Examiners' Report: Preliminary Examination in Mathematics and Philosophy Trinity Term 2023

October 26, 2023

Part I

A. STATISTICS

(1) Numbers and percentages in each class

See Table 1. Overall, 17 candidates were classified.

Table 1: Numbers in each class (Preliminary Examination)

	Numbers					Percentages %				
	2023	(2022)	(2021)	(2019)	(2018)	2023	(2022)	(2021)	(2019)	(2018)
Distinction	4	7	7	7	6	23.53	38.89	35	35	42.86
Pass	12	10	11	11	7	70.59	55.56	55	55	50
Partial Pass	1	0	2	2	1	5.88	0	1	10	7.14
Incomplete	0	1	0	0	0	0	5.56	0	0	0
Fail	0	0	0	0	0	0	0	0	0	0
Total	17	17	20	20	14	100	100	100	100	100

B. NEW EXAMINING METHODS AND PROCEDURES

The methods and procedures reverted to the examining methods used prior to the COVID-19 pandemic.

C. CHANGES IN EXAMINING METHODS AND PROCEDURES CURRENTLY UNDER DISCUSSION OR CONTEMPLATED FOR THE FUTURE

None.

D. NOTICE OF EXAMINATION CONVENTIONS FOR CANDIDATES

The Notice to Candidates, containing details of the examinations and assessments, was issued to all candidates at the beginning of Trinity term. The Examination Conventions in full were made available at

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. GENERAL COMMENTS ON THE EXAMINATION

Acknowledgements

First, the Moderators should like to thank the Undergraduate Studies Administration Team.

We should also like to thank Charlotte Turner-Smith for her invaluable experience with the Mitigating Circumstances Panel, and Matthew Brechin and Waldemar Schlackow for maintaining and running the examination database and their assistance during the final examination board meeting.

We should like to thank the lecturers for their feedback on proposed exam questions; the assessors for their extraordinary assistance with marking; and the team of graduate checkers for their rapid work checking the marks on the papers.

Timetable

The examinations began on Monday 20th June and ended on Friday 24th June.

Marking and marks processing

In Mathematics, the Moderators and Assessors marked the scripts according to the mark schemes and entered the marks. Small adjustments to some mark schemes were made at this stage, and care was take to ensure these were consistently applied to all candidates.

A team of graduate checkers, supervised by Imogen Harbinson-Frith, Haleigh Bellamy and Anwen Amos, sorted all the scripts for each paper and carefully cross checked these against the mark scheme to spot any unmarked parts of questions, addition errors, or wrongly recorded marks. A number of errors were corrected, with each change checked and signed off by a Moderator, at least one of whom was present throughout the process.

In Philosophy all scripts were single marked except for failing scripts, which were double-marked.

Determination of University Standardised Marks

Marks for each individual assessment are reported as a University Standard Mark (USM) which is an integer between 0 and 100 inclusive. For the papers that are common with Mathematics, the same scaling functions as applied for candidates in Mathematics were used.

The scripts of those candidates at the boundaries between outcome classes were scrutinised carefully to determine which attained the relevant qualitative descriptors and changes were made to move those into the correct class.

Mitigating Circumstances were then considered using the banding produced by the Mitigating Circumstances Panel, and appropriate actions were taken and recorded.

Recommendations for Next Year's Examiners and Teaching Committee

There are no recommendations specific to Mathematics & Philosophy. General recommendations are made in the report on the Preliminary Examination in Mathematics.

B. EQUAL OPPORTUNITIES ISSUES AND BREAKDOWN OF THE RESULTS BY GENDER

The breakdown of the final classification by gender is as follows. Here gender is the gender as recorded on eVision.

Table 2: Breakdown of results by gender

Class	Number								
	2023			2022			2021		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	0	4	4	3	4	7	1	6	7
Pass	9	3	12	6	4	10	1	10	11
Partial Pass	1	0	1	0	0	0	2	0	2
Incomplete	0	0	0	1	0	0	0	0	0
Fail	0	0	0	0	0	0	0	0	0
Total	10	7	16	10	8	18	4	16	20

Class	Percentage								
	2023			2022			2021		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	0	57.14	23.53	30	50	38.89	25	37.5	31.25
Pass	90	42.86	70.59	60	50	55.56	25	62.5	43.75
Partial Pass	10	0	5.88	0	0	0	50	0	25
Incomplete	0	0	0	10	0	5.56	0	0	0
Fail	0	0	0	0	0	0	0	0	0
Total	100	100	100	100	100	100	100	100	100

C. STATISTICS ON CANDIDATES' PERFORMANCE IN EACH PART OF THE EXAMINATION

Mathematics I

	Maths and Philosophy		Single School	
Question	Mean	Std Dev	Mean	Std Dev
Q1	12.29	12.29	2.20	17
Q2	5.4	5.4	3.44	15
Q3	8.93	8.93	4.01	14
Q4	4.5	4.5	4.20	4
Q5	11.62	11.62	1.89	16
Q6	11.6	11.6	4.61	5
Q7	9.69	9.69	3.35	13

Mathematics II

Question	Maths and Philosophy		Single School	
	Mean	Std Dev	Mean	Std Dev
Q1	15.82	15.82	2.81	17
Q2	8.22	8.22	4.05	9
Q3	10	10	5.90	8
Q4	9	9	4.53	9
Q5	11.92	11.92	3.55	13
Q6	11	11	3.91	12
Q7	4.76	4.76	2.49	17

Mathematics III(P)

Question	Maths and Philosophy		Single School	
	Mean	Std Dev	Mean	Std Dev
Q1	11.59	11.59	4.44	17
Q2	11	11		1
Q3	11.56	11.56	4.43	16
Q4	12	12	4.50	10
Q5	8.7	8.7	4.35	10
Q6	10.78	10.78	4.26	14

Elements of Deductive Logic

AvgUSM	StdDevUSM
63.41	14.77

Introduction to Philosophy

AvgUSM	StdDevUSM
64.41	5.50

D. COMMENTS ON PAPERS AND ON INDIVIDUAL QUESTIONS

See reports from Mathematics Examiners for Mathematics questions.

Report on Elements of Deductive Logic

Question 1

Generally answered well. Candidates usually gave successful proofs in (a), but the explanations in (b) were often lacking in precision.

Question 2

Generally answered very well, especially (a)-(c). In parts (d)-(f), many answers were a little unclear when laying out their arguments.

Question 3

Generally answered well. Part (b) called for a proof of the DNF theorem, which took a few candidates by surprise. Some answer to (a) and (d) lost marks on rigour or clarity, although most had a rough idea of why the claim should be true. Surprisingly many candidates struggled to give complete answers to (e).

Question 4

Answers varied significantly in quality, with many candidates doing poorly because they did not attempt parts (b) and (c). The system in (b) is complete, because $\phi\ \tau\ \psi$ is both interderivable with and logically equivalent to ϕ – so we can construct a proof for any valid argument in the new language by amending the proof for the corresponding argument in L1. The system in (c) cannot be sound because it allows us to derive, for example, P from Q.

Question 5

Generally answered reasonably well. Common difficulties were: failing to notice that the "union" symbol requires formalization, or omitting from the formalization that something is a member of the union if (not just only if) it is a member of both sets. So (a)(i) should have been rendered as something like $\forall x \forall y \forall z \forall u(Dxy \wedge Dzu \rightarrow \exists v(\forall w(Ewv \leftrightarrow (Ewy \vee Ewu)) \wedge Dxv \wedge Dzv))$.

Misdefining maximal consistent sets

Providing insufficient detail in the proof that every consistent set of L1 sentences can be extended to a maximal consistent one in part (c)(iv).

Question 6

Too few attempts to comment on patterns. The answer to (b) was "no", because we can define Uxy using $\neg Dyxy$. The argument in (c) could be formalized as Paa$\wedge\forall x(\neg Daxa \to a = x)$, $\forall x\forall y\forall z\forall u\forall v\forall w(Pxy \wedge Dzux \wedge Avwz \wedge Cwuy \to Pvw)$, so $\exists x(Pxb \wedge \forall\$yDxyx)$, with b a new constant denoting the conditional $Q \to Q$. For covert premises, we then need Cbb and $\exists x\exists$ y(Dxaa$\wedge Aybx$). With these, the proof is straightforward.

Question 7

Generally done reasonably well. Many candidates missed marks in (a) by failing to spot ambiguities: (a)(i) could mean that there is a journey involving both or that, for each, there is a journey involving only that one; (a)(ii) could mean that London is the only city for which both are options or that London is the only city for which even one is an options. It was disappointing that very few candidates were able to produce the (fairly standard) compactness argument in part (e) to the effect that "you can get from Oxford to Glasgow using only buses" cannot be formalized. The idea was to note that this claim is true only if "you can get from Oxford to Glasgow using exactly n buses" is true for some n. Since the negation of each of these particular sentences can be formalized by part (d), combining an adequate formalization of "only buses" with those negations should be an inconsistent subset. But every finite subset of this set should be consistent, so by compactness the overall set is as well, contradicting the adequacy of the formalization.

Question 8

Too few attempts to comment on patterns.

Question 9

Overall, this question proved quite difficult, although there were also some very good attempts. Good formalizations (especially of the arguments) required quantification over propositions. In this vein, the covert premise in (b)(i) is that the proposition that Moore has hands obviously entails the proposition that Moore is not a handless brain in a vat, formalized as something like Eab. (b)(ii) can be rendered as $\forall x\forall y\forall z(Cxyz \to (Kx \to Ky \wedge Kz)), Kb \to \neg Ka, so \neg Kc$, with a a constant for the proposition P, b a constant for the proposition that P is unknown, and c a constant for their conjunction, and Ccab as a covert premise. The argument in (b)(iii) is valid because it is logically impossible for the premise to be true: if Socrates knew that he knew nothing, he would thereby know something, so it wouldn't be true that he knew nothing, meaning that he couldn't know that he knows nothing.

D. COMMENTS ON PAPERS AND ON INDIVIDUAL QUESTIONS

Report on Mathematics Paper III

Question 1

Part (a) was answered well by most candidates, although a significant minority didn't find y explicitly in terms of x, which was needed to obtain the solution that satisfied the given initial condition. In Part (b) some candidates suggested a variety of unsuccessful substitutions. Again it was expected that the final solution explicitly satisfied the initial condition. There were some really excellent answers to part (c) but too many candidates seemed daunted by the integrations needed, and their solutions weren't simplified sufficiently for them to obtain elegant solutions.

Question 2

This question tested a variety of techniques and overall it was answered very well. Some answers to part (a) and part (b)(ii) were correct but appeared to come out of nowhere; without justification these did not score full marks. In part (d) several solutions had an incorrect range for θ, or, rather, did not justify their non-standard range.

Question 3

Part (a) was answered very well although several candidates did not get the mixed second-order derivative contribution correct. In part (b) the second critical point was sometimes missing or incorrect. Whilst some answers to part (c)(i) were very good, too many students used a Lagrange multiplier without justification, and most solutions did not give the appropriate assumptions requested. Part (c)(ii) was done very well.

Question 4

This question was generally done well. Most people did (a)(i) and (a)(ii) correctly. Common mistakes in (a)(iii) concerned the fact that we need a partition of the sample space Ω rather than the set of values $\mathcal{S}$ taken by the random variable X, and/or restricting to only finite partitions, which loses generality. (b)(i), (b)(ii) and (b)(iii) were again generally done correctly, with the most common errors being calculation slips. Quite a few candidates lost marks for asserting that (iv) was immediate from (iii) by simply forgetting about the conditioning; only those who gave a properly justified argument using the partition theorem for expectations (or equivalent) got full marks here. Most people found (v) straightforward.

Question 5

This seems to have been found harder than the other two probability questions. Many candidates lost the mark in (a)(i) for being too vague: answers which didn't at least specify the probability of a successful trial got 0 marks. (a)(ii) and (a)(iii) were done well, with marks mainly lost for calculation errors. Many students seem to have misunderstood (b). In particular, despite the fact that the question clearly states that N is a random variable, many just assumed that it was always equal to n (which made (b)(iii) particularly confused). Candidates lost marks in (b)(i) for not giving a reasonable justification for their answer. A common error was to say that the parameter of the Bernoulli distribution for X_i was p rather than r. Candidates who did this were not penalised again in the following parts, since it does not render them any easier to set $r = p$. Despite an explicit instruction in the hint that a proof of the random sum formula was not necessary, surprisingly large numbers of candidates opted to waste time proving it regardless (and not always correctly!). However, relatively few candidates gave the clear statement asked for, and many lost marks for not at least mentioning that $X_1, X_2, \ldots$ need to be i.i.d. and also independent of N. (b)(iii) was done correctly by only a small minority, and only an even smaller minority gave a complete justification involving the uniqueness of the p.g.f.; the others lost one mark. Substantial partial credit was awarded in part (c) for spotting how it maps onto the set-up in (b), but only a small number of candidates gave a full and correct justification. Others performed direct calculations to obtain the distribution, which got full marks if correct, but that was the case only for a relatively small number of people.

Question 6

This question was done well. Most people did (a)(i) and (a)(ii) correctly. In (a)(iii), full credit was only given for answers which mentioned that we may use countable additivity because we have a union of a countable number of disjoint events (or equivalent); many people simply ignored the instruction to justify their argument carefully. (b)(i) and (b)(ii) were done correctly by most people, with marks most commonly lost for sign errors in the integration, which might have been caught by sanity-checking: a density function should not be negative, and the expectation of a positive random variable cannot be negative! (b)(iii) was again done well in many cases, with the most common error being in the manipulation of the floor function. Only a tiny minority of students thought to use $\mathbb{E}[\lfloor X \rfloor] = \sum_{k \geq 1} \mathbb{P}(\lfloor X \rfloor \geq k)$ in (b)(iv); the more complicated calculation involving the probability mass function was done correctly by a substantial fraction but far from all candidates; partial credit was given for sensible assertions involving the zeta function but an incomplete calculation. In (b)(iv), many people correctly got some elements of the answer, and received partial credit; a smaller number saw their way through to a complete argument, and full marks were only given if, for example, there was some sensible justification of the statement that $\mathbb{E}[\lceil X \rceil] = \mathbb{E}[\lfloor X \rfloor] + 1$.

Question 7

This question was generally done well. Some people struggled to remember the definitions of bias and MSE in (a)(ii), and this fed through into (b)(ii). Most people calculated the MLE correctly in (a)(iii), and about half checked that their estimator maximised the likelihood. Either graphical or calculus justifications were accepted here, but if they were absent a mark was lost. Many people spent a long time deriving the expectation in (b)(i) by induction, rather than simply using the gamma density given in the question. Unfortunately the inductive method was much slower and had more scope for calculation errors. In (b)(ii), marks were commonly lost for unjustified calculation steps. In (b)(iii), some people struggled to get the confidence interval the right way round, and a substantial number lost marks for not saying they were using the CLT.

Question 8

This question seems to have been perceived as hard. Many seem to have found it difficult to get started, perhaps because of the long preamble. Part (a) was generally done well, though. The main ways in which marks were lost here were either forgetting the factor of $1/\sqrt{n}$ in endpoints of the confidence interval, getting confused about which quantile of $N(0,1)$ to use, or incorrect manipulations of inequalities. In part (b), very few people used the hints given in the preamble to the question. In (b)(i), people who correctly remembered the definition of covariance usually managed to get the independence, but very few people then specified the marginal distributions as required in the question. Most people who reached (b)(ii), knew what they had to show, but got tangled up in the calculations of means and variances. Few people made serious attempts at (b)(iii). Those who were able to do the first step where one extracts $X_1 - \bar{X}$ from the sum found the rest fell quickly into place. Others wasted time expanding the squares and sums, and getting confused in the calculations. Sometimes the final deduction was missing.

Question 9

This seems to have been found hard. In parts (a)(i) and (a)(ii), many people clearly hadn't memorised the estimators, so were deriving them blind; this led to quite a lot of errors which might have been avoided. Many people failed to notice that $\sum_{i=1}^{n} x_{i1} = 0$ is given in the statement of the question, which considerably simplifies the calculations. Quite a few people just set up the simultaneous equations and then said "solve for this" rather than actually solving. This was perhaps a consequence of time pressure, but obviously couldn't be awarded the marks! In (a)(iii), many people correctly stated that the variables could be correlated, but didn't then say why this meant that the interpretation might be problematic; those lost one mark. In (b)(i), most people were unable to give a convincing account of the purpose of PCA, and there was a common confusion between PCA and clustering. Parts (b)(ii) and (iii) were done well. Full marks were only awarded in (b)(ii) for solutions which mentioned the role played by the variance. The reparametrisation in (c)(i) was generally fine, but there were very few convincing answers to (c)(ii): for the advantages, not many people made the connection to (a)(iii) and, for the disadvantages, very few mentioned interpretability.

Report on Introduction to Philosophy

Mean:

Standard deviation:

Section A: General Philosophy

Please see the Examiners' Report for the Preliminary Examination in Philosophy, Politics, and Economics for detailed discussion of individual questions on Section A.

Section B: Frege and the Foundations of Arithmetic

Questions 10 and 11 were the most popular on this part of the exam; the other questions all received between one and three answers. Answers were generally competent; they were primarily differentiated by how successfully they engaged with the details of the question asked. There were, however, a few cases in which candidates displayed quite limited knowledge of the material by failing to acknowledge, for example, Frege's discussion of the argument in Question 9 or the fact that the "direction principle" from Question 12 – unlike Hume's Principle – places no interesting constraints on the size of the domain.

Question 10 *'Frege is right that number-ascriptions assign properties to concepts, rather than to objects. But Mill is right that our knowledge about numbers is ultimately empirical.' Is there any tension in this assessment? Is it correct?*

Candidates were quite able when discussing the two parts of the quoted claim separately, but found it more difficult to directly address the question about potential tensions. Better answers tried to think of some reasons why there might be a tension and evaluate these directly, rather than spending a lot of time surveying Frege's objections to Mill.

Question 11 *'Hume's Principle does not settle whether Julius Caesar is a number. But it doesn't matter to arithmetic whether he is. So this does not impugn the status of Hume's Principle as a foundation for arithmetic.' Is this right?*

This question received quite a range of answers, including quite a few very good ones. Strong answers typically paid some attention to what we might want out of a "foundation for arithmetic", and often showed some awareness of the connection between the attitude in the quote and structuralism about mathematics.

Question 12

Question 13

E. COMMENTS ON PERFORMANCE ON IDENTIFIABLE INDIVIDUALS

E. NAMES OF MEMBERS OF THE BOARD OF EXAMINERS

Prof. Andrew Dancer (Chair), Prof. Adam Caulton, Prof. Bernhard Salow, Prof. Tom Sanders