

EXTRA MATHEMATICS ADMISSIONS TEST

December 2020

Time allowed: 1 hour

Surname	
Other names	

This paper contains 10 multiple choice questions.

Calculators are not permitted.

For each question on pages 2–11 you will be given **five** possible answers, just one of which is correct. Indicate for each question **A–J** which answer (a), (b), (c), (d), or (e) you think is correct with a tick (✓) in the corresponding column in the table below.

	(a)	(b)	(c)	(d)	(e)
A					
B					
C					
D					
E					
F					
G					
H					
I					
J					

A. The distance between opposite corners of a cube is 2. The surface area of the cube equals

- (a) 4, (b) 6, (c) 8, (d) 12, (e) 24.

Please turn over

B. If x is a very large positive real number, then the product

$$2^x \times 3^{-x} \times 4^x \times 5^{-x} \times \cdots \times 18^x \times 19^{-x} \times 20^x \times 21^{-x}$$

is

- (a) very close to zero,
- (b) slightly larger than 1,
- (c) equal to 1,
- (d) very close to 2,
- (e) very large.

Please turn over

C. Using degrees, the number of real solutions x to the equation

$$\cos\left(\frac{240x}{x^2 + 4}\right) = \frac{1}{2}$$

is

- (a) 0, (b) 1, (c) 2, (d) 3, (e) infinite.

Please turn over

D. Let

$$y = 2x + 3x^2 + 5x^3 + \dots$$

so that the coefficient of x^n is the n^{th} prime number. Then the value of $\frac{d^5 y}{dx^5}$ at $x = 0$ is

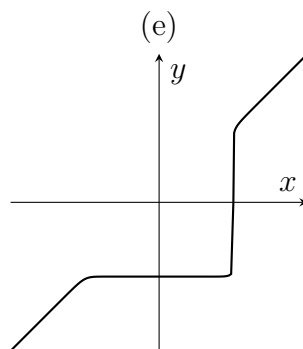
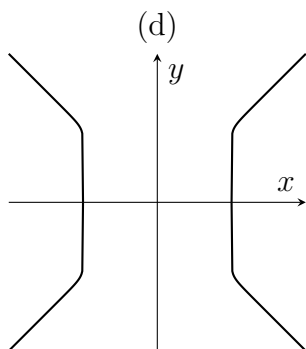
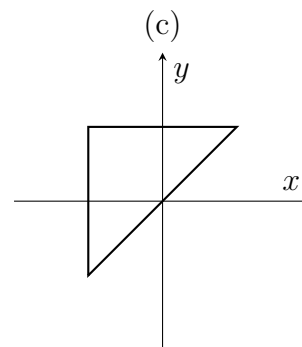
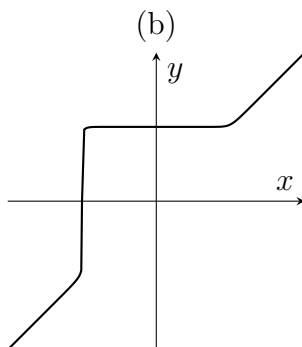
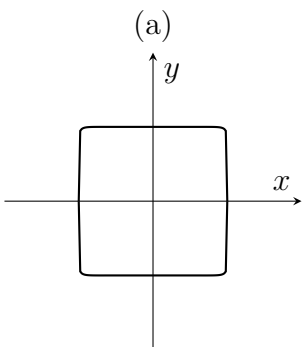
- (a) 0, (b) 120, (c) 840, (d) 1080, (e) 1320.

Please turn over

E. The curve

$$x^{20} - y^{20} = 1$$

is sketched in



Please turn over

F. The following are statements about a real number x .

$$P : \frac{x^2 - 1}{x + 2} < 0, \quad Q : \frac{1 + x}{1 - x} > 0.$$

Then it follows that

- (a) P implies Q but Q does not imply P .
- (b) Q implies P but P does not imply Q .
- (c) P and Q are equivalent.
- (d) If P is true then Q is false.
- (e) If Q is true then P is false.

Please turn over

G. The functions S and T are defined by

$$S(x) = x + 1, \quad T(x) = \frac{1}{2}x - 1.$$

Beginning with $x = 0$ the functions S and T are repeatedly applied in some order. For example, $SSTS(0) = \frac{3}{2}$. The set of possible outputs is

- (a) all positive rational numbers.
- (b) all rational numbers greater than -1 with denominator a power of 2 when written in lowest terms.
- (c) all rational numbers with denominator a power of 2 when written in lowest terms.
- (d) all positive rational numbers with denominator a power of 2 when written in lowest terms.
- (e) all rational numbers greater than -2 with denominator a power of 2 when written in lowest terms.

Please turn over

H. A sequence a_n is defined by $a_0 = A$ and $a_k = (a_{k-1})^2$ for $k > 0$, where $A > 1$. The sequence b_n is defined by $b_n = \log_2 a_n$. The sequence b_n is

- (a) constant
- (b) an arithmetic progression
- (c) a geometric progression
- (d) all of the above
- (e) none of the above

Please turn over

I. An equilateral triangle is drawn in the xy -plane. Two of its vertices are at $(0, 0)$ and $(1000, 0)$. The number of points (x, y) *inside* the triangle, where x and y are both whole numbers, equals

- (a) 866,025, (b) 866,026, (c) 866,027, (d) 432,512, (e) 432,513.

[Note that $\sqrt{3} = 1.7321$ to 4 decimal places.]

Please turn over

J. Let R be the region where all four of the following inequalities hold

$$x^2 < 2 + y, \quad x^2 < 2 - y, \quad y^2 < 2 + x, \quad y^2 < 2 - x.$$

What is the area of R ?

- (a) 0, (b) $\frac{28}{3}$, (c) $4 + 2\pi$, (d) $\frac{4}{3}(8\sqrt{2} - 7)$, (e) infinite.

End of last question