This paper contains 10 multiple choice questions. **Calculators are not permitted.**

For each question on pages 2–11 you will be given five possible answers, just one of which is correct. Indicate for each question A-J which answer (a), (b), (c), (d), or (e) you think is correct with a tick (√) in the corresponding column in the table below.

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Whenever I toss a particular coin, it lands on heads with probability $\cos^2 \alpha$ for some fixed real number $\alpha$ (and the outcome is independent of other tosses). I toss the coin three times. The probability that the coin lands on heads two or more times is equal to

(a) $1 + 3 \sin^4 \alpha - 2 \sin^6 \alpha$,
(b) $1 - 3 \sin^4 \alpha - 2 \sin^6 \alpha$,
(c) $1 + 3 \sin^4 \alpha + 2 \sin^6 \alpha$,
(d) $1 - 3 \sin^4 \alpha + 2 \sin^6 \alpha$,
(e) $1 + 8 \sin^6 \alpha$. 

Please turn over
B. Which of the following graphs is a sketch of $e^{-x/2} - e^{-x}$ for $x > 0$?

(a)  
(b)  
(c)  
(d)  
(e)
C. For precisely which non-zero real values of $x$ is it true that

$$x^2 - 3x + 2 < \frac{x - 1}{x} \quad ?$$

(a) $x < 1 - \sqrt{2}$ or $x > 1 + \sqrt{2},$
(b) $1 - \sqrt{2} < x < 0$ or $1 < x < 1 + \sqrt{2},$
(c) $1 < x < 1 + \sqrt{2},$
(d) $1 - \sqrt{2} < x < 1 + \sqrt{2},$
(e) $1 - \sqrt{2} < x < 0.$
D. Consider the two inequalities

\[ 1 \leq x^2 + y^2 \leq 4 \quad \text{and} \quad x^2 \geq 3y^2. \]

The total area of all regions of the \((x, y)\)-plane where both inequalities hold is

(a) \(\pi \sqrt{3}\),

(b) \(\pi\),

(c) \(2\pi\),

(d) \(\frac{\pi}{2}\),

(e) \(\frac{\pi^2}{6}\).
E. The points $(0,1)$ and $(p,q)$ are on opposite ends of the diameter of circle $C$. The $x$-axis is a tangent to the circle $C$ if and only if

(a) $p = 1 + q,$

(b) $pq = 1,$

(c) $p^2 = 4q,$

(d) $p^2 + (q - 1)^2 = 1,$

(e) $p + q = 1.$
F. The series

\[ 1 + (1 + x - x^2) + (1 + x - x^2)^2 + (1 + x - x^2)^3 + \ldots \]

converges to \( \frac{1}{x(x - 1)} \) for precisely which real values of \( x \)?

(a) If and only if \(-1 < x < 1\),

(b) If and only if we have both \( x \neq 0 \) and \( x \neq 1 \),

(c) If and only if either \(-1 < x < 0 \) or \( 1 < x < 2 \),

(d) If and only if either \(-2 < x < -1 \) or \( 0 < x < 1 \),

(e) For all real \( x \).
Given that \( y = f(x) \) is a solution to \( \frac{dy}{dx} = y^{1/4} \), it follows that one of the following functions is a solution to \( \frac{dy}{dx} = 2y^{1/4} \). Which one?

(a) \( y = (2)^{-4}f(x) \),

(b) \( y = (2)^3f(x) \),

(c) \( y = (2)^{4/3}f(x) \),

(d) \( y = (2)^{-3}f(x) \),

(e) \( y = (2)^4f(x) \).
H. Suppose that a function $f(n)$ on the positive integers is defined such that $f(1) = 1$ and then for $n \geq 1$

$$f(2n) = f(n) \quad \text{and} \quad f(2n + 1) = f(n) + f(n + 1).$$

How many values of $n$ are there such that $f(n) = 3$ and also $n$ is a multiple of 35?

(a) 0,
(b) 1,
(c) 2,
(d) 3,
(e) Infinitely many.
I. The number of positive solutions $x$ to the equation

$$\log_2 x = \log_2(x + a) + b,$$

where $a$ and $b$ are non-zero real numbers, is

(a) zero if $ab < 1$, or one if $ab > 1$,

(b) one if $ab < 1$, or two if $ab > 1$,

(c) one if $ab < 0$, or zero if $ab > 0$,

(d) zero if $ab < 0$, or one if $ab > 0$,

(e) one if $ab < 1$, or zero if $ab > 1$. 
J. Given that \( \int_{1}^{2} \frac{x^2}{1 + x^4} \, dx = A \) where \( A \) is some positive real number (which you should not attempt to determine), it follows that the value of \( \int_{1}^{2} \frac{x^{-2}}{1 + x^4} \, dx \) is equal to

(a) \( 1 - A \),
(b) \( -A \),
(c) \( \frac{1}{A} \),
(d) \( A - 1 \),
(e) \( \frac{1}{2} - A \).

End of last question