

Mathematical Sciences

University of Oxford



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Studying Mathematics at Oxford

University Mathematics

Few people who have not studied a mathematics or science degree will have much idea what modern mathematics involves. Most of the arithmetic and geometry seen in schools today was known to the Ancient Greeks; the ideas of calculus and probability you may have met at A-level were known in the 17th century. And some very neat ideas are to be found there! But mathematicians have not simply been admiring the work of Newton and Fermat for the last three centuries; since then the patterns of mathematics have been found more profoundly and broadly than those early mathematicians could ever have imagined. There is no denying it: mathematics is in a golden age and both within and beyond this university's "dreaming spires", mathematicians are more in demand than ever before.

One great revolution in the history of mathematics was the 19th century discovery of strange non-Euclidean geometries where, for example, the angles of a triangle don't add up to 180°, a discovery defying 2000 years of received wisdom. In 1931 Kurt Gödel shook the very foundations of mathematics, showing that there are true statements which cannot be proved, even about everyday whole numbers. A decade earlier the Polish mathematicians Banach and Tarski showed that any solid ball can be broken into as few as five pieces and then reassembled to form two solid balls of the same size as the original. To this day mathematics has continued to yield a rich array of ideas and surprises, which shows no sign of abating.

Looking through any university's mathematics prospectus you will see course titles that are familiar (e.g. algebra, mechanics) and some that appear thoroughly alien (e.g. Galois Theory, Martingales, Communication Theory). These titles give an honest impression of university mathematics: some courses are continuations from school mathematics, though usually with a sharp change in style and emphasis, whilst others will be thoroughly new, often treating ideas on which you previously had thought mathematics had nothing to say whatsoever.

The clearest change of emphasis is in the need to prove things, especially in *pure mathematics*. Much mathematics is too abstract or technical to simply rely on intuition, and so it is important that you can write clear and irrefutable arguments, which make plain to you, and others, the soundness of your claims. But pure mathematics is more than an insistence on rigour, arguably involving the most beautiful ideas and theorems in all of mathematics, and including whole new areas, such as topology, untouched at school. Mathematics, though, would not be the subject it is today if it hadn't had been for the impact of *applied mathematics* and *statistics*. There is much beautiful mathematics to be found here, such as in relativity or in number theory behind the RSA encryption widely used in internet security, or just in the way a wide range of techniques from all reaches of mathematics might be applied to solve a difficult problem. Also with ever faster computers, mathematicians can now model highly complex systems such as the human heart, can explain why spotted animals have striped tails, and can examine non-deterministic systems like the stock market or Brownian motion. The high technical demands of these models and the prevalence of computers in everyday life are making mathematicians ever more employable after university (see Careers on page 22 for more information).

$$\frac{\pi^2}{6} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots \quad (\text{Euler 1735})$$

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \nabla^2 \psi + V\psi \quad (\text{Schrödinger 1926})$$



The Oxford System

Students at Oxford are both members of the University and one of 29 colleges, and mathematics teaching is shared by these two institutions. Oxford's collegiate system makes both study, and the day-to-day routine, a rather different experience from other universities.

Most of the teaching of mathematics in Oxford, especially in the first two years of a degree, is done in *tutorials*. These are hour long lessons in college between a tutor, who is usually a senior member of the college, and a small group of students (typically a pair). This form of teaching is very flexible and personalized, allowing a tutor time with the specific difficulties of the group and allowing the students opportunities to ask questions. It is particularly helpful for first year mathematicians who naturally begin university from a wide range of backgrounds and syllabi. College tutors follow closely their students' academic progress, guide them in their studies, discuss subject options and recommend textbooks, and are able to answer questions about Oxford generally. Colleges are much more than just halls of residence though, each being a society in its own right, and there will be other students studying mathematics (and other subjects) in college who, invariably, will prove a help with study and often friends during university and beyond.

Mathematicians from across all the colleges come together for lectures which are arranged by the University. This is usually how students first meet each new topic of mathematics. A *lecture* is a 50 minute talk, usually given in the Mathematical Institute, with up to 280 other students present. Unsurprisingly there is less (but, by no means, no) chance to ask questions as the lecturer discusses the material, gives examples,

provides slides and make notes at the boards. The lecturer will, like your college tutors, be a member of the Faculty, but usually a tutor at a different college to your own. For most students the material of a lecture is presented too intensely to take in all at once, and so it falls to a student to review their lecture notes and other textbooks, determine which elements are still causing difficulty, and try to work through these. To help, the lecturer, or a college tutor, will set exercises on the lecture and these problems will typically form the basis of the next tutorial in college.

By the third and fourth years the subject options become much more specialized and are taught in intercollegiate *classes* organized by the University. These are given by a class tutor (usually a member of Faculty or a graduate student who has taken and passed teaching qualifications) and a teaching assistant. They range in size – typically there are 8-12 students – and there is again plenty of chance to ask questions and discuss ideas with the tutors.

College tutors mark their students' tutorial work each week, commenting on progress being made and, at the end of a term, your various tutors will write reports on that term's work and discuss these with you. Most college tutors also set college exams, called *collections*, at the start of each term, to check progress and as practice for later university examinations. The results of collections will not count towards the degree classifications, awarded at the end of the third and fourth years.

See www.ox.ac.uk/ugcolls and www.maths.ox.ac.uk/r/colleges for links to the colleges' webpages.

$$x^3 + mx = n \implies x = \sqrt[3]{\sqrt{\frac{n^2}{4} + \frac{m^3}{27}} + \frac{n}{2}} - \sqrt[3]{\sqrt{\frac{n^2}{4} + \frac{m^3}{27}} - \frac{n}{2}} \quad (\text{dal Ferro c.1500})$$

π is not a root of any polynomial with rational coefficients. (Lindemann 1882)



The Degree Structure

There are three and four year degrees in Mathematics (BA/ MMath) and also in the various joint courses: Mathematics and Statistics (BA/ MMath), Mathematics and Computer Science (BA/ MMathCompSci) and Mathematics and Philosophy (BA/ MMathPhil). There is also a fourth year stream – Mathematics and Theoretical Physics – whereby students study for an MMathPhys. See page 21 for details.

All of these mathematics degrees have a strong reputation, academically and amongst employers. The joint degrees with Philosophy and with Computer Science contain, roughly speaking, the pure mathematics options. The Mathematics and Statistics degrees have the same first year as the Mathematics degrees, before the emphasis in options increasingly moves towards probability and statistics. Each degree boasts a wide range of options, available from the second year onwards. They will train you to think carefully, critically and creatively about a wide range of mathematical topics, and about arguments generally, with a clear and analytical approach.

The degree structures and the assessment of these degrees have much in common. (See later sections for more on the specifics of each degree.) The first year mathematical content of each degree contains core material, covering ideas and techniques fundamental to the later years. At the end of the academic year, in June, there are five university examinations, known as the Preliminary Examination (or “Prelims”). Students taking the examination are awarded a Distinction, Pass or Fail. The vast majority of students pass Prelims at the first attempt. Those who do not pass Prelims in the first instance in June may resit one or more of the examinations in September. Successful students may then continue their degree.

The first term of the second year involves the last of the core, compulsory courses (Linear Algebra, Metric Spaces, Complex Analysis, Differential Equations) and some options. From the second term onwards a wide range of options becomes available. Typically a student takes five or six of nine “long options” in the first two terms and three of nine “short options” in the third term. These vary from pure topics like number theory, algebra and topology, through to applied areas such as fluid dynamics, special relativity and quantum theory. Other options are also available in the joint degrees which reflect the nature of their speciality. Students can choose mainly pure, mainly applied, or a mixture of topics. There are university examinations at the end of the year; no classification is made at that stage though the marks achieved count towards the classification awarded in the third year.

Decisions regarding continuation to the fourth year do not have to be made until the third year. In the third and fourth years there are again a large number of options available, including the chance to write a dissertation and other options which include practical work or projects. Some of these options build on material from earlier courses, whilst others introduce entirely new topics. Some third year courses, and almost all the fourth year courses, bring you close to topics of current research. You may choose a varied selection of options or a more specialized grouping reflecting your future academic or career intentions. There are again university examinations at the end of third year, some of which may be replaced with equivalent project work.

You will receive a classification (First, Upper Second, Lower Second, Third, Pass, Fail) based on the assessment of your examinations, practicals and projects from the second year and

$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ (D’Alembert 1746)

There are infinitely many primes. (Euclid c.300 BC)



third years (i.e. not counting your Prelims results), and a further separate classification based on your fourth year (if applicable).

Libraries

Students normally buy a certain number of basic textbooks, but typically find that libraries cover other more specialist needs. Each college has its own library from which its undergraduates may borrow books. These libraries usually have copies of all recommended books for core courses and many others. The hugely resourced Radcliffe Science Library www.bodleian.ox.ac.uk/science has both a lending-library and a reference library.

IT

If you're buying a computer for university, please do consider a laptop over a desktop/tablet/etc., so that you can take the laptop to classes. If you don't have your own, the department has several spare laptops that you may borrow for the practical classes.

The Invariant Society

The Oxford University Invariant Society is the undergraduate mathematical society. Its primary aim is to host weekly popular mathematically-related talks by notable speakers, on a wide variety of topics. Past speakers have included Benoit Mandelbrot, Sir Roger Penrose, Marcus du Sautoy, and Simon Singh. The Invariants also publishes a termly magazine and runs a puzzle competition. Website: www.invariants.org.uk

The Mirzakhani Society

The Mirzakhani Society is the society for women and non-binary students studying Maths at Oxford University. The society is named after Maryam Mirzakhani, the first female mathematician to win a Fields Medal. Every week we host 'Sip & Solve' where we meet and have drinks and snacks between lectures - it's a great way to meet girls doing maths from other years and colleges and work on problem sheets together! We also have events such as panel discussions, a termly lunch with Mathematrix (our sister postgrad society), talks by female mathematicians, and trips such as our annual exchange with female mathematicians at Cambridge. Website: www.mirzakhanisociety.org.uk

LGBTQubed

LGBTQubed is the student group for all LGBTQ-identifying students in Maths, Statistics, and Computer Science, including undergraduates and graduates, with an aim to provide a friendly space for them to connect. LGBTQubed meets fortnightly during term for tea, biscuits and a chat, and new attendees are always welcome.

MURC and JCCU

The Mathematics Undergraduate Representative Committee (MURC) is a student body representing the interests of students in mathematics and the joint degrees. It consists of an elected representative from each college and an executive committee. This executive committee and members of the Departments form the Joint Consultative Committee for Undergraduates (JCCU), which meets once a term to discuss matters relating to undergraduate teaching in the department. MURC operates a second-hand book scheme, which is particularly useful for first years since they are able to obtain cheaply some of their textbooks as soon as they arrive at Oxford.



Admissions and Preparation for the Course

Admissions

The following applies to prospective students for the Mathematics degree, or for any of the joint degrees, who are considering applying in October 2019 for entry in 2020 or 2021. Much like applying for any other UK university, applications to Oxford are made through UCAS, though the deadline is earlier, on October 15th. Your application may include a preference for one college, or may be an “open” application in which case a college is assigned to you.

The Mathematics Admissions Test (MAT) is sat by candidates in their schools, colleges or at a test centre. The test, which lasts 2½ hours, will be in the same format as in 2007-18 and these past tests, and two further specimen tests, are available with solutions at www.maths.ox.ac.uk/r/mat.

The date of the test is Wednesday 30th October 2019.

All applicants attempt the first ten multiple-choice questions, and then four from six longer questions depending on their proposed degree. Instructions are in the test on which questions to complete. No aids, calculators, formula booklets or dictionaries are allowed. A syllabus for the test is available at the above website; it roughly corresponds to material from the first year of A-level Mathematics, though the questions are devised to test for a deeper understanding of, and imagination with, the syllabus’ methods and material.

The distribution of the test will be administered by Cambridge Assessment Admissions Testing (CAAT) and all applicants must separately register with them by the application deadline (15th October), through their school or college or through a test centre. Schools can register with CAAT to become test centres, but please note that the deadline to become a new centre is 30 September; see www.ox.ac.uk/tests.

All applicants are expected to take the Mathematics Admissions Test and must notify the college and University as soon as possible in the event of any potential difficulties or schedule clashes.

Applicants will be shortlisted for interview in Oxford on the basis of their test marks and UCAS form, with around 3 applicants per place being shortlisted. Currently there are around 9 applicants per place. During your stay (typically being for 2-3 nights), meals and accommodation are provided by the college you applied to, or were assigned if you made an open application. During this time the college arranges for some current students to be available to answer your questions about university mathematics and the college and to give you an alternative view of Oxford. In the event of a shortlisted overseas applicant being unable to travel to Oxford a Skype, video or telephone interview may be arranged.

If you are interviewed in Oxford, you are guaranteed at least one more interview at another college. Typically, interviews last 20-30 minutes with two interviewers, and you may have more than one at a particular college. Applicants for the joint degrees with Philosophy and with Computer Science should expect at least one interview on each discipline. In interview, you may be asked to look at problems of a type that you have never seen before. We want to see how you tackle new ideas and methods and how you respond to helpful prompts,



rather than simply find out what you have been taught. Interviews are academic in nature, essentially imitating tutorials, this being how much of Oxford's teaching is done; feel free to ask questions, do say if unsure of something, and expect hints.

Applicants are informed of their college's decision by mid-January. If your application is unsuccessful with your first college, another may make you an offer; around 25–30% of offers made are not by the applicant's first college. Around 12% of all applicants are currently made offers.

In Mathematics, Mathematics and Statistics, Mathematics and Philosophy, the standard offer for students taking A-levels is A*A*A with A* in Mathematics and in Further Mathematics, unless Further Mathematics is unavailable to you, in which case the standard offer is either A*AAa with A* in Mathematics and a in AS-level Further Mathematics or A*AA with A* in Mathematics, as appropriate. In Mathematics and Computer Science, the standard A-level offer is A*AA with A*A between Maths and Further Maths if taken, and otherwise with the A* in Maths.

For those taking Advanced Highers, the standard offer is AAB/AA, depending on the number of Advanced Highers that you are able to take, and including A in Mathematics. The standard offer for students taking the International Baccalaureate is 39 with 766 at HL including 7 in HL Maths. We will accept candidates who are taking either of the new IB Mathematics courses (HL Mathematics: applications and interpretation or HL Mathematics: analysis and approaches), without preference between the two courses. Information on typical offers involving international qualifications can be found at www.ox.ac.uk/intquals.

Website Links and Email Addresses

Information about admissions, the University, and colleges is on the University website www.ox.ac.uk/admissions or in the University's Undergraduate Prospectus – the University prospectus is circulated to schools, and can be ordered from that website or by writing to The Undergraduate Admissions Office, University Offices, Wellington Square, Oxford OX1 2JD. Each college has a specific prospectus, obtainable by writing to the college's Tutor for Admissions, or online from college websites (see www.ox.ac.uk/ugcolls).

There are two Departmental Open Days on 27th April and 4th May 2019, for which registration is required, and three others on 3rd July, 4th July, and 20th September 2019, when colleges also have open days. There will be talks on Mathematics and each of the joint degrees, and there will be plenty of chances to meet current lecturers and students. See www.maths.ox.ac.uk/open-days for full details. At the July and September open days, registration is not required for the repeated morning and afternoon sessions in the Mathematical Institute. Colleges will be open to visitors without booking and often tours will be available, though you may need to register with a college if you plan to attend their programme of talks. See www.ox.ac.uk/opendays or the University prospectus for full details.

- www.maths.ox.ac.uk – the Mathematical Institute.
- www.stats.ox.ac.uk – the Statistics Department.
- www.cs.ox.ac.uk – the Department of Computer Science.
- www.philosophy.ox.ac.uk – the Philosophy Faculty.
- www.ox.ac.uk/admissions – the Admissions Office's webpage for prospective undergraduates, which includes summaries of all of the colleges.

$a^p \equiv a \pmod p$ (Fermat 1636)

$\nabla \cdot \mathbf{E} = 0, \nabla \cdot \mathbf{B} = 0, \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$ (Maxwell 1864)



- www.ox.ac.uk/feesandfunding – information on student funding and the Oxford Opportunity Bursaries.
- undergraduate.admissions@maths.ox.ac.uk – an email address for any enquiries about admissions relating to Mathematics or its Joint Degrees; copies of this prospectus (for UK addresses) can be requested here.
- undergraduate.admissions@admin.ox.ac.uk – an email address for general enquiries about undergraduate admissions; copies of the University’s prospectus (for UK addresses) can be requested here.
- www.maths.ox.ac.uk/study-here/undergraduate-study – the Mathematical Institute’s page for prospective undergraduates; this includes past admissions tests, information about the courses, lecture notes, and extension material.
- www.ox.ac.uk/opendays – a webpage with the dates and details of college and departmental open days.

The Mathematics Department welcomes applications from disabled students and is committed to making reasonable adjustments so that disabled students can participate fully in our courses. You can find out more about the accessibility of our building at:

www.admin.ox.ac.uk/access/dandt/mps/andrewwilesbuilding/. We encourage prospective disabled students to contact the Department’s Administrator (departmental-administrator@maths.ox.ac.uk) at their earliest convenience, to discuss particular needs and the ways in which we could accommodate these needs. See also: www.admin.ox.ac.uk/eop/disab – the University Disability Office’s website which includes FAQs and further information.

Admissions FAQs

For more FAQs see www.maths.ox.ac.uk/r/faqs and www.ox.ac.uk/ask.

Q: How do I choose a college?

See www.ox.ac.uk/ugcolls

A: There are 29 undergraduate colleges with students taking mathematics, each having 5–10 mathematics students per year. These colleges have tutors and students enough to provide all the support you need. Colleges differ much more in their size, age, location than they do in their teaching of mathematics. Not all colleges, though, take students in the joint degrees. You can find the number of joint degree students and tutors at a college in tables in the university prospectus or at the above website. To help make a choice you could review college prospectuses (usually available to order from college websites) and, if possible, attend a college open day, at which you will have a chance to meet the college’s mathematics tutors and some students. Alternatively you can make an open application and a college will be assigned to you. Remember your chosen or assigned college is simply the first to consider your application, which will be considered by others if you are short-listed for interview. If unsuccessful at the first college, another college may make an offer or you may be made an open offer in which you are guaranteed a place to study at Oxford with your college to be confirmed after your A-level results (or equivalent).

Q: What A-levels do I need?

A: If you are taking A-levels then you need to be taking A-level Mathematics, and Further Mathematics A-level is highly recommended. The standard conditional offer, if you are taking the full A-level in Further Mathematics, is A*A*A with A* grades in Mathematics and Further Mathematics (except for the joint degree with Computer Science where the offer

$\sum_{p \text{ prime}} \frac{1}{p}$ diverges (Euler 1737)

If p and $2p + 1$ are both odd primes, and none of x , y , z is divisible by p , then $x^p + y^p \neq z^p$. (Germain 1823)



Every differentiable symmetry of a system corresponds to a conserved quantity. (Noether 1915)

is A*AA with A*A in some order in Mathematics and Further Mathematics). We encourage students to take what mathematical extension material is available to them (e.g. STEP/AEA), but any offer would not depend on these. We strongly recommend Further Mathematics but recognize that it is not available to many students; single A-level mathematicians successfully study at Oxford, the transition being more difficult, but the tutorial system is especially suited to treating the individual educational needs of students. Philosophy A-level is not required for Mathematics and Philosophy. Recent experience of writing essays, though by no means essential, may be helpful.

Q: How do I prepare for the test?

A: You'll find past papers since 2007 and two specimen tests, with solutions, and a syllabus for the test (which roughly corresponds to the first year of A-level Mathematics) at www.maths.ox.ac.uk/r/mat. You should familiarise yourself with the format of the test.

Q: How do I prepare for interview?

A: Styles differ somewhat, but in interview a tutor will typically discuss problems involving new mathematical ideas, building from a familiar or accessible starting point. The tutor will be interested to see how you respond as the problem is adapted and new ideas introduced, and in how well you can express your arguments. Do share what you're thinking and don't be afraid to admit that you haven't yet covered a topic at school – other questions can be tried, or some help given. As practice you might find it helpful to talk to a school teacher about a favourite area of mathematics or go through a past MAT question with them. The interview will be academic and mathematical in nature. No specialized mathematical knowledge will be expected of you beyond what you have met at school or college, but you may well be expected to employ mathematical techniques with which you should be familiar and so it is always a good idea to revise past material you have already met.

Q: Should I do Additional Further Maths/ more maths modules? Will this make me more likely to be offered a place?

A: If you have the spare time, and are intending to study maths at university, then doing STEP papers would be better preparation than doing more modules for the sake of it. However, if you're particularly interested in the topics covered in later modules then by all means take the extra modules.

Q: I haven't studied philosophy; can I still apply to the Maths and Philosophy degree?

A: Yes, absolutely, studying philosophy is not a prerequisite for applying to the Maths and Philosophy degree, though you should be able to demonstrate an interest in philosophy (e.g. through further reading, suggested on page 18).

Q: Are GCSE grades important? / What are the minimum GCSE requirements?

A: We have no minimum GCSE requirements. However, we do look at the proportion of A* you've achieved at GCSE (where applicable) as part of your overall academic achievement. This is contextualised by the GCSE results of the cohort at your school.

Q: What are your admissions criteria? How do you select students?

A: You can find our formal admissions criteria at www.maths.ox.ac.uk/study-here/prospective-undergraduates/how-apply/admissions-criteria

$$l' = \frac{l}{\sqrt{1 - v^2/c^2}} \quad (\text{Lorentz 1904})$$



Information for International Students

We welcome applications from international students – currently around 25% of our undergraduate students are from outside the EU and around 15% are from within the EU (but from outside the UK). Colleges and the department provide you with many opportunities to socialise with a wide range of people. If you're feeling homesick, there are over 150 University registered clubs and societies dedicated to a wide range of activities and cultures – for example the German Society (which publishes its own guide for new students), the Chinese Society, and the Italian Society all host a variety of events, including film viewings, talks, and food tasting. You can find a complete list at: www.ox.ac.uk/students/life/clubs/list. Many colleges offer storage for international students over the short vacations, and some may also offer accommodation throughout the year.

The application process for international students is the same as for UK students – applicants need to apply via UCAS by the 15th October and also, separately, register to sit the Mathematics Admissions Test (MAT). International schools and colleges can become test centres, so you can sit the MAT at your school – centres need to register by the end of September. Alternatively you can sit the MAT at a local open centre – see www.admissionstesting.org/find-a-centre/

A UCAS application requires you to write a short personal statement focusing on why you are academically suitable for the course you are applying to – this could include mention of extra reading, extra-curricular activities, or extra classes you have taken. It also requires your school (or a teacher at your school) to write you an academic reference. For more information on applying see www.ox.ac.uk/ucasps. Please note that we do not accept additional transcripts, certificates, or references.

Students within the EU will be expected to come in person for interviews in mid-December if shortlisted. Students outside the EU will not be expected to come in person for interview, and Skype interviews will usually be arranged for these students. Candidates who are interviewed over Skype typically have one interview at their first choice college, although some candidates may be interviewed more than once (often by their second college).

If you are made an offer it may be conditional on you achieving particular grades in your qualifications. We accept a wide range of international qualifications – you can find a complete list at www.ox.ac.uk/intquals. If you have not been taught in English for the last three years your offer may also be conditional on you satisfying English Language requirements. English Language requirements must be met by the 31st July in the year after you apply. See www.ox.ac.uk/englang for more information.

There are a number of scholarships available to international undergraduates – most have closing dates in mid-February after you have received your offer. You can search for scholarships at www.ox.ac.uk/feesandfunding/search.

You can find more information about being an international student at Oxford at www.ox.ac.uk/int.

$\pi = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$ (Leibniz 1674)

$\Delta_\psi(P)\Delta_\psi(X) \geq \hbar$ (Heisenberg 1927)



Preparation for the Oxford Mathematics Course

Whilst some courses, early in the degree, have a first-principles approach and assume very little mathematical knowledge, other areas would prove rather difficult without certain ideas and techniques being familiar. The following is a list of topics, largely in pure mathematics, most of which we would expect you to have studied before starting the course (but many students will have a few gaps, especially those who have not taken A-level Further Mathematics or equivalent):

- Polynomials and basic properties of the roots of polynomial equations.
- Partial fractions.
- Simultaneous equations.
- Inequalities and their manipulation.
- Basic properties of triangles and circles.
- Equations of the parabola, ellipse and hyperbola.
- Elementary properties of lines and planes in three dimensions.
- Arithmetic and geometric progressions.
- Product, quotient and chain rules of differentiation.
- Solving simple differential equations.
- Integration by parts.
- Recognition of the shape of a plane curve from its equation, maxima and minima, tangents and normals.
- Binomial Theorem, combinations.
- Taylor series, the binomial series for non-integer exponent.
- Matrices and determinants.
- Induction.
- Complex numbers – their algebra and geometry.
- Exponential and trigonometric expansions and Euler's relation between them.
- Standard integration techniques and spotting substitutions.
- Second-order differential equations with constant coefficients.

As A-level syllabuses contain varying amounts of mechanics, probability and statistics, very little prior knowledge is assumed here. You may find the early parts of some courses repeat material from your A-level whilst other topics may be almost completely new to you. Typically though, even the "old" material will be repackaged and presented with a different emphasis to school mathematics.

After A-level (and other) results come out in the summer, tutors usually write to students joining their college in October, enclosing (with their congratulations) preparatory exercises on topics like the ones above, often with a suggested list of helpful text books. These two months are an important chance for you to read up on any gaps in your knowledge of the topics above or to refresh your knowledge of those that have become "rusty". Similar practice problems are online at www.maths.ox.ac.uk/r/practice.

On the next page is a selection of mathematical texts; some are technical books aimed at bridging the gap between A-level and university mathematics, which will help you fill in those gaps over the summer; others aim to popularize mathematical ideas, the history of a topic or theorem, or are biographies of great mathematicians, which may give you a flavour for how mathematics is discovered and the variety of topics studied at university. Of course, you aren't expected to buy or read all, or any, of them, and the list is far from comprehensive.

You may also find useful a study guide for incoming Oxford mathematicians www.maths.ox.ac.uk/r/study-guide and a set of bridging material www.maths.ox.ac.uk/study-here/undergraduate-study/bridging-gap.

$\chi_A(A) = 0$ (Cayley 1858)

$\forall p(x) \in \mathbb{C}[x] \exists z \in \mathbb{C} : p(z) = 0$ (Gauss 1799)



Recommended Mathematics Reading

Bridging Material

- Alcock, Lara *How to Study for a Mathematics Degree* (2012)
- Allenby, Reg *Numbers and Proofs* (1997)
- Earl, Richard *Towards Higher Mathematics: A Companion* (2017)
- Houston, Kevin *How to Think Like a Mathematician* (2009)
- Liebeck, Martin *A Concise Introduction to Pure Mathematics* (2000)

Popular Mathematics

- Acheson, David *1089 and All That* (2002), *The Calculus Story* (2017)
- Bellos, Alex *Alex's Adventures in Numberland* (2010)
- Clegg, Brian *A Brief History of Infinity* (2003)
- Courant, Robbins and Stewart *What is Mathematics?* (1996)
- Devlin, Keith *Mathematics: The New Golden Age* (1998), *The Millennium Problems* (2004), *The Unfinished Game* (2008)
- Dudley, Underwood *Is Mathematics Inevitable? A Miscellany* (2008)
- Elwes, Richard *MATHS 1001* (2010), *Maths in 100 Key Breakthroughs* (2013)
- Gardiner, Martin *The Colossal Book of Mathematics* (2001)
- Gowers, Tim *Mathematics: A Very Short Introduction* (2002)
- Hofstadter, Douglas *Gödel, Escher, Bach: an Eternal Golden Braid* (1979)
- Körner, T. W. *The Pleasures of Counting* (1996)
- Neale, Vicky *Closing the Gap: the quest to understand prime numbers* (2017)
- Odifreddi, Piergiorgio *The Mathematical Century: The 30 Greatest Problems of the Last 100 Years* (2004)
- Piper, Fred & Murphy, Sean *Cryptography: A Very Short Introduction* (2002)
- Polya, George *How to Solve It* (1945)
- Sewell, Michael (ed.) *Mathematics Masterclasses: Stretching the Imagination* (1997)
- Singh, Simon *The Code Book* (2000), *Fermat's Last Theorem* (1998)
- Stewart, Ian *Letters to a Young Mathematician* (2006), *17 Equations That Changed The World* (2012)

History and Biography

- Burton, David *The History of Mathematics* (2007)
- Derbyshire, John *Unknown Quantity – A Real and Imaginary History of Algebra* (2006)
- Goldstein, Rebecca *Incompleteness – The Proof and Paradox of Kurt Gödel* (2005)
- Gray, Jeremy *Hilbert's Challenge* (2000)
- Hellman, Hal *Great Feuds in Mathematics* (2006)
- Hodgkin, Luke *A History of Mathematics – From Mesopotamia to Modernity* (2005)
- Hodges, Andrew *Alan Turing: The Enigma* (1992)
- Stedall, Jacqueline *The History of Mathematics: A Very Short Introduction* (2012)
- Pesic, Peter *Abel's Proof* (2004)
- Reid, Constance *Julia: A Life in Mathematics* (1996)
- Stillwell, John *Mathematics and Its History* (2002)

$$1 = \frac{\sqrt{8}}{\pi} \sum_{n=0}^{\infty} \frac{(4n)! (1103 + 26390n)}{99^2 (n!)^4 396^{4n}}$$

(Ramanujan 1914)



The Mathematics Course

Mathematics is the language of science and logic the language of argument. Science students are often surprised, and sometimes daunted, by the prevalence of mathematical ideas and techniques which form the basis for scientific theory. The more abstract ideas of pure mathematics may find fewer everyday applications, but their study instils an appreciation of the need for rigorous, careful argument and an awareness of the limitations of an argument or technique. A mathematics degree teaches the skills to see clearly to the heart of difficult technical problems, and provides a “toolbox” of ideas and methods to tackle them.

The first year course currently consists of lectures on the following topics:

- Introduction to University Mathematics
- Introduction to Complex Numbers
- Linear Algebra (I, II)
- Groups and Group Actions
- Analysis (I, II, III)
- Introductory Calculus
- Probability
- Statistics and Data Analysis
- Geometry
- Dynamics
- Constructive Mathematics
- Multivariable Calculus
- Fourier Series and Partial Differential Equations
- Computational Mathematics

The last course involves practical computing classes using *MATLAB* – a popular piece of mathematical software. The course involves introductory sessions in the first term, and two projects in the second term, which count towards Prelims. If you’re buying a computer for university, please do consider a laptop over a desktop, so that you can take the laptop to these classes.

There are no lectures in the second half of third term, so that you can concentrate on revision. The end of year examination consists of five written papers, each between 2–3 hours long; no books, tables or calculators may be taken into the examination room. You are examined on your knowledge of the whole syllabus and your results are overall awarded a Distinction, Pass or Fail. The vast majority of students pass all their papers, but anyone failing one or more papers will need to retake some or all of the papers in September in order to continue on to the second year.

The Mathematics degrees can lead to either a BA after three years or an MMath after four years, though you will not be asked to choose between these until your third year. Both courses are highly regarded: the employability of graduates of both degrees is extremely high, and BA graduates can still go on to second degrees, Masters or PhDs. For the BA, a final classification (First, Upper Second, Lower Second, Third, Pass, Fail) is based on second and third year assessment. MMath students receive this classification and also a similar assessment separately on the fourth year.

First Year (Prelims)

The first year of the course ends with the Preliminary Examination in Mathematics (or “Prelims”). For an example of the current syllabus, see the Course Handbook, available at www.maths.ox.ac.uk/r/handbooks.

On arrival, you will receive a Course Handbook and supplements to this are issued each year which give detailed synopses of all courses and a supporting reading list for each course of lectures.

$x^n + y^n = z^n, n > 2 \implies xyz = 0$ (Wiles 1994)

$3\frac{10}{71} < \pi < 3\frac{1}{7}$ (Archimedes c.250 BC)

Second Year

In the first term of the second year there are currently three compulsory courses totalling 64 lectures:

- Linear Algebra
- Metric Spaces and Complex Analysis
- Differential Equations I

Second year students must also take five or six of the following long options (16 lectures):

- Rings and Modules
- Integration
- Topology
- Differential Equations II
- Numerical Analysis
- Fluids and Waves
- Quantum Theory
- Probability
- Statistics

Students also take three of the following short options (8 lectures):

- Number Theory
- Group Theory
- Projective Geometry
- Introduction to Manifolds
- Integral Transforms
- Calculus of Variations
- Graph Theory
- Special Relativity
- Mathematical Modelling in Biology

At the end of the year, each student sits three core papers and six or seven optional papers.

Third and Fourth Years

In the third and fourth years still more options become available, including material such as the philosophy or the history of mathematics, an extended essay, a structured projects option and the opportunity to be an ambassador. Students choose eight units, with written exams at the end of the year (some of which may be replaced by practicals or projects).

The fourth year range of options is still wider with students taking eight units in all, and exams at the end of the year. Currently an upper second over second

and third year, as well as an upper second in third year alone, is required to progress to fourth year.

A typical list of third year options is below. For a current list see <https://courses.maths.ox.ac.uk>

- Logic
- Set Theory
- Introduction to Representation Theory
- Commutative Algebra
- Galois Theory
- Algebraic Number Theory
- Geometry of Surfaces
- Algebraic Curves
- Algebraic Number Theory
- Topology and Groups
- Functional Analysis (I, II)
- Stochastic Modelling of Biological Processes
- Applied Partial Differential Equations
- Viscous Flow
- Waves and Compressible Flow
- Further Mathematical Biology
- Nonlinear Systems
- Numerical Solution of Differential Equations I and II
- Integer Programming
- Classical Mechanics
- Electromagnetism
- Probability, Measure and Martingales
- Information Theory
- Further Quantum Theory
- Mathematical Models of Financial Derivatives
- Graph Theory
- Structured Projects
- History of Mathematics
- Mathematics Education
- Undergraduate Ambassadors Scheme
- Applied and Computational Statistics
- Foundations of Statistical Inference
- Statistical Machine Learning
- Applied Probability
- Statistical Life-time Models
- Actuarial Science
- Computational Complexity
- Lambda Calculus and Types
- Early Modern Philosophy
- Philosophy of Mathematics
- Philosophical Logic
- Knowledge and Reality

$$\frac{dS}{dt} = -rSL, \quad \frac{dI}{dt} = rSL - bI, \quad \frac{dR}{dt} = bI$$

(Kermack, McKendrick 1927)



The Mathematics and Statistics Course

The twentieth century saw Statistics grow into a subject in its own right (rather than just a single branch of mathematics), and the applicability of statistical analysis is all the more important in the current information age. The probabilities and statistics associated with a complex system are not to be lightly calculated, or argued from, and the subjects contain many deep results and counter-intuitive surprises.

The Mathematics and Statistics degrees (a three year BA or a four year MMath) teach the same rigour and analysis, and many of the mathematical ideas, as the Mathematics degrees and further provide the chance to specialize in probability and statistics, including some courses only available to students on the Mathematics and Statistics degrees. For the BA, a final classification (First, Upper Second, Lower Second, Third, Pass, Fail) is based on second and third year assessment. MMath students receive this classification and also a similar assessment separately on the fourth year.

The course has been accredited by the Royal Statistical Society.

First Year (Prelims)

The first year of the joint degree is identical to the first year of the Mathematics degree and ends with the Preliminary Examination (or "Prelims"), with the joint degree students sitting the same five university examinations at the end of the first year.

The Course Handbook is available at www.stats.ox.ac.uk/current_students/ba_mmath.

On arrival, you will receive a Course Handbook and supplements to this are issued each year which give detailed

synopses of all courses and a supporting reading list for each course of lectures.

The first year course currently consists of lectures on the following topics:

- Introduction to Pure Mathematics
- Linear Algebra (I, II)
- Groups and Group Actions
- Analysis (I, II, III)
- Introductory Calculus
- Probability
- Statistics and Data Analysis
- Geometry
- Dynamics
- Constructive Mathematics
- Multivariable Calculus
- Fourier Series and Partial Differential Equations
- Computational Mathematics

The last course involves practical computing classes using *MATLAB* – a popular piece of mathematical software. The course involves introductory sessions in the first term, and two projects in the second term, which count towards Prelims. If you're buying a computer for university, please do consider a laptop over a desktop, so that you can take the laptop to these classes.

There are no lectures in the second half of third term, so that you can concentrate on revision. The end of year examination consists of five written papers, each 2-3 hours long; no books, tables or calculators may be taken into the examination room. You are examined on your knowledge of the whole syllabus and your results are overall awarded a Distinction, Pass or Fail. The vast majority of students pass all their papers, but anyone failing one or more papers will need to retake some or all of the papers in September in order to continue on to the second year.

$$P\left(\frac{X_1 + X_2 + \dots + X_n - n\mu}{\sigma\sqrt{n}} \leq x\right) \rightarrow \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt \quad (\text{Lyapunov 1901})$$

$$V - E + F = 2 \quad (\text{Euler 1750})$$



Second Year

There are currently five compulsory core lecture courses in the second year totalling 96 lectures:

- Linear Algebra
- Metric Spaces and Complex Analysis
- Differential Equations I
- Probability
- Statistics

with students then choosing three or four of the following “long” (16 lectures) options from the first two terms:

- Rings and Modules
- Integration
- Topology
- Differential Equations II
- Numerical Analysis
- Waves and Fluids
- Quantum Theory
- Simulation & Statistical Programming

and taking three of the following “short” (8 lectures) options from the third term:

- Number Theory
- Group Theory
- Projective Geometry
- Introduction to Manifolds
- Integral Transforms
- Calculus of Variations
- Graph Theory
- Special Relativity
- Modelling in Mathematical Biology

At the end of the year, each student will sit five core papers and four or five optional papers.

Third and Fourth Years

In subsequent years there is a wide choice of topics in mathematics and statistics, including mathematical finance, actuarial science and mathematical modelling. There will be examinations at the end of each year, and a compulsory statistics project for those progressing to the fourth year.

Currently an upper second over second and third year, as well as an upper second in third year alone, is required to progress to fourth year.

In the third year there is currently one mandatory course in

- Applied and Computational Statistics

and at least two must be chosen from

- Statistical Inference
- Statistical Machine Learning
- Applied Probability
- Statistical Lifetime-Models

A typical list of third year options is below, for a current list see www.stats.ox.ac.uk/current_students/bammath

- Actuarial Science
- Martingales Through Measure Theory
- Modelling Financial Derivatives
- Continuous Martingales & Stochastic Calculus
- Logic
- Set Theory
- Representation Theory
- Commutative Algebra
- Geometry of Surfaces
- Topology and Groups
- Algebraic Curves
- Functional Analysis (I, II)
- Applied Partial Differential Equations
- Dynamics and Energy Minimization
- Classical Mechanics
- Electromagnetism
- Viscous Flow
- Waves and Compressible Flow
- Further Quantum Theory
- Introduction to Quantum Information
- Further Mathematical Biology
- Nonlinear Systems
- Galois Theory
- Algebraic Number Theory
- Communication Theory
- Graph Theory
- Integer Programming
- Structured Projects
- Extended Essay
- History of Mathematics
- Undergraduate Ambassadors Scheme

$$\iint_S K \, dA = 2\pi\chi(S) \quad (\text{Bonnet 1848})$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{g} \quad (\text{Navier 1822, Stokes 1842})$$



The Mathematics and Philosophy Course

This course brings together two of the most fundamental and widely applicable of intellectual skills. Mathematical knowledge, and the ability to use it, is the most important means of tackling quantifiable problems, while philosophical training encourages the crucial abilities to analyse issues, question received assumptions and articulate the results clearly. Logic, and the philosophy of mathematics, provide natural bridges between the two subjects.

The Mathematics and Philosophy degrees (a three year BA or a four year course MMathPhil) teach a mixture of these two disciplines during the first three years, with a first year core syllabus and options becoming widely available from the second year. In the third and fourth years students may choose to specialize entirely in mathematics or philosophy or to retain a mixture. You will not need to choose until the end of your third year whether to continue on to the fourth. For the BA, a final classification (First, Upper Second, Lower Second, Third, Pass, Fail) is based on second and third year assessment. MMathPhil students receive this classification and also a similar assessment separately on the fourth year.

The mathematics in the degree essentially consists of the pure mathematics courses from the Mathematics degrees. Whilst the mathematical content is less in quantity, the level is just as demanding: prospective students are expected to be studying A-level Mathematics, or the equivalent, with A-level Further Mathematics highly recommended, if available at your school. Students may find it helpful to study an A-level which involves some essay writing. Note that Philosophy A-level is *not* a requirement, though candidates will be expected at interview to show a strong capacity for

reasoned argument and a keen interest in the subject.

First Year (Prelims)

The first year of the joint degree ends with the Preliminary Examination (or "Prelims").

The first year course covers core material and includes lectures on the following topics in mathematics

- Introduction to Pure Mathematics
- Linear Algebra (I, II)
- Groups and Group Actions
- Analysis (I, II, III)
- Probability
- Introductory Calculus

as well as philosophy courses in

- Elements of Deductive Logic
- Introduction to Philosophy

There are no lectures in the second half of third term, so that you can concentrate on revision. The end of year examination consists of five written papers, three in mathematics (two are 2½ hours long, one is 2 hours long) and two are in philosophy (each lasting 3 hours long); no books, tables or calculators may be taken into the examination room. You are examined on your knowledge of the whole syllabus and your results are overall awarded a Distinction, Pass or Fail. The vast majority of students pass all their papers, but anyone failing one or more papers will need to retake some or all of the papers in September in order to continue on to the second year.

Subsequent Years

The second and third years currently include compulsory courses in each discipline:

- Linear Algebra

$l^2 \geq 4\pi A$ (Weierstrass 1870)

$2^{\aleph_0} = c$ (Cantor 1883)



- Metric Spaces and Complex Analysis
- Set Theory and Logic
- Knowledge and Reality
- Philosophy of Mathematics

Also, in the second year, students will typically choose 4 of the following options (some of which are double (D)):

- Rings and Modules (D)
- Integration (D)
- Topology (D)
- Probability (D)
- Number Theory
- Group Theory
- Projective Geometry
- Introduction to Manifolds
- Graph Theory
- Integral Transforms
- Calculus of Variations
- Special Relativity
- Modelling in Mathematical Biology

There are mathematics examinations at the end of the second year, totally 7½ hours. At the end of the third year, there are six three hour papers (or equivalent), with at least two in mathematics and at least three in philosophy.

Currently an upper second over second and third year, as well as an upper second in third year alone, is required to progress to fourth year.

The fourth year of the course allows you the opportunity to specialize entirely in mathematics, in philosophy or to retain a mixture. The philosophy units and extended essays are approximately equivalent to three units of mathematics. Specifically the pathways in the fourth year involve taking 3, 2, 1 or 0 philosophy units and 0, 3, 6 or 8 mathematics units

respectively. There are examinations at the end of the year with the option of replacing some of these papers with a philosophy thesis or a mathematics dissertation. For a current list of philosophy options see:

www.philosophy.ox.ac.uk/undergraduate

Informal descriptions of the philosophy courses can be found at www.philosophy.ox.ac.uk/undergraduate/course_descriptions

Recommended Philosophy Reading

Prior study of philosophy is in no way a prerequisite for this degree. It is clearly sensible, though, to find out more about the subject first. Here are some recommendations for philosophy and logic reading, to complement the earlier list of mathematical texts. Selected reading from one or more, or similar texts, will help you get a flavour of the degree.

- Simon Blackburn's *Think* (Oxford)
- One or more of the shorter dialogues of Plato such as *Protagoras*, *Meno* or *Phaedo*. (Each widely available in English translation.)
- Bertrand Russell's *The Problems of Philosophy* (Oxford University Press).
- Jonathan Glover's *Causing Death and Saving Lives* (Penguin).
- A.J. Ayer's *The Central Questions in Philosophy* (Penguin).
- Martin Hollis's *Invitation to Philosophy* (Blackwell).
- A.W. Moore's *The Infinite* (Routledge).
- Thomas Nagel's *What Does It All Mean?* (Oxford University Press).
- P.F. Strawson's *Introduction to Logical Theory* (Methuen).

$$\pi(n) \sim \frac{n}{\log n} \quad (\text{Hadamard, de la Vallee Poussin 1896})$$

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K} \right) \quad (\text{Verhulst 1838}), \quad x_{n+1} = rx_n(1 - x_n) \quad (\text{May 1976})$$



The Mathematics and Computer Science Course

Mathematics is a fundamental intellectual tool in computing, but computing is increasingly also a tool in mathematical problem solving. This course concentrates on areas where mathematics and computing are most relevant to each other, emphasizing the bridges between theory and practice. It offers opportunities for potential computer scientists both to develop a deeper understanding of the mathematical foundations of the subject and to acquire a familiarity with the mathematics of application areas where computers can solve otherwise intractable problems. It also gives mathematicians access to both a practical understanding of the use of computers, and a deeper understanding of the limits to the use of computers in their own subject. This training leads to a greater flexibility of approach and a better handling of new ideas in one of the fastest changing of all degree subjects.

The Mathematics and Computer Science degree can lead to either a BA after three years or an MMathCompSci after four years, though you will not be asked to choose between these until your third year. For the BA, a final classification (First, Upper Second, Lower Second, Third, Pass, Fail) is based on second and third year assessment. MMathCompSci students receive this classification and also a similar assessment separately on the fourth year.

First Year (Prelims)

The first year of the course ends with the Preliminary Examination in Mathematics (or "Prelims"). The current syllabus is contained in the Course Handbook, available at www.cs.ox.ac.uk/teaching/handbooks.

On arrival, you will receive a Course Handbook and supplements to this are

issued each year which give a detailed synopsis of all courses and a supporting reading list for each course of lectures.

The first year currently consists of lectures on the following core topics:

- Introduction to Pure Mathematics
- Linear Algebra (I, II)
- Groups and Group Actions
- Analysis (I, II, III)
- Continuous Mathematics
- Probability
- Functional Programming
- Design and Analysis of Algorithms
- Imperative Programming (I, II)

Most of the computer science topics have associated practicals which must be passed in order to progress.

There are no lectures in the second half of third term, so that you can concentrate on revision. The end of year examination consists of five written papers, of between two and three hours in length with three on Mathematics and two on Computer Science; no books, tables or calculators may be taken into the examination room. You are examined on your knowledge of the whole syllabus and your results are overall awarded a Distinction, Pass or Fail. The vast majority of students pass all their papers, but anyone failing one or more papers will need to retake some or all of the papers in September in order to continue on to the second year.

The halting problem is undecidable (Turing 1936)

$P \stackrel{?}{=} NP$ (open Millennium Problem)



The terms $1, -\frac{1}{3}, \frac{1}{5}, -\frac{1}{7}, \frac{1}{9}, -\frac{1}{11}, \frac{1}{13}, -\frac{1}{15}, \frac{1}{17}, -\frac{1}{19}, \frac{1}{21}, -\frac{1}{23}, \frac{1}{25}, -\frac{1}{27}, \frac{1}{29}, -\frac{1}{31}, \frac{1}{33}, -\frac{1}{35}, \frac{1}{37}, -\frac{1}{39}, \frac{1}{41}, -\frac{1}{43}, \frac{1}{45}, -\frac{1}{47}, \frac{1}{49}, -\frac{1}{51}, \frac{1}{53}, -\frac{1}{55}, \frac{1}{57}, -\frac{1}{59}, \frac{1}{61}, -\frac{1}{63}, \frac{1}{65}, -\frac{1}{67}, \frac{1}{69}, -\frac{1}{71}, \frac{1}{73}, -\frac{1}{75}, \frac{1}{77}, -\frac{1}{79}, \frac{1}{81}, -\frac{1}{83}, \frac{1}{85}, -\frac{1}{87}, \frac{1}{89}, -\frac{1}{91}, \frac{1}{93}, -\frac{1}{95}, \frac{1}{97}, -\frac{1}{99}$, summed in the right order, can converge to any limit (Riemann 1854)

Second Year

At the beginning of the second year students take two of the “core” mathematics courses:

- Linear Algebra
- Metric Spaces and Complex Analysis

Also, in the second year, students will typically choose 4 of the following options (some of which are double (D)):

- Rings and Modules (D)
- Integration (D)
- Topology (D)
- Probability (D)
- Numerical Analysis (D)
- Statistics
- Fluids and Waves
- Quantum Theory
- Number Theory
- Group Theory
- Projective Geometry
- Introduction to Manifolds
- Graph Theory

In computing, students are required to take the following two “core” courses

- Models of Computation
- Algorithms
- Practical course and group practical

Students also take additional Computer Science options in either their second or third years, and a full and current listing of these can be found at www.cs.ox.ac.uk/undergradcourses

Other second year mathematical options are also available with a tutor’s consent, but may involve a student needing to catch up on some prerequisite material.

Students will sit exams at the end of the second year, and some of the computer science courses include practicals which must be passed in order to progress.

Third and Fourth Years

In the third year there is a still wider range of options available – students take ten courses in all, at least two of which must be in Mathematics and at least two of which must be in Computer Science. See the link

www.cs.ox.ac.uk/teaching/mcs/PartB/

for further information on all the options available. They will be examined at the end of the year, and in practicals in some cases.

Currently a student must achieve at least an overall upper second in their second and third years to be able to progress to the fourth year.

In the fourth year students may choose to specialize entirely in mathematics or computer science or to retain a mixture. Every student must take either a Mathematics Dissertation (and six further units) or a Computer Science Project (and five further units); each unit is assessed by examination or equivalent.

If you are interested in finding out more about this course, or about the course in Computer Science, details can be obtained from the Academic Administrator, Department of Computer Science, Parks Road, Oxford OX1 3QD.

Or see the website www.cs.ox.ac.uk/

In 2019 the Department of Computer Science will run its own open day on Saturday 11th May. They also run open days overlapping with Mathematics on Wednesday 3rd July, Thursday 4th July, and Friday 20th September, details of which can be found at www.cs.ox.ac.uk/opendays

$H(S) \leq m(S) \leq H(S) + 1$ (Shannon 1948)



The Mathematical & Theoretical Physics 4th Year

This course is a 4th year Masters level course, which unites these two classic disciplines. Theoretical physics utilizes many mathematical techniques, and there are many elegant mathematical proofs to be found in string theory, quantum field theory, and other realms of study usually considered to be applied mathematics.

As this is a 4th year course, you cannot apply for it as a prospective undergraduate. Instead students who are in their 3rd year of Mathematics, Physics, or Physics and Philosophy degrees can apply to transfer onto this 4th year. As with our other joint degrees, in this course you may choose to be highly specialised or gain a broad knowledge of the discipline.

Students must currently choose at least 10 options (16 hour lecture courses) from the following list:

- Quantum Field Theory
- Statistical Mechanics
- Introduction to Quantitative Computing
- Nonequilibrium Statistical Physics
- Kinetic Theory
- General Relativity I and II
- Perturbation Methods
- Numerical Linear Algebra
- Groups and Representations
- Algebraic Topology
- Algebraic Geometry
- Advanced Fluid Dynamics
- Soft Matter Physics
- Advanced Quantum Field Theory
- String Theory I and II
- Networks
- Plasma Physics
- Supersymmetry and Supergravity
- Galactic and Planetary Dynamics
- Cosmology
- Applied Complex Variables
- Differential Geometry

- Geometric Group Theory
- Conformal Field Theory
- Introduction to Gauge-String Duality
- Topics in Soft and Active Matter Physics
- Advanced Quantum Theory
- Quantum Matter
- Turbulence
- Advanced Quantum Computing
- Topics in Quantum Computing
- The Standard Model
- Beyond the Standard Model
- Critical Phenomena
- Geophysical Fluid Dynamics
- Advanced Plasma Physics
- Astrophysical Fluid Dynamics
- Astrophysical Gas Dynamics
- Nonperturbation Methods in Quantum Field Theory
- Astroparticle Physics
- Radiative Processes and High Energy Astrophysics
- Quantum Field Theory in Curved Space
- Dissertation

Students may also choose a maximum of 3 options from Part B and Part C Mathematics courses, or Part C Physics courses.

Currently, a student must achieve at least an overall upper second in their second and third years to be eligible to apply for this fourth year.

Applicants for this course will be assessed on the basis of their academic performance and the compatibility of their previous programme of study. Anyone unsuccessful in the application may choose to continue on their current degree. For more information please see <https://mmathphys.physics.ox.ac.uk>.

$$P = \sum_k \exp\left(-\frac{E_k}{k_B T}\right) \quad (\text{Boltzmann 1884})$$

Every simply connected, closed 3-manifold is homeomorphic to the 3-sphere (Perelman 2003)



Careers

Demand for mathematics graduates has always been strong, but has been growing rapidly with the increased use of highly technical mathematical models and the growing prevalence of computers.

Over 30% of our graduates continue on a course of further study, ranging from a research degree in mathematics to a postgraduate course in teacher training. Mathematics at Oxford has many very active research groups, ranging from Geometry, Group Theory, Topology, and Number Theory to the applied research groups of the Centre of Mathematical Biology, the Oxford Centre for Industrial and Applied Mathematics, Numerical Analysis and Stochastic Analysis. You can find out more at: www.maths.ox.ac.uk/study-here/postgraduate-study.

There are no clearly defined career routes after a mathematics degree, unlike Law or Medicine. However, a degree in mathematics gives you excellent quantitative skills, which are applicable in a wide range of careers. Our graduates have gone into careers as consultants, analysts and a variety of financial roles. Additionally many of our graduates go into academic-related positions, such as research roles in companies, the intelligence services or the civil service.

Not only are there many career options after graduating, the average starting salary for our mathematics graduates, six months after finishing their degree, was £33,000, according to data from the Destination of Leavers from Higher Education (DLHE) survey.

In order to support your future career, the University runs a Careers Service which offers free advice and services such as internship programmes at companies across the world, advice sessions from alumni, and tailored careers advice. This service is available to you for life, so we can support you whenever you need us. See www.careers.ox.ac.uk for more information.

"Studying maths at Oxford gave me the analytical and reasoning skills I use in my job as a Public Health Intelligence Officer, as well as teaching me a great deal about communicating difficult mathematical/statistical concepts and how to translate public health questions (e.g. "Does this service work?") into questions that can be answered well by data -- and translating the answers back out again."

"Oxford has given me the opportunities to get where I am today through two main areas in my personal development: academia, as the drive and discipline required to complete a degree at Oxford have to come from yourself; and the inter-personal skills developed through sport, student politics, and relaxing in the bar with very bright and interesting people."

*All material and course details are correct at the time of writing.

$$\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i \quad (\text{Lagrange 1760})$$



Puzzles

Puzzle 1

The *Lotka-Volterra equations* show the interaction of a population of predators (F, for foxes) and a population of prey (R for rabbits). For positive $\alpha, \beta, \gamma, \delta$ the interactions are given by:

$$\frac{dF}{dt} = \alpha RF - \beta F$$
$$\frac{dR}{dt} = \gamma R - \delta RF$$

- What do you think α , β , γ , and δ represent?
- The population is at equilibrium if $\frac{dF}{dt} = 0$ and $\frac{dR}{dt} = 0$. Find and interpret all possible population equilibrium states in terms of α, β, γ , and δ .
- By considering the signs of $\frac{dF}{dt}$ and $\frac{dR}{dt}$ near each equilibrium state, describe what happens if the populations are close to their equilibrium values.

Puzzle 2

Fibonacci numbers are numbers defined by the recurrence relation $F_n = F_{n-1} + F_{n-2}$, with $F_1 = 1$ and $F_2 = 1$. Zeckendorf's Theorem states that every positive integer can be represented by the sum of non-consecutive Fibonacci numbers.

- Write 400 as the sum of non-consecutive Fibonacci numbers.
- Prove Zeckendorf's Theorem.
- The positive integers $a_1 \leq a_2 \leq a_3 \leq \dots$ have the property that, even when any one of the a_n is removed from the list, every positive integer can be written as a sum of the remaining a_n without repeat. Show that $a_n \leq F_n$ for $n \geq 1$.

Puzzle 3

Imagine a traveler on the integers between 0 and 5 inclusive. At each time step the traveler either moves up to the next integer or down to the previous integer with equal probability. However, if the traveler reaches 0 or 5, then they stop moving.

- If the traveler starts at 2, what is the probability that they are at 0 after five steps? What is the probability that the traveler is at 5?
- Now consider an infinite number line, without the stopping condition at 0 or 5. What does the distribution of possible locations after N steps look like? How does this vary with N?

Puzzle 4

I have a flock of chickens. For each pair of chickens C_1 and C_2 , either C_1 pecks C_2 or C_2 pecks C_1 . If C_1 pecks C_2 and C_2 pecks C_3 , then I say that C_1 *threatens* C_3 . A *king chicken* is one which either pecks or threatens every other chicken.

- Consider the chicken that pecks the most other chickens (if there's a tie, consider one of them). Prove that it's a king chicken.
- Prove that if a chicken is pecked at all, then one of the chickens that pecks it must be a king chicken.
- Could there be exactly two king chickens in my flock?

The Equations

The equations in the margins of this prospectus represent a wide range of mathematics, dating from ancient times to the present day. Here is a brief summary of their various meanings and importance. Further details can be found on them in some of the books recommended on page 12.

p.2 – A famous infinite series of Euler (its summing was known as the Basel problem), more generally he calculated the sum of reciprocals of all even powers. – Schrodinger's equation describing a wave function in quantum theory. **p.3** – Dal Ferro's (and later Tartaglia's) formula for the roots of a cubic equation. – π is transcendental, i.e. not the solution of any polynomial with integer coefficients; Lindemann's result finally showed that a circle cannot be squared. **p.4** – D'Alembert's wave equation for small transverse vibrations of a string, an early example of a mathematical model. – Euclid's proof of an infinity of prime numbers (from his *Elements*) is one of the most aesthetic in mathematics. **p.5** – Newton's proof that the planets' elliptical orbits were a consequence of the inverse square law of gravity was the first significant application of calculus. – Cauchy's Theorem is the fundamental theorem of complex analysis; complex analysis can tell us much about real integrals and series by means of path integrals in the complex plane. **p.6** – The area of a triangle in non-Euclidean (hyperbolic) geometry in terms of its angles. – Hales' recent proof of Kepler's conjecture on sphere packing; the proof takes up 250 pages in total and 3Gb in programs and data. **p.7** – Fermat's Little Theorem (p is prime) is an early theorem from Number Theory. – Maxwell's equations of electromagnetism relating the magnetic field B and electrical field E . **p.8** – The sum of the reciprocals of primes diverges. – Germain made the first major progress on Fermat's Last Theorem after Fermat's death. **p.9** – Noether's Theorem links symmetries of a system to conserved quantities; for example, rotational symmetry in the laws of motion gives the conservation of angular momentum. – Lorentz's formula for the contraction in lengths an observer traveling at speed v will perceive, where c is the speed of light. **p.10** – A famous infinite series of Leibniz. – Heisenberg's Uncertainty Principle showing momentum and position cannot be simultaneously observed. **p.11** – The Cayley-Hamilton Theorem states that a matrix satisfies its characteristic polynomial $\chi_A(x) = \det(xI - A)$. – The Fundamental Theorem of Algebra states that a polynomial with complex coefficients has a complex root. **p.12** – This series converges rapidly; the first term gives π to six d.p. **p.13** – Wiles' proof of Fermat's Last Theorem (x, y, z are whole numbers) was the biggest headline in 20th century mathematics. – Archimedes' early estimate for π used a polygon with 96 sides inscribed in a circle. **p.14** – There is a formula for finding the roots of a quadratic equation, and similar ones for cubics (see p.3) and quartics; Abel and Galois showed quintics cannot be solved simply using $+, -, \times, \div$ and taking roots. – The SIR model for the spread of an epidemic amongst a fixed population; individuals are either susceptible (S), infected (I) or have recovered (R), with the constants r and b relating to the disease's contagion and recovery rates. **p.15** – The Central Limit Theorem from probability, here the random variables X_i have the same distribution. – Euler's formula relates the number of vertices, edges and faces on a polyhedron (with no holes), it was one of the first topological results in mathematics. **p.16** – The Gauss-Bonnet Theorem shows that the total curvature of a closed surface relates only to its topology. – The Navier-Stokes equations describe the behaviour of viscous flow; their solution is the subject of a Millennium prize. **p.17** – The isoperimetric inequality relates the length l of a closed curve and the area it bounds A , with equality only in the case of a circle. – Cantor's result relating the infinity of the whole numbers (\aleph_0 , read aleph-null) with the infinity of the real numbers (denoted c). **p.18** – The Prime Number Theorem is an estimate for the number of primes $\pi(n)$ less than a given number n ; it was first conjectured by Gauss in 1792. – Verhulst modelled the *continuous* growth of a population of size $N(t)$ at time t with (intrinsic) growth rate r and population capacity K ; May then investigated a *discrete* version of the model involving the logistic map with r again denoting growth rate; the population can show widely varying behaviour depending on r – extinction, stable growth, alternating populations or chaos. **p.19** – Turing's result that there is no algorithm to decide if a program will stop. – The open Millennium Problem that NP-hard problems (ones where solutions can be checked in polynomial time) are the same as P-hard problems (ones that can be solved in polynomial time). **p.20** – Rearranging Leibniz's series from p.10 can give any answer. – Shannon's Noiseless Coding Theorem states that the minimum average codeword length $m(S)$ exceeds the entropy $H(S)$ of the source S ; this was one of the first results in Information Theory. **p. 21** – The canonical partition function describes the states of a system of fixed composition which is thermal equilibrium with a heat bath; each state has a varying energy and is given a different probability depending on its total energy; this equation is used in statistical mechanics, and describes behaviour in areas as diverse as magnetism and voting. – The Poincaré Conjecture is the only Millennium Problem to be solved; it claims that a three-dimensional space which is *connected*, finite, has no boundary, and has the property that each loop in the space can be shrunk to a point, is *homeomorphic* (identical) to a sphere. **p.22** – A famous integral which can be proved using complex analysis, as can Euler's sum on p.2. – Lagrange's equations governing a mechanical system; here L is the Lagrangian (kinetic energy minus potential energy) and the q_i are co-ordinates.