PROBLEM SHEET 10

10.1 For the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, show

(a) $|\boldsymbol{a}+\boldsymbol{b}|^2$ + $|\boldsymbol{a}$-$\boldsymbol{b}|^2 = 2(|\ \boldsymbol{a}|^2 + 2(|\ \boldsymbol{b}|^2)$

(b) $\boldsymbol{a}\cdot\boldsymbol{b}= \frac{1}{4}(|\boldsymbol{a}+\boldsymbol{b}|^2$ - $|\boldsymbol{a}$-$\boldsymbol{b}|^2)$

where $|\boldsymbol{a}|$ denotes the modulus of the vector $\boldsymbol{a}$ etc.

10.2 In component form let $\boldsymbol{a} = (1, -2, 2)$, $\boldsymbol{b}= (3, -1, -1)$, and $\boldsymbol{c}= (-1, 0, -1)$. Evaluate the following.

(a) $\boldsymbol{a} \times \boldsymbol{b}$

(b) $\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c})$

(c) $\boldsymbol{c} \cdot (\boldsymbol{a} \times \boldsymbol{b})$

10.3 What is the geometrical significance of $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{0}$?

10.4 Show that the vectors $\boldsymbol{a}= 2\hat{\boldsymbol{\imath}} + 3\,\hat{\boldsymbol{\jmath}} + 6\,\hat{\boldsymbol{k}}$ and $\boldsymbol{b}= 6\,\hat{\boldsymbol{\imath}} + 2\,\hat{\boldsymbol{\jmath}} - 3\,\hat{\boldsymbol{k}}$ are perpendicular. Find a vector which is perpendicular to $\boldsymbol{a}$ and $\boldsymbol{b}$.

10.5 Let $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ be three non-coplanar vectors, and $\boldsymbol{v}$ be any vector. Show that $\boldsymbol{v}$ can be expressed as $\boldsymbol{v}= X\boldsymbol{a}+Y\boldsymbol{b}+Z\boldsymbol{c}$, where $X, Y, Z,$ are constants given by $X = \boldsymbol{v} \cdot (\boldsymbol{b} \times \boldsymbol{c})/\mathrm{D}$, $Y = \boldsymbol{v} \cdot (\boldsymbol{c} \times \boldsymbol{a})/\mathrm{D}$, $Z = \boldsymbol{v} \cdot (\boldsymbol{a} \times \boldsymbol{b})/\mathrm{D}$, where $D = \boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c})$. (Hint: start by forming, say, $\boldsymbol{v} \cdot (\boldsymbol{b} \times \boldsymbol{c})$).