox.ac.uk/study-here/postgraduate-study

There are no clearly defined career routes after a mathematics degree, unlike Law or Medicine. However, a degree in mathematics gives you excellent quantitative skills, which are applicable in a wide range of careers. Our graduates have gone into careers as consultants, analysts and a variety of financial roles. Additionally many of our graduates go into academic-related positions, such as research roles in companies, the intelligence services or the civil service.

Not only are there many career options after graduating, the average starting salary for our mathematics graduates, six months after finishing their degree, was £31,300 compared with the national average of £23,400, according to data from the Destination of Leavers from Higher Education (DLHE) survey.

In order to support your future career, the University runs a Careers Service which offers free advice and services such as internship programmes at companies across the world, advice sessions from alumni, and tailored careers advice. This service is available to you for life, so we can support you whenever you need us. See www.careers.ox.ac.uk for more information.

"Studying maths at Oxford gave me the analytical and reasoning skills I use in my job as a Public Health Intelligence Officer, as well as teaching me a great deal about communicating difficult mathematical/statistical concepts and how to translate public health questions (e.g. "Does this service work?") into questions that can be answered well by data -- and translating the answers back out again."

"Oxford has given me the opportunities to get where I am today through two main areas in my personal development: academia, as the drive and discipline required to complete a degree at Oxford have to come from yourself; and the inter-personal skills developed through sport, student politics, and relaxing in the bar with very bright and interesting people."

*All material and course details are correct at the time of writing.

$$\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i \quad (\text{Lagrange 1760})$$

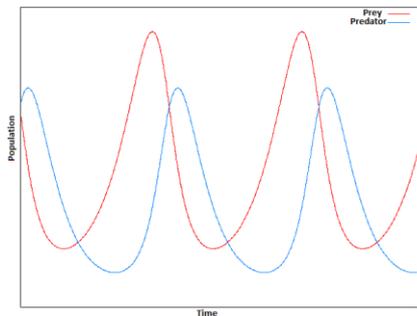
Puzzle Number 1

The *Lotka-Volterra equations* show the interaction of a population of predators (F, for foxes) and a population of prey (R for rabbits). For positive $\alpha, \beta, \gamma, \delta$ the interactions are given by:

$$\frac{dF}{dt} = \alpha RF - \beta F$$

$$\frac{dR}{dt} = \gamma R - \delta RF$$

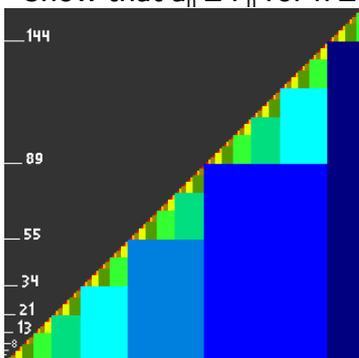
- What do you think $\alpha, \beta, \gamma,$ and δ represent?
- What is the population equilibrium (i.e. when $\frac{dF}{dt} = \frac{dR}{dt} = 0$)? Is there an interpretation of either of the equilibrium states?
- If we let $\alpha = 1.5, \beta = 1, \gamma = 3, \delta = 1$, plot how F changes with R. What shape is traced out? What does this tell us about the system?



Puzzle Number 4

Fibonacci numbers are numbers defined by the recurrence relation $F_n = F_{n-1} + F_{n-2}$, with starting values $F_1 = 1, F_2 = 1$. *Zeckendorf's Theorem* states that every positive integer can be represented by the sum of non-consecutive Fibonacci numbers F_n where $n \geq 1$.

- Verify this for the number 400.
- Can you prove Zeckendorf's Theorem for any positive integer?
- The positive integers $a_1 \leq a_2 \leq a_3 \leq \dots$ have the property that, even when any one of the a_n is removed from the list, every positive integer can be written as a sum of the remaining a_n without repeat. Show that $a_n \leq F_n$ for $n \geq 1$.



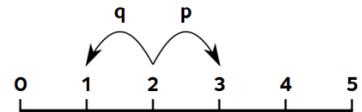
Puzzle Number 2

Imagine a traveller on an integer number line who travels between adjacent integers. At each time step the traveller either moves one to the right (increasing the integer) or move one to the left (decreasing the integer) with equal probability. Let us restrict the traveller to moving between 0 and 5 inclusive, and if the traveller ever reaches 0 or 5 the walk ends.

- If the traveller starts on 2, what is the probability that the traveller ends up at 0 after 5 steps? What is the probability that the traveller ends up at 5?

Now consider an infinite number line.

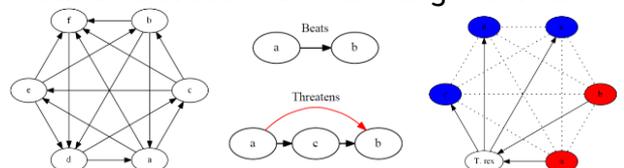
- After 100 steps, what is the mean number of steps to the left? What is the mean number of steps to the right?
- What does the distribution of possible locations after N steps look like? How does this vary with N ?



Puzzle Number 3

Imagine a flock of chickens where every chicken pecks every other chicken. Pecking isn't transitive in general, so just because a chicken C_1 pecks a chicken C_2 which pecks another chicken C_3 , it doesn't necessarily mean that C_1 also pecks C_3 . However, if $C_1 \rightarrow C_2 \rightarrow C_3$ (where arrows represent pecking), then we say that C_1 threatens C_3 . A *king chicken* is a king, K, if it either pecks every other chicken directly or threatens every other chicken.

- Start with the chicken that pecks the most other chickens. Can you prove that that chicken must always be a king chicken?
- If you take a chicken, C, in a flock can you prove that if that chicken is pecked by other chickens then one of the chickens that pecks C must be a king chicken?
- Can a flock ever have two king chickens?



The Equations

The equations in the margins of this prospectus represent a wide range of mathematics, dating from ancient times to the present day. Here is a brief summary of their various meanings and importance. Further details can be found on them in some of the popular reading recommended later or on the internet through search engines.

p.2 – A famous infinite series of Euler (its summing was known as the Basel problem), more generally he calculated the sum of reciprocals of all even powers. Schrodinger's equation describing a wave function in quantum theory.

p.3 – Dal Ferro's (and later Tartaglia's) formula for the roots of a cubic equation. π is transcendental, i.e. not the solution of any polynomial with integer coefficients; Lindemann's result finally showed that a circle cannot be squared.

p.4 – D'Alembert's wave equation for small transverse vibrations of a string, an early example of a mathematical model. Euclid's proof of an infinity of prime numbers (from his *Elements*) is one of the most aesthetic in mathematics.

p.5 – Newton's proof that the planets' elliptical orbits were a consequence of the inverse square law of gravity was the first significant application of calculus. Cauchy's Theorem is the fundamental theorem of complex analysis; complex analysis can tell us much about real integrals and series by means of path integrals in the complex plane.

p.6 – the area of a triangle in non-Euclidean (hyperbolic) geometry in terms of its angles. Hales' recent proof of Kepler's conjecture on sphere packing – the proof takes up 250 pages in total and 3Gb in programs and data.

p.7 – Fermat's Little Theorem (p is prime) – an early theorem from Number Theory. Maxwell's equations of electromagnetism relating the magnetic field \mathbf{B} and electrical field \mathbf{E} .

p.8 – the sum of the reciprocals of primes diverges.

p.9 – Lorentz's formula for the contraction in lengths an observer traveling at speed v will perceive, here c is the speed of light.

p.11 – a famous infinite series of Leibniz. Heisenberg's Uncertainty Principle showing momentum and position cannot be simultaneously observed.

p.12 – the Cayley-Hamilton Theorem states that a matrix satisfies its characteristic polynomial $\chi_A(x) = \det(xI - A)$. The Fundamental Theorem of Algebra states that a polynomial with complex coefficients has a complex root.

p.13 – Wiles' proof of Fermat's Last Theorem (x, y, z are whole numbers) was the biggest headline in 20th century mathematics. Archimedes' early estimate for π used a polygon with 96 sides inscribed in a circle.

p.14 – there is a formula for finding the roots of a quadratic equation, and similar ones for cubics (see page two) and quartics; Abel and Galois showed quintics cannot be solved simply using $+, -, \times, \div$ and taking roots. The SIR model for the spread of an epidemic amongst a fixed population; individuals are either susceptible (S), infected (I) or have recovered (R), with the constants r and b relating to the disease's contagion and recovery rates.

p.15 – the Central Limit Theorem from probability, here the random variables X_i have the same distribution. Euler's formula relates the number of vertices, edges and faces on a polyhedron (with no holes), it was one of the first topological results in mathematics.

p.16 – the Gauss-Bonnet Theorem shows that the total curvature of a closed surface relates only to its topology. The Navier-Stokes equations describe the behaviour of viscous flow; their solution is the subject of a Millennium prize.

p.17 – the isoperimetric inequality relates the length l of a closed curve and the area it bounds A , with equality only in the case of a circle. Cantor's result relating the infinity of the whole numbers (\aleph_0 , read aleph-null) with the infinity of the real numbers (denoted c).

p.18 – The Prime Number Theorem is an estimate for the number of primes $\pi(n)$ less than a given number n ; it was first conjectured by Gauss in 1792. Verhulst modelled the *continuous* growth of a population of size $N(t)$ at time t with (intrinsic) growth rate r and population capacity K ; May then investigated a *discrete* version of the model involving the logistic map with r again denoting growth rate; the population can show widely varying behaviour depending on r – extinction, stable growth, alternating populations or chaos.

p.19 – Turing's result that there is no algorithm to decide if a program will stop. The open Millennium Problem that NP-hard problems (ones where solutions can be checked in polynomial time) are the same as P-hard problems (ones that can be solved in polynomial time).

p.20 – rearranging Leibniz's series from page ten can give any answer. Shannon's Noiseless Coding Theorem states that the minimum average codeword length $m(S)$ exceeds the entropy $H(S)$ of the source S ; this was one of the first results in Information Theory.

p. 21 The canonical partition function describes the states of a system of fixed composition which is thermal equilibrium with a heat bath. Each state has a varying energy and is given a different probability depending on its total energy. This equation is used in statistical mechanics, and is different from classical thermodynamics in considering uncertainty. Statistical mechanics describes behaviour in areas as diverse as magnetism and voting. The Poincaré Conjecture is the only Millennium Problem to be solved. It claims that a three-dimensional space which is *connected*, finite, has no boundary, and has the property that each loop in the space can be shrunk to a point, is *homeomorphic* (identical) to a sphere.

p.22 – a famous integral which can be proved using complex analysis, as can Euler's sum on page one. Lagrange's equations governing a mechanical system; here L is the Lagrangian (kinetic energy minus potential energy) and the q_i are co-ordinates.