Gapless SPT and SSB Phases with Non-Invertible Symmetries in (1+1)d

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Phases with non-invertible symmetry

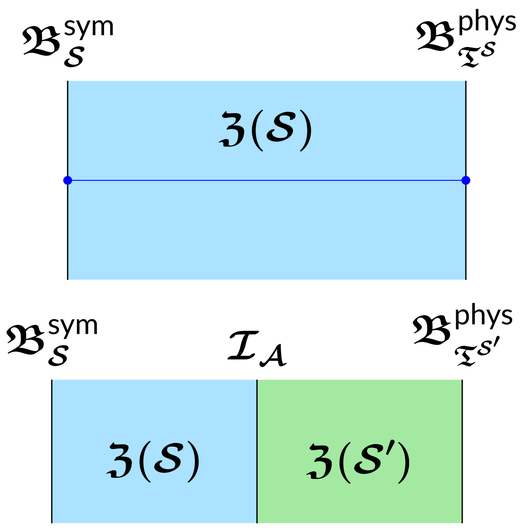
What: study **gapped** and **gapless phases** in (1+1)d with non-invertible (categorical) symmetry $\mathcal{S}$

$$a \otimes b = \sum_c N_{ab}^c \, c, \qquad a, b, c \in \mathcal{S}$$

Why: to understand exotic new phases and their phase transitions $\Rightarrow$ Categorical Landau paradigm!

How: using the **SymTFT** which is a (2+1)d TFT $\mathfrak{Z}(\mathcal{S})$

- Anyons: topological lines
- Condensable algebras of anyons $\leftrightarrow$ phases
- Maximal condensation of anyons on both boundaries $\Rightarrow$ **gapped phases**



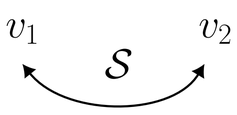
- Non-maximal condensation of anyons: interface $I_{\mathcal{A}}$ $\Rightarrow$ **gapless phases**

Symmetric gapped phases

$n =$ number of anyons ending on both $\mathfrak{B}_{\mathcal{S}}^{\text{sym}}$ and $\mathfrak{B}_{\mathfrak{T}^{\mathcal{S}}}^{\text{phys}}$

- **SPT**: symmetry protected topological phase $n = 1$ symmetric ground state

- **SSB**: spontaneous symmetry breaking phase $n > 1$ ground states related by $\mathcal{S}$



Non-invertible Rep(D_8) symmetry

$$D_8: \quad \langle a, b \mid a^4 = b^2 = 1,\ bab = a^3 \rangle$$

$$\text{Rep}(D_8): \quad 1,\ 1_a,\ 1_b,\ 1_{ba},\ 2$$

$$2 \otimes 2 = 1 \oplus 1_a \oplus 1_b \oplus 1_{ba} \quad \leftarrow \textbf{ non-invertible fusion}$$

Anyons are labelled by $([g], \boldsymbol{R})$.

The symmetry algebra for Rep(D_8) is:

$$\mathcal{A}_{\text{Rep}(D_8)} = ([1],1) \oplus ([a],1) \oplus ([a^2],1) \oplus ([b],1) \oplus ([ba],1)$$

We computed all possible condensations of anyons for $\mathcal{S} =$ Rep(D_8). There are 38 possible phases!

Gapped Rep(D_8) phases

There are 11 gapped phases:

- 3 SPTs with $n = 1$
- 3 $\mathbb{Z}_2$ SSBs with $n = 2$
- 1 Rep(D_8)/($\mathbb{Z}_2 \times \mathbb{Z}_2$) SSB with $n = 2$
- 3 $\mathbb{Z}_2 \times \mathbb{Z}_2$ SSBs with $n = 4$
- 1 Rep(D_8) SSB with $n = 5$

Symmetric gapless phases

Gapless phases $\leftrightarrow$ **non-maximal** anyon consensation on $\mathcal{I}_{\mathcal{A}}$

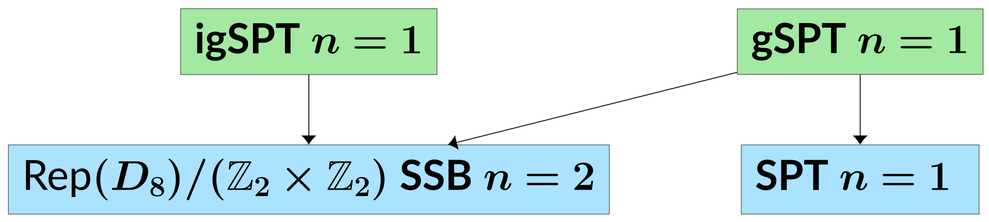
$n =$ number of anyons condensing on both $\mathfrak{B}_{\mathcal{S}}^{\text{sym}}$ and $\mathcal{I}_{\mathcal{A}}$

- **gSPT**: gapless symmetry preserving phase $n = 1$
- **gSSB**: gapless spontaneous symmetry breaking phase $n > 1$
- **Intrinsically gapless** (igSPT or igSSB) phases: can only flow to gapped phases by spontaneously breaking symmetry

gSPT vs. igSPT for Rep(D_8)

There are 16 gSPT phases + 1 igSPT phase

- A **gSPT** is given by a subalgebra of both SSB and **SPT** phases $\Rightarrow$ it can become gapped while preserving the full Rep(D_8)
- An **igSPT** can only flow to an **SSB**

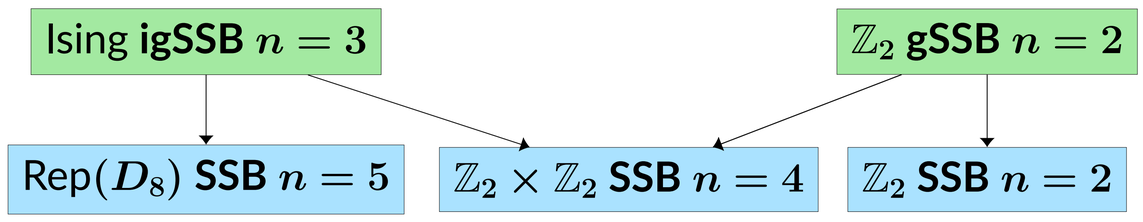


gSSB vs. igSSB for Rep(D_8)

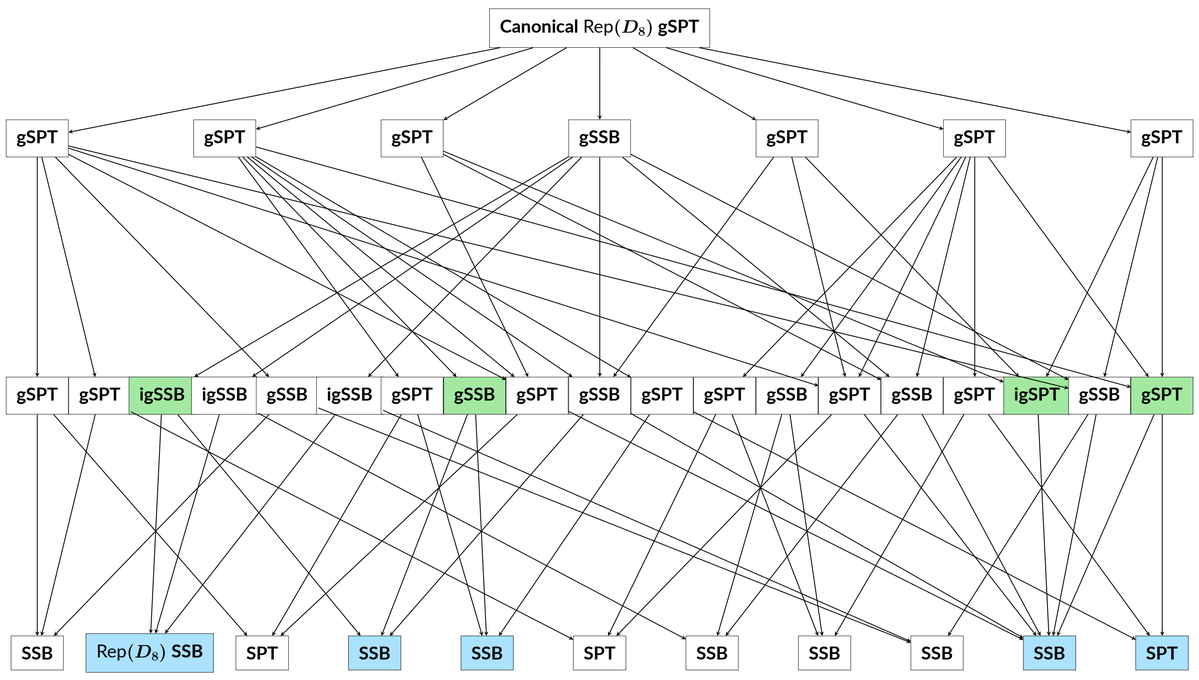
- 3 $\mathbb{Z}_2$ gSSBs with $n = 2$
- 4 Rep(D_8)/($\mathbb{Z}_2 \times \mathbb{Z}_2$) gSSB with $n = 2$
- 3 Ising igSSBs with $n = 3$

all gapped phases have $n \neq 3 \Rightarrow n$ must increase

only these phases can flow to the Rep(D_8) SSB with $n = 5$



Full Rep(D_8) phase diagram



References

[1] L. Bhardwaj, D. Pajer, S. Schäfer-Nameki, and A. Warman `arXiv:2403.00905`.

[2] L. Bhardwaj, L. E. Bottini, D. Pajer, and S. Schäfer-Nameki `arXiv:2310.03786`, `2310.03784`, `2312.17322`.

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[5] R. Thorngren and Y. Wang *JHEP 04 and 07* (2024) , `arXiv:1912.02817`, `2106.12577`.