PROBLEM SHEET 7

7.1 The matrix $A = (a_{ij})$ is given by

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 2 & -2 & 4 \\ 1 & 5 & -3 \end{pmatrix}$$

Identify the elements a_{13} and a_{31}.

7.2 Given that

$$A = \begin{pmatrix} 1 & 3 & 0 \\ 2 & 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ -1 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 1 \\ -1 & 1 \\ -0 & 1 \end{pmatrix},$$

verify the distributive law $A(B+C) = AB + AC$ for the three matrices.

7.3 Let

$$A = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & -1 \\ 4 & 2 \end{pmatrix}.$$

Show that $AB = 0$, but that $BA \neq 0$.

7.4 A general $n \times n$ matrix is given by $A = (a_{ij})$. Show that $A + A^T$ is a symmetric matrix, and that $A - A^T$ is skew-symmetric.

Express the matrix

$$A = \begin{pmatrix} 2 & 1 & 3 \\ -2 & 0 & 1 \\ 3 & 1 & 2 \end{pmatrix}.$$

as the sum of a symmetric matrix and a skew-symmetric matrix.

7.5 Let the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ a & -1 & 0 \\ b & c & 1 \end{pmatrix}.$$

Find A^2. For what relation between a, b, and c is $A^2 = I$ (the unit matrix)? In this case, what is the inverse matrix of A? What is the inverse matrix of A^{2n-1} (n a positive integer)?

7.6 Using the rule for inverses of 2×2 matrices, write down the inverse of

$$\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$$

7.7 If A and B are both $n \times n$ matrices with A non-singular, show that

$$(A^{-1}BA)^2 = A^{-1}B^2A.$$