PROBLEM SHEET 9

9.1 The figure $ABCD$ has vertices at $(0, 0), (2, 0), (3, 1)$ and $(1, 1)$.

Find the vectors $\overrightarrow{AC}$ and $\overrightarrow{BD}$. Find $\overrightarrow{AC} \cdot \overrightarrow{BD}$.

Hence show that the angles between the diagonals of $ABCD$ have cosine $-1/\sqrt{5}$.

9.2 Show that the vectors $\mathbf{a} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$ are perpendicular.

Obtain any vector $\mathbf{c} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k}$ which is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$.

9.3 Find the value of λ such that the vectors $(\lambda, 2, -1)$ and $(1, 1, -3\lambda)$ are perpendicular.

9.4 Find a constant vector parallel to the line given parametrically by

$$x = 1 - \lambda, y = 2 + 3\lambda, z = 1 + \lambda.$$

9.5 A circular cone has its vertex at the origin and its axis in the direction of the unit vector $\hat{\mathbf{a}}$. The half-angle at the vertex is α. Show that the position vector $\mathbf{r}$ of a general point on its surface satisfies the equation

$$\hat{\mathbf{a}} \cdot \mathbf{r} = |\mathbf{r}| \cos \alpha.$$

Obtain the cartesian equation when $\hat{\mathbf{a}} = (2/7, -3/7, -6/7)$ and $\alpha = 60°$.