

KMS STATES ON UNIFORM ROE ALGEBRAS

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Abstract: Given a flow $\{\sigma_t\}_{t \in \mathbb{R}}$ on a C^* -algebra A , and a real parameter β , one defines a (σ, β) -KMS state on A to be any state φ satisfying the relation

$$\varphi(a\sigma_{i\beta}(b)) = \varphi(ba),$$

for every a and b in A , with b analytic (meaning that the map $t \in \mathbb{R} \mapsto \sigma_t(b) \in A$ extends to an analytic function on the whole complex plane). Such states were introduced by Kubo, Martin, and Schwinger in the late 1950's to generalize the celebrated Gibbs *grand canonical ensembles* to infinite particle system.

Given a uniformly locally finite metric space X , we consider a class of naturally occurring flows on the uniform Roe algebra $C_u^*(X)$ and initiate a study of the corresponding KMS states. We show that the study of those states splits into understanding the strongly continuous ones and those which vanish on the ideal of compact operators. The strongly continuous KMS states are always unique when they exist and we give explicit formulas for them. We link the study of KMS states which vanish on the compacts to the Higson corona of X and provide lower bounds for the cardinality of the set of extreme KMS states. Lastly, we apply our theory to n -branching trees and to the free groups on n -generators.