Hybrid Lattice Surgery: Non-Clifford Gates via Non-Abelian Surface Codes

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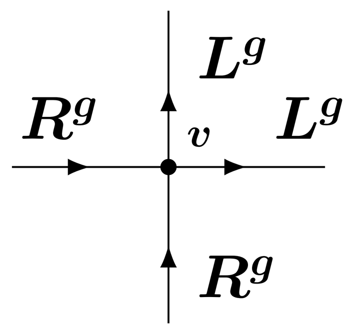
Introduction

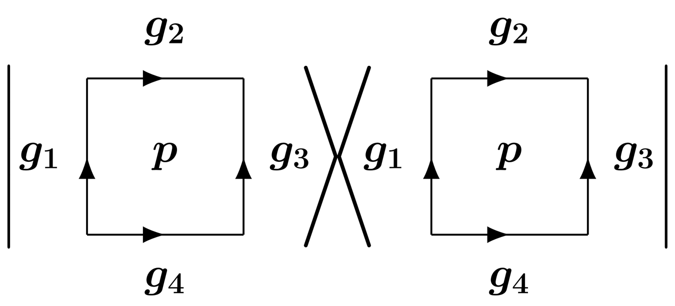
- A set of unitaries for universal quantum computing is Clifford (map Paulis to Paulis) + $T = \text{diag}(1, e^{i\pi/4})$ gate
- T requires substantial resources $\Rightarrow$ we propose a new implementation via merge and split of surface codes
- Advantages: local measurements along interfaces, fault-tolerance and decoder, protocols for T and $T^{1/n}$ gates

Group surface codes

Local Hilbert space with basis $\{\,|h\rangle \; : \; h \in G\}$
and multiplication operators $L^g\,|h\rangle = |gh\rangle\,, \quad R^g\,|h\rangle = |hg^{-1}\rangle$

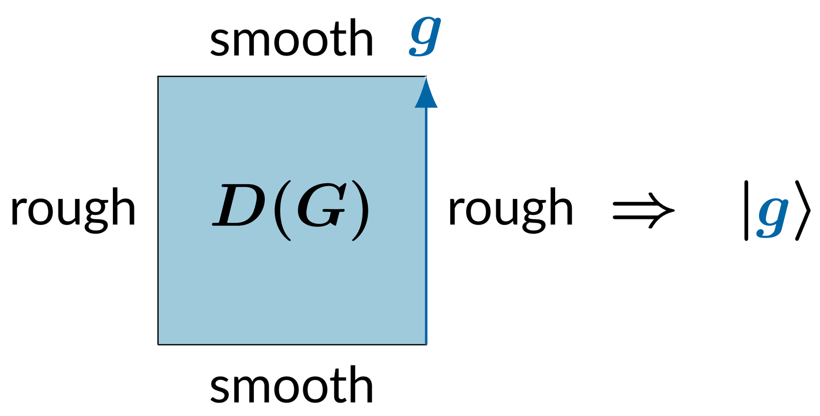
$A_v^{(g)} =$ 

vertex and plaquette operators

$B_p^{(g)} = \sum_{g_1, g_2, g_3, g_4} \delta_{g,\, g_1 g_2 g_3^{-1} g_4^{-1}}$ 

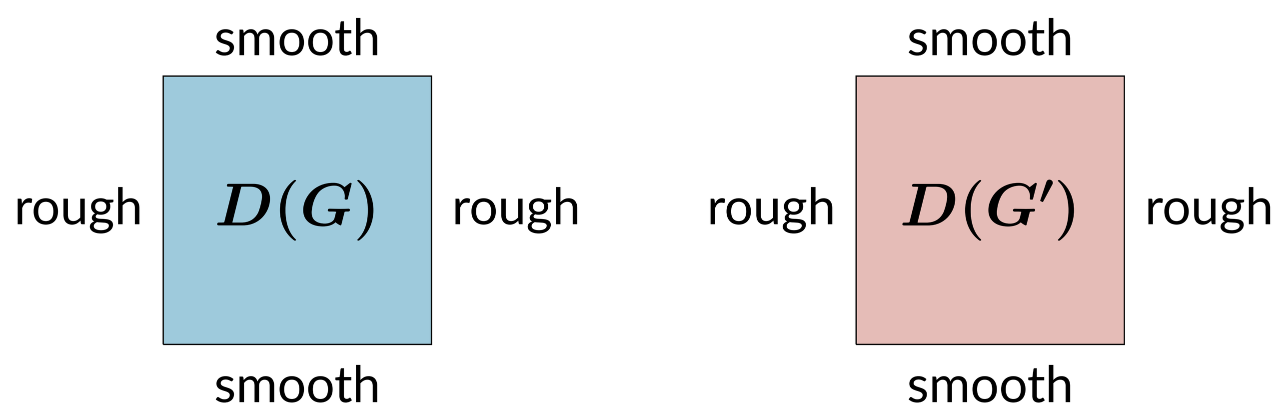
$$H_G = -\frac{1}{|G|}\sum A_v^{(g)} - \sum B_p^{(\text{id})} \quad \text{Hamiltonian}$$

Logical computational basis
$\{|g\rangle : g \in G\}$
from magnetic anyons



Rough Merge and Split

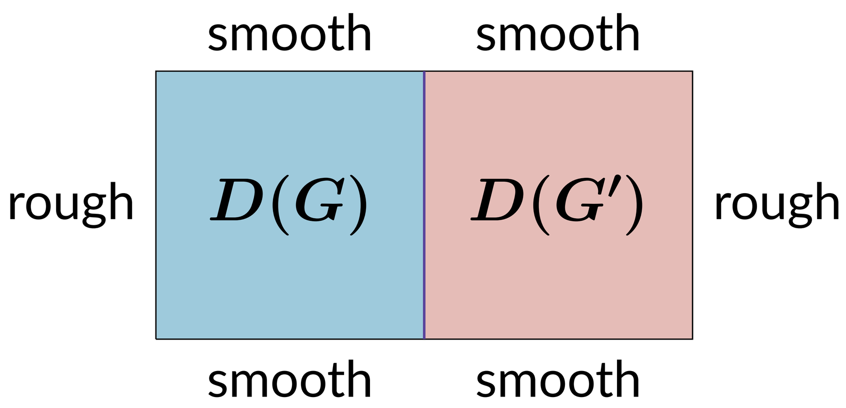
Start with $D(G)$ and $D(G')$ code patches

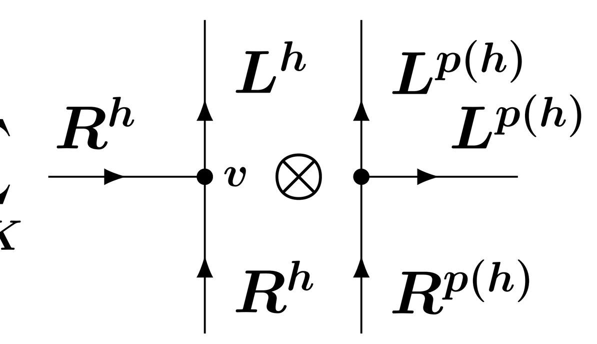


- $K \subseteq G$ is a subgroup
- $N \triangleleft K$ is a normal subgroup and $p : K \to K/N \subseteq G'$

$\Rightarrow K^{\text{diag}} = \{(h, p(h)) \in G \times G' \;\mid\; h \in K\} \cong K$

$\Rightarrow$ Merge $D(G)$ and $D(G')$ by gauging K^{diag} along interface

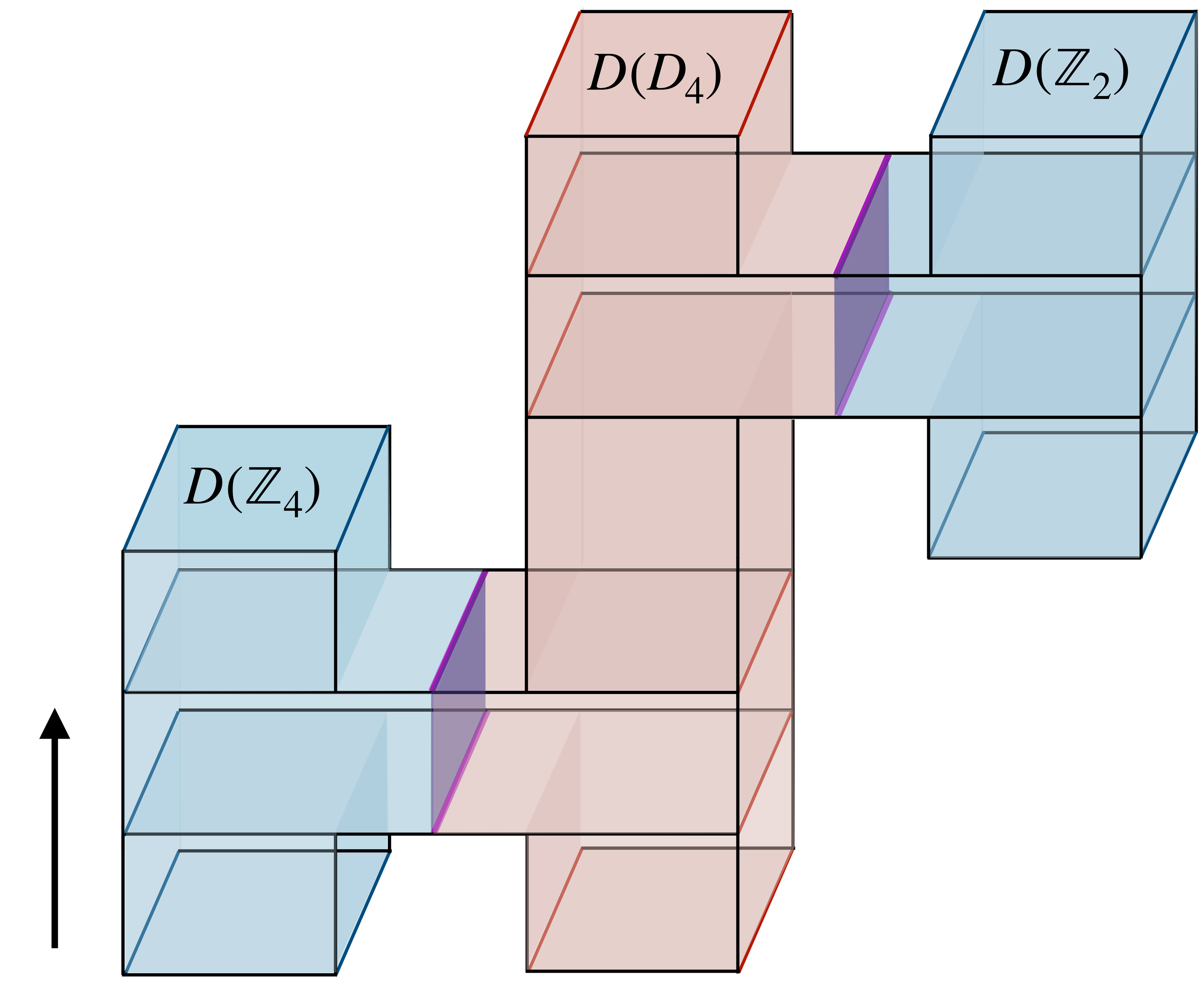


$A_v^{K^{\text{diag}}} = \frac{1}{|K|}\sum_{h \in K}$ 

Split by measuring all the vertical edges on the interface in the computational basis $\{|g, g'\rangle \;\mid g \in G,\, g' \in G'\}$

T-gate protocol

Non-Abelian group $D_4 = \langle r, s \,|\, r^4 = s^2 = \text{id}\,,\; srs = r^{-1}\rangle$



- Step 1: input $|S\rangle_{\mathbb{Z}_4} = \frac{1}{2}(|\text{id}\rangle + e^{i\pi/4}\,|m\rangle - |m^2\rangle + e^{i\pi/4}\,|m^3\rangle)$
  merge $D(\mathbb{Z}_4)$ and $D(D_4)$ by gauging $\mathbb{Z}_4^{\text{diag}} = \langle (m, r)\rangle$
  $\Rightarrow |S\rangle_{D_4} = \frac{1}{2}(|\text{id}\rangle + e^{i\pi/4}\,|r\rangle - |r^2\rangle + e^{i\pi/4}\,|r^3\rangle)$ then split
- Step 2: merge $D(D_4)$ and $D(\mathbb{Z}_2)$ by gauging
  $\mathbb{Z}_2 \times \mathbb{Z}_2^{\text{diag}} = \langle (rs, \text{id}),\, (r^2,\, m)\rangle$ then split
- Step 3: gauge $\mathbb{Z}_2 = \langle s \rangle$ on $D(D_4)$ boundary
  then measure $\langle \text{id}|_{D_4} \Rightarrow$ qubit gate

$$\begin{aligned} U_{k,l} = \langle \psi_k|\,\langle \text{id}|_{D_4} &(\mathbb{I}\otimes\mathbb{I} + R^s \otimes \mathbb{I})\times \\ &(\mathbb{I}\otimes\mathbb{I} + R^{rs} \otimes \mathbb{I})\times \\ &(\mathbb{I}\otimes\mathbb{I} + R^{r^2} \otimes L^m)\,|S_{D_4}\rangle\,|\psi_l\rangle \end{aligned}$$

$$U = (HX)\,T\,(XH) \;\cong\; T = \text{diag}(1, e^{i\pi/4})$$

Generalization: $T^{1/n}$-gates

Non-Abelian group $D_{4n} = \langle r, s \,|\, r^{4n} = s^2 = \text{id}\,,\; srs = r^{-1}\rangle$

- Step 1: input $|S\rangle_{\mathbb{Z}_{4n}} = \frac{1}{2\sqrt{n}}\sum_{j=0}^{4n-1} e^{\frac{i\pi}{4n}j^2}\,|m^j\rangle\,,$
  merge $D(\mathbb{Z}_{4n})$ and $D(D_{4n})$ by gauging $\mathbb{Z}_{4n}^{\text{diag}} = \langle (m, r)\rangle$
  $\Rightarrow |S\rangle_{D_{4n}} = \frac{1}{2\sqrt{n}}\sum_{j=0}^{4n-1} e^{\frac{i\pi}{4n}j^2}\,|r^j\rangle\,,$ then split
- Step 2: merge $D(D_{4n})$ and $D(\mathbb{Z}_2)$ by gauging
  $\mathbb{Z}_2 \times \mathbb{Z}_2^{\text{diag}} = \langle (rs, \text{id}),\, (r^{2n},\, m)\rangle$ then split
- Step 3: gauge $\mathbb{Z}_2 = \langle s \rangle$ on $D(D_{4n})$ boundary
  then measure $\langle \text{id}|_{D_{4n}} \Rightarrow$ qubit gate

$$\begin{aligned} U_{k,l} = \sqrt{n}\,\langle \psi_k|\,\langle \text{id}|_{D_{4n}} &(\mathbb{I}\otimes\mathbb{I} + R^s \otimes \mathbb{I})\times \\ &(\mathbb{I}\otimes\mathbb{I} + R^{rs} \otimes \mathbb{I})\times \\ &(\mathbb{I}\otimes\mathbb{I} + R^{r^{2n}} \otimes L^m)\,|S_{D_{4n}}\rangle\,|\psi_l\rangle \end{aligned}$$

$$U = (HX^n)\,T^{1/n}\,(X^nH) \;\cong\; T^{1/n} = \text{diag}(1, e^{i\pi/4n})$$