

Twin Algebras and Intrinsic Deconfined Quantum Criticality

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**Based on [2605.31601] and [2605.31602]
with Yuhan Gai and Sakura Schafer-Nameki**

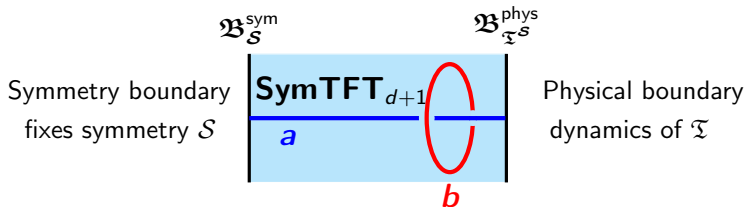


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Motivation

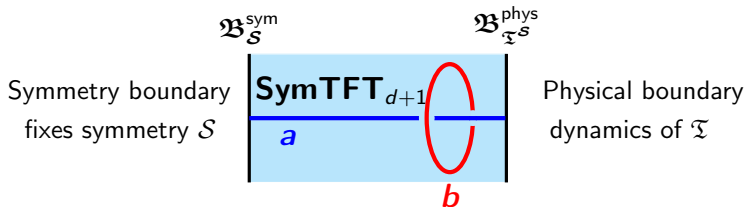
- The **Symmetry Topological Field Theory (SymTFT)** is a powerful tool for systematically studying generalized symmetries and their breaking patterns



- The **boundary conditions** capture the symmetry and the symmetry-breaking properties
- Boundary conditions are actually **algebras**
Does the algebra structure play a physical role?

Motivation

- The **Symmetry Topological Field Theory (SymTFT)** is a powerful tool for systematically studying generalized symmetries and their breaking patterns



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- Boundary conditions are actually **algebras**
Does the algebra structure play a physical role?

YES

⇒ **“twin” theories without hidden-symmetry-breaking**

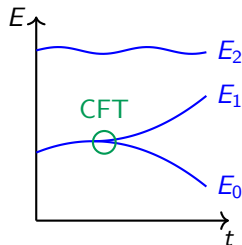
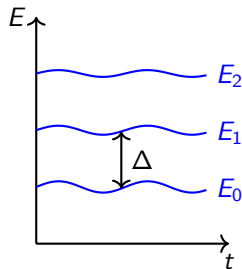
Quantum Phases with Generalized Symmetries from the SymTFT

Topological (gapped) phases of quantum systems

- TQFT $\longleftrightarrow$ ground states of (symmetric) local Hamiltonians with energy gap $\Delta > 0$
- Equivalence relation $|\psi\rangle \sim |\psi'\rangle$
if there exists a path of (symmetric) gapped $H(t)$
between H and $H' \Rightarrow U = \mathcal{T}\left[e^{-i \int_0^1 dt H(t)}\right]$
such that $U|\psi\rangle = |\psi'\rangle$

[X. Chen, Z.-C. Gu, X.-G. Wen, '10]

- If $|\psi\rangle \approx |\psi'\rangle$ they are in different phases
 $\Rightarrow$ CFT $\longleftrightarrow$ continuous phase transition point

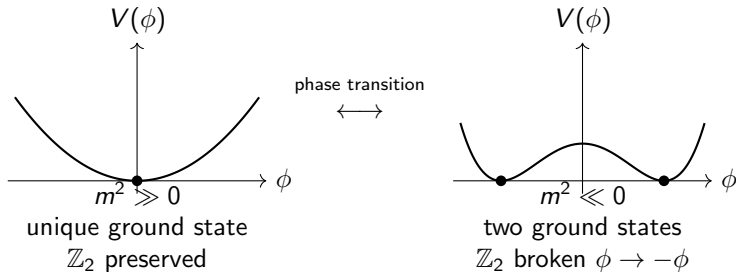


Example: $\mathbb{Z}_2$ in $(1+1)d$

- Consider the $\mathbb{Z}_2$ -invariant ϕ^4 theory

$$\mathcal{L} = (\partial\phi)^2 + m^2\phi^2 + \lambda\phi^4 \quad \lambda > 0$$

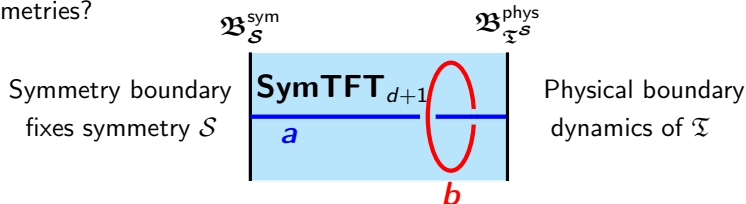
- Large positive $m^2 \Rightarrow$ the theory is gapped with $\mathbb{Z}_2$ preserved
- Large negative $m^2 \Rightarrow$ the theory is gapped with $\mathbb{Z}_2$ spontaneously broken



The transition is second order and is described by the 1+1d Ising CFT $c = \frac{1}{2}$

Symmetry Topological Field Theory (SymTFT)

- What can we use to systematically compute phases with generalized symmetries?



- Generalized charges of operators: from linking in the SymTFT bulk
- Example: $\mathbb{Z}_N$ SymTFT from $U(1)$ BF-theory:

$$S_{BF} = \frac{N}{2\pi} \int_{d+1} b \wedge da$$

Bulk topological line operators with commutation relation:

$$e^{i \int_{\Sigma} b} e^{i \int_M a} = e^{\frac{2\pi i \text{Link}(\Sigma, M)}{N}} e^{i \int_M a} e^{i \int_{\Sigma} b}$$

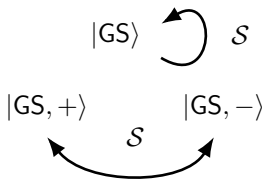
[Freed, Moore, Teleman], [Apruzzi, Bonetti, García Etxebarria, Hosseini, Schafer-Nameki], [Kaidi, Ohmori, Zheng], [Chatterjee, X-G Wen], [R. Wen, Potter], [Bhardwaj, Pajer, Schäfer-Nameki, AW], ...

Gapped (topological) phases from the SymTFT

$$\begin{array}{c} \mathfrak{B}_S^{\text{sym}} \\ \vdots \\ \mathfrak{B}_S^{\text{phys}} \end{array} = \begin{array}{c} \mathfrak{T}^S \\ \mathcal{O}_1 \\ \vdots \\ \mathcal{O}_n \end{array}$$

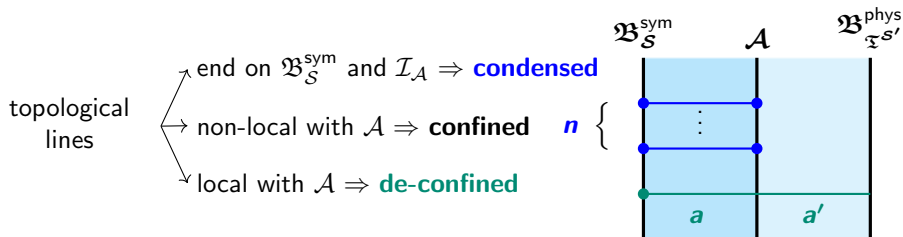
The diagram shows an equality between two representations. On the left, a light blue rectangular region is bounded by two vertical black lines. The left boundary is labeled $\mathfrak{B}_S^{\text{sym}}$ and the right boundary is labeled $\mathfrak{B}_S^{\text{phys}}$. Inside the rectangle, there are horizontal blue lines with dots between them, and a brace on the left labeled n . On the right, a vertical black line is labeled $\mathfrak{T}^S$ at the top. Below the line are two blue dots labeled $\mathcal{O}_1$ and $\mathcal{O}_n$, with vertical dots between them. An equals sign is placed between the two diagrams.

- **SPT**: symmetry protected topological phase
 $n = 1$ symmetric ground state
- **SSB**: spontaneous symmetry breaking
 $n > 1$ ground states related by $\mathcal{S}$
- Systematic classification for any $\mathcal{S} \Rightarrow$ **Categorical Landau paradigm**
- This talk will focus on 1+1d: TQFTs are characterized by
 $n = \text{number of ground states} = \sum_a n_{\text{sym}}^a n_{\text{phys}}^a$ and symmetry action on them



Gapless (conformal) phases from the SymTFT

- **Gapless** phases from **non-maximal** SymTFT condensable algebra $\mathcal{A}$
 $\text{SymTFT}(\mathcal{S})/\mathcal{A} = \text{SymTFT}(\mathcal{S}')$ for a reduced symmetry $\mathcal{S}'$



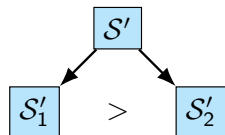
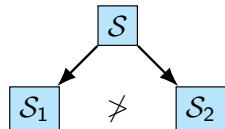
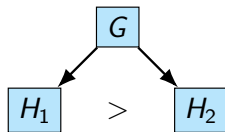
- New phase transitions from old:

$$\begin{array}{ccccc}
 \mathfrak{I}_B^{S'} & \xleftarrow{-\mathcal{O}'} & \mathfrak{I}_{\text{CFT}}^{S'} & \xrightarrow{+\mathcal{O}'} & \mathfrak{I}_A^{S'} \\
 & & \text{map across the interface} & & \\
 & & \Downarrow & & \\
 \mathfrak{I}_B^S & \xleftarrow{-\mathcal{O}} & \mathfrak{I}_{\text{CFT}}^S & \xrightarrow{+\mathcal{O}} & \mathfrak{I}_A^S
 \end{array}$$

Twin Algebras and Twin Phases

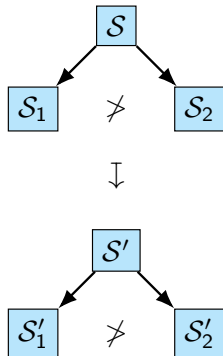
Are all phase transitions dual to SSB transitions?

- Landau theory: continuous phase transitions are spontaneous symmetry breaking (SSB) transitions
- Many 1+1d transitions appear beyond Landau but become Landau for a dual symmetry obtained by (partial) gauging
- Examples: $\mathcal{S} = \mathbb{Z}_2^a \times \mathbb{Z}_2^b$
 - 1) $\text{Triv} \not> \text{SPT} \longmapsto \text{Triv} > \mathbb{Z}_2^2 \text{ SSB}$
[Kennedy, Tasaki] “Hidden Symmetry Breaking”
 - 2) With mixed anomaly $\mathbb{Z}_2^a \not> \mathbb{Z}_2^b \longmapsto \text{Triv} > \mathbb{Z}_4 \text{ SSB}$
[Zhang, Levin]



Phase Transitions Without Hidden Symmetry Breaking

- Are all phase transitions dual to symmetry-breaking transitions?
- No [AW, Gai, Schäfer-Nameki]
- Why? Because we can have **twin phases**: a pair of inequivalent phases with the same number of ground states for all (partial) gaugings $\Rightarrow$ no relative SSB
- Mathematically, they come from **twin algebras**: condensable algebras with same object but different algebra structures
- Direct and stable **phase transitions between twin phases are intrinsically beyond Landau**: not dual to any SSB transition
- Deconfined Quantum Critical Point (DQCP) is a direct stable transition between incompatible subgroup SSB. **Twins $\Rightarrow$ Intrinsic DQCP Transitions**



- Condensable Algebras in $D^\omega(G)$ are classified by

① $H \subseteq G$ a subgroup (preserved symmetry)

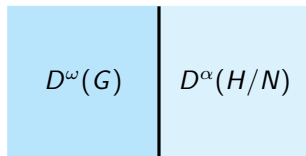
② $N \triangleleft H$ a normal subgroup (symmetry acting trivially)

③ $\gamma : N \times N \rightarrow U(1)$ such that $d\gamma = \omega|_N$

$\mathcal{O}_{n_1} \mathcal{O}_{n_2} = \gamma(n_1, n_2) \mathcal{O}_{n_1 n_2}$ (N -projective representation)

④ $\epsilon : H \times N \rightarrow U(1)$ (H -action on N) $h \mathcal{O}_n h^{-1} = \epsilon(h, n) \mathcal{O}_{hnh^{-1}}$

$$\mathcal{I}_{\mathcal{A}(H, N, \gamma, \epsilon)}$$



[Davydov, Simmons], [Ostrik], [Natale], [Gai, Schafer-Nameki, AW]

Twin Algebras and Twin Phases

- The simple objects in condensable algebras are topological lines (anyons) that end at the interface/boundary with $n_{\mathcal{A}}^a \in \mathbb{N}$, $\mathcal{A} \cong_{\text{obj}} \sum_a n_{\mathcal{A}}^a a$
- Condensable algebras are defined with an algebra structure on the anyons
- **Twin algebras:** two algebras $\mathcal{A}_1$ and $\mathcal{A}_2$ with the same anyon decomposition but inequivalent algebra structures [Gai, Schäfer-Nameki, AW]
- **Twin phases:** the theories obtained from the SymTFT with the twin algebras $\mathcal{A}_k$ ($k = 1, 2$) on $\mathfrak{B}^{\text{phys}}$ [AW, Gai, Schäfer-Nameki]

$$\begin{array}{c}
 \mathfrak{B}_S^{\text{sym}} \quad \mathfrak{B}_{\mathfrak{T}^S}^{\text{phys}} = \mathcal{A}_k \\
 \begin{array}{|c|} \hline \text{[Diagram: A light blue rectangular region with two vertical black lines on the left and right. Inside, there are two horizontal blue lines with dots at their ends, and a vertical ellipsis in the center. A brace on the left is labeled 'n'.] \\ \hline \end{array}
 \end{array}
 =
 \begin{array}{c}
 \mathfrak{T}^S \\
 \begin{array}{|c|} \hline \text{[Diagram: A vertical black line with two blue dots. The top dot is labeled O_1^{(k)} and the bottom dot is labeled O_n^{(k)} in blue.] \\ \hline \end{array}
 \end{array}$$

Physical properties of twin phases

- Twin phases are very similar:
 - ① they share the same anyon decomposition $\Rightarrow$ same number of ground states $n = \sum_a n_{\text{sym}}^a n_{\text{phys}}^a$ and order parameters are part of the same generalized charge
 - ② gauging i.e. change $\mathfrak{B}^{\text{sym}}$ doesn't modify the twin algebras on $\mathfrak{B}^{\text{phys}}$
 $\Rightarrow$ no relative SSB for any symmetry $\mathcal{S}$
- But they are distinct phases:
 - ① G twin SPT phases can be distinguished by generalized string order parameters [Pollmann, Turner], [Chavda, Delcamp, Turzillo, You]
 - ② Twin SSB phases can be distinguished by different symmetry action or by different OPE [Kobayashi, Watanabe], [Gai, Schäfer-Nameki, AW]
 - ③ Twin gapless phases give rise to different maps of anyons across the SymTFT interface [Gai, Schäfer-Nameki, AW]
- Direct and stable **phase transitions between twin phases are intrinsically beyond Landau**: not dual to any SSB transition [AW, Gai, Schäfer-Nameki], [Gai, Schäfer-Nameki, AW]

Example

- The quantum double of $GL(2, 3) \cong Q_8 \rtimes S_3$ admits Lagrangian twin algebras: $\mathcal{A}_{S_3^{(1)}}$ and $\mathcal{A}_{S_3^{(2)}}$ with $S_3^{(1)} \not\sim S_3^{(2)}$

- They have the same anyon decomposition (objects):

$$\mathcal{A}_{S_3^{(k)}} \cong_{\text{obj}} 1 \oplus \rho_3 \oplus \rho_4 \oplus [a]_{++} \oplus [a]_{+-} \oplus [h]_{++} \oplus [h]_{+-}$$

where ρ_n are n -dimensional irreps of G and $S_3^{(k)} = \mathbb{Z}_3^{(a)} \rtimes \mathbb{Z}_2^{(c^{k-1}h)}$

- For G symmetry, the twin phases have eight ground states labeled by the cosets $S_3^{(k)} q$ with $q \in Q_8$ but inequivalent preserved symmetry $S_3^{(k)}$
- $\rho_4|_{S_3^{(k)}}$ decomposes into S_3 irreps as:

$$\rho_4|_{S_3^{(1)}} = 1 \oplus \rho_{1-} \oplus \rho_2 \quad \text{and} \quad \rho_4|_{S_3^{(2)}} = \rho_{1-} \oplus 1 \oplus \rho_2$$

the component that has a vev is the trivial irrep 1, which appears in entry $(k, k) \Rightarrow$ twin order parameters are different components of the same irrep ρ_4

The non-maximal algebra for the phase transition

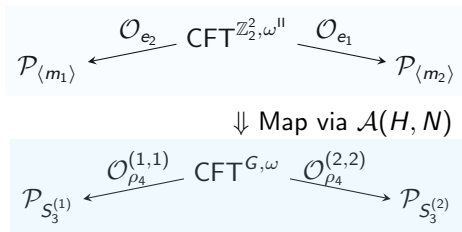
- The transition is obtained from a non-maximal condensable algebra $\mathcal{A}(H, N)$
 $H = \langle S_3^{(1)} \cup S_3^{(2)} \rangle \cong S_3 \times \mathbb{Z}_2$, $N = \langle S_3^{(1)} \cap S_3^{(2)} \rangle \cong \mathbb{Z}_3$, $\mathcal{A}_{(H, N)} \cong_{\text{obj}} 1 \oplus \rho_3 \oplus [a]_{++}$
- Without anomaly the gapped phase preserving H would allow the two consecutive Landau transitions $S_3^{(1)} < H > S_3^{(2)}$
- With mixed anomaly the H -preserving gapped phase is absent
 $\Rightarrow$ direct and stable transition between twin gapped phases
- $\mathcal{A}(H, N) \Rightarrow$ interface to reduced TO $D^{\omega_{\text{II}}}(\mathbb{Z}_2^2)$ with map of anyons

$$\mathcal{A}(H, N)$$

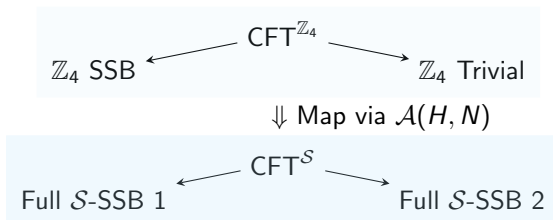
$D^{\omega}(G)$	$D^{\omega_{\text{II}}}(\mathbb{Z}_2^2)$
ρ_4	e_1, e_2
$[h]_{++}$	m_1, m_2

Twin phase transitions

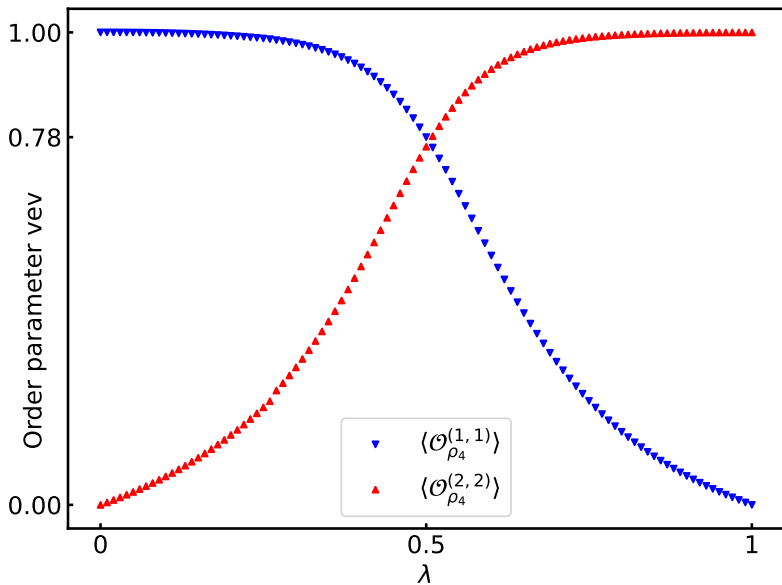
- Input $\text{CFT}^{\mathbb{Z}_2^2, \omega_{\parallel}}$ (the $c = 1$ compact boson with $\omega_{\parallel}$ between shift and winding) on $\mathfrak{B}^{\text{phys}}$ of the Reduced TO $D^{\omega_{\parallel}}(\mathbb{Z}_2^2)$ and map via $\mathcal{A}(H, N)$



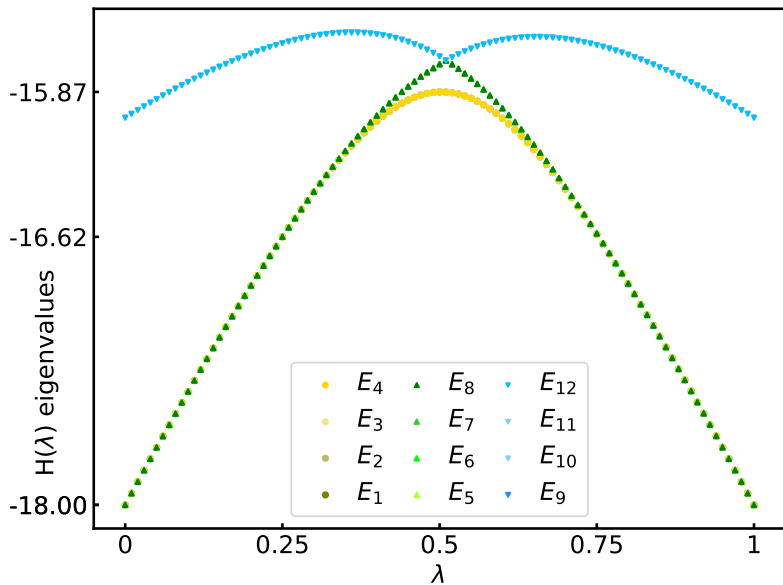
- It remains a DQCP transition even after changing $\mathfrak{B}^{\text{sym}}$ by gauging



Order parameter VEVs

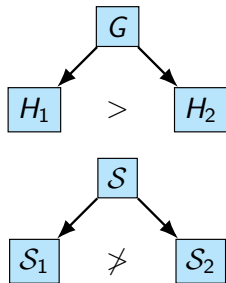


Energy levels





- Landau theory: continuous phase transitions are spontaneous symmetry breaking (SSB) transitions
- Twin phases** for a symmetry $\mathcal{S}$: inequivalent phases whose order parameters are part of the same generalized charges under $\mathcal{S}$



- Twin phases $\Rightarrow$ no relative SSB even after (partially) gauging $\mathcal{S}$

Stable and direct transitions between twin phases are
not (dual to) symmetry-breaking transitions

- Extend beyond the group-theoretical 1+1d symmetries to:
 - 1) twin algebras in higher dimensions (difficulty in precise definitions, algebra equivalences and decomposition into simple objects)
 - 2) twin phases for continuous, higher-form and intrinsically non-invertible $\mathcal{S}$
- Explore the features of the critical theories of twin-phase transitions both from continuum CFT and numerical signatures from lattice models
- Understand the role of anomalies in stabilizing the twin-phase transitions
- When do finite global symmetries enhance to continuous symmetries?
- What is the complete general framework for phase transitions beyond symmetry breaking?

Thank you