

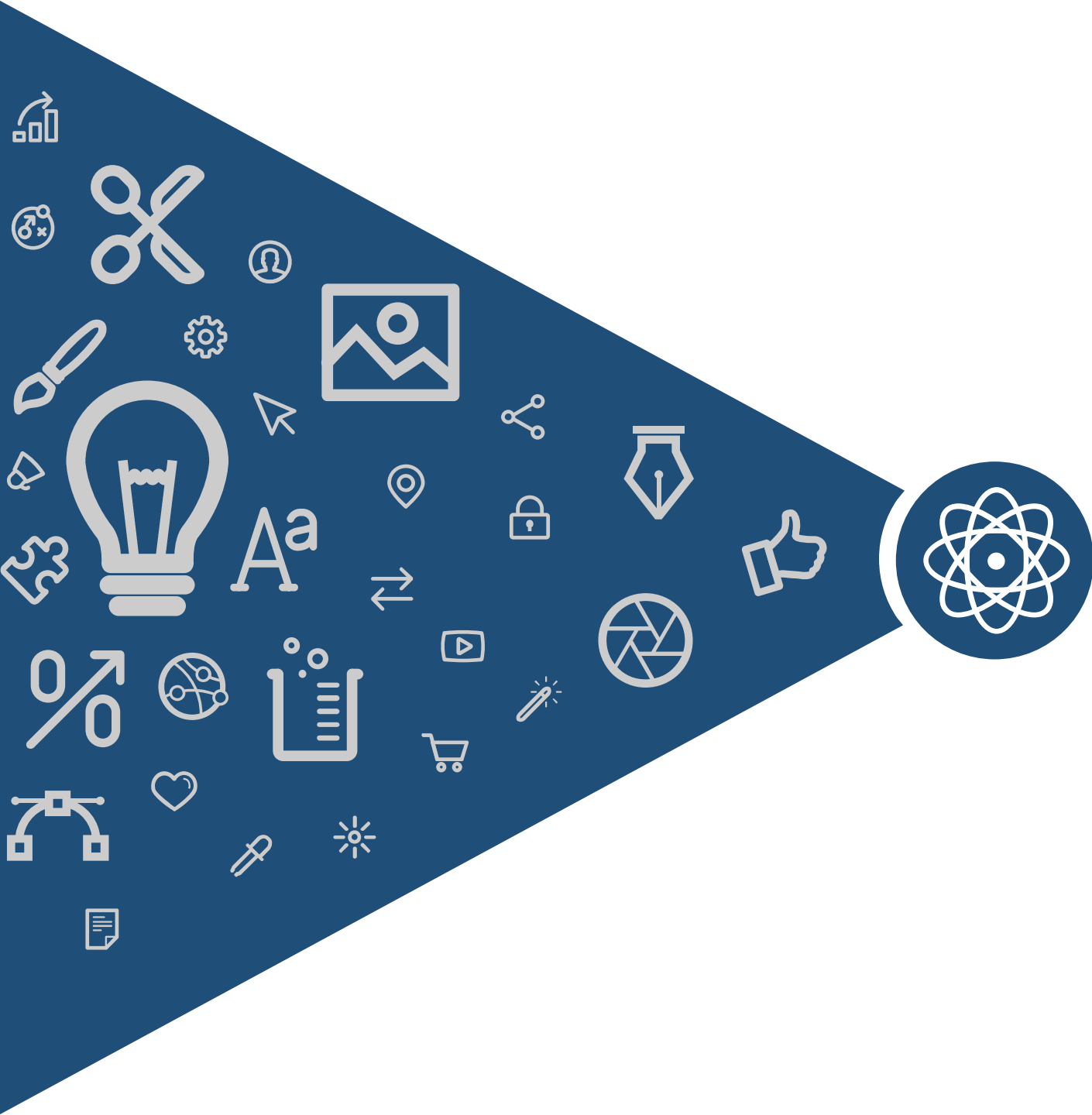


# Revisiting Anonymity in Post-Quantum Public Key Encryption

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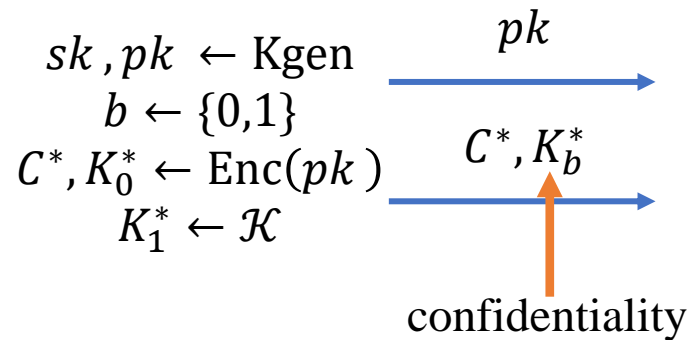
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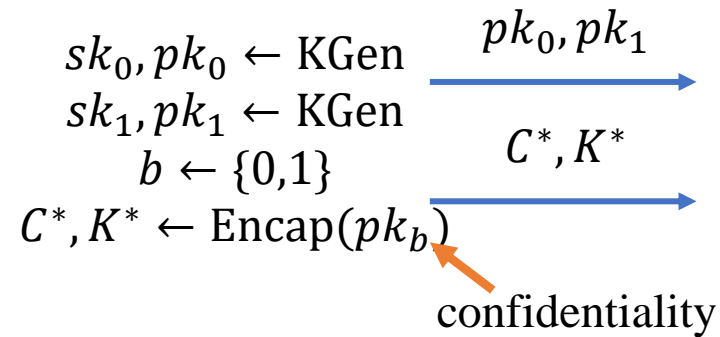
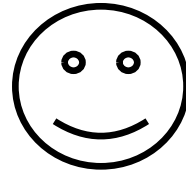
# Backgrounds

# Anonymity of KEM

**Indistinguishability (IND):**



**Anonymity (ANO):**



**Applications:**

Zcash, anonymous credential systems

auction protocols, anonymous AKE

UPDATES

# PQC Standardization Process: Announcing Four Candidates to be Standardized, Plus Fourth Round Candidates

NIST is announcing four Post-Quantum Cryptography candidates for standardization, plus candidates for a fourth round of analysis.

July 5, 2022

Public-Key Encryption/ key encapsulation mechanism	Digital Signatures
CRYSTALS-KYBER(Standardization) BIKE、 Classic McEliece、 HQC、 SIKE (candidates)	CRYSTALS-Dilithium FALCON SPHINCS <sup>+</sup>



FO Transformations

ANO-CPA  $\sim$  ANO-CCA

# Previous Works(ANO-CCA KEM)

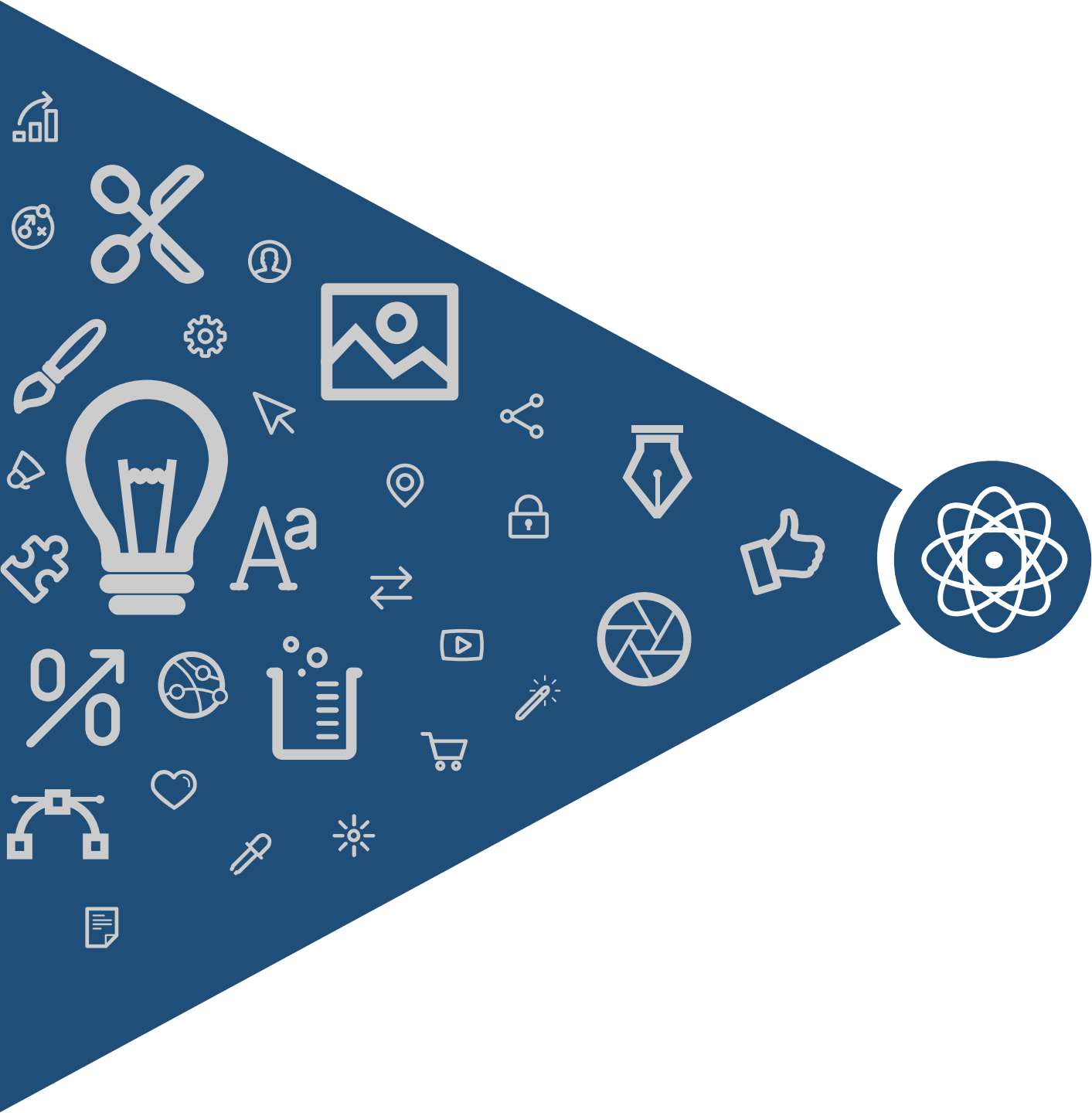
FO	Works	Underlying Security			Additional Hash
		wANO-CPA	OW-CPA	Additional Property	
Implicit	[GMP22]	✓	✓	SCFR-CPA	×
	[Xagawa22]	✓	✓	SDS-IND	×
Explicit	[Xagawa22]	✓	✓	SDS-IND	✓

## Problems:

Hard to simulate: two dec oracles

Additional hash in the explicit case





# Our Results

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# Results

FO	Underlying Security			Additional Hash
	wANO-CPA	OW-CPA	Additional Property	
Implicit	✓	✓	$\gamma = \text{spreadness}$	×
Explicit	✓	✓	$\gamma = \text{spreadness}$	×

KEM-DEM	Underlying Security			
	KEM			DEM
Implicit	wANO-CCA	IND-CCA	$\eta$ -randomness	INT-CTXT
Explicit	wANO-CCA	IND-CCA		INT-CTXT
[Mohassel10]	ANON-CCA		WROB-CCA	INT-CTXT





# Modular Analysis: T-trans & U-trans

$$FO^\times = U^\times \circ T, FO^\perp = U^\perp \circ T$$

[HHK17]

- $U^\times: OW-qPCA \rightarrow IND-CCA$
- $U^\perp: OW-qPCVA \rightarrow IND-CCA$
- $T: OW-CPA \rightarrow OW-qPCA$

*Our Works*

- $U^\times: wANO-CCOA \rightarrow ANO-CCA$
- $U^\perp: wANO-CCOVA \rightarrow ANO-CCA$
- $T: wANO-CPA \rightarrow wANO-CCA$

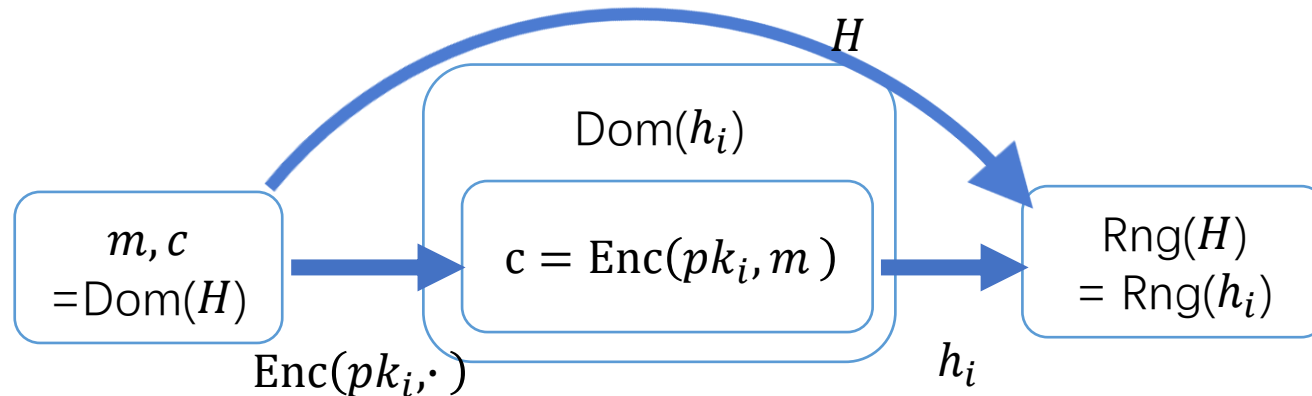
$Gen^\times:$	$Encaps^\times(pk):$	$Decaps^\times(sk, c):$
1: $(pk, sk') \xleftarrow{\$} Gen_1$	1: $m \xleftarrow{\$} \mathcal{M}, c \leftarrow Enc_1(pk, m)$	1: Parse $sk = (sk', s)$
2: $s \xleftarrow{\$} \mathcal{M}, sk := (sk', s)$	2: $K := H(m, c)$	2: $m' := Dec_1(sk', c)$
3: <b>return</b> $(pk, sk)$	3: <b>return</b> $(c, K)$	3: <b>if</b> $m' \neq \perp$
The transformation $U^\times$		
		4: <b>return</b> $K := H(m, c)$
		5: <b>else return</b> $K := H(s, c)$

$Gen^\perp:$	$Encaps^\perp(pk):$	$Decaps^\perp(sk, c):$
1: $(pk, sk) \xleftarrow{\$} Gen_1$	1: $m \xleftarrow{\$} \mathcal{M}, c \leftarrow Enc_1(pk, m)$	1: $m' := Dec_1(sk, c)$
2: <b>return</b> $(pk, sk)$	2: $K := H(m, c)$	2: <b>if</b> $m' \neq \perp$
The transformation $U^\perp$		
	3: <b>return</b> $(c, K)$	3: <b>return</b> $K := H(m, c)$
		4: <b>else return</b> $\perp$

$Gen_1:$	$Enc_1(pk, m):$	$Dec_1(sk, c):$
1: $(pk, sk) \xleftarrow{\$} Gen_0$	1: $r := G(m)$	1: $m' := Dec_0(sk, c)$
2: <b>return</b> $(pk, sk)$	2: $c \leftarrow Enc_0(pk, m, r)$	2: <b>if</b> $Enc_0(pk, m', G(m')) = c$
The transformation $T$		
	3: <b>return</b> $c$	3: <b>return</b> $m'$
		4: <b>else return</b> $\perp$

# $U : wANO-CCOA/wANO-CCOVA \rightarrow ANO-CCA$

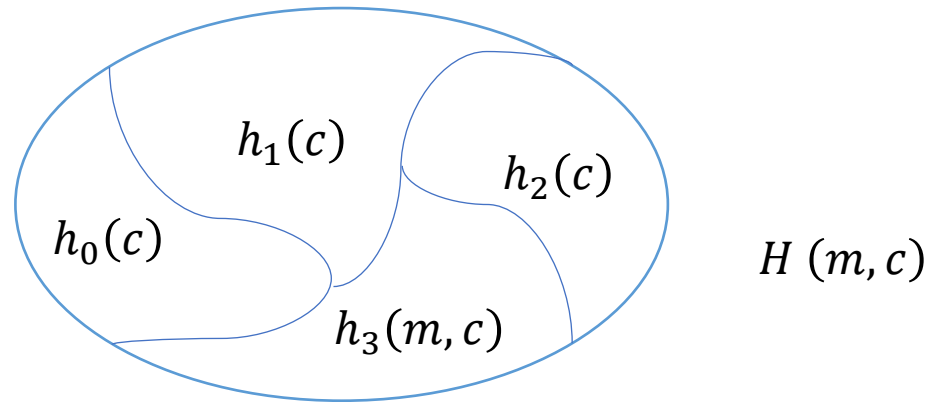
$\text{Dec}(c \neq c^*)$	$\text{CVO}(c \neq c^*)$	$\text{CCO}(c \neq c^*)$
1 : <b>return</b> $\text{Dec}(sk, c)$	1 : $m := \text{Dec}(sk, c)$ 2 : <b>return</b> $\llbracket m \in \mathcal{M} \rrbracket$	1 : <b>return</b> $\llbracket \text{Dec}(sk_0, c) = \text{Dec}(sk_1, c) \neq \perp \rrbracket$



# $U : wANO-CCOA/wANO-CCOVA \rightarrow ANO-CCA$

➤ The problem of responding  $Decaps(sk_i, c)$  with  $h_i(c)$

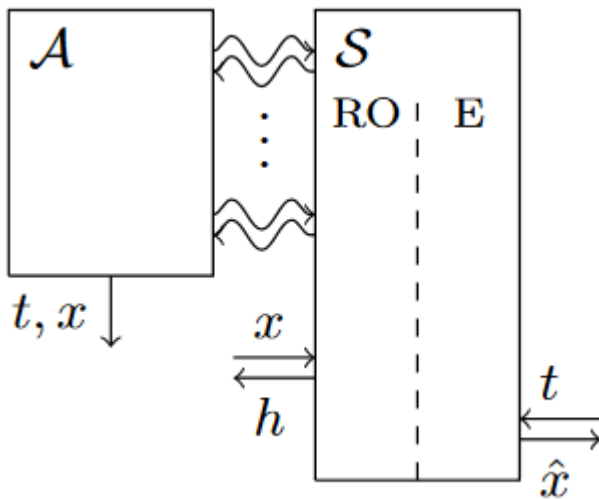
$$CCO(pk_0, pk_1, c) = 1? \quad [Dec(sk_0, c) = Dec(sk_1, c) \neq \perp]$$



$$\left\{ \begin{array}{l} Enc(pk_0, m) = c, Enc(pk_1, m) \neq c: h_0(c) \\ Enc(pk_0, m) \neq c, Enc(pk_1, m) = c: h_1(c) \\ Enc(pk_0, m) = c, Enc(pk_1, m) = c: h_2(c) \\ Enc(pk_0, m) \neq c, Enc(pk_1, m) \neq c: h_3(m, c) \end{array} \right.$$

# $T : wANO-CPA \rightarrow wANO-CCA$

$\text{Dec}(c \neq c^*)$	$\text{CVO}(c \neq c^*)$	$\text{CCO}(c \neq c^*)$
1 : <b>return</b> $\text{Dec}(sk, c)$	1 : $m := \text{Dec}(sk, c)$ 2 : <b>return</b> $\llbracket m \in \mathcal{M} \rrbracket$	1 : <b>return</b> $\llbracket \text{Dec}(sk_0, c) = \text{Dec}(sk_1, c) \neq \perp \rrbracket$



$$\text{Enc}(pk_0, m; G(m)) = c \quad \text{Enc}(pk_1, m; G(m)) = c$$

**Definition 2** ( $\gamma$ -spread [17]). We say  $\text{PKE} = (\text{Gen}, \text{Enc}, \text{Dec})$  is  $\gamma$ -spread if

$$\max_{m \in \mathcal{M}, c \in \mathcal{C}, pk} \Pr[c = \text{Enc}(pk, m)] \leq 2^{-\gamma},$$

where the probability is over the randomness of the encryption. For a  $(pk, sk)$  pair sampled by  $\text{Gen}$ , we also define parameter

$$\gamma(pk, sk) := \max_{m \in \mathcal{M}, c \in \mathcal{C}} \Pr[c = \text{Enc}(pk, m)].$$

[Zhandry20]: compressed oracle technique

[DFM+22]: online extraction technique

# Q&A

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