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Superconformal anomalies from superconformal Chern-Simons polynomials

Based on [C.Imbimbo, D.Rovere, A.Warman, *JHEP* 05 (2024) 277]

Women in Theoretical Physics
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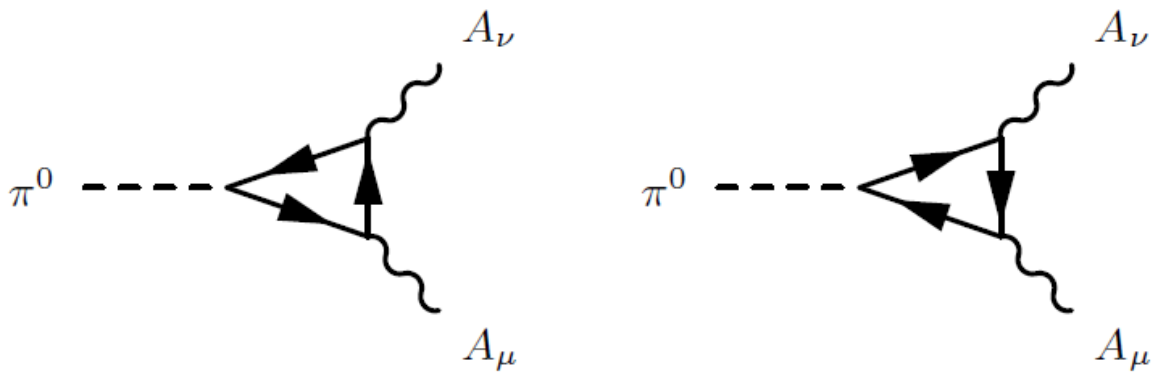
Symmetries and anomalies

- An ordinary continuous global symmetry of an action implies, at the classical level, conservation of the corresponding Noether current, e.g. in massless QED

$$\partial_\mu j^\mu = 0 \quad , \quad \partial_\mu j^\mu_A = 0$$

- However, quantum effects can break a symmetry $\Rightarrow$ there is an anomaly

$$\partial_\mu j^\mu_A = \mathcal{A} \neq 0$$



Motivation: why study superconformal anomalies?

- Anomalies give constraints on possible theoretical models, on phases of matter, and explain the violation of certain classical selection rules
- SCFTs are quantum field theories with:
- conformal symmetry (invariance under scale transformations), that has many applications both in condensed matter (e.g. phase transitions) and in high energy physics (e.g. the AdS/CFT correspondence);
- supersymmetry, which exchanges bosons and fermions. It is useful for studying models of strongly coupled QFTs and provides candidate particles for Beyond Standard Model physics

Technical background

- One can couple a theory with global symmetries to background (classical) gauge fields A
- In the BRST formalism gauge parameters are replaced by ghosts c with unphysical statistics. s is the BRST operator
- To determine Yang-Mills anomalies, Stora and Zumino defined

$$\boldsymbol{A} = A + c\,, \qquad \delta = \mathrm{d} + s\,, \qquad \boldsymbol{F} = \delta \boldsymbol{A} + \boldsymbol{A}^2$$

- Anomalies (in $d = 4$) are computed from the exact relations

$$\begin{cases} P_6 = \delta Q_5 \\ P_6 = 0 \end{cases} \quad \Rightarrow \quad \delta Q_5 = 0$$

where P_6 is an invariant polynomial cubic in $\boldsymbol{F}$ and Q_5 is the associated Chern-Simons class

$$\text{e.g.} \quad P_6 = \mathrm{tr}_R(\boldsymbol{F}^3)\,, \quad Q_5 = \mathrm{tr}_R(\boldsymbol{A}\boldsymbol{F}^2 - \tfrac{1}{2}\boldsymbol{A}^3\boldsymbol{F} + \tfrac{1}{10}\boldsymbol{A}^5)$$

Original results

- We generalized the Stora-Zumino framework to $d = 4$, $\mathcal{N} = 1$ SCFTs and showed that (certain) superconformal anomalies can be obtained from superconformal Chern-Simons polynomials
- This revealed the topological nature of these conformal and supersymmetry anomalies, providing a non-perturbative formulation which enabled us to calculate them exactly
- A possible future direction is to provide a holographic (AdS/CFT) interpretation of our results

Conformal supergravity BRST transformations

- Conformal supergravity is a field theory whose local symmetries include all known gauge symmetries
- The BRST transformations for the ghosts and gauge fields

$$\hat{s}c = -\tfrac{1}{2}[c, c] + \gamma^{\mu} A_{\mu}$$
$$\hat{s}A = -\mathrm{d}c - [A, c] + \lambda_0$$

- include contributions from the full superconformal algebra where $[\cdot, \cdot]$ is the super-Lie bracket
- and extra terms coming from local supersymmetry
  $\Rightarrow$ possible obstruction to the anomaly mechanism

The superconformal Chern-Simons polynomial

- To determine superconformal anomalies we need

$$P_6 = d_{ijk} \boldsymbol{F}^i \boldsymbol{F}^j \boldsymbol{F}^k$$

- This is obtained by solving a superconformal algebra
  invariance equation for d_{ijk} $\Rightarrow$ $P_6 = \delta Q_5$
- We show that, despite the extra terms, $P_6 = 0$
- This gives the anomaly consistency condition $\delta Q_5 = 0$,
  where Q_5 is the superconformal Chern-Simons polynomial
  from which we compute the exact type-a superconformal anomalies
- Details in [C.Imbimbo, D.Rovere, A.Warman, *JHEP* 05 (2024) 277]

Current research: Non-Invertible Symmetries

- I am currently at the University of Oxford supervised by Prof. Sakura Schäfer-Nameki, supported by the UKRI Frontier Research Grant, underwriting the ERC Advanced Grant "Generalized Symmetries in Quantum Field Theory and Quantum Gravity"
- Non-invertible symmetries are generated by topological operators with fusion rules $a \otimes b = \sum_c N_{a,b}^c \, c$
- They can describe dualities not captured by groups like the Kramers–Wannier duality of the Ising model
- We studied phases with non-invertible symmetries in (1+1)d [arXiv:2403.00905] and in (2+1)d [arXiv:2408.05266]
- And have many future projects planned!

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Thank you!

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