Liquid metal behaviour inside a tokamak fusion reactor

Oliver Bond
oliver.bond@maths.ox.ac.uk
Mathematical Institute, University of Oxford
Supervisors: *Prof. Peter Howell, Prof. John Ockendon (Oxford), Dr. Peter Buxton (Tokamak Energy).*

THERMOELECTRIC MAGNETOHYDRODYNAMICS (TEMHD)

Magnetohydrodynamics (MHD) is the coupling of two sets of well-known equations: **Maxwell's equations** of electromagnetism, and the **Navier-Stokes equations** of (incompressible) fluid dynamics. Thermoelectric magnetohydrodynamics (TEMHD) combines these principles with thermal effects; namely, those given by the **Seebeck effect**. This modifies the Ohm and Fourier laws to give

$$\mathbf{J} = \sigma\left(\mathbf{E} + \mathbf{u} \times \mathbf{B} - S\boldsymbol{\nabla}T\right), \qquad \mathbf{q} = -k\boldsymbol{\nabla}T + ST\mathbf{J},$$

where S is a number called the **Seebeck coefficient**, which is unique for each electrically conducting material, and $\boldsymbol{\nabla}T$ is the temperature gradient. The Seebeck effect is an essential driving mechanism for the flow of the liquid metal in the trenches considered by Tokamak Energy.

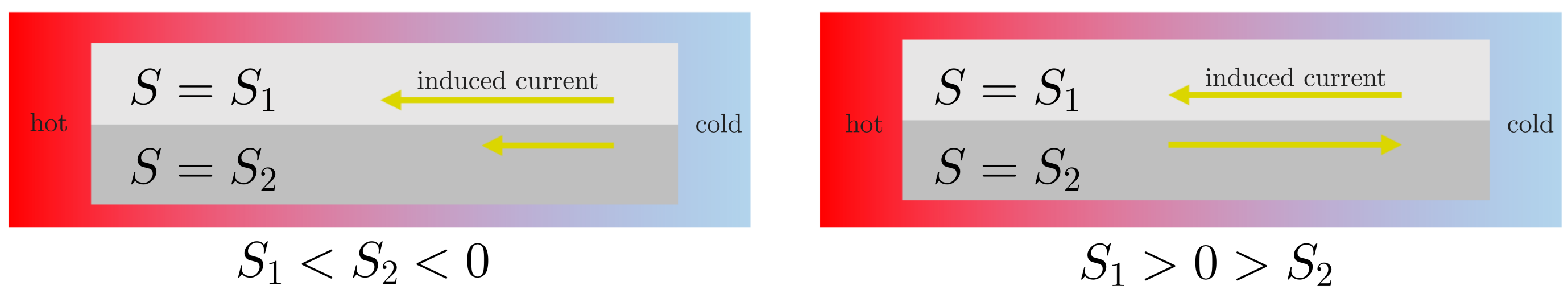


Figure 1 – A simple illustration of the Seebeck effect (adapted from [1]) and how a temperature gradient induces an electrical current across an interface between two electrical conductors with Seebeck coefficients S_1 and S_2 respectively.

TOKAMAK ENERGY'S "ST40"

Tokamak Energy, a fusion company based near Oxford, has built a device known as a *tokamak* (derived from the Russian word, meaning "toroidal chamber with magnetic field"). It is called ST40 (`www.tokamakenergy.co.uk/st40`) and has successfully reached *100 million degrees Celsius*; temperatures under which nuclear fusion can take place, just like in the core of the Sun.

At the top and bottom of ST40 is a component called the *divertor*. A simple illustration of the divertor, as well as ST40 in operation, are shown in Figure 2. Tokamak Energy are currently exploring ways to incorporate a layer of liquid lithium as a regenerative surface coating the divertor plate, but they want to avoid any regions where the divertor plate is left exposed to the plasma and therefore prone to being damaged in a phenomenon known as **dryout**.

ST40 currently has a piecewise flat divertor plate but Tokamak Energy are investigating using one comprised of a sequence of radial channels or "trenches", inspired by **Liquid Metal Infused Trenches (LiMIT)**, a design created at the University of Illinois at Urbana-Champaign (UIUC) [2]. Combined with a toroidal magnetic field, the temperature gradient created by the plasma creates a Lorenz force down the trenches which pushes the liquid lithium down them. Although the focus of this poster is on trenches, the engineers at UIUC are also interested in divertor plates with periodic arrays of posts. This research project is inspired by their designs, which Tokamak Energy envision that they will use in designing the divertor of the ST40.

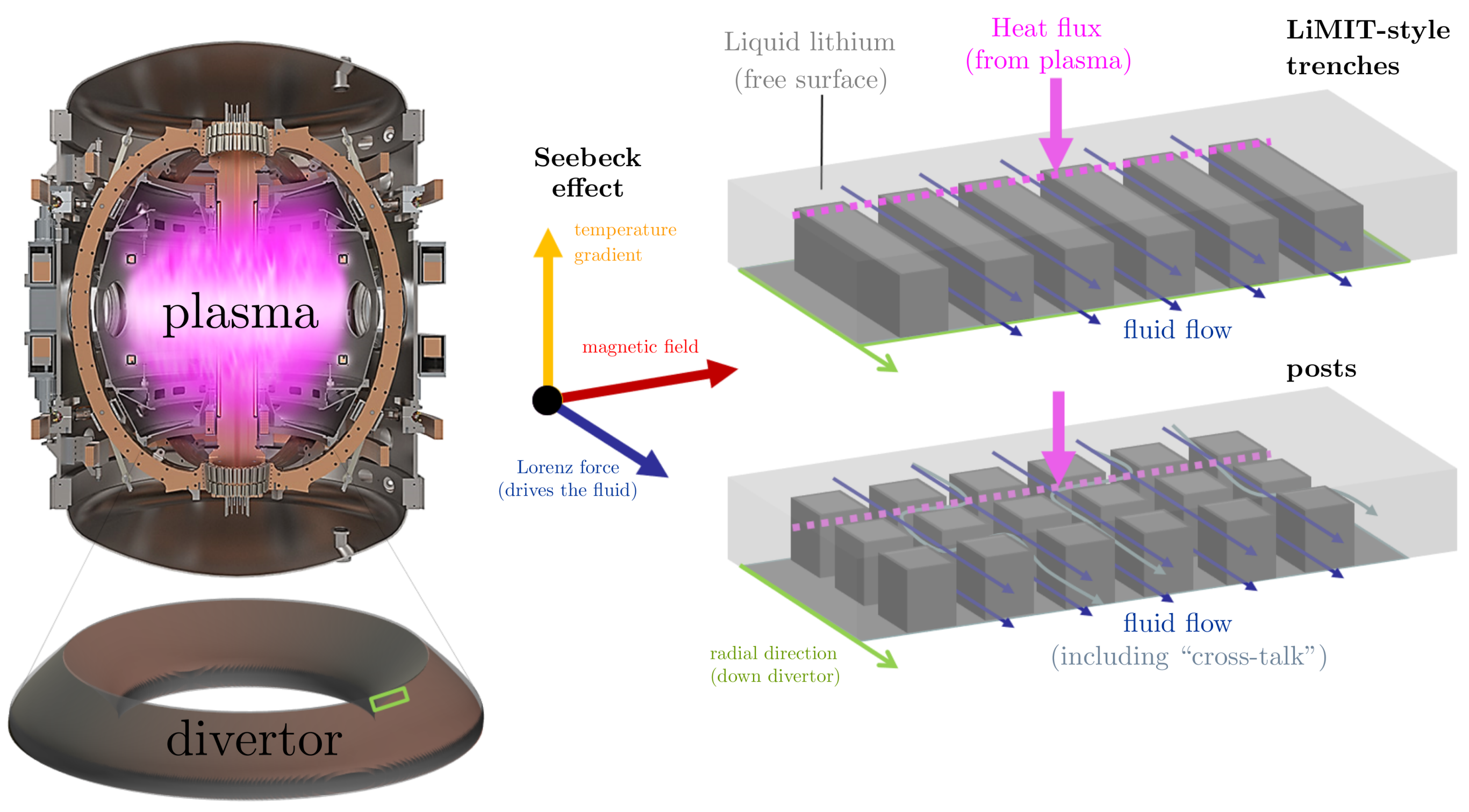


Figure 2 – Tokamak Energy's "ST40", different suggested divertor texture designs and application of the Seebeck effect.

2D TRENCH MODELLING

In this project, we have primarily focused on modelling 2D TEMHD flow in a single trench.

Modelling assumptions

1. The flow is **steady**, so all variables are taken to be *independent of time, t*.
2. The trench is much longer than it is wide, so the flow is treated as **unidirectional**, so all variables are taken to be *independent of distance x along the trench.*

   To this end, the **fluid velocity u** is given by $\mathbf{u} = w(x, y)\mathbf{k}$, and the **induced magnetic field** $\mathbf{B}_\mathrm{k}$ is given in terms of an *axial induced magnetic field* $b(x, y)$ and an *induced magnetic field potential* $A(x, y)$, by $\mathbf{B}_\mathrm{i} = b(x, y)\mathbf{k} + \boldsymbol{\nabla} \times (A(x, y)\mathbf{k})$.
3. The applied magnetic field $\mathbf{B}_\mathrm{a}$ has a strength of $\mathcal{B}_\mathrm{a}$ applied at an angle ψ to the horizontal, so that $\mathbf{B}_\mathrm{a} = \mathcal{B}_\mathrm{a}(\mathbf{i}\cos\psi + \mathbf{j}\sin\psi)$ where $\mathcal{B}_\mathrm{a}$ is a constant.
4. The trench is **one of many**, so all variables are taken to be *periodic in the x-direction.*

Nondimensionalisation

$$[w] = \frac{\mathcal{Q}\Delta S}{k_\mathrm{f}\mathcal{B}_\mathrm{a}}, \quad [p] = \frac{\gamma}{\mathcal{W}}, \quad [b_x] = \epsilon\mathrm{Rm}\mathcal{B}_\mathrm{a}, \quad [A] = \chi_\mathrm{f}\mathcal{W}\mathcal{B}_\mathrm{a}, \quad [T] = \frac{\mathcal{Q}\mathcal{W}}{k_\mathrm{f}}.$$

The fluid velocity is scaled such that **the flow is driven by the impinging heat flux** $\mathcal{Q} \approx 10\mathrm{MW/m}^2$ via the Seebeck effect.

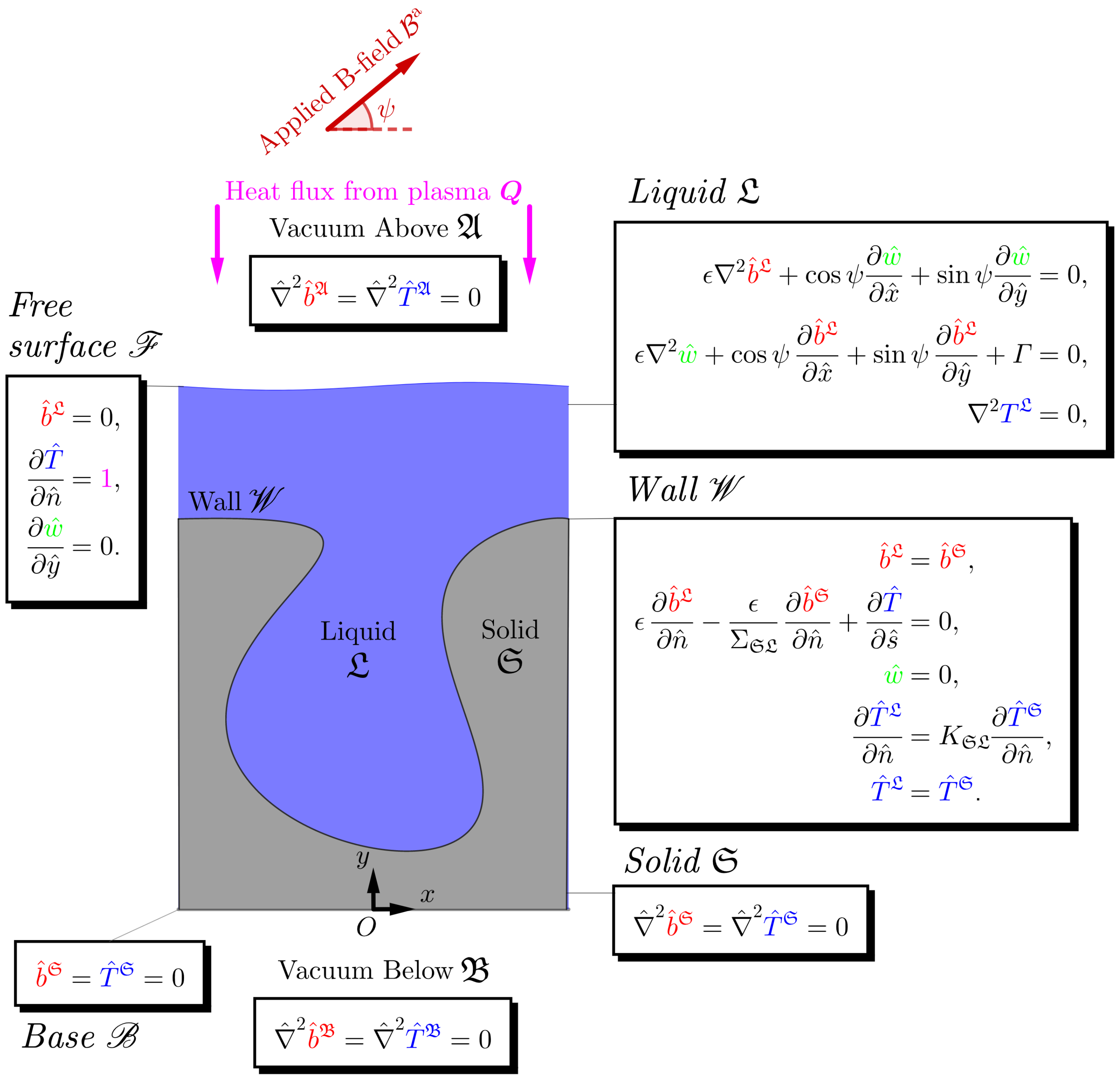
REFERENCES

[1] J A Shercliff. "Thermoelectric magnetohydrodynamics". In: *J. Fluid Mech.* 91.02 (1979), p. 231.

[2] Wenyu Xu, Davide Curreli, and David N. Ruzic. "Computational studies of thermoelectric MHD driven liquid lithium flow in metal trenches". In: *Fusion Eng. Des.* 89.12 (2014), pp. 2868–2874.

PROBLEM STATEMENT

A comprehensive representation of a dimensionless TEMHD model of a single LiMIT-style trench (at leading-order) along with boundary conditions and governing equations, is shown in Figure 4.



Applied B-field $\mathcal{B}^a$

ψ

Heat flux from plasma $\boldsymbol{Q}$

Vacuum Above $\mathfrak{A}$

$$\hat{\nabla}^2\hat{b}^{\mathfrak{A}} = \hat{\nabla}^2\hat{T}^{\mathfrak{A}} = 0$$

Liquid $\mathfrak{L}$

$$\epsilon\nabla^2\hat{b}^{\mathfrak{L}} + \cos\psi\frac{\partial\hat{w}}{\partial\hat{x}} + \sin\psi\frac{\partial\hat{w}}{\partial\hat{y}} = 0,$$
$$\epsilon\nabla^2\hat{w} + \cos\psi\frac{\partial\hat{b}^{\mathfrak{L}}}{\partial\hat{x}} + \sin\psi\frac{\partial\hat{b}^{\mathfrak{L}}}{\partial\hat{y}} + \mathit{\Gamma} = 0,$$
$$\nabla^2 T^{\mathfrak{L}} = 0,$$

Free surface $\mathscr{F}$

$$\hat{b}^{\mathfrak{L}} = 0, \quad \frac{\partial\hat{T}}{\partial\hat{n}} = 1, \quad \frac{\partial\hat{w}}{\partial\hat{y}} = 0.$$

Wall $\mathscr{W}$

Liquid $\mathfrak{L}$

Solid $\mathfrak{S}$

y x O

Wall $\mathscr{W}$

$$\hat{b}^{\mathfrak{L}} = \hat{b}^{\mathfrak{S}},$$
$$\epsilon\frac{\partial\hat{b}^{\mathfrak{L}}}{\partial\hat{n}} - \frac{\epsilon}{\Sigma_{\mathfrak{S}\mathfrak{L}}}\frac{\partial\hat{b}^{\mathfrak{S}}}{\partial\hat{n}} + \frac{\partial\hat{T}}{\partial\hat{s}} = 0,$$
$$\hat{w} = 0,$$
$$\frac{\partial\hat{T}^{\mathfrak{L}}}{\partial\hat{n}} = K_{\mathfrak{S}\mathfrak{L}}\frac{\partial\hat{T}^{\mathfrak{S}}}{\partial\hat{n}},$$
$$\hat{T}^{\mathfrak{L}} = \hat{T}^{\mathfrak{S}}.$$

Solid $\mathfrak{S}$

$$\hat{\nabla}^2\hat{b}^{\mathfrak{S}} = \hat{\nabla}^2\hat{T}^{\mathfrak{S}} = 0$$

$$\hat{b}^{\mathfrak{S}} = \hat{T}^{\mathfrak{S}} = 0$$

Base $\mathscr{B}$

Vacuum Below $\mathfrak{B}$

$$\hat{\nabla}^2\hat{b}^{\mathfrak{B}} = \hat{\nabla}^2\hat{T}^{\mathfrak{B}} = 0$$

Figure 3 – A visual summary of the leading-order problem with an arbitrary wall shape.

We can exploit the fact that the Hartmann number Ha (relating electromagnetic forces to viscous forces) is large, so $\epsilon = 1/\mathrm{Ha} \ll 1$. In simple cases, this problem can be solved using the **method of matched asymptotic expansions**. To leading-order, we make several deductions. Firstly, $A(x, y)$ and $T(x, y)$ both decouple from the problem, and the fluid pressure p and free-surface curvature $\varkappa$ are constant. Given periodic boundary conditions, the free surface is given by $y = h(x) =$ constant and is therefore flat.

RECTANGULAR TRENCHES

The mathematical model above was implemented in COMSOL and solved using a finite element method for a rectangular trench wall. Plots of the velocity, induced magnetic field and temperature are shown below, where physical constants have been taken from material properties and typical trench dimensions likely to be used in ST40.

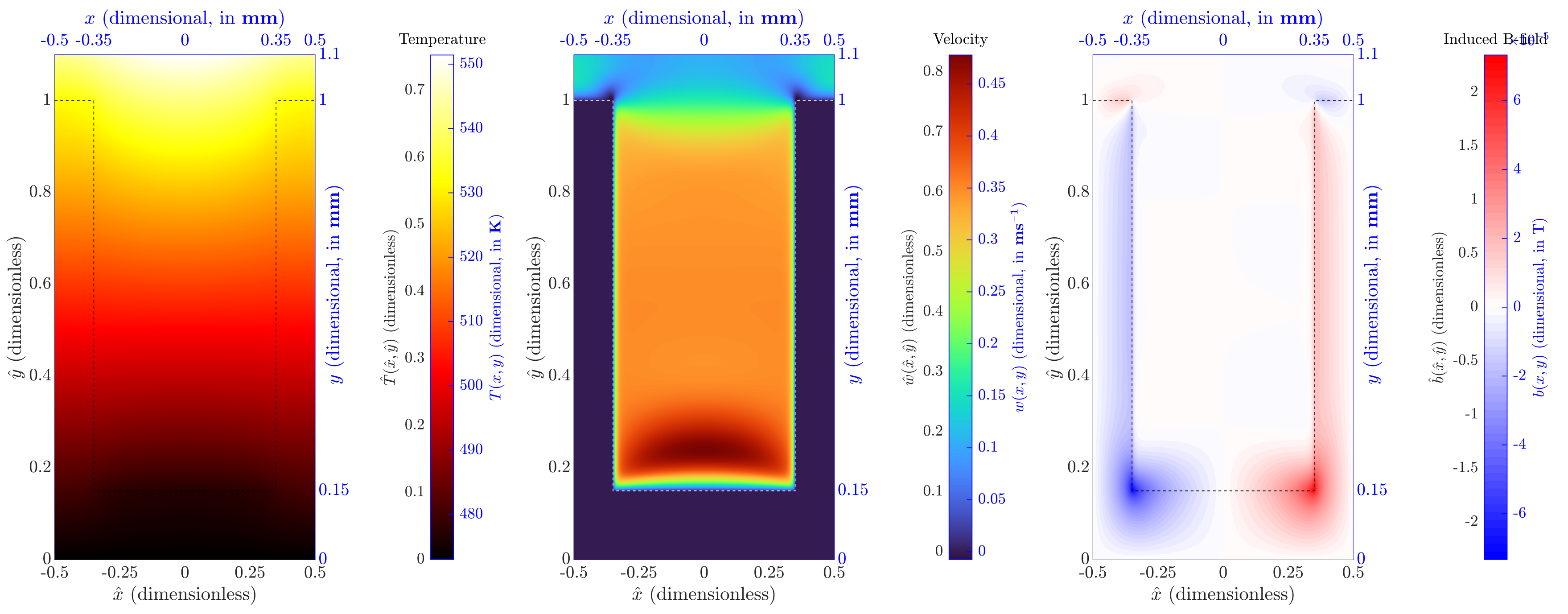


Figure 4 – Representative plots of numerical solutions to the 2D trench problem posed above, with parameters relevant to ST40 and an applied magnetic field *pointing to the right* ($\longrightarrow$).

Key observations

1. There are **Hartmann layers** of thickness $\mathcal{O}(\epsilon)$ perpendicular to the applied field, and a weaker **"side" layer** of thickness $\mathcal{O}(\epsilon^{1/2})$ parallel to the applied field.
2. There is **no** boundary layer at the free surface.
3. The flow is slower nearer to the free surface than between the walls.

Interesting questions

1. Why does the lithium flow more slowly near the free surface? How does the difference in speed depend on model parameters?
2. Can we find an "optimal" trench wall shape that offers the best performance?
3. What does the model look like in three dimensions? Can we make progress when the geometry and variables are "slowly varying" in the z-direction?

ACKNOWLEDGEMENTS

Tokamak Energy
EPSRC
Engineering and Physical Sciences Research Council