

Twin Phases and Phase Transitions Without Hidden Symmetry Breaking

Based on [2605.31601] and [2605.31602]

Alison Warman, Yuhan Gai and Sakura Schäfer-Nameki (University of Oxford)



Mathematical
Institute

Are All Phase Transitions Symmetry-Breaking Transitions?

- Landau theory: continuous phase transitions are spontaneous symmetry breaking (SSB) transitions

- Many 1+1d transitions become Landau for a dual symmetry obtained by partial gauging

Examples: $\mathcal{S} = \mathbb{Z}_2^a \times \mathbb{Z}_2^b$

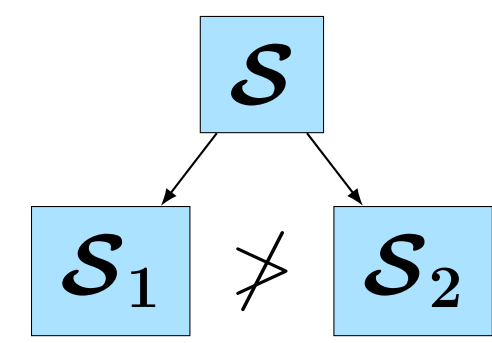
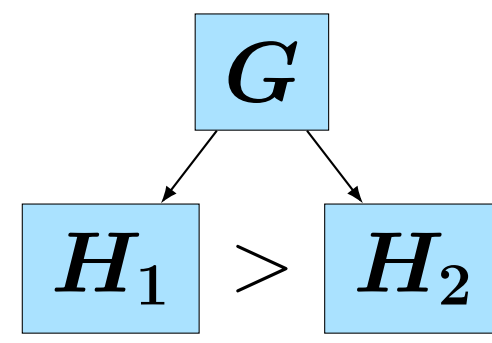
1) Trivial $\not\sim$ SPT $\mapsto$ Trivial $> \mathbb{Z}_2^2$ SSB

[Kennedy, Tasaki] "Hidden Symmetry Breaking"

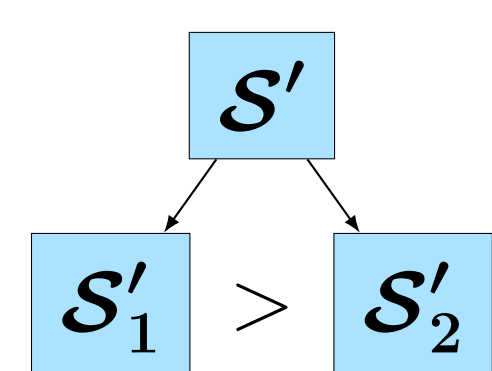
2) With mixed anomaly $\mathbb{Z}_2^a \not\sim \mathbb{Z}_2^b \mapsto$ Trivial $> \mathbb{Z}_4$ SSB

[Zhang, Levin]

- Are all phase transitions (dual to) symmetry-breaking transitions? No!



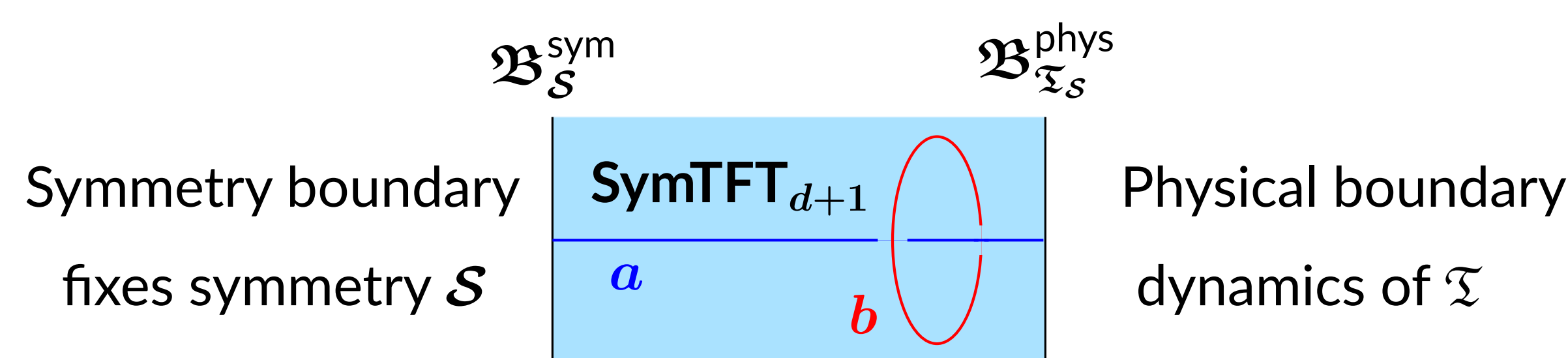
$\Downarrow ?$



Twin phases have no relative SSB in any symmetry frame
 $\Rightarrow$ **direct stable transitions between twins are always beyond-Landau**

[AW, Gai, Schäfer-Nameki], [Gai, Schäfer-Nameki, AW]

Symmetry Topological Field Theory (SymTFT)



- Generalized charges of operators: from linking in the SymTFT bulk
- Example: $\mathbb{Z}_N$ SymTFT from $U(1)$ BF-theory:

$$S_{BF} = \frac{N}{2\pi} \int_{d+1} b \wedge da$$

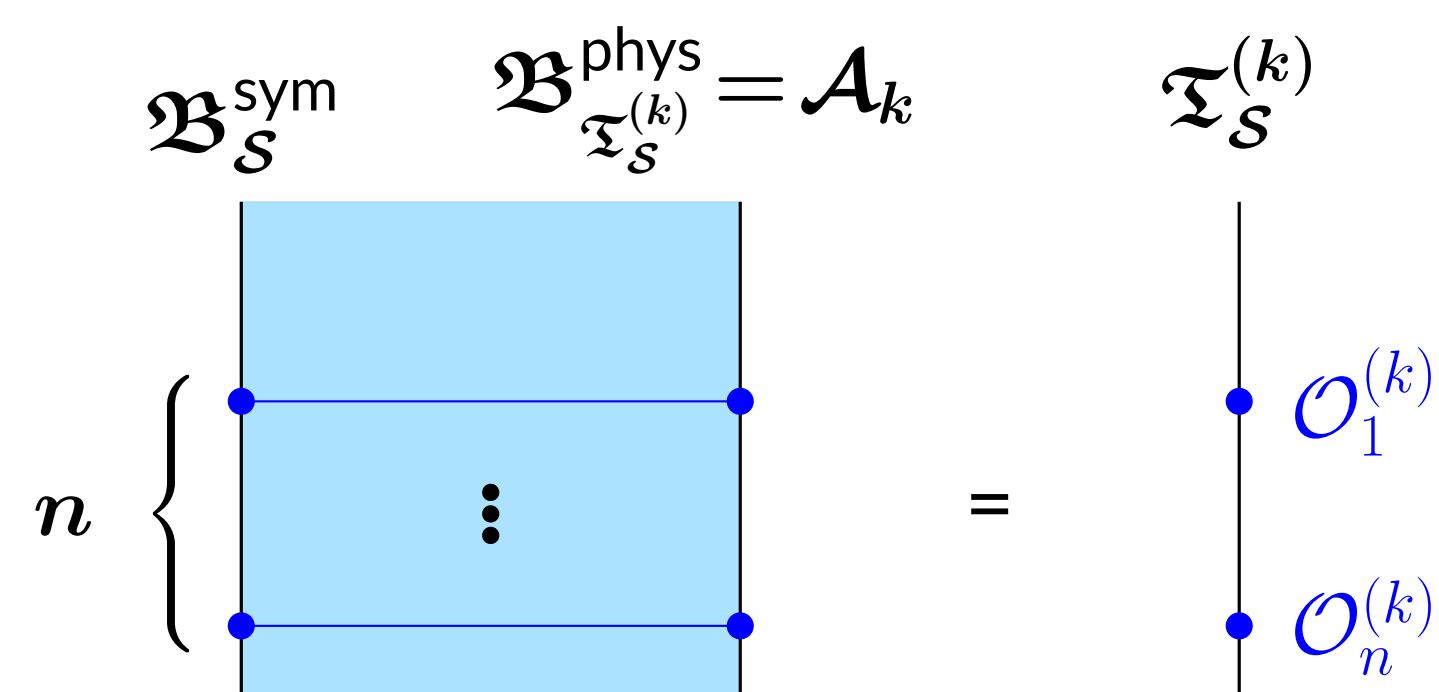
Bulk topological line operators with commutation relation:

$$e^{i \int_{\Sigma} b} e^{i \int_M a} = e^{\frac{2\pi i \text{Link}(\Sigma, M)}{N}} e^{i \int_M a} e^{i \int_{\Sigma} b}$$

[Freed, Moore, Teleman], [Kaidi, Ohmori, Zheng], [Schäfer-Nameki et al.], [X.-G. Wen et al.]

Twin Algebras and Twin Phases in (1+1)d

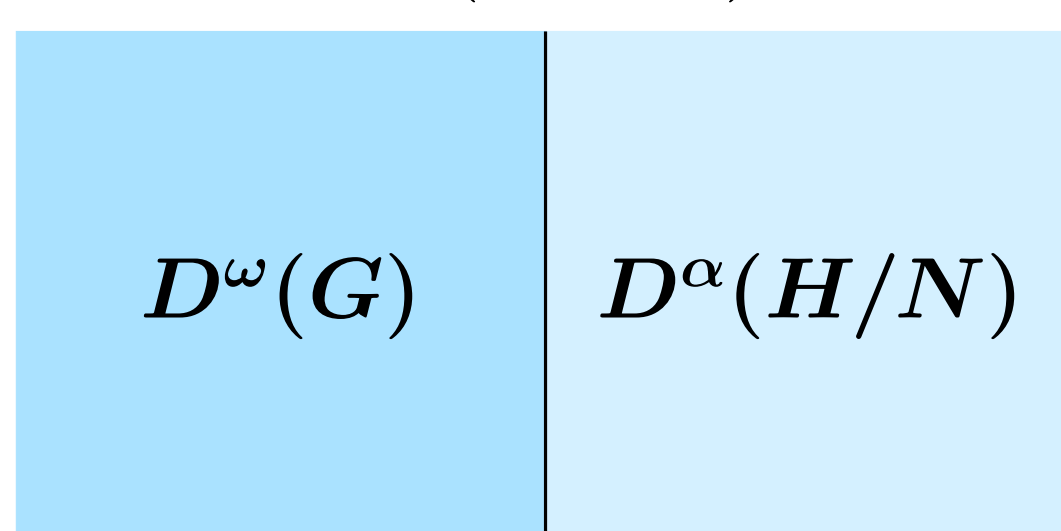
- Twin algebras:** two algebras $\mathcal{A}_1$ and $\mathcal{A}_2$ with the same anyon decomposition $\mathcal{A} \cong_{\text{obj}} \sum_a n_a a$ but inequivalent algebra structures [Gai, Schäfer-Nameki, AW]
- Twin phases:** the theories obtained from the SymTFT with the twin algebras $\mathcal{A}_k$ ($k = 1, 2$) on $\mathfrak{B}^{\text{phys}}$ [AW, Gai, Schäfer-Nameki]



Algebra Data for Boundaries and Interfaces

- Condensable Algebras in $D^{\omega}(G)$ are classified (with equivalences) by:
 - $H \subseteq G$ a subgroup (up to conjugation)
 - $N \triangleleft H$ a normal subgroup (up to conjugation)
 - $\gamma : N \times N \rightarrow U(1)$ such that $d\gamma = \omega|_N$ and $\mathcal{O}_{n_1} \mathcal{O}_{n_2} = \gamma(n_1, n_2) \mathcal{O}_{n_1 n_2}$
 - $\epsilon : H \times N \rightarrow U(1)$ (H -action on N) $h \mathcal{O}_n h^{-1} = \epsilon(h, n) \mathcal{O}_{hnh^{-1}}$

$\mathcal{I}_{A(H, N, \gamma, \epsilon)}$



[Davydov, Simmons], [Ostrik], [Natale], [Gai, Schafer-Nameki, AW]

Physical Properties of Twin Phases

Twin phases share the same anyon decomposition $\Rightarrow$ they are very similar:

- same number of ground states $n = \sum_a n_{\text{sym}}^a n_{\text{phys}}^a$
- order parameters are part of the same generalized charge
- gauging (changing $\mathfrak{B}^{\text{sym}}$) doesn't modify the twin algebras on $\mathfrak{B}^{\text{phys}}$
 $\Rightarrow$ no relative SSB for any symmetry $\mathcal{S}$

But they are distinct phases:

- Twin SPTs can be distinguished by generalized string order parameters
- Twin SSB phases can be distinguished by different symmetry action or OPE
- Twin gapless phases give rise to different maps of SymTFT anyons

Example of Twin Gapped Phases

- The quantum double of $GL(2, 3) \cong Q_8 \rtimes S_3 \subset$ Clifford group on one qubit admits two Lagrangian twin algebras: $\mathcal{A}_{S_3^{(1)}}$ and $\mathcal{A}_{S_3^{(2)}}$ with $S_3^{(1)} \not\sim S_3^{(2)}$

$$\mathcal{A}_{S_3^{(k)}} \cong_{\text{obj}} 1 \oplus \rho_3 \oplus \rho_4 \oplus [a]_{++} \oplus [a]_{+-} \oplus [h]_{++} \oplus [h]_{+-}$$

ρ_n are n -dimensional irreps of G and $S_3^{(k)} = \mathbb{Z}_3^{(a)} \rtimes \mathbb{Z}_2^{(c^{k-1}h)}$

- $\rho_4|_{S_3^{(k)}}$ decomposes into S_3 irreps as:

$$\rho_4|_{S_3^{(1)}} = 1 \oplus \rho_{1-} \oplus \rho_2 \quad \text{and} \quad \rho_4|_{S_3^{(2)}} = \rho_{1-} \oplus 1 \oplus \rho_2$$

the component that has a vev is the trivial irrep 1, which appears in entry $(k, k) \Rightarrow$ twin order parameters are different components of the same irrep

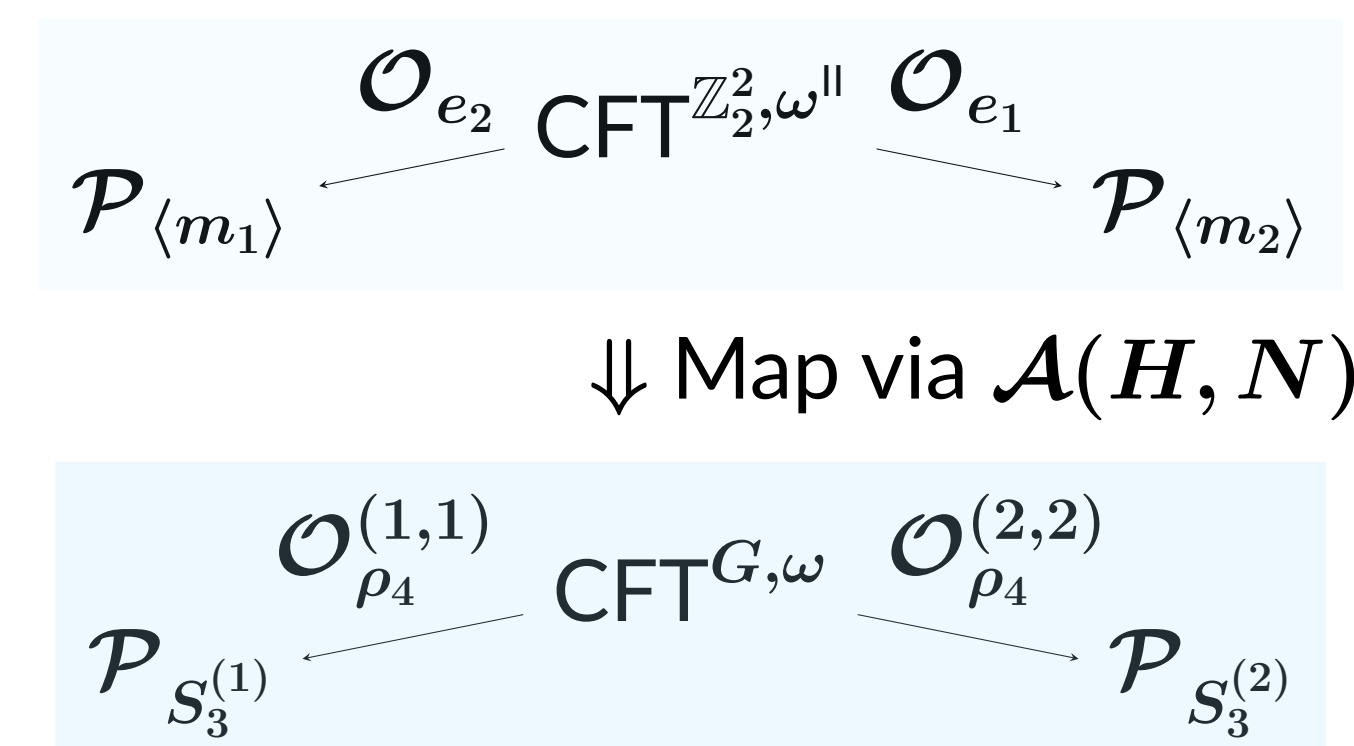
Example of Twin Phase Transition

- The transition is obtained from the algebra $\mathcal{A}_{(H, N)} \cong_{\text{obj}} 1 \oplus \rho_3 \oplus [a]_{++}$
 $H = \langle S_3^{(1)} \cup S_3^{(2)} \rangle \cong S_3 \times \mathbb{Z}_2$, $N = \langle S_3^{(1)} \cap S_3^{(2)} \rangle \cong \mathbb{Z}_3$
- Without anomaly the gapped phase preserving H would allow the two consecutive Landau transitions $S_3^{(1)} < H > S_3^{(2)}$
- With mixed anomaly the H -preserving gapped phase is absent
 $\Rightarrow$ direct and stable transition between twin gapped phases

$\mathcal{I}_{A(H, N)}$

$D^{\omega}(G)$	$D^{\omega_{\parallel}}(\mathbb{Z}_2^2)$
ρ_4	e_1, e_2
$[h]_{++}$	m_1, m_2

- Input CFT $^{\mathbb{Z}_2^2, \omega_{\parallel}}$ the $c = 1$ compact boson with $\omega_{\parallel}$ between shift and winding symmetries on $\mathfrak{B}^{\text{phys}}$ of the Reduced TO $D^{\omega_{\parallel}}(\mathbb{Z}_2^2)$ and map via $\mathcal{I}_{A(H, N)}$



- It remains a beyond-Landau transition even after changing $\mathfrak{B}^{\text{sym}}$ by gauging

Lattice Realization and Outlook

- 1+1d lattice model with local Hilbert space $\{|f\rangle : f \in G\}$ and operators $L_j^{g, \omega}, R_j^{g, \omega}$ (disordering) and $Z_{n, m}^{\rho} |g\rangle = \rho(g)_{n, m} |g\rangle$ (ordering)
- $H(\lambda) = (1 - \lambda)H_{S_3^{(1)}} + \lambda H_{S_3^{(2)}}$ interpolating between the twin phases
- The fixed-point twin gapped phases occur at $\lambda = 0$ and $\lambda = 1$ respectively
- The gapless phase transition is at $\lambda = 1/2$ and an intrinsically gapless SSB

What is the complete general framework for phase transitions beyond symmetry breaking?