

Precision

General Advice

We can use logic and proof to make an idea precise.

- The TMUA Content Specification has sections on logic and proof, reproduced below.
- For TMUA, if statements A and B are both true, then the statement “A or B” is considered to be true.
- The statement “A if B” can be rewritten as “if B then A”.
- The statement “A only if B” can be rewritten as “if A then B”.
- The statement “A if and only if B” can be rewritten as “if A then B, and if B then A”.
- The statement “A is necessary for B” can be rewritten as “if B then A”.
- The statement “A is sufficient for B” can be rewritten as “if A then B”.
- The statement “A is necessary and sufficient for B” can be rewritten as “if A then B, and if B then A”.
- The converse of “if A then B” is “if B then A”.
- The contrapositive of “if A then B” is “if (not B) then (not A)”.

TMUA Specification (April 2025, Section 2 §“The Logic of Arguments”)

- Understand and be able to use mathematical logic in simple situations:
 - The terms **true** and **false**;
 - The terms **and**, **or** (meaning **inclusive or**), **not**;
 - Statements of the form:
$$\text{if A then B,} \quad \text{A if B,} \quad \text{A only if B,} \quad \text{A if and only if B.}$$
 - The **converse** of a statement;
 - The **contrapositive** of a statement;
 - The relationship between the truth of a statement and its converse and its contrapositive.

Note: candidates will not be expected to recognise or use symbolic notation for any of these terms, nor will they be expected to complete formal truth tables.

- Understand and use the terms **necessary** and **sufficient**.
- Understand and use the terms **for all**, **for some** (meaning for at least one), and **there exists**.
- Be able to negate statements that use any of the above terms.

TMUA Specification (April 2025, Section 2 §“Identifying Errors in Proofs”)

- Identifying errors in purported proofs.
- Be aware of common mathematical errors in purported proofs; for example, claiming “if $ab = ac$, then $b = c$ ”, or assuming “if $\sin A = \sin B$, then $A = B$ ” neither of which are valid deductions.

Warm-up

What’s wrong with the following claims?

- if $ab = ac$, then $b = c$.
- if $\sin A = \sin B$, then $A = B$.
- if $\sqrt{x^2} = \sqrt{y^2}$ then $x = y$.
- for all real x , if $x^2 > x$ then $x > 1$.
- the equation $a(x - p)(x - q) = 0$ has two real solutions for x .
- if $b^2 - 4ac > 0$ then $ax^2 + bx + c = 0$ has two real solutions for x .
- for all integers N , if N^2 is a multiple of 100 then N is a multiple of 100.
- for all integers n , the integer $n^2 + n + 41$ is prime.
- for all positive real numbers a and for all x and y , if $a^x = a^y$ then $x = y$.

Questions

TMUA 2021 Paper 2 Question 10

The first seven terms of a sequence of positive integers are:

$$u_1 = 15, \quad u_2 = 21, \quad u_3 = 30, \quad u_4 = 37, \quad u_5 = 44, \quad u_6 = 51, \quad u_7 = 59.$$

Consider the following statement about this sequence:

(*) **If** n is a prime number, **then** u_n is a multiple of 3 **or** u_n is a multiple of 5.

What is the smallest value of n that provides a **counterexample** to (*)?

A 1 B 2 C 3 D 4 E 5 F 6 G 7

TMUA 2022 Paper 2 Question 5

A straight line L passes through $(1, 2)$.

Let P be the statement

if the y -intercept of L is negative, **then** the x -intercept of L is positive.

Which of the following statements **must** be true?

- I P
- II the converse of P
- III the contrapositive of P

- A none of them
- B I only
- C II only
- D III only
- E I and II only
- F I and III only
- G II and III only
- H I, II and III

TMUA 2021 Paper 2 Question 5

On which line is the first error in the following argument?

- A $\sin^2 x + \cos^2 x = 1$ for all values of x .
- B Therefore $\cos x = \sqrt{1 - \sin^2 x}$ for all values of x .
- C Hence $1 + \cos x = 1 + \sqrt{1 - \sin^2 x}$ for all values of x .
- D Thus $(1 + \cos x)^2 = \left(1 + \sqrt{1 - \sin^2 x}\right)^2$ for all values of x .
- E Substituting $x = \pi$ gives $0 = 4$.

TMUA 2022 Paper 2 Question 6

A list consists of n integers. Consider the following statements:

P: n is odd.

Q: The median of the list is one of the numbers in the list.

Which one of the following is true?

- A P is **necessary and sufficient** for Q.
- B P is **necessary** but **not sufficient** for Q.
- C P is **sufficient** but **not necessary** for Q.
- D P is **not necessary** and **not sufficient** for Q.

MAT 2021 Q1J

Four distinct real numbers a , b , c , and d are used to define four points

$$A = (a, b), \quad B = (b, c), \quad C = (c, d), \quad D = (d, a).$$

The quadrilateral $ABCD$ has all four sides the same length

- (a) if and only if $(a - b)^2 = (c - d)^2$,
- (b) if and only if $(a - c)^2 = (b - d)^2$,
- (c) if and only if $(a - d)^2 = (b - c)^2$,
- (d) if and only if $a - b + c - d = 0$,
- (e) for no values of a , b , c , d .

Part of an Interview

*Adapted from an interview question used by James Munro for Maths interviews at Oxford.
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I'd like my computer to calculate the sum $1 + 2 + 3 + 4 + 5$.

My computer can only add two numbers at a time, and it's expensive!

The cost for replacing $a + b$ with c is $\pounds a \times b \times c$.

For example, $\begin{array}{c} 2 \quad + \quad 3 \\ \hline \downarrow \\ 5 \end{array}$ costs $\pounds 30$, and I'm not done; my sum is now $1 + 5 + 4 + 5$.

The calculation ends when there's only a single number.

There are lots of ways that we could order the operations.

What's the minimum total cost?

(If you have a theory but you can't prove it, it can help to look at cases that are simpler but more general; try experimenting with the different ways to get my computer to calculate the sum $a + b + c$.)

(It's also a good idea to try to understand $a^2b + ab^2$. Where have you seen terms like that?)