Examiners' Report: Preliminary Examination in Mathematics Trinity Term 2025

October 14, 2025

Part I

A. STATISTICS

179 candidates in Mathematics and Mathematics & Statistics were awarded an overall year outcome. Candidates on both degrees submit the same assessments and no distinction is made between the two groups in this document.

Table 1: Numbers in each outcome class

	Numbers					Percentages				
	2025	2024	(2023)	(2022)	(2021)	2025	2024	(2023)	(2022)	(2021)
Distinction	52	(57)	(52)	(53)	(60)	29.05	(31.67)	(29.21)	(29.78)	(30.61)
Pass	115	(109)	(113)	(116)	(124)	64.25	(60.56)	(63.48)	(65.17)	(63.27)
Partial Pass	9	(10)	(11)	(6)	(7)	5.03	(5.56)	(6.18)	(3.37)	(3.57)
Incomplete	0	(0)	(0)	(0)	(2)	0.00	(0.00)	(0.00)	(0.00)	(1.02)
Fail	3	(4)	(2)	(3)	(3)	1.68	(2.22)	(1.12)	(1.69)	(1.53)
Total	179	(180)	(178)	(178)	(196)	100	(100)	(100)	(100)	(100)

B. NEW EXAMINING METHODS AND PROCEDURES

None.

C. CHANGES IN EXAMINING METHODS AND PROCEDURES CURRENTLY UNDER DISCUSSION OR CONTEMPLATED FOR THE FUTURE

None.

D. NOTICE OF EXAMINATION CONVENTIONS FOR CANDIDATES

The Notice to Candidates, containing details of the examinations and assessments, was issued to all candidates at the beginning of Trinity term. The Examination Conventions in full were made available at

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. GENERAL COMMENTS ON THE EXAMINATION

Acknowledgements

First, the Moderators should like to thank the Undergraduate Studies Administration Team.

We should also like to thank Charlotte Turner-Smith for her invaluable experience with the Mitigating Circumstances Panel, and Matthew Brechin and Waldemar Schlackow for maintaining and running the examination database and their assistance during the final examination board meeting.

We should like to thank the lecturers for their feedback on proposed exam questions; the assessors for their extraordinary assistance with marking; Patrick Farrell for coordinating the Computational Mathematics assessments, and the team of graduate checkers for their rapid work checking the marks on the papers.

Timetable

The examinations began on Monday 23rd June and ended on Friday 27th June.

Setting and checking of papers

The Moderators set and checked the questions, model solutions, and mark schemes. Every question was carefully considered by at least two moderators, and feedback was sought from lecturers. In a small number of cases feedback from lecturers was not available, and those were discussed in more detail until the Board of Moderators was satisfied that all questions were appropriate.

The questions were then combined into papers which were considered by the Board of Moderators and small changes were made to satisfy the Board that the papers were appropriate. After this a final proof-reading of the papers was completed before the Camera Ready Copies (CRCs) were produced. The whole Board of Moderators signed off the CRCs which were submitted to Examination Schools.

Marking and marks processing

The Moderators and Assessors marked the scripts according to the mark schemes and entered the marks. Small adjustments to some mark schemes were made at this stage, and care was take to ensure these were consistently applied to all candidates.

A team of graduate checkers, supervised by the Academic Admin Team sorted all the scripts for each paper and carefully cross checked these against the mark scheme to spot any unmarked parts of questions, addition errors, or wrongly recorded marks. A number of errors were corrected, with each change checked and signed off by a Moderator, at least one of whom was present throughout the process.

Determination of University Standardised Marks

Marks for each individual assessment are reported as a University Standard Mark (USM) which is an integer between 0 and 100 inclusive. The Moderators used their academic judgment to map the raw marks on individual assessments to USMs using a process similar to previous years. In coming to this judgement the board followed the advice from the Mathematics Teaching Committee that the percentages awarded for each USM range of the examination should be in line with recent years. This alignment can be seen in Table 1; in more detail, for Papers I–V, a piecewise linear map was constructed as follows:

1. Candidates' raw marks for a given paper were ranked in descending order.
2. The default percentages p_1 of Distinctions and p_2 of Nominal Upper Seconds were selected.
3. The candidate at the p_1 percentile from the top of the ranked list was identified and assigned a USM of 70, and the corresponding raw mark denoted R_1.
4. The candidate at the $(p_1 + p_2)$ percentile from the top of the list was assigned a USM of 60 and the corresponding raw mark denoted R_2.
5. The line segment between $(R_1, 70)$ and $(R_2, 60)$ was extended linearly to USMs of 72 and 57 respectively, and the corresponding raw marks denoted C_1 and C_2 respectively.

6. A line segment through $(C_2, 57)$ was extended towards the vertical axis, as if it were to join the axis at $(0, 10)$, but the line segment was terminated at a USM of 37 and the raw mark at the termination point was denoted C_3.

With these data a piecewise linear map was constructed with vertices at $\{(0,0),\ (C_3,37),\ (C_2,57),\ (C_1,72),\ (100,100)\}$.

Reports from the Assessors describing the apparent relative difficulty and the general standard of solutions for each question were then considered, and the Board decided that the values of $p_1 = 31\%$ and $p_2 = 48\%$ were suitable for all papers.

In line with previous years, for the Computational Mathematics assessment the linear map with gradient 2.5 was used to map from raw marks to USMs.

The vertices of the final maps used in each assessment are listed in Table 2.

Table 2: Vertices of final piecewise linear model

Paper	Vertices				
I	0;0	22;37	35;57	66.4;72	100;100
II	0;0	28;37	49.4;57	82.4;72	100;100
III	0;0	33.5;37	60.3;57	88.8;72	120;100
IV	0;0	18.9;37	32.9;57	58.4;72	80;100
V	0;0	19.47;37	33.9;57	59.4;72	80;100
CM	0;0	40;100			

With the USMs, a provisional outcome class for each candidate was produced as follows: Write MI, MII, $MIII$, MIV and MV for the USMs on Papers I–V respectively, and CM for the USM on the Computational Mathematics assessment. Write Av_1 and Av_2 for the quantities

$$\frac{MI + MII + \frac{6}{5}MIII + MIV + \frac{4}{5}MV + \frac{1}{3}CM}{5\frac{1}{3}}$$

and

$$\frac{MI + MII + \frac{6}{5}MIII + MIV + \frac{4}{5}MV}{5}$$

respectively, symmetrically rounded. With these auxiliary statistics the provisional outcome class was determined by the definitions:

Distinction: both $Av_1 \geq 70$ and $Av_2 \geq 70$ and a USM of at least 40 on each paper and for the Computational Mathematics assessment;

Pass: not a Distinction and a USM of at least 40 on each paper and for the Computational Mathematics assessment;

Partial Pass: not a Pass or Distinction and a USM of at least 40 on three or more of Papers I–V;

Fail: not a Partial Pass, Pass, or Distinction, and a USM of less than 40 on three or more of Papers I–V.

The scripts of those candidates at the boundaries between outcome classes were scrutinised carefully to determine which attained the relevant qualitative descriptors and changes were made to move those into the correct class.

Mitigating Circumstances were then considered using the banding produced by the Mitigating Circumstances Panel, and appropriate actions were taken and recorded.

Table 3 gives the rank list ordered by the average of Av_1 and Av_2 (as defined above), showing the number and percentage of candidates with USM greater than or equal to each value.

Table 3: Rank list of average USM scores

		Candidates with USM $\geq x$	
USM (x)	Rank	Number	Percentage
92.08	1	1	0.56
91.56	2	2	1.12
89.6	3	3	1.68
88.7	4	4	2.23
87.44	5	5	2.79
86.76	6	6	3.35
86	7	7	3.91
84.04	8	8	4.47
83.88	9	9	5.03
83.72	10	10	5.59
83.11	11	11	6.15
82.6	12	12	6.7
81.36	13	14	7.82
81.36	13	14	7.82
80.56	15	15	8.38
79.84	16	16	8.94
79.44	17	18	10.06
79.44	17	18	10.06
79.31	19	19	10.61
79.24	20	20	11.17
78.6	21	21	11.73
77.6	22	22	12.29
75.84	23	23	12.85

Table 3: Rank list of average USM scores (continued)

		Candidates with average USM $\geq x$	
USM (x)	Rank	Number	Percentage
75.21	24	24	13.41
74.84	25	25	13.97
74.76	26	26	14.53
73.64	27	27	15.08
73.56	28	28	15.64
73.4	29	29	16.2
73.24	30	31	17.32
73.24	30	31	17.32
73.12	32	32	17.88
72.72	33	33	18.44
72.56	34	34	18.99
72.52	35	35	19.55
72.24	36	36	20.11
72.12	37	37	20.67
72	38	38	21.23
71.84	39	39	21.79
71.76	40	41	22.91
71.76	40	41	22.91
71.72	42	42	23.46
71.56	43	43	24.02
71.44	44	44	24.58
71.16	45	45	25.14
71.12	46	46	25.7
70.64	47	47	26.26
70.61	48	48	26.82
70	49	49	27.37
69.84	50	50	27.93
69.64	51	51	28.49
69.56	52	52	29.05
69.48	53	53	29.61
69.44	54	54	30.17
69.2	55	55	30.73
68.84	56	57	31.84
68.84	56	57	31.84
68.74	58	58	32.4
68.48	59	59	32.96
68.4	60	60	33.52
68.04	61	61	34.08
67.92	62	62	34.64
67.84	63	63	35.2

Table 3: Rank list of average USM scores (continued)

USM (x)	Rank	Candidates with average USM $\geq x$ Number	Percentage
67.64	64	64	35.75
67.24	65	65	36.31
67.16	66	66	36.87
67.08	67	67	37.43
67.01	68	68	37.99
67	69	69	38.55
66.96	70	70	39.11
66.88	71	71	39.66
66.84	72	72	40.22
66.8	73	73	40.78
66.64	74	74	41.34
66.16	75	75	41.9
66.08	76	76	42.46
65.76	77	77	43.02
65.68	78	78	43.58
65.64	79	79	44.13
65.56	80	80	44.69
65.44	81	81	45.25
65.3	82	82	45.81
65.28	83	83	46.37
65.16	84	84	46.93
65.15	85	85	47.49
65.12	86	87	48.6
65.12	86	87	48.6
65.04	88	88	49.16
64.96	89	89	49.72
64.88	90	90	50.28
64.68	91	91	50.84
64.52	92	92	51.4
64.48	93	93	51.96
64.4	94	95	53.07
64.4	94	95	53.07
64.24	96	96	53.63
64.12	97	97	54.19
64.08	98	98	54.75
64.04	99	99	55.31
63.95	100	100	55.87
63.88	101	102	56.98
63.88	101	102	56.98
63.84	103	103	57.54

Table 3: Rank list of average USM scores (continued)

		Candidates with average USM $\geq x$	
USM (x)	Rank	Number	Percentage
63.8	104	104	58.1
63.64	105	106	59.22
63.64	105	106	59.22
63.52	107	108	60.34
63.52	107	108	60.34
63.44	109	109	60.89
63.4	110	110	61.45
63.36	111	111	62.01
63.04	112	112	62.57
62.92	113	113	63.13
62.88	114	114	63.69
62.84	115	115	64.25
62.68	116	116	64.8
62.32	117	117	65.36
62.12	118	118	65.92
61.92	119	119	66.48
61.84	120	120	67.04
61.8	121	122	68.16
61.8	121	122	68.16
61.76	123	123	68.72
61.44	124	125	69.83
61.44	124	125	69.83
61.29	126	126	70.39
61.24	127	127	70.95
61.2	128	128	71.51
60.72	129	129	72.07
60.68	130	130	72.63
60.56	131	131	73.18
60.48	132	132	73.74
60.32	133	134	74.86
60.32	133	134	74.86
60.28	135	135	75.42
60.16	136	136	75.98
59.84	137	137	76.54
59.8	138	138	77.09
59.56	139	140	78.21
59.56	139	140	78.21
58.88	141	141	78.77
58.72	142	142	79.33
58.6	143	143	79.89

Table 3: Rank list of average USM scores (continued)

		Candidates with average USM $\geq x$	
USM (x)	Rank	Number	Percentage
58.44	144	144	80.45
58.32	145	145	81.01
58.16	146	146	81.56
58.12	147	147	82.12
58	148	148	82.68
57.9	149	149	83.24
57.76	150	150	83.8
57.72	151	151	84.36
57.6	152	152	84.92
56.6	153	154	86.03
56.6	153	154	86.03
56.16	155	155	86.59
56.1	156	156	87.15
56.04	157	157	87.71
55.4	158	158	88.27
55.04	159	160	89.39
55.04	159	160	89.39
54.56	161	161	89.94
54.25	162	162	90.5
54.19	163	163	91.06
52.92	164	164	91.62
52.6	165	165	92.18
52.28	166	166	92.74
51.88	167	167	93.3
51.4	168	168	93.85
51.08	169	169	94.41
51.04	170	170	94.97
47.32	171	171	95.53
46.08	172	172	96.09
45.48	173	173	96.65
41.64	174	174	97.21
41.48	175	175	97.77
37.64	176	176	98.32
34.76	177	177	98.88
27.53	178	178	99.44
9.48	179	179	100

B. EQUAL OPPORTUNITY ISSUES AND BREAKDOWN OF THE RESULTS BY GENDER

Table 4 shows the performances of candidates by gender. Here gender is the gender as recorded on eVision.

Table 4: Breakdown of results by gender

Outcome	Number											
	2025			2024			2023			2022		
	Female	Male	Total	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	3	49	52	4	53	57	9	43	52	8	45	53
Pass	37	78	115	35	74	109	38	75	113	43	73	116
Partial Pass	3	6	9	2	8	10	4	7	11	2	4	6
Incomplete	0	0	0	0	0	0	0	0	0	0	2	2
Fail	1	2	3	1	3	4	1	1	2	0	3	3
Total	44	135	179	41	138	180	52	126	178	53	125	178
Outcome	Percentage											
	2025			2024			2023			2022		
	Female	Male	Total	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	6.82	36.3	29.05	9.52	38.41	31.67	17.23	34.15	29.21	15.09	36.00	29.78
Pass	84.09	57.78	64.25	83.33	53.62	60.56	73.08	59.52	63.53	81.13	58.40	65.17
Partial Pass	6.82	4.44	5.03	4.76	5.8	5.56	7.69	5.56	6.17	3.77	3.20	3.37
Incomplete	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Fail	2.27	1.48	1.68	2.38	2.17	2.22	1.92	0.79	1.12	0.00	2.40	1.69

C. STATISTICS ON CANDIDATES' PERFORMANCE IN EACH PART OF THE EXAMINATION

Table 5: Numbers taking each paper

Paper	Number of Candidates	Average raw mark	Std dev of raw marks	Average USM	Std dev of USMs
I	178	52.53	16.05	65.06	11.25
II	178	66.89	17.46	65.62	12.63
III	178	75.61	17.81	65.44	11.94
IV	179	46.9	13.48	64.81	12.72
V	179	47.55	13.18	65.27	11.95
CM	179	32.87	5.51	82.02	14.11

Tables 6–11 give the performance statistics for each individual assessment, showing for each question the average mark, first over all attempts, and then over the attempts used; the standard deviation over all attempts; and finally the total number of attempts, first those that were used, and then those that were unused.

Table 6: Statistics for Paper I

Question	Average mark		Std	No. of Attempts	
Number	All	Used	dev	Used	Unused
Q1	12.58	12.58	4.42	173	0
Q2	12.81	12.81	4.41	159	0
Q3	9.53	9.55	4.35	137	1
Q4	9.37	9.37	3.77	65	0
Q5	9.17	9.17	3.54	166	0
Q6	9.36	9.36	4.24	109	0
Q7	8.56	8.56	4.64	79	0

Table 7: Statistics for Paper II

Question	Average mark		Std	No. of Attempts	
Number	All	Used	dev	Used	Unused
Q1	13.9	13.9	3.86	112	0
Q2	12.53	12.52	3.58	87	1
Q3	15.13	15.13	3.91	157	0
Q4	10.28	10.39	5.42	71	1
Q5	14.82	14.87	4.05	145	1
Q6	13.98	13.98	4.86	139	0
Q7	11.83	11.83	5.17	173	0

Table 8: Statistics for Paper III

Question	Average mark		Std	No. of Attempts	
Number	All	Used	dev	Used	Unused
Q1	13.59	13.65	4.16	158	1
Q2	14.74	14.74	3.12	58	0
Q3	14.86	14.94	3.74	139	1
Q4	10.64	10.64	4	107	0
Q5	11.18	11.18	3.24	144	0
Q6	12.98	12.98	3.78	104	0
Q7	13.77	13.77	5.01	133	0
Q8	11.13	11.16	5.04	156	1
Q9	12.26	12.26	4.07	57	0

Table 9: Statistics for Paper IV

Question Number	Average mark All	Used	Std dev	No. of Attempts Used	Unused
Q1	10.56	10.79	4.3	43	2
Q2	12.14	12.14	3.78	155	0
Q3	10.35	10.36	3.81	159	1
Q4	10.03	10.16	5.88	85	2
Q5	14.91	14.91	3.92	171	0
Q6	10.75	10.75	5.17	92	0

Table 10: Statistics for Paper V

Question Number	Average mark All	Used	Std dev	No. of Attempts Used	Unused
Q1	9.72	9.82	4.97	133	2
Q2	12.75	12.8	4.3	137	1
Q3	12	12.15	5.55	88	2
Q4	14.6	14.6	3.03	179	0
Q5	9.66	9.66	5.13	148	0
Q6	10.75	11.33	6.36	30	2

Table 11: Statistics for Computational Mathematics

Question Number	Average mark All	Used	Std dev	No. of Attempts Used	Unused
Project A	17.4	17.4	2.55	144	0
Project B	16.48	16.48	3.66	50	0
Project C	15.67	15.67	3.19	163	0

D. COMMENTS ON PAPERS AND ON INDIVIDUAL QUESTIONS

Paper I

Question 1. The bookwork questions part (a)(i) and (a)(ii) were attempted by all students that chose Question 1 and were generally correct, with very few exceptions. Part (a)(iii) had a good number of correct solutions. Many solutions went through some unnecessary steps, for example checking that $\det(C) \neq 0$ even after having exhibited a matrix D such that $CD = DC = I$. The most common mistake was using that det(A + B) = det(A) + det(B), which is not a true property of the determinant.

Part (b)(i) had attempts of solutions by a great number of students, and around half of those attempts were correct proofs. The main challenge of the question was proving that S (or T) was invertible. Many candidates assumed this without any proof, or provided an incorrect proof of it. This question also unveiled some lack of understanding of fundamental concepts in linear algebra by some candidates, that should be addressed before the next linear algebra course. In particular, one (concerningly common) mistake was confusing id_V with the 0 map, and providing a (wrong) proof of the (false) statement that $TS = 0$ implies $ST = 0$. Part b(ii) had many solutions or partial solution, with a wide range of different methods. The main two strategies were finding an inverse directly using the factorization of $(1 - L^n)$, and using (a)(iii) with the polynomial $p(x) = (x - 1)^n$. With the first approach, several students had some struggles with the signs, sometimes dividing in two (almost identical) cases, depending if n was even or odd, but sometimes only providing a solution to one of the two. The main mistake for the second approach was forgetting to verify that the constant term of the polynomial was not 0 (which was sometimes non-trivial, depending on how the polynomial was defined). Another common approaches to prove the invertibility of $L+I$ (without finding the inverse) were to verify it has trivial kernel, ultimately proving that an element of the kernel of $L + I$ would be a (-1)-eigenvector for L, which is in contradiction to the fact that $L^n = 0$.

Question 2. Part (a) was generally done well. Some candidates did not write a correct definition of a subspace and sometimes the nonemptiness condition was omitted from an otherwise correct definition. Some candidates only stated and proved the subspace test in one direction. Part (b) was also mostly done well with the exception of part (b)(i) with a large number of candidates incorrectly writing that it is a basis of V . Often in part (b)(iii), candidates did not justify linear independence or that the set spans, often only writing that it is clear.

Question 3. There were perhaps more issues than expected with the bookwork, with many ommitting details in (a)(i) such as invertibility of P. Occasional confusion would arise between being diagonalisable and being *orthogonally* diagonalisable, which caused issues in proving the equivalent condition in (a)(ii). The most common missing detail in this part was a justifying why the columns of P were linearly independent, or why P is invertible in the other direction. Some candidates mentioned Gram–Schmidt, which was irrelevant here but did highlight a general misunderstanding about orthogonality of vectors in different eigenspaces. The final part of (a) was generally done well, with most opting for a determinantal approach via the characteristic polynomial.

In (b), a fairly common oversimplification was to deduce $A = \pm\lambda I$ by factorising $A^2 - \lambda^2 I = 0$. Similarly, the most frequent misunderstanding was that any $v \in V$ would lie in precisely one of the kernels, despite most vectors actually not lying in either. In this way many claimed that V was the *disjoint union* of the two eigenspaces, which is not a vector space in general. Most found the intersection of the kernels to be trivial (or even empty, with the above confusion), but applying the hint was less consistent. Those who started by writing out the definition of a direct sum were less prone to the above misunderstandings. The conclusion regarding the diagonalisabiltiy of A was quite often ommitted.

There were two recurring misunderstandings in (c)(i), a question in which many candidates did not make much of an attempt. The first was to continue asumming that $A^2 = \lambda^2 I$ on the entirety of V, as opposed to observing a similar property held after restricting to each eigenspaces following the hint. With more subtlety, the second was to start with $P^{-1}A^2P = (P^{-1}AP)^2 = D$ and then "square root" to obtain a diaognal form for $P^{-1}AP$. However this does not neccesarily follow, as there are non-diagonal matrices M for which M^2 is diagonal. For (c)(ii), candidates' intuition was generally correct, but not all provided justified counterexamples, and it was common to see proposed invertible matrices A that were actually diagonalisable.

Question 4. The bookwork in (a) was carried out generally well, with some rare ommissions of orthogonality for P in the Spectral Theorem and a few more cases of writing $W_1 \cup W_2$ instead of $W_1 + W_2$ in the dimension formula, the former not being a vector space in general. The dimension formula was almost always applied in the correct way, however. For (a)(iii), a common approach was to use $\operatorname{rank} A = \operatorname{rank} D$ from the diagonalisabiltiy of A, but the details on why the ranks coincided were inconsistent. Indeed, a misconception arose that similar matrices are row equivalent, which is not the case in general. Some saw the use of the term "multiplicity" and included a discussion on algebraic and geometric multiplicity for the eigenvalues, which was not really required here. The final part of (a) was fairly strong, especially in the converse direction where the candidates could use any nonzero vector to gain information. For the other direction, one technique was to write $x^T Ax = x^T \lambda x = \lambda x^T x$, but not all gave full detail on why restricting to eigenvectors was sufficient, or why this was *strictly* positive.

Whilst the start of (b) was done well and many applied (a)(iii) correctly, the hint in (b)(i) was frequently disregarded, with some opting to diagonalise A and B separately, leading to confusing notation. In (b)(ii), imprecise terms such as "negative eigenvectors" were common, and defintions for the space N_A were often unclear. Those who defined it directly as a span had the most success, with other candidates attempting to prove that there was such a subspace N_A by checking the vector space axioms. Almost no candidates found a suitable choice of W_2, with the closest being a space P_B spanned by the eigenvectors of B with positive eigenvalues, stopping short of "pulling this back" to the level of A.

Question 5. This was the most popular question. Good answers on the bookwork in part (a) including the variation on the subgroup test in (a)(iii). Many candidates struggled in computations with permutations in part (b) despite this part appearing on problem sheet 2. Some candidates attempted induction which can be made to work but very few gave a correct proof in this way.

Part (c)(i) was generally answered well using the hint. Many students gave explicit elements b and c with $bc = (12 \cdots n)$ but arguing with D_{2n} considered as a subgroup of S_n and deducing the existence of b, c with a simple picture of reflections was sufficient.

Part (c)(ii) had very few complete solutions, all using the order of the cyclic subgroup $\langle ab \rangle < S_n$ in some form. A common error was to assume that a and b must be transpositions of S_n.

Question 6. A popular question. Part (a) was done well by most. In part (b)(ii) many candidates just wrote the map f but did not check that it is indeed a homomorphism. Part (c)(i) was answered well by many who observed that an inner automorphism of D_n must send r to $r^{\pm 1}$ and hence chose $f(r) \notin \{r, r^{-1}\}$. Part (c)(ii) had few rigorous answers, some candidates tried to argue vaguely with conjugacy classes of D_6 but did not consider all possibilities for $f \in \mathrm{Aut}(G)$.

Question 7. This was the least popular question of Section B. Part (a) was well done. Surprisingly few managed to answer part (b) despite observing that different subgroups of size p must intersect only at the identity.

Part (c)(i) had a few good answers. One approach was to argue that if G has two different subgroups $H_1 \neq H_2$ of size p then $|H_1 H_2| = p^2 > |G|$, contradiction.

Part (c)(ii) had a handful of correct solutions, some following the model solution, while others considered the conjugation action of a subgroup K of size q on the normal subgroup H with $|H| = p$ from (c)(i).

Paper II

Question 1. (143 attempts): The general level of solutions was good. In (a)(i), many candidates either missed the assumption that $x >$ -1, or did not mention that multiplying by $x + 1 > 0$ preserved the inequality. In (a)(ii), several candidates did not work out N such that $c^n < \epsilon$ for $n \geq$ N. Some solutions relied on existence and properties of the logarithm function, which did not follow the structure of the course (this part was bookwork). Solutions for (a)(iii) were generally correct. In part (b), most candidates got the main ideas of applying (a)(iii) and the integral test. However, many solutions did not check or mention that loglog2 < 0.

Question 2. (141 attempts): The general level of solutions was good. In (a)(i), some candidates missed out the possibility that A could be finite, or, equivalently, assumed there was a bijection with $\mathbb{N}$ instead of an injection. In (a)(ii), many candidates failed to mention that $\mathbb{R}$ was infinite and how to obtain a bijection with $\mathbb{N}$ from an injection. Furthermore, some candidates did not properly deal with the possibility of real numbers having different decimal expansions. Solutions for (a)(iii) were typically correct, though many candidates assumed that algebraic and transcendental numbers were real, as opposed to complex. In part (b)(i), some candidates incorrectly concluded that $A \setminus (x - \epsilon,\ x + \epsilon) = \{a1, ..., a_{N-1}\}$. Part (b)(ii) proved to be difficult. There were several correct solutions. However, many solutions only produced countably many sets, with some indexed infinitely many times.

Question 3. 3(a)(i) – no problems in general. Some students wrote $\sum a_n$ converge iff $\sum x_n$ and $\sum y_n$ converge, where $a_n = x_n + iy_n$.

3(a)(ii) – also no problems in general. Again, some use the decomposition $a_n = x_n + iy_n$ unnecessarily and unsuccessfully.

3(a)(iii) – quite a few students decided to get a lower bound $0 < \sin a_k$ by taking a sufficiently small a_k ($|a_k| < 1$ was a popular choice somehow) instead of working with $|\sin a_k|$.

3(b)(i) – almost everyone get the radius of convergence but the most common mistake was to forget the $k = 0$ term of the series and get e^{4z} instead of $e^{4z} - 1$.

3(b)(ii) – a lot of students unnecessarily computed the radius of convergence. Many did not use the differentiation theorem.

Question 4. 4.(a) This was answered by essentially everyone using the ϵ, δ-definition.

(b) This question was also answered correctly by a vast majority. Most students showed that the sequence must have a convergent subsequence to a point ζ in the interval $[a, b]$ and using the fact f is continuous the result follows. Some people did not have the full 6 marks because they forgot to say why the convergent point of the subsequence is in $[a, b]$ (as it is closed). Very few argued that since $[a, b]$ is connected then the image by a continuous function is also a connected set so it must be an interval. Moreover, the image of a compact set is compact as f is continuous, so it attains a maximum and a minimum, so $\ell \in f([a, b])$.

(c) Essentially everyone knew heuristically why f had to be bounded. Many tried and some succeed to prove it directly. I did not give full marks if they did not know how to reduce the problem to applying the previous question as a black box (as requested).

(d) This question stumped many of the students. Few were able to argue completely correctly why (i) was true. I would say thirty-five percent realized that the sequence $x_n = 2^-n$ satisfies $x_{n+1} \geq x_n + 1/2$, provided n is large enough, but failed to explain why does this, per se, shows that $f \to \infty$ as $x \to 0^+$.

(ii) This caused even more challenges to most students. A large proportion did not attempt it at all. Many who attempted gave wrong counterexamples (f was not continuous). Those who solved correctly constructed a very slowly growing function f i.e. $f(x) = loglog(1/x)$ or more explicitly using the divergence of $\sum_i 1/i$ e.g. Interpolating with linear functions between the points $(2^{-n}, \sum_{i \leq n} 1/i)$

Question 5. This question was very popular and was generally done very well. In (b)(i) several students noted that $\cos(\sqrt{x})$ was in fact Lipschitz continuous (proved either by differentiation, or by using various trig identities and inequalities), and then stated that this implies uniform continuity. This is more complicated than the model solution, but very much acceptable as a solution. Part (c)(ii) was probably the most difficult part with a number of students stating incorrectly that a power series converges uniformly inside its radius of convergence. In part (d), students often did not make it clear that $n \geq N$ needs to be chosen before δ. Some even seemed to assume that the same δ in the definition of uniform continuity applied to all the f_n. Quite a few students did not do part (e), or just guessed the answer without providing the counterexample.

Question 6. This question was also very popular and was generally done well. In part (a) the main mistake was to misstate Rolle's Theorem, either (unnecessarily) requiring $f(a)$ and $f(b)$ to be zero, or only stating that $\xi \in [a, b]$ rather than $\xi \in (a, b)$. While most students did (b)(i) correctly, there were quite a few students who tried to go back to the definition of a derivative and just stated that $\lim_{x \to x_0}(f(x) - f(x_0))/(x - x_0) \geq 0$ implied that $(f(x) - f(x_0))/(x - x_0)$ is always non-negative. Students lost one mark if they assumed without proof that $f' \geq 0$ on an interval implied f is increasing, as it should be clear that this was part of what needed to be proved. Quite a few students had difficulty applying (b)(i) to (b)(ii), often trying to bound both sides separately. Many students had problems with the algebra in part (c), although usually the strategy employed was correct.

Question 7. Part (a) was largely done well. Part (b) was also done well with the exception that most candidates did not clearly state the results they used, either just referring to them by name or not at all. For part (c), some candidates used an infinite geometric series. Part (d) was done very well. Part (e) caused a lot of difficulties. Some candidates wrote that the series does not converge uniformly and most candidates that did write that it does converge did not correctly justify it.

Paper III

Question 1. Many candidates attempted Question 1, but a significant proportion did not complete it. The parts that were attempted were generally done well. Most of the mistakes were minor computational errors in the first and second subquestions. Several students performed extremely well on this question, with some achieving full marks.

Question 2. Nearly all candidates were able to correctly complete part (a). In part (b), curves (i) and (ii) were usually handled correctly in parabolic coordinates, though curve (iii) proved more challenging, often due to algebraic difficulties. The final part of the question caused some confusion: many candidates did not realise it was asking for the area enclosed by all three curves, and instead attempted to compute areas related to each curve individually. This approach often led to results that didn't align with the intent of the question.

Question 3. Candidates had few problems on their bookwork; nearly all who attempted to solve part (a) could identify critical points by the simultaneous vanishing of partial derivatives and that they could characterize each critical point by inspecting the Hessian. The most common issues arose from numerical errors and failing to include the maximal value of the function.

Most candidates were able to successfully set up the optimization problem for part (b) via Lagrange multipliers and recognize the role of the Lagrange multiplier in implementing the constraint $x^2 + y^2 + z^2 = a^2$. Aside from numerical errors, a large fraction of candidates implicitly assumed the non-vanishing of one or more coordinate functions and then proceeded to solve the critical point equations, thereby missing the critical points for vanishing Lagrange multiplier $\lambda = 0$ which lie at the poles $x = y = 0$. With that in mind, candidates were largely successful in solving for the critical points with non-vanishing Lagrange multiplier $\lambda \neq 0$ but many included solutions with imaginary values of x, y, i.e. when $z > 0$. As with part (a), numerous candidates stopped at the identification of the critical points and failed to include the critical values of the function.

Question 4. While this question was chosen by a substantial number of students, overall, students found this question harder than anticipated leading to a very wide spread of marks including a small number of (almost) perfect scores, but also many single-digit scores. It looked like some spent too long writing out detailed justifications in (a) leaving too little time for (b).

While most students scored the two marks in (a)(i), a significant number of students missed the fact that changing the order of pairs or the order of colleges within pairs does not change the pairing in (a)(ii) and some lost further marks in the later parts of (a)(ii). Also a significant number of students did not spot the connection of (a)(iii) to counting the number of ways of choosing n colleges among the given $2n$ colleges, the RHS effectively counting those with j Oxford and $n-j$ Cambridge colleges, then summing over j. A handful of students correctly used the Binomial Theorem to evaluate the coefficient of x^n in $(1+x)^{2n} = (1+x)^n(1+x)^n$ and also scored full marks. Those who attempted an inductive proof wasted time.

In (b)(i), a variety of mistakes led to lost marks: the winner has to win $k+1$ matches, for the best college independently, each with probability p; for the randomly chosen college, explicit calculations did not succeed, but a number of candidates either spotted the symmetry or argued that the uniform pick in particular picks the winning college with probability $1/2n$ (regardless of which it is!), others introduced notation for probability that the ith college wins and nicely used this in an application of LTP. In (b)(ii) many scored some marks for the definition of expectation and the identification of some or all of the relevant probability mass function, or indeed for correct manipulations of geometric sums (and their derivatives), but few put this together correctly to find the expected number of matches of the best college. Some scored the further two marks for the random college by working out the average number of matches per college!

Question 5. This was the most popular question, but the spread of marks was rather wide again particularly towards the bottom. A significant number of students gave an incomplete statement of the LTP in (a). (b) was mostly done well, with some mistakes in the variance calculation and missing justification in (ii). In (c) many students did not justify their answer, and some seemed unsure about the concept of order in (i), but (ii) was often done well up to the point where repeated roots (notably 1) were to be discussed. In (iii), many had the right idea but struggled with the argument. (d) was done by a good number of students, but some left this blank or were unsure what quantity satisfies the recurrence relation.

Question 6. This question was also chosen by a substantial number of students and obtained a good spread of marks with quite a few perfect or near-perfect answers. Some students lost a mark or two failing to state the assumptions of the Weak Law of Large Numbers or getting the statement muddled. (b) was done well by the vast majority of students, and (c) was mostly done well, too – those who failed to acknowledge independence where needed lost a mark. The spread of marks arose from (d) and (e) mainly. Some candidates completed one, some the other, and there were often some marks lost, in (d) for not checking the assumptions of the WLLN carefully or for applying it with $\varepsilon = 0$, which is false, in (e) candidates often got stuck calculating the probability depending on U_1 and U_2, where an argument referring to the clearly specified area of a region in $[0,1]^2$ was the most convincing, but full marks were available for all those who wrote the correct double integral. Partial answers also received partial credit.

Question 7. Together with question 8 this was the most popular statistics question. Students performed overall very well. (a) (i),(ii) and (iii) were answered correctly by the vast majority of students. Some students lost marks in parts (iv) and (vi) by considering the wrong likelihood function. (v) was answered correctly by most students. In part (b)(i) the majority of students was able to state the central limit theorem, some marks were lost by not clearly stating the iid and finite variance assumption. Most students also managed to construct the confidence interval for the Poisson distribution whereby most of the time a mark was lost by not substituting an estimator for the variance.

Question 8. Together with question 7 the most popular statistics question. This question seemed to be more challenging for the students. In part (a) the definition of the covariance was merely a problem. However, a lot of students did not get the correct formula for the variance in part (ii). In part (b) most students derived correctly the maximum likelihood estimators whereby marks were lost mostly because students did not realise that a lot of terms cancel out during the computations. Some students also struggled with the computations of the biases and especially with the computation of the covariance in part (iii). (c) was the most challenging part in this question with students having problems with the computation of the covariance and the correct application of the variance formula from part (a).

Question 9. This was the least popular question in the Statistics section, but was chosen by about a third of the candidates. It achieved a good spread of marks for those who seriously attempted it, including some almost perfect answers. (a) was mostly done well, except that most students did not identify the parameters in (a)(i). In (b), students struggled with the sample correlation matrix in (ii), which has to be based on the sample rather than the theoretical parameters, using the sample variance, which should be written explicitly in terms of the data matrix, correctly mean-centred by the sample mean, not the mean. Most students knew what to do in (iii), but were uncomfortable with the guidance in (iv), with some reverting to the vector differentiation method used in lectures (scoring all but one mark if correctly completed), while expressing the Lagrange function in the form given (with i fixed!) gives an alternative approach. (c) was often done well, although some did not label the vertical axis of the scree plot by percentages, and some were distracted by the explicit values of λ_i while the reason why they sum to 6 results from two different ways of calculating the trace of R.

Paper IV

Question 1. This was the least frequently attempted, with a somewhat bimodal distribution that appears to contain very confident students on the upper end, and those that were generally lost on the other.

Question 2. Most had no trouble identifying conic sections from their equations, and the vast majority received full marks on (a), which was marked leniently, and mainly errors of oversight in (c). (b) caused more difficulty: many students struggled to find a convenient parametrisation, or else struggled to apply it successfully in the second part, or else omitted the second part entirely. I also became somewhat concerned about integrity in (b)(i), as many appeared to just add (=πab) on the end of a computation that was not equal to π*ab. Part (d) seemed to show some discomfort – most could generate an appropriate basis, but reparametrised incorrectly. This part may also have suffered from the time crunch (see below).

Question 3. No concerns with (a) apart from the sloppiness and lack of detail associated with limited time. Some completed (b)(i) by working backward from the question. I would have liked to see some citation of uniform convergence to justify taking the transpose individually on infinite summands in (iii), which very few did. Almost none thought to apply the previous results to the 3-by-3 matrix, most tried computing powers and then gave up. Lastly, in (c), apart from computational errors, a significant group used the limit cos(a) $\rightarrow$ 1-$a^2/2$, without noting that higher-degree terms are dominated as the limit approaches 0.

Question 4. This was an unpopular question, though part (c) is essentially the same as a problem sheet question for a bead on a circular hoop rotating about a vertical axis.

Part (a) was mostly done well by those who attempted it.

Few candidates attempted part (b). The system was often sketched with the rod above the horizontal, rather than hanging below the horizontal so the spring force pulls the rod upwards against gravity. Most attempts had sign errors in the torques, and some lost factors of 2. The overall gravitational force acts through the centre of the rod (as one can verify by integrating the gravitational force along the rod) but the spring force acts on the far end.

The angular momentum equation $\dot{\mathbf{L}} = \boldsymbol{\tau}$ gives an equation for $\ddot{\theta}$. Multiplying by $\dot{\theta}$ and integrating with respect to time leads to the energy given in the question. Almost all candidates who attempted this part did so by differentiating the energy in the question instead.

Most candidates who attempted part (c) could find the equilibria at $\theta = \pm\pi/2$. When $mg < 2g\ell$ there is a third equilibrium at $\theta = \sin^{-1}(mg/2k\ell)$.

Candidates who found a formula for this third solution often did not consider whether it actually exists, *i.e.* whether $\sin\theta = mg/2k\ell$ has a real solution for θ.

Some candidates tried to find equilibria by putting $\dot{\theta} = 0$ into the undifferentiated energy equation. Many wrote down conditions for stability, or instability, without any obvious linearisation. The equilbrium at $\theta = \pi/2$ with the rod hanging vertically downwards is stable if $mg > 2k\ell$, *i.e.* gravity is stronger than the spring force, and otherwise unstable. The equilibrium at $\theta = -\pi/2$ with the rod hanging vertically upwards is always unstable. The third equilibrium at $\theta = \sin^{-1}(mg/2k\ell)$ is stable whenever it exists.

Question 5. This question was very popular, with many near-complete solutions. In part (a) almost all candidates could show that $h = r^2\dot{\theta}$ is constant, but many did not even try to show that the motion lies in a plane using conservation of the angular momentum vector. The force is directed radially outwards from the origin, so in general it has a z component as well as an r component in cylindrical polar coordinates. Some candidates incorrectly asserted that $\ddot{z} = 0$ because the force had no z component. Moreover, the basis vector $\mathbf{e}_r$ depends on θ in cylindrical polar coordinates, so $\mathbf{r} = r\mathbf{e}_r + z\mathbf{e}_z$ does not automatically lie on a plane.

Almost all candidates got full marks for part (b).

Finding h was found most difficult in part (c). One can start from $r\dot{\theta} = v\sin\theta$ by resolving the initial velocity into r and θ components. Alternatively, one can say that $\dot{r} = 0$ at closest approach following the trajectory with no force, so $r = d$ and $r\dot{\theta} = v$. Full marks were given for either, provided they were adequately explained. Some candidates did not realise that $\dot{r} \approx -v$ with a minus sign as the particle is initially moving *towards* the origin.

In part (d) many candidates made sign errors when substituting $F(r)$. Quite a few candidates took the constant part of the solution for u to be k/h^2 instead of $k/(h^2 + \epsilon)$. It is easier to consider a general solution involving $\sin(\omega\theta)$ and $\cos(\omega\theta)$ with $\omega = \sqrt{1 + \epsilon^2/h}$ but some candidates successfully found the solution in the form $\cos(\omega\theta + \text{constant})$ instead.

Almost all candidates who attempted part (e) found a condition for the orbit to be *bounded*. The bounded orbits are ellipses, with u involving $\sin(\omega\theta)$ and $\cos(\omega\theta)$. The solution u is 2π-periodic, so the ellipses are closed, in the Newtonian case with $\epsilon = 0$. The solution u is not 2π-periodic when $\epsilon \neq 0$, so the orbits precess. The orbits are only *closed* if $\omega = p/q$ is a rational number, so increasing θ by $2q\pi$ increases $\omega\theta$ by $2p\pi$.

Question 6. Students generally performed well on the bookwork: most candidates who attempted part (a) answered it correctly. The unseen part of the question proved more challenging. In part (b), several students confused the definitions of acceleration in inertial and non-inertial frames of reference, which led to incorrect application of Newton's Second Law. This, in turn, resulted in many not correctly identifying why the Coriolis acceleration does not appear in the derived equation. Some students also omitted the normal force in the force balance. In part (c), most students successfully derived the given equation for the function $f(r)$ and the value of the integration constant, but only a few attempted to solve for $f(r)$, and even fewer arrived at the correct solution.

Paper V

Question 1. The first sub-question (a), involving relatively simple integrations, was generally well done. Parts (b) and (c) involved three-dimensional integrations to determine volumes bounded by surfaces. Students struggled similarly with both questions, despite the fact that part (c) was supposed to be a little bit harder. Many students failed to adopt the right coordinate system in order to facilitate the integrations. Part (d) involved similar calculations but got on average better results as the candidates could identify the solutions in certain limits using geometrical arguments.

Question 2. This question was overall easier than Question 1. Some students struggled to finish the question in time, though. In part (a), most of the students gave a full solution, but sometimes lacking rigour. The next questions involved finding the work done in a vector field. In part (b) and (c), most of the students made good progress, but sometimes with calculation mistakes. Part (d) was more challenging, and several students failed to find the work done by the rotating circle, in particular by the vertical component of the force.

Question 3.

This question was relatively unpopular. Part (a) was done well, though for full marks candidates needed to write something about the relative orientations of the line and surface integrals in Stokes' theorem: the right hand rule for $\mathrm{d}\mathbf{r}$ and $\mathbf{n}$, or alternatively that $\mathrm{d}\mathbf{r} \wedge \mathbf{n}$ should point away from $\mathcal{S}$ across its boundary.

Part (b) can be done by taking $\phi = x_i x_j$ in part (a). Many candidates used Green's theorem in the plane for the first part with x_1 and x_2. This received full marks, though one needs to replace the 3D curve $\mathcal{C}$ by its projection onto the $x_1 x_2$ plane. If arguing the second part for general i and j by analogy, it is necessary to consider the cases with $i \neq j$ and $i = j$ separately. Some candidates just wrote down properties of the elements of an antisymmetric matrix M. They did not show that $m_{ij} = \int_{\mathcal{C}} x_i \, \mathrm{d}x_j$ satisfied these properties.

Almost all candidates could apply the vector triple product formula in part (c). This gives the required integral $\int_{\mathcal{C}} \mathbf{r} \cdot \mathrm{B}\, \mathrm{d}\mathbf{r}$ minus another integral $\mathrm{B} \int_{\mathcal{C}} \mathbf{r} \cdot \mathrm{d}\mathbf{r}$. Many candidates claimed that $\mathrm{d}\mathbf{r}$ is perpendicular to $\mathbf{r}$, which is false, instead of using part (a) with $\phi = |\mathbf{r}|^2/2$ to show that the second integral vanishes.

Part (d) caused more difficulty. The easiest approach applies the vector triple product formula to the given expression involving $(\mathbf{r} \wedge \mathrm{d}\mathbf{r}) \wedge \mathrm{B}$. This gives $\frac{1}{2}\mathbf{T}$ from part (c) and a second integral involving $-\mathrm{r}\,\mathrm{B} \cdot \mathrm{d}\mathbf{r}$. This becomes the integral of $+\,\mathrm{r} \cdot \mathrm{B}\, \mathrm{d}\mathbf{r}$ using part (b).

Alternatively, one can derive the given result by writing $m_{ij} = \frac{1}{2}(m_{ij} - m_{ji})$ in part (c) using the antisymmetry from part (b), then using the vector triple product formula backwards:

$$\int_{\mathcal{C}} \mathbf{r} \cdot \mathrm{B}\, \mathrm{d}\mathbf{r} = \sum_{i,j} m_{ij} B_i \mathbf{e}_j = \frac{1}{2} \sum_{i,j} (m_{ij} - m_{ji}) B_i \mathbf{e}_j = \frac{1}{2} \int_{\mathcal{C}} (\mathbf{r} \cdot \mathrm{B}\, \mathrm{d}\mathbf{r} - \mathrm{d}\mathbf{r} \cdot \mathrm{B}\, \mathbf{r}) \,.$$

Question 4. This was an extremely popular question, with all students choosing it. The students did very well, with a high percentage of them solving fully parts a and b. While this was expected for part a (bookwork), perhaps part b should have been more challenging. Students also found the first bit of part c quite easy, while the second half of part c (worth 5 marks) was quite challenging. In particular, there where basically three distinct ways to solve the second half of part c, but one of them (the one proposed in the model solutions) was much simpler

Question 5. This question was also very popular, with more than 2/3 of students choosing it. Students found this question quite challenging, although it was clear some students attempted this question with little time left (and hence got no marks in many cases). Surprisingly, many students failed to answer properly part a, which was bookwork. Overall Students did well in part b, while part c was more challenging, as expected.

Question 6. Few candidates attempted Q6. Part (a) was done very well. Some candidates only attempted part (a). Those that attempted part (b) mostly did so successfully. Few candidates completed part (c), those that did often made good progress.

E. COMMENTS ON PERFORMANCE OF IDENTIFIABLE INDIVIDUALS

Prizes

The Departmental Prize was awarded to:

Aakash Shankar, Mansfield College.

F. MODERATORS AND ASSESSORS

Moderators: Prof. Paul Balister (Chair), Prof. Fernando Alday, Prof. Paul Dellar, Prof. Andras Juhasz, Prof. Renaud Lambiotte, Prof. Nikolay Nikolov, Prof. Matthias Winkel.

Assessors: Dr. Giovanni Italiano, Dr. William Hide, Dr. Samuel Lewis, Dr. Fedor Pavutnitskiy, Dr. Antonio Girao, Dr. Gillian Grindstaff, Dr. Giulia Celora, Dr. Adrian Fischer, Dr. Romain Ruzziconi, Dr. Akshay Srikant, Dr. Niklas Garner.