Examiners' Report: Final Honour School of Mathematics Part C Trinity Term 2022

October 3, 2022

Part I

A. STATISTICS

- **Numbers and percentages in each class**

  See Table 1, page 1.

- **Numbers of vivas and effects of vivas on classes of result.**
  As in previous years there were no vivas conducted for the FHS of Mathematics Part C.

- **Marking of scripts.**
  The dissertations and mini-projects were double marked. The remaining scripts were all single marked according to a pre-agreed marking scheme which was very closely adhered to. For details of the extensive checking process, see Part II, Section A1.

- **Numbers taking each paper.**
  See Table 7 on page 8.

Table 1: Numbers in each class (post-2021 classification)

	Number		Percentages %	
	2022	(2021)	2022	(2021)
Distinction	46	(60)	58.23	(60)
Merit	19	(20)	24.05	(20)
Pass	14	(18)	17.72	(18)
Fail	0	(2)	0	(20)
Total	79	(100)	100	100

Table 2: Numbers in each class (pre-2021 classification)

	Number					Percentages %				
	2020	(2019)	(2018)	(2017)	(2016)	2020	(2019)	(2018)	(2017)	(2016)
I	63	(58)	(53)	(48)	(44)	67.74	(57.43)	(56.99)	(57.14)	(50.57)
II.1	30	(40)	(26)	(23)	(31)	32.26	(39.6)	(27.96)	(27.38)	(35.63)
II.2	0	(2)	(13)	(12)	(9)	0	(1.98)	(13.98)	(14.29)	(10.34)
III	0	(1)	(1)	(1)	(3)	0	(0.99)	(1.08)	(1.19)	(3.45)
F	0	(0)	(0)	(0)	(0)	0	(0)	(0)	(0)	(0)
Total	93	(101)	(93)	(84)	(87)	100	(100)	(100)	(100)	(100)

B. Changes in examining methods and procedures currently under discussion or contemplated for the future

C. Notice of examination conventions for candidates

The first notice to candidates was issued on 12th April 2022 and the second notice on 11th May 2022. These contain details of the examinations and assessments.

All notices and the examination conventions for 2022 examinations are on-line at http://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments.

Part II

A1. General Comments on the Examination

The examiners would like to convey their grateful thanks for their help and cooperation to all those who assisted with this year's examination, either as assessors or in an administrative capacity. The chair would like to thank Anwen Amos, Clare Sheppard, Charlotte Turner-Smith, Waldemar Schlackow, Matt Brechin and the rest of the academic administration team for their support of the Part C and OMMS examinations.

In addition the internal examiners would like to express their gratitude to Prof Alan Champneys and Prof James Robinson for carrying out their duties as external examiners in a constructive and supportive way during the year, and for their valuable input at the final examiners' meetings.

Timetable

The examinations began on Monday 10th May and finished on Friday 10th June.

Mitigating Circumstances Notice to Examiners and other special circumstances

A subset of the examiners (the 'Mitigating Circumstances Panel') attended a pre-board meeting to band the seriousness of the individual notices to examiners. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate. See Section E for further details.

Setting and checking of papers and marks processing

Following established practice, the questions for each paper were initially set by the course lecturer, with the lecturer of a related course involved as checker before the first draft of the questions was presented to the examiners. The course lecturers also acted as assessors, marking the questions on their course(s).

The internal examiners met in early January to consider the questions on Michaelmas Term courses, and changes and corrections were agreed with the lecturers where necessary. The revised questions were then sent to the external examiners. Feedback from external examiners was given to examiners, and to the relevant assessor for each paper for a response. The internal examiners met a second time late in Hilary Term to consider the external examiners' comments and assessor responses (and also Michaelmas Term course papers submitted late). The cycle was repeated for the Hilary Term courses, with the examiners' meetings in the Easter Vacation. Before questions were submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions.

Exams returned to in-person and were held in the Exams Schools. Papers were collected by the Academic Administration team and made available to assessors approximately half a day following the examination. Assessors were made aware of the marking deadlines ahead of time and all scripts and completed mark sheets were returned, if not by the agreed due dates, then at least in time for the script-checking process.

A team of graduate checkers, under the supervision of Anwen Amos, Clare Sheppard and Charlotte Turner-Smith, reviewed the mark sheets for each paper of this examination, carefully cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Sub-totals for each part were also checked against the mark scheme, noting correct addition. In this way a number of errors were corrected, with each change approved by one of the examiners who were present throughout the process.

Determination of University Standardised Marks

The Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average in each class over the last four years, together with recent historic data for Part C, the MPLS Divisional averages, and the distribution of classifications achieved by the same group of students at Part B.

The examiners followed established practice in determining the University standardised marks (USMs) reported to candidates. This leads to classifications awarded at Part C broadly reflecting the overall distribution of classifications which had been achieved the previous year by the same students.

We outline the principles of the calibration method.

Firstly, note that the Department's algorithm to assign preliminary USMs for consideration and review by the examiners has been updated this year to decrease processing times and the overall workload and these changes will be highlighted below. In addition, papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part B classification of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage).

The algorithm converts raw marks to USMs for each paper separately (in each case, the raw marks are initially out of 50, but are scaled to marks out of 100). For each paper, the algorithm sets up a map $R \to U$ (R = raw, U = USM) which is piecewise linear and automatically contains the points $(0, 0)$, $(100, 100)$. While the previous scaling process assigned two vertices for the preliminary scaling map using Part B marks, the new algorithm only uses the number of firsts achieved at Part B to assign one vertex. This vertex, P_3, is placed at 72 USM with an associated raw mark that ensures that the number of 1st class Part C on the paper after scaling is the same as the number of Part C candidates taking the paper with a 1st class in Part B. The vertex P_3 is then joined to (0,40) by a line segment, with a further vertex, P_2, placed at 57 USM on this line segment. The vertex P_2 is then joined by a new line segment to (0,10), and an additional vertex, P_1, is placed at 37 USM on the new line segment. The points $(0, 0)$, P_1, P_2, P_3, $(100, 100)$ provide the piecewise linear map for each paper's preliminary scaling map. The results of the algorithm with the above default settings of the parameters provide the starting point for the determination of USMs.

Following customary practice, a preliminary, non-plenary, meeting of examiners was held two days ahead of the plenary examiners' meeting to assess the results produced by the algorithm alongside the reports from assessors. The examiners reviewed each paper and report, and discussed the preliminary scaling maps and the preliminary class percentage figures. The examiners have scope to make changes, usually by adjusting the position of the vertices P_1, P_2, P_3 by hand, so as to alter the map raw $\to$ USM, to remedy any perceived unfairness introduced by the algorithm, in particular in cases where the number of candidates is small. They also have the option to introduce additional vertices. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low. These revised USM maps provided the starting point for a review of the scalings, paper by paper, by the full board of examiners.

Table 3: Vertex positions for the piecewise linear scaling maps, P_1, P_2, P_3 with the raw marks rescaled to be out of 50. The entries N_3, N_2, N_1 respectively give the number of incoming firsts, II.1s, and II.2s and below from Part B for that paper.

Paper	P_1	P_2	P_3	Additional corners	N_3	N_2	N_1
C1.1	12.7;37	22.1;57	41.6;72	50;100	1	4	2
C1.2	14;37	20;50	24.37;57	41;70	1	2	2
C1.3	13;50	20.4;57	35;72	50;100	3	7	2
C1.4	11.39;37	19.83;57	31;70	50;100	1	2	2
C2.1	10;50	19;65	24;70	50;100	2	3	0
C2.2	16;57	28.8;72	50;100		8	3	0
C2.3	9.44;37	26;65	30;70	50;100	0	1	0
C2.4	11.39;37	19.83;57	36;72	50;100	3	3	0
C2.5	20;50	30;65	35;70	50;100	0	0	0
C2.6	7.81;37	13.6;57	25.6;72	50;100	5	1	0
C2.7	9.77;37	17;57	35;72	50;100	10	7	0
C3.1	12;50	34;72	50;100		9	7	0
C3.2	11.39;37	19.83;57	37.33;72	50;100	4	3	1
C3.3	9.77;37	17;57	32;72	50;100	5	6	1
C3.4	9.77;37	17;57	32;72	50;100	12	6	0
C3.5	11.07;37	19.27;57	36.27;72	50;100	3	2	1
C3.7	11.39;37	19.83;57	37.33;72	50;100	12	8	0
C3.8	8.46;37	14.73;57	33;72	50;100	10	7	1
C3.10	9.11;37	15.87;57	31;70	50;100	5	6	0
C3.11	14.65;37	25.5;57	42;72	50;100	1	3	0
C3.12	13.35;37	23.23;57	40;70	50;100	2	2	0
C4.1	7.49;37	15;55	25;70	50;100	5	4	0
C4.3	9.11;37	23;60	29.87;72	50;100	5	2	0
C4.6	12.37;37	21.53;57	37;72	50;100	3	2	0
C4.8	5.86;37	15;55	18;65	21;70	2	1	0
C4.9	8.79;37	18;50	25;65	30;70	2	0	0

Paper	P_1	P_2	P_3	Additional corners	N_1	N_2	N_3
C5.1	12.7;37	22.1;57	38;72	50;100	6	4	0
C5.2	8.79;37	15;50	28.8;72	50;100	8	4	0
C5.3	11.39;37	26;55	34;70	50;100	0	2	0
C5.5	10.42;37	20;52	30;70	50;100	12	10	0
C5.6	14.32;37	29;60	42;70	50;100	11	5	0
C5.7	12.37;37	21.53;57	40.53;72	50;100	8	8	0
C5.9	12.37;37	25;55	37;72	50;100	3	2	0
C5.11	12.37;37	23;50	40.53;72	50;100	9	12	1
C5.12	11.39;37	22;50	37.33;72	50;100	13	9	1
C6.1	8.79;37	16;50	28.8;72	50;100	8	9	4
C6.2	11.07;37	22;50	36.27;72	50;100	8	10	1
C6.3	12.37;37	20;50	38;72	50;100	5	5	0
C6.4	11.07;37	25;55	33;70	50;100	0	3	1
C7.4	14.32;37	29;50	44;70	50;100	1	3	3
C7.5	12;50	22;60	30;70	50;100	0	1	0
C7.6	15;50	30;70	50;100		0	1	0
C7.7	13.02;37	25;52	37;70	50;100	2	4	1
C8.1	9.77;37	18;50	32;72	50;100	5	10	0
C8.2	9.44;37	15;50	30.93;72	50;100	2	3	0
C8.3	12.7;37	22;55	35;70	50;100	7	15	3
C8.4	18;50	22.67;57	35;70	50;100	3	15	1
C8.6	25;54	30;64	34;70	50;100	0	1	0
SC1	12.7;37	22.1;55	41.6;72	50;100	14	18	1
SC2	12.37;37	23;50	40.53;72	50;100	15	27	2
SC4	16;50	19.27;57	33;70	50;100	13	17	4
SC5	9.77;37	17;57	32;72	50;100	12	16	0
SC6	14;50	19.27;57	36.27;72	50;100	11	14	1
SC9	10.09;37	23;62	33.07;74	50;100	8	7	0
SC10	12.7;37	26;57	36;70	50;100	5	5	2

B. Equality and Diversity issues and breakdown of the results by gender

Table 5: Breakdown of results by gender

Class	Number					
	2022			2021		
	Female	Male	Total	Female	Male	Total
Distinction	10	36	46	15	45	60
Merit	9	10	19	8	12	20
Pass	5	9	14	5	13	18
Fail	0	0	0	1	1	2
Total	24	55	79	29	71	100

Class	Percentage					
	2022			2021		
	Female	Male	Total	Female	Male	Total
Distinction	40	65.45	58.23	51.72	63.38	60
Merit	36	18.18	24.05	27.59	16.9	20
Pass	20	16.36	23.73	17.24	18.31	18
Fail	0	0	0	3.45	1.41	2
Total	100	100	100	100	100	100

Table 6: Breakdown of results by gender (pre-2021 classification)

Class	Number								
	2020			2019			2018		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	16	47	63	8	50	58	6	47	53
II.1	4	26	30	9	31	40	7	19	26
II.2	0	0	0	0	2	2	3	10	13
III	0	0	0	0	1	1	1	0	1
F	0	0	0	0	0	0	0	0	0
Total	20	73	93	17	84	101	17	76	93

Class	Percentage								
	2020			2019			2018		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	80	64.38	72.19	47.06	59.52	57.43	35.29	61.84	56.99
II.1	20	35.62	27.81	52.94	36.9	39.6	41.18	25	27.96
II.2	0	0	0	0	2.38	1.98	17.65	13.16	13.98
III	0	0	0	0	1.19	0.99	5.88	0	1.08
F	0	0	0	0	0	0	0	0	0
Total	100	100	100	100	100	100	100	100	100

C. Detailed numbers on candidates' performance in each part of the exam

Data for papers with fewer than six candidates are not included in the public versions of the examiners' report.

Table 7: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
C1.1	7	29.43	8.12	61.43	8.16
C1.2	-	-	-	-	-
C1.3	12	29.08	9.55	67.17	11.89
C1.4	-	-	-	-	-
C2.1	-	-	-	-	-
C2.2	11	29.91	9.64	73.55	12.8
C2.3	-	-	-	-	-
C2.4	6	33.17	4.36	69.67	4.41
C2.6	6	27.17	11.18	72.17	15.77
C2.7	17	31.47	8.37	70.76	10.06
C3.1	16	30.81	11.62	71.12	14.97
C3.2	8	35.38	4.81	72.12	7.41
C3.3	12	27	8.06	67.08	10.54
C3.4	18	31	9.19	71.72	13.17
C3.5	6	30.83	8.13	68.33	9.31
C3.7	20	33.3	8.52	70.15	10.91
C3.8	16	26.06	9.72	66.38	11.69
C3.10	11	27.36	4.88	67.36	5.18
C3.11	-	-	-	-	-
C3.12	-	-	-	-	-
C4.1	9	22.89	6.45	66.44	8.99
C4.3	7	30.14	5.79	72	9.4
C4.6	-	-	-	-	-
C4.8	-	-	-	-	-
C4.9	-	-	-	-	-
C5.1	10	38.5	5.76	76	8.93
C5.2	12	30.33	8.22	74.36	11.73
C5.3	-	-	-	-	-
C5.5	19	31.21	5.91	72.44	9.06
C5.6	16	42.75	7.33	81.31	14.18
C5.7	16	37.75	5.42	71.88	8.44
C5.9	-	-	-	-	-
C5.11	19	36.95	6.1	69.05	10.11
C5.12	18	36.83	5.77	72.89	10.29
C6.1	11	25.45	5.84	67	8.83
C6.2	12	29.58	6.14	63.36	9.1
C6.3	8	37.62	8.45	78	10.6
C6.4	-	-	-	-	-

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
C7.4	7	38.14	5.58	62.29	7.52
C7.5	-	-	-	-	-
C7.6	-	-	-	-	-
C7.7	-	-	-	-	-
C8.1	9	25.22	7.87	61.56	12.53
C8.2	-	-	-	-	-
C8.3	18	34.17	6.44	71	10.37
C8.4	14	31.64	7.19	67.93	10.22
C8.6	-	-	-	-	-
SC1	9	37.56	6.33	72.25	8.48
SC2	8	34.62	4.34	66.14	5.37
SC4	7	30	7.77	68.86	8.97
SC5	-	-	-	-	-
SC6	-	-	-	-	-
SC9	6	31.83	4.96	72.83	6.31
SC10	-	-	-	-	-
SC8	-	-	-	-	-
CCS1	-	-	-	-	-
CCS2	-	-	-	-	-
CCS4	-	-	-	-	-
C3.9	-	-	-	-	-
C5.4	10	-	-	74.44	8.78
C6.5	7	-	-	62.57	5.62
CCD	75	-	-	74.44	8.78
COD	-	-	-	-	-

The tables that follow give the question statistics for each paper for Mathematics candidates. Data for papers with fewer than six candidates are not included in the public version of the report.

Paper C1.1: Model Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	9.6	11.75	6.99	4	1
Q2	17.14	17	3.34	7	0
Q3	13	13	8.72	3	0

Paper C1.3: Analytic Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.67	11.67	6.15	12	0
Q2	17.42	17.42	4.7	12	0
Q3	2		2.83	0	2

Paper C2.2: Homological Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.73	16.73	4.98	11	0
Q2	12.83	12.83	7.13	6	0
Q3	13.6	13.6	8.56	5	0

Paper C2.4: Infinite Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	2.5	15	6.12	1	5
Q2	19.67	19.67	2.94	6	0
Q3	13.2	13.2	2.59	5	0

Paper C2.6: Introduction to Schemes

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.5	16.5	5.75	6	0
Q2	8.5	8.5	5.92	4	0
Q3	15	15	8.49	2	0

Paper C2.7: Category Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.2	13.2	3.19	10	0
Q2	14.07	14.07	3.88	15	0
Q3	19.8	21.33	5.63	9	1

Paper C3.1: Algebraic Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14	14	7.21	15	0
Q2	13	16	8.23	8	2
Q3	15.5	17.22	7.49	9	1

Paper C3.2: Geometric Group Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.5	17.71	6.46	7	1
Q2	7.63	20.33	10.53	3	5
Q3	14	16.33	7.37	6	1

Paper C3.3: Differentiable Manifolds

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.1	15.1	4.75	10	0
Q2	12.1	12.1	3.98	10	0
Q3	11.2	13	5.89	4	1

Paper C3.4: Algebraic Geometry

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.33	16.33	4.12	18	0
Q2	14.67	14.67	5.41	15	0
Q3	8	14.67	7.9	3	4

Paper C3.5: Lie Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.67	19.67	4.89	6	0
Q2	11.17	11.17	4.54	6	0

Paper C3.7: Elliptic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.44	17.44	4.03	16	0
Q2	14.88	14.88	2.95	8	0
Q3	16.75	16.75	6.65	16	0

Paper C3.8: Analytic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.06	12.87	5.42	15	1
Q2	11.63	13.29	7.71	14	2
Q3	2.63	12.67	5.3	3	13

Paper C3.10: Additive and Combinatorial Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.33	15.33	3.54	9	0
Q2	12.25	13.57	6.45	7	1
Q3	11.33	11.3	3.14	6	0

Paper C4.1: Further Functional Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.75	10.75	5.12	8	0
Q2	11.25	12.14	3.37	7	1
Q3	11.67	11.67	3.06	3	0

Paper C4.3: Functional Analytical Methods for PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.57	15.57	2.57	7	0
Q2	15	15	6	4	0
Q3	14	14	3.46	3	0

Paper C5.1: Solid Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.89	19.89	2.2	9	0
Q2	16	18.86	6.96	7	2
Q3	12.57	18.5	9.11	4	3

Paper C5.2: Elasticity and Plasticity

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.2	18.2	4.32	5	0
Q2	13.92	14.73	5.3	11	1
Q3	12.67	13.88	6.2	8	1

Paper C5.5: Perturbation Methods

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.31	16.31	2.68	16	0
Q2	12	14.09	5.28	11	4
Q3	16.09	16.09	3.18	11	0

Paper C5.6: Applied Complex Variables

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	22.4	22.4	2.61	15	0
Q2	13.17	14.6	5.46	5	1
Q3	22.92	22.92	2.75	12	0

Paper C5.7: Topics in Fluid Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.71	17.71	1.89	7	0
Q2	20.83	20.83	2.29	12	0
Q3	17.69	17.69	4.23	13	0

Paper C5.11: Mathematical Geoscience

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16	16	5.29	7	0
Q2	19.11	19.11	3.03	19	0
Q3	18.92	18.92	3.5	12	0

Paper C5.12: Mathematical Physiology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.17	18.06	4.79	17	1
Q2	14	14	4.43	6	0
Q3	20.92	20.92	3.2	13	0

Paper C6.1: Numerical Linear Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	9.8	12	5.17	4	1
Q2	13.36	13.36	3.75	11	0
Q3	10.56	12.14	4.9	7	2

Paper C6.2: Continuous Optimization

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.73	15.3	3.13	10	1
Q2	12	13.57	6.05	7	1
Q3	16.13	15.29	4.82	7	1

Paper C6.3: Approximation of Functions

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21	21	3.06	7	0
Q2	13	13	5.1	5	0
Q3	22.25	22.25	2.22	4	0

Paper C7.4: Introduction to Quantum Information

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.71	17.71	3.9	7	0
Q2	20.6	20.6	4.22	5	0
Q3	20	20	2.83	2	0

Paper C8.1: Stochastic Differential Equations

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	9.71	9.71	5.47	7	0
Q2	15.11	15.11	5.25	9	0
Q3	11.5	11.5	2.12	2	0

Paper C8.3: Combinatorics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.53	19.21	4.34	14	1
Q2	15.19	16.2	5.22	15	1
Q3	14.44	14.71	2.74	7	2

Paper C8.4: Probabilistic Combinatorics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.46	17.46	3.55	13	0
Q2	12	12	8.54	3	0
Q3	15	15	3.19	12	0

Paper SC1: Stochastic Models in Mathematical Genetics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.2	17.2	5.07	5	0
Q2	17.71	20	6.92	6	1
Q3	18.86	18.86	2.91	7	0

Paper SC2: Probability and Statistics for Network Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.17	18.17	4.17	6	0
Q2	14.88	15.86	3.87	7	1
Q3	19	19	5.2	3	0

Paper SC9: Interacting Particle Systems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.67	15.67	4.08	6	0
Q2	16.17	16.17	4.62	6	0

D. Recommendations for Next Year's Examiners and Teaching Committee

Projects, academic poor practice and plagiarism. The process for dealing with prospective academic poor practice and plagiarism should be formalised. The inclusion of a review of potential cases of academic poor practice and plagiarism should be included in the tasks for the examiners that are agreed at the first examiners' meeting, with a deadline date allowing for advice from assessors and in good time for Proctorial referral if necessary. Chairs from other examination boards should be involved with cases involving students from their programme, as the Chair of the relevant board is designated as responsible for referring prospective plagiarism to the Proctors.

Scaling. The new scaling algorithm worked as well as one could expect, and is a fundamental improvement from previous years. In the examiners' meeting scatterplots of candidates' mean USM from Part B vs their preliminary USM mark on the paper under consideration, together with a linear fit, proved to be most useful in refining the scalings. If possible given the pressures on the academic administration during the examination process, having this data available for the scaling pre-meeting would be beneficial.

Summary sheets Discussions among the examiners indicated an overall opinion that one page summary sheets for examinations should be discontinued.

Examiners' plenary meeting There is significant time pressure in the examiners' meeting given it is a single day and the need to consider scalings and academic poor practice, together with external examiner enquiries. Timetabling a contingency that allows for the prospect of the meeting continuing briefly into the next day without immediately conflicting with other scheduled commitments, in particular other examination boards, would be welcome.

Calculators. In one module where calculators were allowed, some candidates were reported to have given general expressions rather than exact answers to a part of one question due to not possessing a calculator. Thus, further highlighting which, if any, courses are allowed calculators to candidates in any given year may be beneficial.

On the distinction/merit/pass/fail system. The MMath Part C recently moved to distinction/merit/pass/fail rather than I/II.1/II.2/III/pass/fail. The rule that candidates who fail one paper (USM < 50) are not eligible for a merit or distinction would merit review.

The class boundaries, in particular the Merit and Fail boundaries still should continue to be expressly highlighted to assessors when setting their examinations.

Auto-plagiarism. Auto-plagiarism, or the analogous poor practice, between the undergraduate dissertation and the mini-project was noted with multiple candidates this year. Explicit guidance was distributed to students this year that auto-plagiarism would be unacceptable, including within the documentation for both the projects and mini-projects; however, this point should be further highlighted in the project and mini-project documentation.

Project Deadlines As noted by one of the external examiners, there are discrepancies in the treatment of deadline penalties between the Mathematical Institute and the Computer Science department, generating an uneven treatment of late penalties across the cohort, which should be corrected if possible.

E. Comments on papers and on individual questions

The following comments were submitted by the assessors.

C1.1: Model Theory

1) Surprisingly basic mistakes dominated. In a number of papers, the distinction between types, partial types, and even formulas was not understood. It was unfortunately di cult to recover from this, as parts (a,b) are mostly about classifying types (though some did manage to recover.) Another confusion was between types or formulas in n variables, and in a single variable; for example in the definition of an atomic model, and in the Ryll-Nardjewski theorem, it does not suffice to check the condition for 1-types.

In a large number of papers, the types were correctly described but with no indication of why they are indeed (complete) types. There are two ways to approach this: via direct constructions of automorphisms or via quantifier-elimination; in the latter case, it is legitimate to explain that quantifier elimination is inherited from the known case of DLO. Both these routes were taken successfully in some papers.

2) Proved to be easier than (1), and well done on the whole. Requires understanding the proof of the Los-Vaught test and basic concepts on omitting and realising types. (b) hinges on the fact that a non-principal type for a theory T remains non-principal for any $T' \supset T$; this is very easy but must be stated in some form, as was not always done.

3) Problem 3 was less popular. (a) was a 'book' question, while (b) required adapting the idea the proof of primality of atomic models to a more general setting. Both were generally done well by those who chose to answer this question. (c) requires, in addition, recognizing the assumption as asserting that there is exactly one 1-type.

C1.2: Gödel's Incompleteness Theorems

Question 1. is about Gödel-Löb logic, and many candidates had a good deal of success with it.

In part (a), about fixed points for syntactic operators, parts (i), (ii) and (iii) can be done more or less mechanically using an algorithm described in lectures. There's a short cut in part (iii): because of the Gödel-Löb version of the Second Incompleteness Theorem, $A(p)$ is provable, so $\top$ is a fixed point. Part (iv) can be done by noting that $A(p)$ is a propositional contradiction, so $\perp$ is a fixed point.

Part (b) is about a semantics of modal logic in which one of the axioms of Gödel-Löb logic is not true, and about how much of a difference that makes. It was vital, then, not to assume the axioms of Gödel-Löb logic; the set-up was designed to make that error as untempting as possible (a few candidates misunderstood it in various ways). In parts (i),

(ii) and (iii) we find that the rules and all but one of the axiom schemes of Gödel-Löb logic work in this semantics; in part (iv) we find that the final axiom scheme does not; and we note in part (v) that the Gödel-Löb version of the First Incompleteness Theorem is not true.

In question 2., part (a) is looking for weak conditions under which some version of the First Incompleteness Theorem can be proved. Various candidates encountered difficulties of various sorts. Rosser's Theorem can be adapted to this setting.

Many candidates had reasonable success with part (b). The last part is really about the Gödel sentence, which can be taken to be Π_2, so that its negation is Σ_2.

Question 3. is largely about n-consistency and ω-consistency. People had a great deal of success with parts (a) and (b), and found (c) to be rather harder. Part (c)(i) is about how to express 1-consistency, and proved tricky. Part (ii) proved to be very tricky, and the solution is counterintuitive: if we add to S a statement ϕ expressing that $S \cup \{\phi\}$ is 1-*in*consistent, then $S \cup \{\phi\}$ is 1-consistent (note the switch of polarity), but false in the natural numbers, and so not ω-consistent (because ϕ can be taken to be Σ_2).

After two years of online teaching, it was business as usual this year (with the addition of a bit of technology). I cannot speak on behalf of the students, but for this member of staff, it was a breath of fresh air, and a reminder not to take things for granted.

C1.3: Analytic Topology

Question 1 gives parts of the proof of the Hahn-Mazurkiewicz Theorem, which characterises Hausdorff continuous images of the closed interval $[0,1]$. (This theorem was, many years ago, part of the Analytic Topology syllabus.)

Part (a) contained parts that many students tackled with success. The most difficult part was second-countability: the trick here is that if U is an open subset of $[0,1]$, then $f^*(U)$ will be open but may also be empty. What one needed to do was to choose a countable basis $\mathcal{B}$ for $[0,1]$, and the countable basis for X then contains sets $f^*(U)$ where U is a finite union of elements of $\mathcal{B}$: by properness, some such U can be found such that for any $x \in X$ and open $V \ni x$, $f^{-x}(\{x\}) \subseteq U \subseteq f^{-1}(V)$, so that $x \in f^*(U) \subseteq V$.

Part (b) was found to be difficult, but many candidates at least spotted approximately what had to be done.

Question 2. was about the Wallman compactification of T_4 spaces, which is equivalent to the Stone-Čech compactification (though candidates were not asked to prove this).

In part (a)(i), most people spotted that a minimal adaptation of the proof that every filter can be extended to an ultrafilter would do the trick. There were also many successful attempts at parts (ii) and (iii). A few people assumed that if p is a closed ultrafilter, then $A \in p$ or $X \setminus A \in p$, which only works if A is clopen.

Part (b) was found to be harder. In part (i), it is necessary to use normality of X, because if γX is Hausdorff, then because it is compact it is normal, then from part (ii) we can deduce that X is normal. And Hausdorffness was perhaps the most difficult condition to prove.

Very few people attempted question 3., which is about the Tychonoff corkscrew, a T_3 topological space that is not Tychonoff.

After two years of remote teaching and online exams, we now have teaching more or less as normal and exams as normal with a significant change (which I will come to). I hope the experience has been as transformative for the students as it has been for the staff (or at least this member of it). The pandemic has not gone away; the world has not become a much easier place to live in; but we appreciate a bit more things we once took for granted.

The significant change is that candidates were allowed to take in an A4 sheet of revision material.

As far as I could tell (on a small sample size), this did not make much difference to people's performance in exams. If this impression is correct, I can think of three explanations, which are not mutually exclusive. (1) You cannot fit much on an A4 piece of paper. (2) Revision has always been about understanding, and ability to use methods, than about rote learning. Or (3): efficient ways of compressing information onto an A4 sheet of paper need to be identified and taught.

C1.4: Axiomatic Set Theory

Questions 2 and 3 were more popular than question 1, but the marks on question 1 were not systematically lower than on the other questions.

A common oversight at various points was lack of mention of absoluteness, especially if this was not explicit.

Question 1 The bookwork was generally well done with some candidates not saying why the Mostowski Collapse was well-defined and injective.

A number of candidates were stumped by the tricky direction of part (b), assuming (incorrectly) that $V = L$ and trying to use $L_\kappa = H_\kappa$ instead of adjusting the proof of (a) to the new situation. Most candidates were not sufficiently clear why we want the transitive closure of the parameters in the elementary submodel obtained from the Löwenheim-Skolem Theorem or forgot to Mostowski collapse the submodel.

There were a number of good ideas for (c) but often the formula given involved cardinality and it was not made clear why 'being a cardinal' and hence $\aleph_1$ was absolute between ghe classes.

Question 2 In part (a) a lot of candidates did not mention absoluteness (for the basic axioms) or forgot to relativize the formula for **Separation** and **Replacement**. In the proof that A satisfies **Infinity** candidates did not mention that the successor operation was absolute: this is not relevant if they showed that $\omega^V \in A$, but if they took an inductive (in V) z and tried to show that $z \cap A$ is inductive in A then it is needed.

In part (b) finding the right formula to show that **Induction** implies **Foundation** was trickier than expected given that the obvious 'every non-empty subset of x has an $\in$-minimal element' cannot (easily) be shown to satisfy the premise of the Induction scheme. Most successful attempts in this direction were via showing that Induction implies $\forall x\ x \in V$, although some candidates had other approaches which could be made to work.

As expected, the **Foundation** implies **Induction** direction was easier, but not all candidates ensured that they took a minimal element of a set of counterexamples (as opposed to the class of counterexamples).

In part (c) most candidates struggled to find a correct pairing function, often given a formula which did not satisfy the second condition.

Question 3 Part (a)(i) and (ii) were generally well done, although sometimes the approaches were very lengthy. In (a)(iii) the main oversights were not remarking that $On^A \subseteq On$ (so that the statement makes sense) or not remarking that 'being a limit' and 'being the successor of α' are absolute: this is necessary as the construction of V_α in A depends of course on whether α is a successor or limit in A. Most candidates did remark that the union in the limit step was absolute.

In part (b) a number of candidates tried to give a proof without using that C is a proper class which is unlikely to work.

In part (c) the correct formalization why $C \neq V$ was tricky, although a number of candidates described the correct approach that if $C = V$ satisfies **ZF** then a 'diagonal' set which should exist by **Replacement** does not exist in C.

C2.1: Lie Algebras

Overall the paper appears to have been challenging for candidates, but most managed to demonstrate good understanding of the subject through the answers they submitted.

Question 1 : The question was attempted by most students. The part a) was well answered, and in part b) most candidates established that $\overline{\mathfrak{h}}$ was nilpotent, but fewer managed to show that it is self-normalizing. The final part caused difficulties for most students, and while a number of students was how to use part b) to show that, if $\overline{\mathfrak{h}}$ is a Cartan in $\mathfrak{b}_n$, then $\overline{\mathfrak{h}} = \mathfrak{b}_n/\mathfrak{n}_n$, almost none could deduce from this that h contained a regular element.

Question 2 : This question was also attempted by many students, but proved more challenging than Q.1. Part a) was answered well, but almost no students thought to use Cartan's criterion in part b) and thus only established the easy implication. Part c) was generally well-answered, as were d)ii) and d)iii). On the other hand, d)i) was only tackled successfully by one candidate.

Question 3 : This was the least popular question. Parts a) and b) were well answered, but parts c) and d) caused more difficulties, with no candidate making much progress on part d).

C2.2: Homological Algebra

Question 1 was by far the most popular, having been attempted by all candidates. Question 3 was the least popular.

Question 1. Question 1 was mostly done well by the students. The task of showing that left derived functors are additive functors was somewhat ambiguous: some students interpreted it as asking to show that it's a functor, while some others interpreted it as asking to show that it's additive.

Question 2. Many candidates forgot to check in (a)(iii) the categorical equivalence on morphisms (not just on objects). The most difficult part was (c), but also the implication 3 to 1 in (b) caused difficulties.

Question 3. Part (a) and (b) were mostly done well, but parts (c) and (d) turned out to be rather dichotomic: 3 students solved them very well, while the others did essentially nothing.

C2.3: Representation Theory of Semisimple Lie Algebras

This exam was quite hard and long. Many parts were not attempted by the students – possibly also by lack of time.

Question 1 was done well, with the exception of the calculations involving the Weyl dimension formula, which were quite messy.

Question 2 was mixed. The last part, which relied on understanding that the weight diagram of an irreducible representation is always contained in some translate of the root lattice, was missed by all the students who attempted the question.

Question 3 was the hardest: the structure of submodules of Verma modules caused a lot of difficulties, and the later questions were not even attempted.

C2.4: Infinite Groups

On the whole, the candidates performed less well than expected, considering that two of the three questions were bookwork rather than new, since they were simplified versions of the two key theorems of the course, Wolf's Theorem and Milnor's Theorem, discussed in detail and with worked examples in lectures and classes.

Question 1. This question was attempted by only one candidate, with moderate success, despite the fact that it contained a proportion of bookwork considerably larger than usual. Possibly, this is due to the fact that it required an in depth knowledge of the proof of Wolf's Theorem.

Question 2. Attempted by all candidates, quite successfully on the whole. In the proof of the fact that $GL(n, \mathbb{Z})$ is residually finite, a number of candidates disregarded the case of diagonal elements. The Ping-Pong Lemma for the copy of F_2 in $SL(2, \mathbb{Z})$ was not always well explained.

Question 3. This question was attempted by most candidates, with mixed results however. In answering (b), (ii), many candidates seemed to believe that the limit γ_S is either the same, or the same up to bi-Lipschitz equivalence, no matter what the choice of the generating set S is. Many candidates misunderstood both in (a) and, more importantly, in (b), (v), that the normal subgroup N would be normally generated by finitely many elements in N, instead of in the larger group G. As a result, there were very few correct answers to (b), (v), rather surprisingly, as this is a simplified version of Milnor's Theorem.

C2.5: Non-Commutative Rings

Generally good results, the unseen material was both manageable and sufficient to challenge all candidates.

Question 1 had one nearly complete solution. In part (c) it was critical to pay care to the coefficients of the element a_1 in terms x_1; x_2; x_3 and use this to define an epimorphism f from A to the free algebra $\mathbb{Q}\langle z_1, z_2\rangle$ such that $f(y_1) = 0$.

Question 2 was attempted by all candidates, with good results on parts (a) and (c). In part (b) a common error was a lack of justification why f induces an injective homomorphism $S^{-}1R^n \to S^{-}1R^m$.

Question 3 was least popular. In part (c) the proof that $\dim_k M = p$ requires showing that y_1^p and x_1^p are central in A1(k) and therefore act on M as scalars by Schur's Lemma.

C2.6 Introduction to Schemes

All candidates did exercise 1, and then the candidates were roughly split 50-50 in choosing exercise 2 or 3. The average raw marks for exercises 2 and 3 were roughly equal, both being 3 raw marks less than the average for exercise 1.

Exercise 1: (b) many candidates only proved the property at the level of topological spaces, without considering sheaves (in particular, without using the assumption that an open subscheme structure was chosen). Many students did not remember the bookwork for (d)(i), essentially all candidates got the first counterexample for (d)(ii) but only a few managed to find the second.

Exercise 2: (a) some confusion in writing down the complex correctly, or losing one raw mark for not saying why it suffices to just consider a cover by two basic open sets; (b) was mostly fine; (c) not many candidates explained how the maps in the short exact sequence were defined; (d)(iii) only very few candidates considered the $O(k)$ bundles for suitable k.

Exercise 3: (b)(i) most candidates were not careful about the issue that epimorphisms are not necessarily surjective on sections, here the key is to consider a generator of $L(U)$ as a free $O(U)$-module on a small enough neighbourhood U; (c) was mostly fine; (d)(i) again identifying $L(U)$ with $O(U)$ by a choice of generator makes this part easier; (d)(iv) not many candidates checked that the two open subsets cover X, and that the two maps agree on the overlap; (d)(v) not all candidates noticed that the two required functors were already constructed in (c) and (d)(iv), so it was sufficient to say that one checks the constructions are natural and inverse to each other.

C2.7 Category Theory

All questions were attempted in roughly equal proportions, there was no clear preference for one question versus the others.

Question 1. (a) Most candidates did this correctly, sometimes with some details missing in (a)(ii), or forgetting specify what a functor in (iii) is explicitly.

(b) The right adjoint was generally fine, but the left adjoint proved difficult.

(c) (i,ii) were solved correctly by most candidates, but the new material in (iii) differentiated between the candidates.

Question 2. (a) (i,ii) were bookwork, but there were mistakes in the coequaliser for vector spaces, and often the solutions for (iii) were incomplete.

(b) was generally fine, but some candidates forgot to verify that the Yoneda functor on morphisms is the inverse of the canonical restriction map.

(c) (i) implies (ii) and (iii) implies (i) were the more difficult parts of this equivalence.

Question 3. (a) was mostly bookwork except for (iii). Here the implication "third to first" was the most difficult part.

(b) mostly OK, but some candidates stated the bijections between Homs without properly checking it.

(c) turned out not to pose major difficulties, the hypotheses in Barr-Beck Theorem were not difficult to check given the results in the rest of the question.

C3.1: Algebraic Topology

Essentially everyone chose Exercise 1, and there was a 50-50 split in choosing ex.2 or 3. The raw averages were roughly 13.5, 15, 17.5 respectively for exercises 1,2,3.

Exercise 1: (a) candidates often just wrote down the answer for the cup product for T^2, without explaining how they used (if they did) the Künneth theorem to compute it; (b) many candidates did not spot that it was enough to apply the projection to the homotopy, because they never wrote down the actual homotopy map; (c) after showing injectivity of the pull-back of the projection map; candidates sometimes did not explain why the SES splits; for the cup product part the key was to consider the unit; for the last part not many candidates realised that the non-commutative cup product from part (1)(a) (using projection to one circle factor) provided a counterexample.

Exercise 2: (a) generally fine; (b) some minor slips e.g. not noticing that $G/2G = 0$ or $Hom(Z/2, G) = 0$; (c) all candidates wrote a correct fundamental cycle, but very few finished the exercise: some candidates did not draw the barycentric subdivision correctly, some candidates guessed a chain that works but miscalculated the cap product (which needs to be separately calculated, by linearity, for each of the two faces).

Exercise 3: (a) several candidates wrote down Poincaré duality as a cap product involving homology and cohomology, instead of the requested bilinear form on cohomology (working modulo torsion, in complementary dimensions, and using cup product); acceptable was also to use cup product using coefficients in a field, although the answer to the second half of the question was then harder or incomplete); (b) some candidates forgot to reduce the homology of the quotient when calculating the relative homology; (c) many good answers here, the last part can be done in several ways (either by course methods considering the pull-back on cohomology using that H^2 of the torus is generated by H^1 classes which pull-back to zero in $H^2(S^2)$, or using homotopy methods: the second homotopy group of a torus is zero or more explicitly: lifting a map from S^2 to the torus to the universal cover R^2 of T^2 and then using that R^2 is contractible).

C3.2 Geometric Group Theory

The quality and completeness of the answers was unusually high. A pleasant surprise was that many candidates answered the third question, involving the most elaborate part of the course, the last part.

Question 1. This question was attempted by most candidates. The uniqueness of the amalgamated product was not always well justified. There were a few mistakes too in the statement of Kurosh's Theorem, for instance some candidates stated that all subgroups must have as many free factors as there are conjugates of the free factors in the given group.

Most candidates were unable to provide a complete answer to question (c), (iii), however a few wrote nice solutions to it.

Question 2. This question was attempted by one third of the candidates only, but those who attempted it provided very good answers, with in particular clean proofs that the solvability of the word problem is independent of the presentation. Most candidates got confused when attempting to prove that a cyclically reduced word representing an element of finite order has length bounded by that of the relators, if the presentation is Dehn.

Question 3. This question was likewise attempted by most candidates. They all displayed a good knowledge of the fundamental group of a graph of groups and, surprisingly, most of them answered the question about the non-triviality of a reduced word very accurately, both with a well explained induction for finite graphs and with the argument required for infinite graphs. The majority also answered the three questions in (c), all new, which shows a genuine understanding of the topic.

C3.3: Differentiable Manifolds

Question 1. This was a popular choice of question. Part (a) was bookwork and done well. Part (b) was usually done well by students, with marks usually only being lost due to lack of justification as to why the map is a submersion. Part (c) was bookwork and typically done well. There was a mixed response to part (d). In (d)(i), the most common reason for losing marks was not justifying sufficiently why the maps are embeddings (and not just immersions) and omitting the argument as to why the curves are disjoint. In (d)(ii), the usual approach was to draw a diagram, which yielded the correct minimal distance, but often students failed to correctly identify the points where the minimum is attained. In (d)(iii), the main issue was in showing the linking number is positive, which few students succeeded in justifying.

Question 2. This was a popular choice of question. Part (a) was bookwork and usually done well. Part (b) was bookwork or seen material and usually done well. Part (c) was challenging for students. Some recognized they should use the definition of the Lie derivative and the earlier parts of the question, but they did not reach the desired conclusion. Part (d) had a mixed response from students. Part (d)(i) was either done very well or was found to be difficult. The main issues were that students were not able to spot the solutions to the ODEs defining the flow, and that they missed out the part about the curves preserved by the flow. Part (d)(ii) was understood conceptually, but often led to computation errors.

Question 3. This was the least popular question. Part (a)(i) was bookwork and done well. Part (a)(ii) produced a mixed response. Students who attempted it usually got the key idea, but marks were lost in justifying the argument. For part (a)(iii), the "if" part of the statement was done well, but the "only if" part proved challenging. Students who attempted it followed the hint for that part, but were not able to reach the desired conclusion. Most students missed out the final part of (a)(iii) entirely. Part (b) was bookwork and usually done well, with marks only typically lost for not justifying why the pullback induces linear maps on cohomology. Part (c) was again bookwork and done well. Part (d)(i) was either done well with students only losing marks in justification or it was challenging, which was the case for the majority of students. Students who attempted (d)(ii) did it correctly, only losing marks in showing that the classes are linearly independent.

C3.4: Algebraic Geometry

Question 1. All students answered this question, the average mark on this question being 16.4. There were no issues with the bookwork in (a) and (b). For (c), the standard examples included unions of hypersurfaces or of disjoint points, but a full solution needed the comment that as the base field is algebraically closed, it has infinite cardinality, so an arbitrarily large finite set of disjoint points or hypersurfaces can be found. (d) was largely done well; some candidates failed to realise that the quadric is also isomorphic to the affine line as an abstract variety. (e) caused more issues; the plane component was found by many candidates but the identification of the other component was less straightforward.

Question 2. Most candidates answered this question, with the average mark being 14.8. In (a), one issue that lead to the loss of a mark was if students failed to explain that projective morphisms are defined locally, or if they failed to mention that the homogeneous polynomials defining them should not have common zeros. (b) was done well. In (c), many candidates found the right open sets, but sometimes failed to argue that they are affine or the proof that the relevant morphisms give an isomorphism had gaps. (d) and (e)(i) were generally done well. For (e)(ii), many candidates thought the answer was yes, though the strongest answers gave a full argument for the fact that S is singular so cannot be isomorphic to the projective plane.

Question 3. Only 4 candidates answered this question, the average mark being 17.2. From the scripts it was clear that several others attempted this question, but got stuck in (b)(i) so moved to the other questions. Of the 4 students who carried on, 3 gave substantially complete answers, whereas one did not get very far.

Overall, the average mark on this paper was 63/100. This is lower than in previous years, indeed the questions were a little harder than in some previous years, leading to better differentiation of candidates, the highest mark being 96/100 and the lowest 26/100.

C3.5: Lie Groups

Question 1 contained some familiar material coming from early in the course, and there were many very good attempts, with one candidate producing a perfect solution and several others providing excellent answers. Quite a few candidates gave much longer solutions to parts of the question (especially the early parts) than was necessary to obtain full marks, and this probably meant that they did not have enough time to spend on their other question.

Question 2 was equally popular (like the first question it was attempted by all candidates) but attracted only one very good solution. It was a long question and (as for the first question) some candidates spent too long on the standard material in (a). Very few candidates were able to calculate correctly the Killing form for SO(n), though some remembered that the answer (without details) was given in the lectures and in the lecture notes. The first two parts of (c) were answered well, but very few made serious attempts at (c)(iii) and there was only one attempt at (c)(iv), perhaps because the question was so long.

Question 3 was focussed on material from the last section of the course (on the structure and representations of compact Lie groups). The second part of the question (on the exceptional group G_2) was an application of this material which had not been touched on in the lectures (though the question provided all the necessary background information). Probably for this reason there were no attempts at this question.

C3.7: Elliptic Curves

Overall I felt this paper turned out a little harder than average, compared to earlier years.

Question 1: This was quite a popular question and on the whole was done fairly well. I was quite pleased to see that many candidates got both the unfamiliar point addition computation in (a)(ii) and also the argument in (b)(i).

Question 2: This was not attempted by that many candidates. The bookwork in (a) was done very well, and candidates found (c) quite straightforward. They were less successful with (b), and indeed no one figured out the connection between (b)(iii) and the rest of part (b). The key in (b), which made the computation in (b)(i) easier and (b)(ii) possible, was to realise that $[p](T) = (1+T)^p - 1$, and thus the inverse in (b)(ii) is given from a functional inverse of this polynomial. (Several students made this observation.)

Question 3: A very popular question done by most of the candidates. Most though had difficulty with (b)(ii). This could be done by a rather nice reduction modulo p argument, and many students got this (or partly got it). (My intended solution in fact involved looking at a certain equation modulo 16, which was a longer argument and no one tried this.)

C3.8 Analytic Number Theory

Overall the questions appeared to work reasonably well, with a good spread of marks separating candidates. The biggest issue was that Question 3 was not popular with candidates - only 4 candidates really attempted the question, although those that did attempt it did reasonably well.

Question 1 was attempted by the vast majority of candidates. The easier early parts were generally answered correctly, but the harder later sections distinguished between candidates pretty well. No candidate gave a suitable answer for the nal part of the question, which required a slightly different perspective. Perhaps with hindsight the question was slightly on the long side, but overall worked well.

Question 2 was also attempted by the vast majority of candidates. In general this was answered reasonably well, although there were still several candidates who struggled with estimating a sum by switching the order of summation, despite this appearing many times in past questions and examples sheets. The most challenging final part of the question proved to be a bit too di cult, and I had the impression that the question was a bit longer than it should have been (a couple of candidates appeared to struggle for time). Despite these issues, it did distinguish between candidates quite well.

Question 3 was very unpopular, focussing on a part of the course which does not come up regularly in past papers. I think the fact that it was on part of the course which is not emphasised so much, as well as it requiring a slightly more non-standard approach put out many candidates. Those candidates that did attempt it did reasonably well, although the bound at the end of part (b) caused more difficulty than expected.

Overall the exam worked pretty well, but I think with hindsight the removal of bookwork because of the reminder sheets meant that too much content was added which demanded more thought, and this hurt slower candidates a bit more than it ideally would have done.

C3.10: Additive and Combinatorial Number Theory

Q1. (a) surprisingly few students managed the lower bound using the Cauchy-Schwarz inequality.

(b) This bookwork question was done well, though quite a number of candidates wasted time supply a proof of Schnirel'man's inequality, which was not asked for.

(c) This proved to be an effective discriminator, with some complete solutions and various partial ones.

(d) No candidate got near this and a great many made claims which they should have known were false.

Q2. This question seemed to work fairly well, with attempts spanning the range from little beyond the bookwork, to full marks.

Q3. Most candidates handled part (a) fairly well, though rather few manage (iv). No candidates were able to do (b) (ii), which was actually a piece of bookwork in disguise. Only one candidate managed part (c), and that candidate provided a solution rather easier than the official one, without needing the Cauchy-Schwarz inequality.

C3.11: Riemannian Geometry

Question 1. Part (a) was bookwork and typically done well. Though part (b) was seen and usually done well, marks were sometimes lost because students did not show that the second fundamental form is symmetric. Part (c) was either done almost perfectly or else students found it very challenging. The common issue was to use the minimality condition correctly and compute the appropriate second fundamental form terms arising from the Gauss equation.

Question 2. Part (a) was either done well or the common issue was to try to use the first variation formula and assume the curve was a geodesic, rather than use the hint and the minimizing property. Part (b)(i) was essentially bookwork and typically done well. Students who attempted (b)(ii) invariably had little or no problems with it.

Question 3. Part (a) was bookwork and usually fine. Most students understood well what to do in (b). The only common issues in (b) were not explaining why the exponential map is surjective (using Hopf–Rinow) and why the metric defined in (ii) is complete (again, using Hopf–Rinow). Part (c) proved challenging, with most students not realizing that they had to look at Jacobi fields on the round sphere and relate them to Jacobi fields on the product.

C3.12: Low-Dimensional Topology and Knot Theory

Solutions for Question 1 were generally good, with some candidates failing to require cobordisms to be compact in 1(a)(i). Solutions for 1(a)(ii) were essentially all correct. In 1(a)(iii), several candidates failed to check inverses. In (b)(i), candidates usually had the right idea. Part (b)(ii) proved to be more difficult, but there were several different correct approaches among the solutions, including doubling and the long exact sequence of a pair. Solutions for (b)(iii) were typically correct. There were essentially no complete solutions for part (c), but many partial results. Showing that the connected sum of an even number of copies of the projective plane is null-cobordant was usually missing.

Overall, there were lots of good solutions for Question 2. In part (a), some people forgot to require a Seifert surface to be compact. In part (b), many candidates got the wrong Seifert matrix, due to miscalculating linking numbers. This did not affect the marks given for (b)(iii) and (c). Most solutions for (c) were correct.

There was just one solution for Question 3, which was essentially correct.

C4.1: Further Functional Analysis

Question 1 This question was attempted by all candidates. (a) was often well done, though often with rather convoluted answers in (ii), and lengthy arguments in (iii). Many candidates missed that given $x + Y \in \mathrm{B}^0_{x/y}$ in (ii), the definition of the norm gives $y \in Y$ with $\|x + y\| < 1$ which has $T(x + y) = x + Y$. A number of candidates gave good answers to (b)(i), either using Hahn-Banach to obtain norming functionals to directly show that $\{x : \|x\| > a\}$ is weakly open, and others quoting the result from the course that norm closed convex sets are weakly closed to see that $\{x : \|x\| \leq 1\}$ is weakly closed.

Part (b)(ii) caused difficulties. Few candidates used the result from the course that 0 is in the closure of S_X if X is in finite dimensional, which quickly shows that if the norm is weakly continuous, then the space must be finite dimensional.

(c)(i) proved challenging for many, but a number of strong and creative answers where produced showing excellent functional analytic skills. It was intended that candidates might take $x \in B_x$ and then consider $B_X(2) + (x + Y)$ which is a weakly closed (as it is norm closed and convex) subset of $B_X(2)$ which is weakly compact as X is reflexive. Then taking an element which attains the in mum of the norm on this set does the job. One very nice alternative answer noted that as

$$T(B_X) \subset \overline{T(B_X)} \subset \overline{\overline{T(\mathrm{B}_X^0)}} = \overline{T(\mathrm{B}_{x/y}^0}$$

(using continuity at the first equality) it suffices to show that $T(B_X)$ is norm closed. They then did this as B_X is weakly compact (by reflexivity) and T is weakly continuous. While a number of candidates noted that a non-reflexive space, such as ℓ^1 , would be needed in (c)(ii), few turned this into a counter example. (d)(ii) saw few attempts, probably as candidates where short of time.

Question 2 (a)(i) and (ii) where well done (though often with slightly longer than expected answers for (ii) - few candidates noted that if $p_C(x) \geq 1$, then $x \in C$ as C is closed). In (iii) candidates could have saved time by simultaneously showing the required equivalence along side demonstrating that p_C is a norm (and many candidates did not use the conditions in the course for when p_C is a norm). (iv) caused difficulties to many candidates, with only few making much progress with the Hahn-Banach separation argument (or the converse, where one should use the failure of the condition to give an explicit weak*-open neighbourhood of some f with $|||f|||x* > 1$).

Part (b)(i) was generally well done, and many candidates showed that $|||f|||x*$ is not strictly convex. Few noticed that it was also necessary to check that the ball $\{f \in X^* : |||f||| \leq 1\}$ is weak*-closed in the topology coming from the original norm on X^* so that (a)(iv) could be applied to see that this is the dual norm of the associated norm p_C on X.

Question 3 This question was not popular. Part (a) was typically well done, as was part (b)(i), but the later parts of the question proved difficult for many candidates. Many struggled to extract the relevant ingredient from the proof of the Fredholm alternative to show that T is bounded below on a complement of ker$T$ in (b)(ii)(II). Some candidates put the argument together well in (b)(iii), but others found this challenging. (c)(ii) is a modification of a problem sheet question on complete continuity to work with the weak*-topology, but a number of candidates did not note that the (f_n) will be uniformly bounded by the principle of uniform boundedness so $(T^* f_n)$ has a convergent subsequence.

C4.3: Functional Analytic Methods for PDEs

Q1. This question was attempted by all but one candidates. (a)(i) and (b)(i) were handled well. Most candidates realised that (a)(ii) involves Hölder's and Gagliardo-Nirenberg–Sobolev's inequalities, but a small proportion made errors such as $\|v\varphi_\varepsilon\|_{L^2} \leq \|v\|_{L^2}\|\varphi_\varepsilon\|_{L^2}$! Those who attempted (a)(iii) did well with the choice $v \equiv 1$ or a more complicated choice involving logarithm. (b)(ii) was handled well in the case $n \geq 3$. A small proportion of candidates attempted (c) with fair level of success.

Q2. This question was attempted by about two third of the candidates. Most candidates handled (a) reasonably well with minor exceptions. About half of those who attempted Q2 attempted (b) with reasonable level of success.

Q2. This question was attempted by about one third of the candidates. (a) was handled mostly well. (b)(i) was handled reasonably well, in particular by those who realised the given ζ satisfies $-\frac{1}{\zeta}\Delta\zeta > \frac{1}{(1-|x|^2)^2}$. Only a couple of candidates had some ideas what (b)(ii) entails but could not follow through their ideas.

C4.6 Fixed Point Methods for Nonlinear PDEs

Question 1 was attempted by 1/4 of the students. It was overall done quite well even if no-one was able to fully solve the most challenging part (part(b)(ii)).

Part (a) was done quite well by everyone, some minor issues have been to not define the counterexample of (a)(iii) with values in the unit ball, and a few candidates were not very familiar with manipulating the liminf in part (iv) (let me mention that Oxford students learn to manipulate the liminf at the very first module of analysis, i.e. Analysis I of the first year Prelims). Part (b) (i) was bookwork and was done well by everyone. Part (b)(ii) was probably the most challenging question of the exam and it was only partially solved: students have been able to simplify the expression using that the determinant is a null Lagrangian, but no-one managed to give a full solution. The full solution needed to have very clear in mind the structure of the proof that the determinant is a null Lagrangian, one of the most tricky proofs given in the course. Part (b)(iii) was a direct application of (b)(ii) when combined by results of the course, and overall it was done well.

Question 2 has been the most popular question, as it was attempted by all the students but one. The solutions have been overall very good. Part (a) was bookwork and was done well by everyone. Part (b) and (c) were variations of material covered in the class and was done well overall. A common mistake by a few students has been to show uniqueness in 2 (d) without showing existence (note that in part (c) one shows existence, but the assumptions of part (c) are stronger than the ones of part (d), so existence in (d) does not follow from (c)). Part (e) was not very hard but new, and there has been a range in the quality of solutions, as usual for unseen questions.

Question 3 has been a very popular question, as it was attempted by all the students but two. The solutions have been overall very good. Part (a) was bookwork and was done well by everyone. Part (b) was a variations of material covered in the class with some new twists and was done well overall. A common mistake by a few students was not to show that the operator A is hemicontinuous (when showing it is a monotone operator). Another point that was missed by some of the students was the last one about uniqueness of the solution, the key difficulty being that the PDE is nonlinear.

C4.8 Complex Analysis: Conformal Maps and Geometry

1 (a) This is a typical problem that is based on Schwartz Lemma. The question is to prove the function is conformal and 1-1. All the students attempted to show 1-1, none of them checked the map is conformal. (b) (i) is book work. For (ii), one may use Koebe's 1/4 theorem to argue by contradiction. All the students could write down Koebe's 1/4 theorem. But none of them could apply it to make progress. (iii) The problem is adapted from notes. But most of the students struggled with constructing h, and failed to follow the main scheme in the notes to solve the problem. (iv) and (v) Most students figured out the correct substitution for doing the problem, but got lost in analysis such as applying Holder's inequality, etc.

2 (a) (b) Book work, appeared in the notes. Only one student can solve it completely. (c) (di) Some students can analyse the images of the maps, and apply maximum principle. (dii)None of the students got this part right.

3(a) Seen problems, adapted from problem sheets, but not everyone who attempted can solve them completely. (b) Some of the students can draw the right picture, but their understanding is not precise enough to solve the problem completely. (c) nearly no one attempted this part.

The exam paper is not really easy. The book work parts are not completed to the most satisfactory level. The unseen questions are adapted from notes or seen problems, requiring a further step such as arguing by contradiction or applying Holder's inequality. These tricks have been used in the notes frequently. I suppose it would be less challenging to solve the unseen ones if not sitting in an exam.

C4.9: Optimal Transport & Partial Differential Equations

Question 1 was taking by most of the students and apart from the most challenging part of the characterization of the mid points, it was properly answered by most of them.

Question 2 was attempted with a good success and what it was answered by the students was mostly correct. Some students skip details to check the Lipschitz property of the velocity field using optimal transport, that needed an easy but properly done argument. This is a very typical question for this course material, and then it is clear that it was a lack of time for most of them not to finish question 2.

Question 3 was attempted by fewer students, a common mistake is a proper Taylor expansion up to order 2 of the potential for the first item. It was also clear that they lack time to properly discuss the other parts of the question.

C5.1: Solid Mechanics

Q1: This question was attempted by all but one candidates. It was generally well done, though surprisingly few candidates gave an explicit condition for the inversion in part (b)(iii) in terms of the w_i. Very few candidates were able to use the positive definiteness of $\mathbf{B}$ in either part (b)(iv) or (c)(ii).

Q2: This question was fairly popular, but was not particularly well done. Most students tried to use a boundary condition at $r(A) = a$ to determine the unknown constant in part (a)(ii), rather than the observation that $r(0) = 0$ (since material is at the origin and must remain there). Similarly, students did not, in general, realize that $T_{rr} = T_{\theta\theta}$ in this geometry, meaning they were not able to solve the differential equation for T_{rr} required in part (b)(ii). Finally, for part (b)(iv), relatively few students calculated the integral required for the total load, and so did not derive the equation for $\hat{W}$.

Q3: This question was not especially popular, but received relatively high marks on the whole. Part (a) followed lecture material very closely and was done well. Part (b) followed similar lines with a twist; here candidates were generally able to find the sixth order polynomial required for part (b)(ii) but dealing with the different stretches in part (b)(iii) proved more challenging.

C5.2: Elasticity and Plasticity

Question 1 This was the least popular question, but many of those who attempted it managed to get good marks. Some candidates really struggled with the basic geometric identities needed in part (a), but then the non-dimensionalisation and the manipulations needed for parts (b)–(d) were mostly handled well. In part (e), no-one got the point about the curvatures of the beam and the wall matching when $\lambda = 4\pi^2$.

Question 2 This was the most popular question, but had the lowest average mark. The bookwork in parts (a) and (c) was done quite well, although often laboriously and with important steps omitted. In part (b), many students were confused by the fact that $\mathrm{Re}[f]$ was not the same as the stress function used in lectures for a standard Mode III crack problem. In part (d), almost no-one successfully posed and solved the problem in the ζ-plane to determine $f(z)$, although some anyway managed to spot that $f\big(z(\zeta)\big) = -c^2 e^{2\epsilon} / \big(4\zeta^2\big)$ works. Part (e) was generally fine, although with a lot of minor algebraic slips.

Question 3 This question was a generalisation of a problem sheet question and was reasonably popular, but the average mark was rather low. The solutions were often over-complicated, leading to students getting lost in the algebra. In part (a), several students fallaciously imposed $\tau_{rr} = \tau_{\theta\theta} = 0$ at $r = b$ and then found themselves with too many boundary conditions. In part (b), many students did not clearly state and apply the correct conditions at the elastic–plastic free boundary $r = s$, and so were unable to close the problem for s. In part (c), few students correctly imposed a purely elastic response on the residual stress to describe the unloading. In part (d), very few students understood that $2P_{c1} < P_{c2}$ is required for the described behaviour to be possible, and almost no-one managed to deduce the given bounds on the parameter β.

C5.3: Statistical Mechanics

Question 1 was done by only one candidate and they did well.

Question 2 was done by both candidates, and one of them got the key points of the question and did well.

Question 3 was done by all candidates but both struggled with the concept of how to find the critical point, despite the hint in the question part a) urging the candidates to consider setting the first AND the second derivative with respect to v to zero. This made it difficult to get any further marks in part c). However, marks in b) would still have been accessible but candidates did not realise that the leading order behaviour in $T - T_c$ was $O((T - T_c)^{-1})$ i.e. not a positive integer power.

C5.4: Networks

Overall, the students had done very well. The quality of the submitted is even more remarkable, given the short time available for the project, and its openness. Several works were excellent, showing that the student had explored thoroughly the literature, had a good command of the material, and even developed their own research direction. Typical weaknesses appeared in the description of the analytical and numerical parts of the work, e.g. with vague justifications or insufficient details. This year, two projects showed a worrying level of plagiarism in Turnitin, a situation that should be considered with care in the future.

C5.5: Perturbation Methods

Overall Question 1 was popular. The first part of the question and the path of steepest descent were generally tackled very well. For the next part of the question, the choice of the appropriate contour for the use of the steepest descent method proved to be a genuine hurdle in the question for a number of candidates, while only the best attempts successfully expanded about the location of the dominant contribution to the steepest descent integral. Many candidates parametrised the steepest descent curve with respect to ζ, without properly accounting for the fact the steepest descent curve has an infinite gradient with respect to ζ at the location of the dominant contribution or recognising that an alternative parametrisation may have been more convenient.

Question 2 appeared to be the least favourite question. A few attempts gathered difficulties early and these candidates generally moved onto the other questions, while there was also a number of very high scoring solutions. Many candidates did not apply the method of intermediate variable matching correctly, while the most successful solutions recognised which terms had to balance when matching via the intermediate variable method.

Question 3 started with bookwork concerning important definitions that candidates knew well. An occasional candidate used a different method than requested in part (b), and a few candidates struggled, but on the whole part (b) was executed very well. The final part differentiated most attempts. In particular keeping track of the level of approximation and the terms that need to cancel between the two integral contributions in the use of the domain splitting method to more than leading order typically, but not always, proved problematic for the candidates.

C5.6: Applied Complex Variables

Question 1

This question was very popular, and there were a lot of good answers. Despite the relative complexity of the setup, all but one candidate identified the correct domains in the potential and hodograph planes (though not all fully justified their figures). Most managed to work through the question successfully. The final equation should have read

$$\phi = \frac{2}{\pi} log|sec2\alpha|$$

since $sec2\alpha < 0$. Most candidates did not notice this; those that did correctly assumed that the question was in error.

Question 2 This was a very unpopular question, probably because the set up was slightly less familiar than that in Q1 and Q3. There were only a handful of answers, and very few good ones. Only one candidate realised that the correct approach was to look for a Cauchy integral representation of $w'(z) + \pi i W(z)$. Ironically if instead of asking them to show

$$w'(z) + \pi i W(z) = \frac{1}{2\pi i} \int_\Gamma \frac{d(\zeta)}{\zeta - z} \qquad (\dagger)$$

I had told them that

$$w'(z) + \pi i W(z) = \frac{1}{2\pi i} \int_\Gamma \frac{G(\zeta) d\zeta}{\zeta - z}$$

where $G(\zeta)$ was to be determined then they have found the question much easier. Despite not being able to prove (†) most candidates could use it to solve for w and deduce f.

Question 3 This was a very popular question and there were a lot of good answers. Most candidates successfully identified domains on which $\bar{f}_+, \bar{g}_+$ and $\bar{h}_-$. are holomorphic, and the correct values of and α and β.

C5.7: Topics in Fluid Mechanics

The range of marks out of 50 was 28-48 for 16 M. Math. candidates.

Question 1 was straightforward and well done, except for the very last part (the one-dimensional 'phase plane') which baffled everybody.

Question 2 was straightforward and well done until the last part, where drawing the graph of $f(\phi)$ and then $\sum$ was challenging.

Question 3 was straightforward and well done. The ability of candidates to make their way through the algebra of part (c) was encouraging.

In summary, the paper appeared to work well.

C5.9: Mechanical Mathematical Biology

Q1. This question was attempted by most candidates. Part (a) was done well. A few computational errors were made in parts (b) and (c) (i), but most candidates had the right idea. Some candidates did not define the configuration space correctly, for instance missing the integration over the angles, or missing the $\mathrm{b}_i^2 \sin\theta_i$ that arises in spherical coordinates. The argument in part (c)(ii) - seeing that taking $\frac{\partial}{\partial \lambda_1}$ of the partition function gives the numerator in the quantity $\langle R_z \rangle$ - was very similar to previous examples seen in the course, although a few mistakes appeared in the details, for instance trying to cancel everything but the ith integral, which does not work in this problem.

Q2 This problem was attempted by nearly all candidates, and was done to a high standard. The bookwork derivation in part (a) was done well, and generally efficiently also, with very few mistakes. The first parts of (b) were nearly identical to an example worked out in the lectures, and this was done generally well also. Parts (iv)-(vi) were trickier, and served to separate class boundaries. The calculation in (iv) is actually very simple for a sphere – one only needs to arrive at an equation for the energy in terms of the radius a, which can be obtained in a straightforward manner, and it is a simple calculus exercise after that. Parts

(v) and (vi) required a good conceptual understanding to interpret the descriptions, and tended to be hit or miss.

Q3 This question was only attempted by two candidates, neither of which attained very high marks, though this seems to be connected to running out of time as much as difficulty of the question. Part (a) was straightforward and done well. Part (b) and (c) were quite similar to examples done in lecture/problem sheets, but with some small differences that may have caused difficulties. Small mistakes on part (b) carried through to part (c). One candidate did not realise that n_x and n_y also need to be expanded as variables. No candidate finished part (c) or (d).

C5.11: Mathematical Geoscience

qn1: Climate. This was the least popular question.

(a) Some candidates mixed up the Keeling curve with Saturated Vapour Pressure. (b) Some candidates did not have calculators, so only gave rough estimates of the time scales. (c) Lack of calculators meant candidates gave general expressions for the quantities asked for.

qn 2: Rivers. Answered by all candidates. Some very good solutions. (a) Fine (b) Non-dimensionalisation not as clearly expressed as it could have been. (c) Derivation of the dispersion relation was fine in general, apart from some errors in derivation. Some candidates did not explain that $\sigma_r > 0$ meant unstable wave growth. Most obtained σ_i but mixed up upstream propagation for $F > 1$ with downstream propagation.

qn. 3: Ice

This question proved reasonably popular and attracted a range of marks.

Most candidates managed the first part quite well, although some seemed to get quite confused by the non-dimensionalisation, including providing answers that were clearly inconsistent dimensionally.

For the second part, only one or two candidates gave compelling explanations of the fact that accumulation would likely depend on surface altitude, and that that is primarily controlled by r or h, depending on whether lambda is large or small. The integration to find $h(r)$ was done reasonably well, although many candidates made this much more complicated than necessary by not immediately noting the fact that h needs to be bounded at $r = 0$, which fixes the constant of integration.

The third part had some decent partial attempts, with one or two people getting the correct expression for $h(r)$. A common difficulty was realising that r needed to be re-scaled to get the equation in the form of Bessel's equation (though this was hinted at by the given solution for r_m).

C5.12: Mathematical Physiology

Question 1 was most popular. It was straightforward and mostly well done, except for the last part on solitary waves, which flummoxed almost everyone.

Question 2 was straightforward and well done until the last part, which required some thought, and stymied everyone.

Question 3 was straightforward, but the last part fooled many, who forgot that σ can be complex.

In summary, the paper appeared to work well.

C6.1: Numerical Linear Algebra

Q2 was the most popular, and was attempted by over 80% of the candidates. Q1 was attempted by slightly fewer candidates than Q3.

Q1: some struggled to use the Courant-Fischer theorem properly in a(i) to get the desired inequality. a(iv) was a new problem requiring some guessing and computation, and seemed to be very challenging. In (b) some failed to note the assumption $k(A) \gg 1$; when $k(A) = O(1)$ it is easy to come up with examples, but the inequality is not very interesting.

Q2(b): while most correctly used the connection between the power method and QR algorithm to discuss the convergence of the latter, very few noted the requirement in the power method convergence that the initial vector has nonzero components in the dominant eigenvector. Q2(c)(ii): some presented examples that are triangular or diagonal; while the QR algorithm may not change these matrices much (or at all), this is not a good example as such matrices have already converged! (d)(i) appears to have been very challenging. One needs to use the backward stability of QR factorisation and orthogonal matrix multiplication to prove one step of QR algorithm is backward stable.

Q3(a) (i,ii): A fair number of candidates wrote H^{-1}; this is inappropriate as H is not even square. Some answered (iii)(c) by noting that once the exact solution is found GMRES stops making progress; this is technically correct (and received marks) but the intended solution was to note that GMRES can stagnate even before the solution is found; a fact indicated in a question in the problem sheets. (b) appears to have been challenging, even though it is pretty similar to the discussion in lectures and a question in problem sheets. Most attempts failed to use the orthogonal invariance of Gaussian matrices together with a QR (or SVD) of A.

C6.2: Continuous Optimisation

The students have done well on the C6.2 exam this year. Problem 1 was easier but carefully marked. Students coped well with all questions typically, and all subparts were achievable/have been achieved by some students, given sufficient time. It was an accessible exam.

C6.3 Approximation of Functions

Everything went smoothly with the exam. Students found problem 2, related to complex variables, harder than the other two problems.

There was a typographical error on 3(d), which no students noticed just as none of us writing and checking the exam had noticed it: 9^{-15} was stated to be around 5×10^{-16}, when in fact it is obviously bigger than that (in fact around 5×10^{-15}). This had no consequences.

C6.4: Finite Element Methods for Partial Differential Equations

Q1: This question was attempted by all but one student. It revealed a good spread of abilities across those who attempted it. Q1(a)(i) was answered correctly by every candidate. Q1(a)(ii) was generally answered incorrectly; candidates failed to observe that the basis function ϕ_4 associated with evaluation at the barycentre was zero on the boundary of the cell, and hence did not contribute to the value of the function across a shared edge. Q1(b)(i) was generally answered well, with most candidates invoking the Sobolev embedding theorem as expected; some candidates responded with irrelevant bookwork. In Q1(b)(ii) some candidates did not recall the definition of a direct sum of two vector spaces, or forgot to show that the sum was direct. Q1(b)(iii) and (iv) were answered well by those who attempted them.

Q2: This question was attempted by half of all candidates. This question attracted the stronger students, and was generally answered well. Q2(a) was standard bookwork and was answered well. Q2(b) was again answered well, with marks lost only for minor slips. Q2(c)(ii) challenged some candidates; they did not realise to use the Poincaré inequality on the first term of the right-hand side, and wound up with formulae for ε that were not valid (e.g. requiring $\sqrt{1-K^2}$, where $K > 1$ generally). Q2(c)(iii) was well-answered by those who attempted it.

Q3: This question revealed a good spread of abilities across those who attempted it. Q3(a)(i) was answered correctly by every candidate. In Q3(a)(ii), some candidates failed to justify that $k(x) < M$ for some M; this follows because k is continuous on a compact domain. For the last part of Q3(a)(ii), some candidates invoked characteristic functions to explain why the bilinear form would not be coercive, but such functions are not in H^1; one should instead use bump functions on the subset S of Ω (with nonzero measure) where $k(x) < 0$. Q3(b)(i) was mostly answered well, but every candidate applied integration by parts to the $\nabla \times E$ term to shift the curl operator onto the test function; this is not valid, because the test function for this equation is drawn from $H(\mathrm{div})$, and in general does not have a square-integrable curl. In Q3(b)(ii), candidates sometimes neglected that the question had $\Omega \subset \mathbb{R}^3$, not $\Omega \subset \mathbb{R}^2$, or stated finite elements with scalar-valued spaces instead of vector-valued ones. Q3(b)(iii) was answered well by those who attempted it.

C6.5: Theories of Deep Learning

- The effectiveness of how specific tasks were approached within the mini-project:

  The course is designed around the students gaining increased familiarity with reading recent journal articles in the field of mathematical theories of deep learning. During the third and fourth tutorial they have also began writing brief summaries of specific articles covered in lecture. This training is designed to facilitate the students ability to complete the mini-project which is primarily a longer (five page) summary and analysis of two to three conference proceedings. The students approach this culminating task well and based upon submissions appear to have learned the skills necessary to complete a successful mini-project.

- The candidates' overall standard of answers:

  The scores had a good overall distribution with mean around 60, the majority of scores between 50 and 70, and a few outlier scores of around 80. The students have clearly learned how to read and assimilate information from recent journal articles. They also are familiar with the core mathematical material taught in the course and general tenor of the mathematical theories of deep learning.

- Common difficulties/areas of improvement for candidates:

  It appears that the main areas in which students struggle are on writing and computer programming skills. Students would benefit from a Part B course where they learned principles of writing mathematical reports and generally how to professionally and succinctly convey their ideas. The students would also benefit from greater exposure to computer programming throughout their mathematical curriculum. While the core material of the course isn't focusing on computer programming or applications, it is extremely insightful to programme the algorithms being analysed and to probe their inner workings. While computer code is provided for all tutorials, and students are able to find code on-line for associated conference proceedings being reviewed, it does seem that some students who have avoided computer programming find using the code challenging which limits their ability to understand the core theory. These limitations are not specific to the course C6.5, rather they are issue associated with the current curriculum. There have been proposals from within the department indicating faculty viewing writing, projects, and programming skills as important skills that the should be further included in the curriculum.

C7.4: Introduction to Quantum Information

Question 1. It was by far the most popular question, attempted by *all* of the students. Perhaps not so surprising, given that the question was based on the mainstream material. The book-work in part (a) was very well answered. In part (b) the students showed a good grasp of the Born rule but many of them struggled with calculations that led to the $\mathrm{Tr}(U)$ expression. Part (c), again, was almost perfectly answered, most likely because the question did not explicitly ask for a detailed calculation of the posterior probabilities. Those few who attempted to calculate the posterior probabilities made minor mistakes, even though they managed to arrive at the right conclusion. Part (d) turned out to be the most difficult one, with many students failing to use the fact that the eigenvalues of a unitary matrix are of the form $e^{i\theta}$. Instead many attempted to obtain the eigenvalues from the constraints on the trace and the determinant. This is a good alternative approach, but most of the students who took this route could not see the relevance of the *real* part of $\mathrm{Tr}(U)$ when deriving the probability from which the eigenvalues of U are then obtained.

Question 2. In general, the question was well answered and students scored well. Part

(a) was book-work but, surprisingly, many students couldn't succinctly justify the answers; part (b) was done in a few different ways, but almost always successfully; in parts (c) and (d) most students dropped a few marks, having struggled with upper bounds; part (e) was usually answered correctly using mathematical induction; part (f) was unproblematic and very few students got it wrong.

Question 3. At first glance this question might have looked difficult for it contained new topics (encryption of quantum states), hence it was not very popular, but those who attempted it did quite well. Part (a) was similar to one of the class problems and most students provided correct answers, but only few supplemented it with geometric interpretation. Students knew how to handle parts (b) and (c) but most did it by analysing specific cases, rather than using general notation. Showing that compositions of Clifford gates are Clifford gates in part (d) posed no problems. Most students noticed that part (e) is a generalisation of part (c) and provided a reasonable description of delegated quantum computation based on Clifford gates. Part (f) was well answered but hardly anyone commented on the need to go beyond the Clifford gates.

C7.5: General Relativity I

Question 1: This question was very popular and attempted by most students. The majority were able to do parts *a-c* without too much difficulty, although a surprising number of students assumed that the curve γ was a geodesic, despite the question explicitly saying that this may not be the case. Those students who struggled with parts *b* and *c* also often seemed to be under the impression that all curves are geodesics. Part *d* required some more algebra, and the ability to convert between abstract tensor expressions and concrete expressions for derivatives of functions along curves – this proved a challenge to a number of students. A frequent error here was believing that the t derivative of the t-component of a vector is always 1, while in fact, in this question, the t-component of the vector in question is a constant (and so its t derivative vanishes). Finally, part *e* should really have been approached as a system of linear ODEs, but almost no students did this. Instead, the majority of students who attempted part *e* derived a second order ODE for one component of Y, and in doing so showed that this component undergoes periodic oscillation – though they rarely went on to show that the other components also oscillate periodically. Overall, most students scored well in the parts of the question they attempted, and low scoring students most often offered partial answers to only a few parts of the questions (the "bookwork" parts) and spent time copying out parts of the question, while leaving other parts of the question completely untouched.

Question 2: This was by far the least popular question, and was only attempted by a handful of students, probably because it was the least familiar in style (compared with past exam questions). Most of the students who did attempt it did well, however, scoring slightly higher on average than the other two questions. Part *a* was not completely straightforward but almost all students were able to do it well, and part *b* required an understanding of normal coordinates and special relativity which was also demonstrated by almost all students. Part *c* was the most difficult part of the question, requiring some fairly intricate algebraic manipulation, and in fact no student was able to completely solve this part of the question, though some came very close. Part *d* was generally done fairly well, even by those students who could not complete part *c*, although no student made explicit the crucial fact that the coordinate vector fields are parallel-transported in Minkowski space.

Question 3: This was a very popular question, with the vast majority of students attempting it together with question 1. Part *a* was done very successfully by almost all students, with only a small minority forgetting that the Lagrangian itself is a conserved quantity (when the curve is parametrised by an affine parameter). Part *b*, however, was generally not done successfully – in fact, no student completely solved this part of the question, though some came very close. A very common error was to assume that a geodesic which is *emitted* radially will always remain radial, whereas in fact (since the spacetime is rotating) the geodesic will itself start to rotate. The key point was to realise that the conserved angular momentum is zero: noticing this made the rest of the algebra considerably easier. Even taking this fact into consideration, the resulting integral was not accurately solved by any student – the easiest way to solve it is to first make a substitution of variables to remove the hyperbolic cosine, and then to remember the formulae for derivatives of inverse trig functions, and while some students were able to perform one of these operations, no student did both. In retrospect this integral is probably too difficult without a hint. Finally, students generally faired better on part *c*, although a surprisingly large number of students made algebraic mistakes in solving the quadratic inequality in part *c (i)* (perhaps they were running out of time when trying this question), and some students made the common mistake of believing that every curve is a geodesic. Finally, most answers to part *c (ii)* were nonsense, and while some students said something true but trivial (e.g. that there are timelike circular orbits only in the interior region – although even this statement is true only if "circular" is interpreted in a coordinate-relative manner), only one student identified the closed timelike curves.

C7.6: General Relativity II

Problem 1 This problem exploring the redshift formula, the stress-energy and the Ricci tensors in the (unnamed) Janis-Newman-Winicour metric was attempted by the majority of the candidates. The typical issues were the following.

- Confusing the coordinate time with the proper time, namely taking the velocity vector of an observer following a curve $\gamma^\mu = (t, r_0, \theta_0, \phi_0)$ with constant r_0, θ_0, ϕ_0 to be simply (1, 0, 0, 0), which is off by a factor of $(g_{tt}|_\gamma)^{-1/2}$.
- Taking the wave vector of a null ray $\gamma^\mu(\lambda)$ to be the velocity vector, as opposed to $\dot{\gamma}^\mu$, where k is the wave number.
- Ignoring or not taking full advantage of the trace reversal in the Einstein field equations.

Judging by the candidates' performance, this problem may have been the most challenging.

Problem 2 This problem on Einstein's quadrupole formula was only tackled by one candidate - and with a very decent level of success. The unpopularity of the problem may indicate the propensity of the students attempting the exam towards more typical problems involving exact metrics.

Problem 3 This problem exploring the Killing horizon of the (unnamed) extremal Kerr solution was attempted by all candidates. The typical mistakes were the following.

- In the context of a hypersurface $\sum$ defined by $r = \text{Const}$, identifying normal vector N with ∂_r instead of taking the normal covector to be $n \propto dr$, as implied by the regular-value theorem.
- Having observed the normal vector to be $N = aT|_{\sum} + bL|_{\sum} =: K|_{\sum}$, where both T and L are Killing vector fields, extending the Killing vector field K away from $\sum$ with non-constant a and b. This reflects the complexity of the concept of Killing horizon, which relies on the non-trivial combination of vector fields defined on and away from it.

C7.7: Random Matrix Theory

Question 1 was attempted by most of the candidates. Parts (a), (b) and (c) were straightforward and were in general answered well. Most candidates found part (d) difficult. Only a few calculated the variance, as asked for; many only established an order estimate for it, but could then still prove almost sure convergence successfully. Only a few candidates correctly identified the paths that give a non-zero contrition in the limit and that the contributions from these can be evaluated using the information given in the question.

Question 2 was attempted by roughly half the candidates. Parts (a), (b) and (d) were straightforward. In answering part (b), some candidates failed to say that the random variables need to be paired with their complex conjugates. Many candidates did well on part (c)(i), but some attempted a more general calculation than was asked for. Part (c)(ii) was challenging and only a few candidates scored well on it. Many didn't see that the permutations fall into two classes, with permutations in each class giving the same contribution. Part (e) was also challenging and only a few candidates scored well on it. Many failed to take advantage of the fact that the matrix entries were stated to be Gaussian random variables and so to use Wick's theorem, despite the question saying to do this.

Question 3 was attempted by roughly half the candidates. Most of those who did attempt it gained high marks. Part (a) was straightforward. Many candidates saw that using the Fourier expansion for the ratio of sine functions considerably simplifies the calculation, but not all did. Most found parts (b) and (c) straightforward too, although some failed to apply Gaudin's Lemma correctly. Several candidates did part (d) well, but some failed to spot the connection to the two-point correlation function, which simplifies the calculation considerably.

C8.1: Stochastic Differential Equations

*Q*uestion 1. This is a popular question (due to the first question on the paper I believe) attempted by most candidates, while unfortunately there are few good solutions. The first part (a)(i) turns out to be the most challenging part, and very few candidates have idea how to argue independence of Gaussian random variables. By part (a)(i), the part a(ii) should be easy and follows from the martingale property of $M^2 - \langle M \rangle_t$, but still many candidates had no idea how to do it. While most candidates got the marks for a(iii) by using Itô's formula. Candidates find (b) also very challenging, and most of candidates tried to answer this part by using Itô's formula, which is not the direct to do yet though it is possible. While most candidates could not argue properly.

*Q*uestion 2. This is again a question attempted by most candidates. There are good answers for part (a) which may be answered following step by step (i) - (iv). While candidates had difficult to show the Lipschitz continuity of the coefficients, which is required a bit Prelims analysis. Part (b)(i) is an easy exercise for the exponential martingales, so most candidates got a fair marks, while (b)(ii) seems challenging, some candidates tried to use Itô's formula though the correct way should be the one most elementary: writing down the expectations of both sides in terms of normal distributions.

*Q*uestion 3. Several candidates attempted this question, but I saw no near complete solutions unfortunately. Parts (a) and (b) are mainly book, including a simple application Lévy's characterisation of Brownian motion. To calculate the pdf in part (c), one should apply Cameron-Martin formula to work out the weak solution, then apply (b). But most candidates who attempted this question just applied Tanaka's formula to the solution directly which leads to a wrong formula.

C8.2: Stochastic Analysis and PDEs

Questions 1 and 3 were the most popular ones. A surprising number of candidates had trouble with 1a which was a simple calculation with conditional expectations; most managed to use the hint for 1b; however, very few made substantial progress on 1c. For question 2a most candidates used continuity and symmetry in the Gaussian density but very few managed to successfully exploit the resulting equi-continuity of p_t^D. For 2b(i) candidates noticed that this is an immediate consequence of the explicit formula, but little progress was made on 2b(ii) and 2c. For 3a nearly every attempt made substantial progress, and in 3b nearly all attempts correctly calculated the second derivative but a common error was in solving the resulting ODE (e.g. not correctly identifying the constants). Few candidates managed to attempt 3c but a common hurdle seemed to be to find a simple change of variables for the integration.

C8.3: Combinatorics

Q1 was the most popular question. Most candidates did well on (a) and (b), and the majority picked up the idea of moving sets up to the kth layer in part (c), either by taking upper shadows or by using symmetric chains. Part (d) was more difficult; although, even without a complete answer, candidates could gain marks by finding an example showing that equality can be attained, or noting that the bound holds if both families are intersecting.

Q2 was also popular. (a)(i) and (b)(i) were bookwork, and most candidates had a good go at (a)(ii). The last two parts of (b) were more difficult, and there were fewer successful attempts.

Q3 was slightly less popular. (a)(i) was bookwork, and (a)(ii) was a slight variant. It was necessary here for candidates to note what field they were working over. Parts (b)(i) and (b)(ii) were relatively straightforward. The last two parts had fewer successful answers: several candidates tried to bound the VC-dimension by showing that a specific set is not shattered, which does not rule out that some other large set is shattered.

C8.4: Probabilistic Combinatorics

Question 1 was by far the most popular, and was with hindsight a bit too straightforward. (a) (i) and (ii) are bookwork, though it is quite a tricky proof to get right if you do not understand it. This year most candidates did - perhaps they had useful notes on their summary sheets! (iii) is a very simple test of understanding of the definitions. (b)(i) was generally well done, though you do need to explain why G itself is valid as the relevant dependency digraph. For (ii) many candidates got in a mess with the calculations, although they can be done very simply. One mistake was to try to bound $\binom{k}{k/2}$ by using a standard bound for $\binom{n}{r}$, not noticing that 2^k is a better (and simpler) upper bound in this case.

Question 2 was the least popular. The first part was mostly well done by those that attempted it, though there were a number of scripts that blindly reproduced relevant (or not so relevant) bookwork without the (rather minor) modifications needed to the setting of the question, thus demonstrating a lack of understanding. Part (b) turned out to be rather tricky, though there were some good, mostly complete attempts.

Question 3 was the second most popular. Part (a) was generally well done, though some candidates failed to explain why the induction hypothesis can be applied in the key step. In part (b), an asymptotic equality is required, not just an inequality one way. Janson gives one bound (most candidates managed this). Rather few managed the reverse, which follows from Harris's Lemma applied to the complements of the relevant events. This is disappointing, since this idea is explicitly mentioned in the lecture notes when Janson's inequality is introduced. Part (c) proved to be tricky, though the idea is just to use the 1st and 2nd moment methods on triangle free-sets of a certain size.

C8.6: Limit Theorems and Large Deviations in Probability

Question 1. Part (a) is rather standard, while still I saw no good answer for (a)(i), which requires some knowledge from the courses in Part A papers (A4, A8). Part (b) requires a good understanding about tightness, candidates gave good solutions to b(i-iii). To answer (b)(iv), one has to appeal some estimates by using BDG inequality from papers B8 or C8.1, which proves demanding for some candidates.

Question 2. This question covers the core material of this paper. The candidates answered parts (a) and (b) quite well, but lost some marks for part (c) for which one has to construct a continuous mapping in order to apply the contraction principle for LDPs.

Question 3. Candidates did Part (a) about Cramér's theorem, though no one is able to work out the rate function for two points distribution correctly, so only partial marks were gained. All candidates who attempted this question realized that part (b) can be answered by using Cramér's LDP together with the rate function calculated in part (a), but again some candidates have difficulty for applying the LDP to justify (i) and (ii), and give proper answer for (iii), and therefore only partial marks were awarded.

Statistics Units

Reports on the following courses may be found in the Mathematics and Statistics examiners' report.

SC1 - Stochastic Models in Mathematical Genetics
SC2 - Probability and Statistics for Network Analysis
SC4 - Advanced Topics in Statistical Machine Learning
SC5 - Advanced Simulation Methods
SC7 - Bayes Methods
SC9 - Interacting Particle Systems
SC10- Algorithmic Foundations of Learning

Computer Science

Reports on the following courses may be found in the Mathematics and Computer Science examiners' report.

Quantum Computer Science
Categories, Proofs and Processes

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