Phases with Generalized Symmetries

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Generalized Symmetries

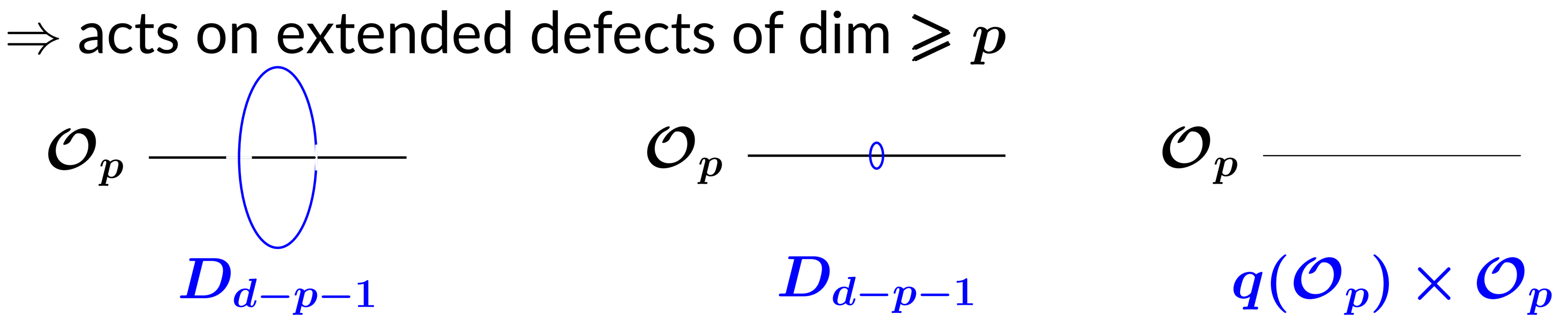
What are the possible global symmetries of a Quantum Field Theory (QFT)?

Generalized global symmetries
$\updownarrow$
Topological defects in QFT

Higher-form symmetries:

A p-form symmetry is a $d-p-1$-dim operator

$\Rightarrow$ acts on extended defects of dim $\geqslant p$



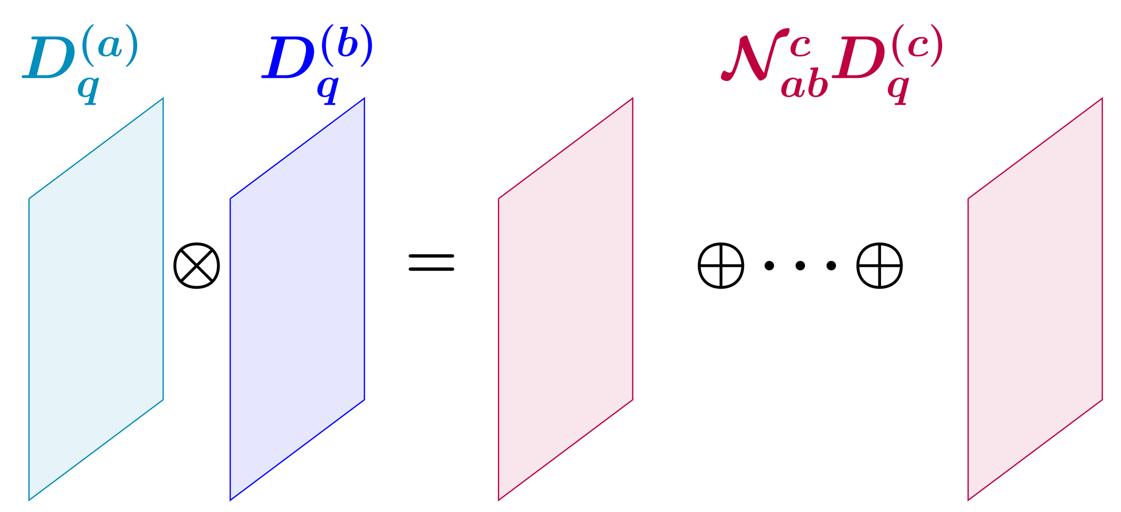
Example: pure Maxwell theory in 4d: $d \star F = 0$ and $dF = 0$

$U_e = \exp\left(i \int \frac{\star F}{g^2}\right)$ and $U_m = \exp\left(i \int \frac{F}{2\pi}\right)$ are topological

$\Rightarrow$ Electric and magnetic $U(1)$ one-form symmetries

Non-Invertible Symmetries:

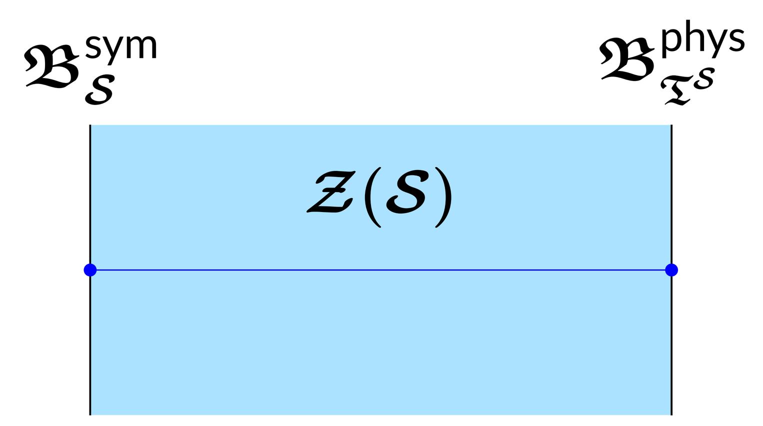
Fusions of topological defects do not have an inverse



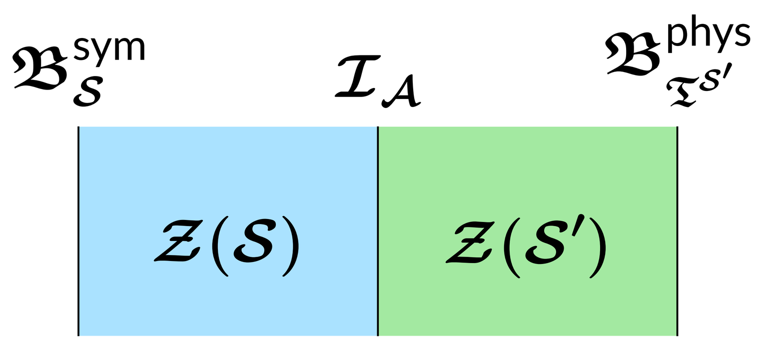
Symmetry Topological Field Theory (SymTFT)

The SymTFT is a (d+1)-dim TQFT with:

- Topological defects $\mathcal{Z}(\mathcal{S})$
- Topological symmetry boundary $\mathfrak{B}^{\text{sym}}_{\mathcal{S}}$
- Physical boundary $\mathfrak{B}^{\text{phys}}_{\mathfrak{T}^{\mathcal{S}}}$ (dynamics of the theory)



- Interfaces $\mathcal{I}_{\mathcal{A}}$ specified by condensable algebras



Example: $\mathbb{Z}_N$ SymTFT from $U(1)$ BF-theory:

$$S_{BF} = \frac{N}{2\pi} \int_{M_3} b_1 \wedge da_1 \,.$$

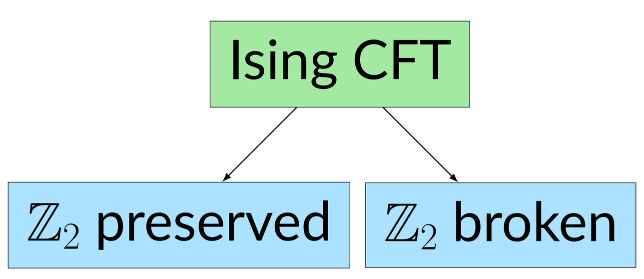
Bulk topological line operators with commutation relation:

$$e^{i\int_\ell b_1}\, e^{i\int_{\ell'} a_1} = e^{\frac{2\pi i L(\ell,\ell')}{N}}\, e^{i\int_{\ell'} a_1}\, e^{i\int_\ell b_1}$$

[F. Apruzzi, F. Bonetti, I.G. Etxebarria, S.S. Hosseini, S. Schafer-Nameki '21], [D. Freed, G. Moore, C. Teleman, '22], [J. Kaidi, K. Ohmori, Y. Zheng, '22]

Gapped and gapless phases

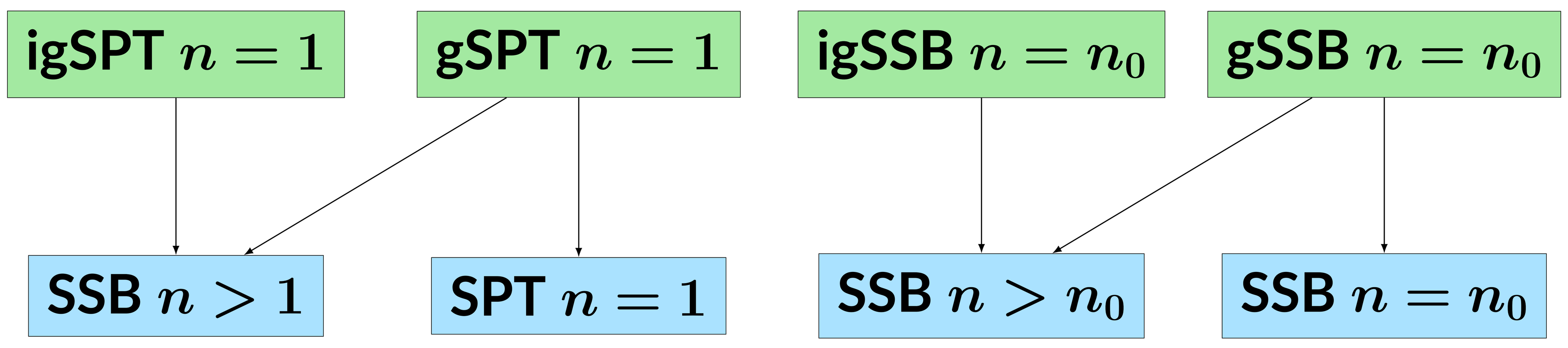
- **Gapless** Phases: Conformal Field Theories (**CFT**)
- **Gapped** Phases: Topological Field Theories (**TQFT**)



Symmetric phases

n = number of lines ending on both $\mathfrak{B}^{\text{sym}}_{\mathcal{S}}$ and $\mathfrak{B}^{\text{sym}}_{\mathcal{S}}$

- **(g)SPT**: gapless symmetry preserving phase $n = 1$
- **(g)SSB**: gapless spontaneous symmetry breaking phase $n > 1$
- **Intrinsically gapless (ig)** phases: can only be deformed to gapped phases by spontaneously breaking symmetry



[L.Bhardwaj, D.Pajer, S.Schäfer-Nameki, **A.W.**, *SciPost* '25]

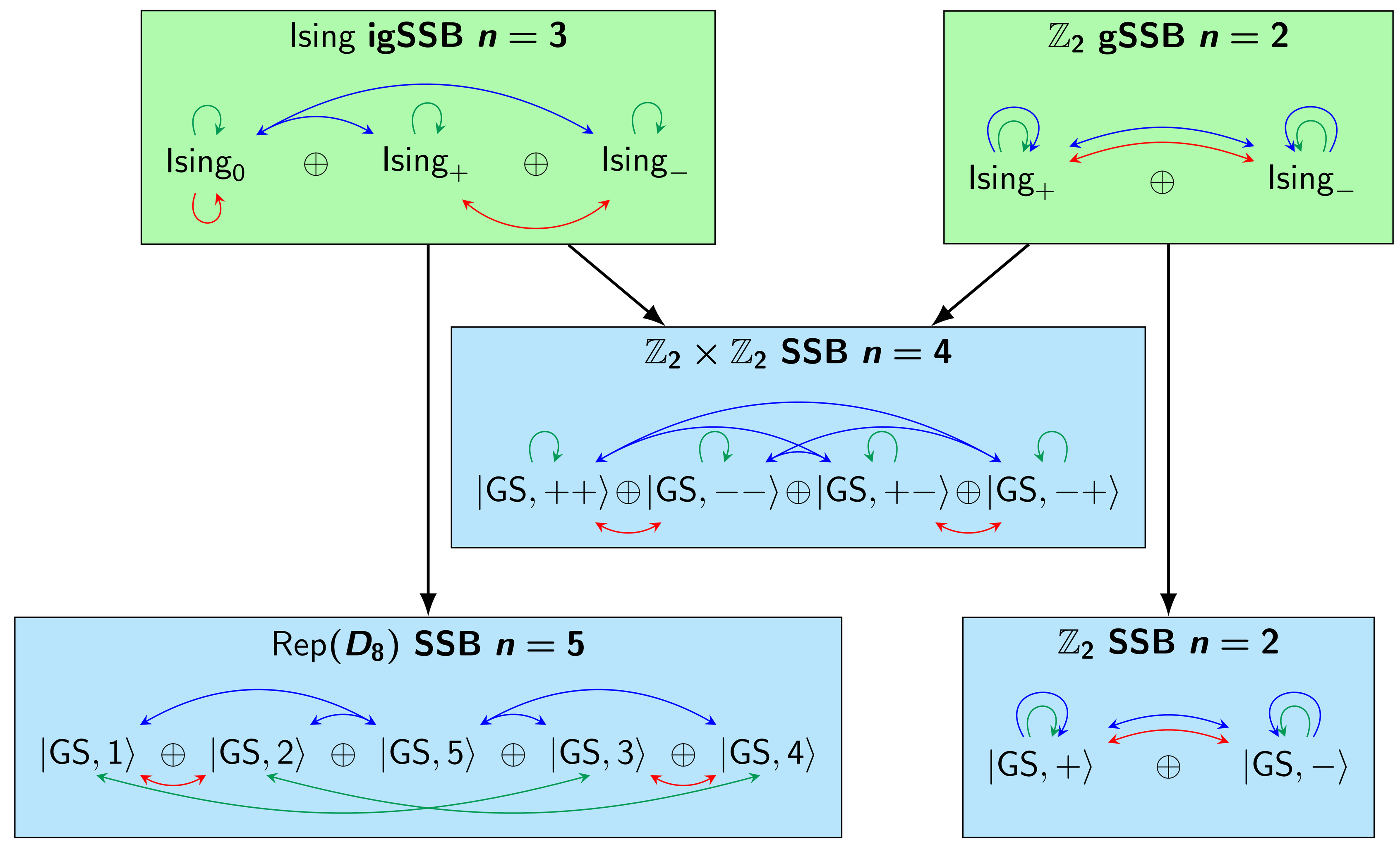
Example in (1+1)d: igSSB vs. gSSB for Rep(D_8)

$D_8 = \mathbb{Z}_4^{(r)} \rtimes \mathbb{Z}_2^{(s)}$ group of symmetries of a square

Rep(D_8) : $\quad 1,\ 1_{rs},\ 1_s,\ 1_{rs}1_s,\ E$

$E \otimes E = 1 \oplus 1_{rs} \oplus 1_s \oplus 1_{rs}1_s \quad \leftarrow$ **non-invertible fusion**

[L.Bhardwaj, D.Pajer, S.Schäfer-Nameki, **A.W.**, *SciPost* '25]
[**A.W.**, F. Yang, A. Tiwari, H. Pichler, S. Schäfer-Nameki, *Phys.Rev.Lett.* '25]



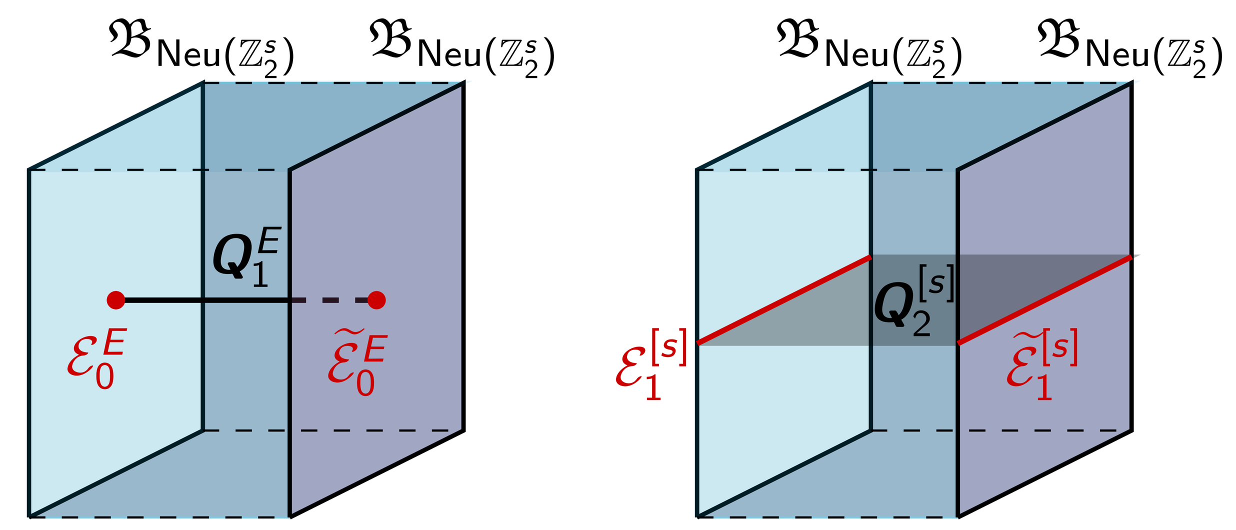
Example in (2+1)d: 2Rep$(\mathbb{Z}_4^{(1)} \rtimes \mathbb{Z}_2^{(0)})$ SSB Phase

- SymTFT lines ending $\Rightarrow$ local charges $\{1, \mathcal{O}_{1_s}, \mathcal{O}_E\}$

$\Rightarrow D_2^{[r]}, D_2^{r^2}$ 0-form SSB

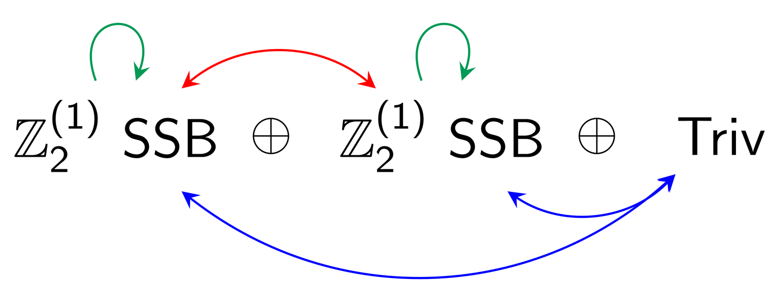
- SymTFT surfaces ending $\Rightarrow$ line charges $\{1, \mathcal{L}^s\}$

$\Rightarrow D_1^{\hat{s}}$ 1-form symm SSB



- Non-inv 0-form symm relates vacua with different 1-form symm!

[L. Bhardwaj, S. Schäfer-Nameki, A. Tiwari, **A.W.**, '25]



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