Versatility

General Advice

Sometimes an idea isn't working, and you need to decide whether to salvage or start over.

- Lay out your work in a way that makes sense to you, so that if you need to re-read it later you can follow what you were doing.
- It's helpful if you can read your own handwriting!
- Don't scribble out rough work so much that you can't read it, because there's some chance you'll need to come back to it.
- If you're in a non-calculator test and everything has got very complicated (for example, a quadratic equation with three-digit coefficients), then it's possible that a different method exists that is easier to work with.
- There's a clear link to the Fluency worksheet; if you can predict what's going to happen when you try a method, then you can decide ahead of time whether to go down that route or not.

Warm-up

A student is attempting this question:

> Given $y = x-1$, find the minimum value of $\sqrt{(x-1)^2+(y-2)^2}+\sqrt{(x-2)^2+(y-5)^2}$.

The student has performed the following calculations, which you are welcome to read if you like. You may assume that the student has not made any mistakes.

- Make the substitution $y = x - 1$ to get $\sqrt{(x-1)^2+(x-3)^2}+\sqrt{(x-2)^2+(x-6)^2}$.
- Differentiate this with the chain rule and set the derivative equal to zero. This gives

$$\frac{1}{2} \times \frac{2(x-1)+2(x-3)}{\sqrt{(x-1)^2+(x-3)^2}} + \frac{1}{2} \times \frac{2(x-2)+2(x-6)}{\sqrt{(x-2)^2+(x-6)^2}} = 0.$$

- This simplifies to

$$\frac{x-2}{\sqrt{(x-1)^2+(x-3)^2}} + \frac{x-4}{\sqrt{(x-2)^2+(x-6)^2}} = 0.$$

Decide whether you would like to continue with this method, or start again with a different method.

Work on the question yourself and consider whether you've made the right decision. It's not too late to change your mind!

Questions

TMUA 2020 Paper 1 Question 19

Find the lowest positive integer for which $x^2 - 52x - 52$ is positive.

A 26 **B** 27 **C** 51 **D** 52 **E** 53 **F** 54

[See the next page for a hint]

TMUA 2021 Paper 1 Question 16

Consider the expansion of

$$(a + bx)^n$$

The third term, in **ascending** powers of x, is $105x^2$
The fourth term, in **ascending** powers of x, is $210x^3$
The fourth term, in **descending** powers of x, is $210x^3$
Find the value of $\left(\frac{a}{b}\right)^2$

A $\frac{1}{4}$ **B** $\frac{4}{9}$ **C** $\frac{25}{36}$ **D** $\frac{5}{6}$ **E** 1

[See the next page for a hint]

Hints

TMUA 2020 Paper 1 Question 19

- If you've started evaluating that quadratic for each of the values (for example, you've worked out $26^2 - 52 \times 26 - 52 = -728$), that's a lot of work. Stop and think about simplifying the quadratic first.

TMUA 2021 Paper 1 Question 16

- If you've written out equations

$$\binom{n}{2} a^{n-2} b^2 = 105, \quad \text{and} \quad \binom{n}{3} a^{n-3} b^3 = 210 \quad \text{and} \quad \binom{n}{n-3} a^3 b^{n-3} = 210$$

  and you're trying to solve those simultaneously, stop and think of an easier way to get to the value of n.

TMUA 2021 Paper 2 Question 19

The angle θ can take any of the values $1°, 2°, 3°, \ldots, 359°, 360°$.
For how many of these values of θ is it true that

$$\sin\theta\sqrt{1+\sin\theta}\sqrt{1-\sin\theta}+\cos\theta\sqrt{1+\cos\theta}\sqrt{1-\cos\theta}=0$$

A 0 **B** 1 **C** 2 **D** 4 **E** 93 **F** 182 **G** 271 **H** 360

[See the next page for a hint]

TMUA 2022 Paper 1 Question 16

The solutions to $7x^4 - 6x^2 + 1 = 0$ are $\pm\cos\theta$ and $\pm\cos\beta$.
Which one of the following equations has solutions $\pm\sin\theta$ and $\pm\sin\beta$?

A $7x^4 - 8x^2 - 5 = 0$
B $7x^4 - 8x^2 + 2 = 0$
C $7x^4 - 6x^2 - 2 = 0$
D $7x^4 - 6x^2 + 1 = 0$
E $7x^4 + 6x^2 - 1 = 0$
F $7x^4 + 6x^2 + 5 = 0$

[See the next page for a hint]

Hints

TMUA 2021 Paper 2 Question 19

- If you've squared that whole expression, it's probably got quite messy. Can you start by simplifying any of the square-roots in the expression, before you do anything complicated like squaring?

TMUA 2022 Paper 1 Question 16

- If you've solved the equation for the four values of x then well done for spotting that you could do that, but your nested square-roots are going to be a little tricky to deal with. Proceed with care, or consider whether you needed a solution for x anyway.

MAT 2019 Q1I
The positive real numbers x and y satisfy $0 < x < y$ and

$$x2^x = y2^y$$

for

(a) no pairs x and y.

(b) exactly one pair x and y.

(c) exactly two pairs x and y.

(d) exactly four pairs x and y.

(e) infinitely many pairs x and y.

Part of an Interview
Adapted from an interview question used by James Munro for Maths interviews at Oxford. Reproduced here with permission.

Suppose that A, B, and C are points on the parabola $y = x^2$, with x-coordinates a, b, and c. Suppose that A and C are fixed points, and that B lies somewhere on the curve in between A and C.

I would like to maximise the area of triangle ABC.

In terms of a and c, where should I put the point B?

(If you have a guess based on something that you know to be between a and c, that's fine, but let's try to replace the guesswork with a calculation and/or some reasoning!)

(You know lots of different formulas for the area of a triangle, perhaps including $\frac{1}{2}ab$ for a right-angled triangle, or $\frac{1}{2}bh$, or $\frac{1}{2}ab\sin C$, or maybe you've even seen more exotic formulas like $\sqrt{s(s-a)(s-b)(s-c)}$ or $\frac{abc}{4R}$.
For this question, some of these formulas are not good ideas! If you've picked something that's not working, then the interviewer could help you to pick another direction.)