PROBLEM SHEET 1

1.1 Find the radius and centre of the circle described by the equation

$$x^2 + y^2 - 2x - 4y + 1 = 0$$

by writing it in the form $(x-a)^2 + (y-b)^2 = c^2$ for suitable a, b and c.

1.2 Find the equation of the line perpendicular to $y = 3x$ passing through the point $(3, 9)$.

1.3 Given

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \quad \text{and} \quad \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B,$$

show that

$$\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)] \quad \text{and} \quad \sin^2 A = \frac{1}{2}[1 - \cos 2A].$$

1.4 Show that

$$4\cos(\alpha t) + 3\sin(\alpha t) = 5\cos(\alpha t + \phi)$$

where $\phi =$arc $\tan(-3/4)$.

1.5 Show that, for $-1 \leq x \leq 1$,

$$\cos\left(\sin^{-1} x\right) = \pm\sqrt{1 - x^2}.$$

1.6 Given

$$\sinh(A \pm B) = \sinh A \cosh B \pm \cosh A \sinh B \quad \text{and} \quad \cosh(A \pm B) = \cosh A \cosh B \pm \sinh A \sinh B,$$

show that

$$\cosh A \cosh B = \frac{1}{2}[\cosh(A+B) + \cosh(A-B)] \quad \text{and} \quad \sinh^2 A = \frac{1}{2}[\cosh 2A - 1].$$

1.7 Given that

$$\sinh x = \frac{1}{2}[e^x - e^{-x}],$$

show that

$$\sinh^{-1} x = \ln\left[x + \sqrt{1 + x^2}\right].$$

1.8 Express

$$\frac{x}{(x-1)(x-2)}$$

in partial fractions.

1.9 If $a_n = \frac{1}{n}$, find $\sum_{i=1}^{5} a_n$ as a fraction.

1.10 If $S = \sum_{i=0}^{N} x^i$, show that $xS = \sum_{i=1}^{N+1} x^i$. Hence show that $S - xS = 1 - x^{N+1}$ and therefore that

$$S = \frac{1 - x^{N+1}}{1 - x}.$$

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