

PERK

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Overview

PERK is a signature scheme based on the **PER**muted **K**ernel problem

- ◇ Fiat-Shamir based signature along with a Zero-Knowledge Proof of Knowledge (ZK PoK)
- ◇ Underlying PoK built using Multi-Party Computation in the Head (MPCitH)
- ◇ Relies on the hardness of the relaxed Inhomogeneous Permuted Kernel Problem (r-IPKP)

Agenda

1 - Signature from ZK PoK

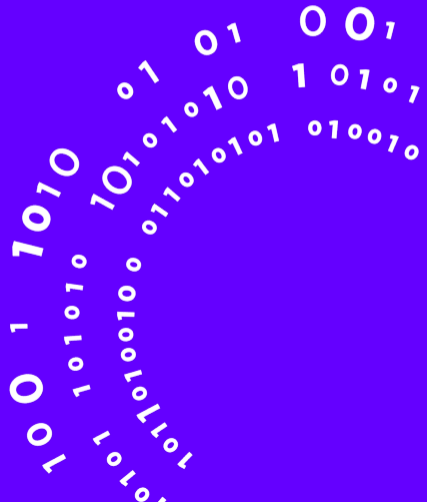
2 - ZK PoK from MPC

3 - ZK PoK for r-IPKP

4 - Sizes & Performances

5 - Advantages & Limitations

Signature from ZK PoK



Zero-Knowledge Proof of Knowledge (informal)

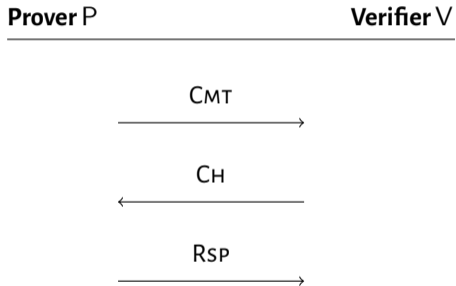
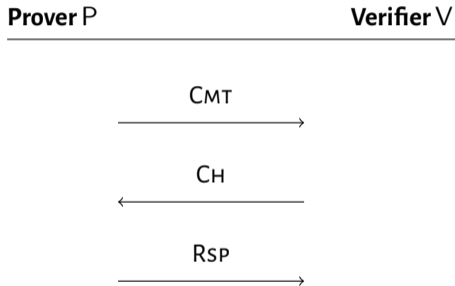


Figure 1.1: 3-rounds ZK PoK

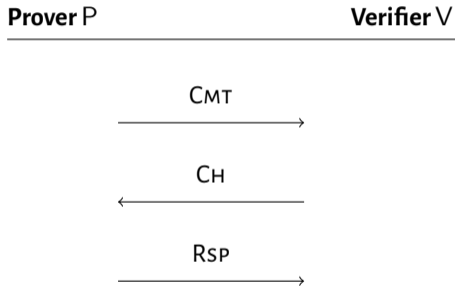
Zero-Knowledge Proof of Knowledge (informal)



Correctness - Honest prover P can always convince a verifier that he knows some secret s

Figure 1.1: 3-rounds ZK PoK

Zero-Knowledge Proof of Knowledge (informal)



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Soundness - Malicious prover \tilde{P} can't convince a verifier that he knows the secret s except with negligible probability ϵ

Figure 1.1: 3-rounds ZK PoK

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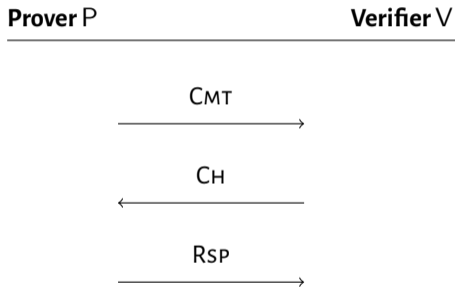


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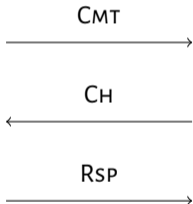
Soundness - Malicious prover \tilde{P} can't convince a verifier that he knows the secret s except with negligible probability ϵ

Honest-Verifier ZK - Honest-Verifier does not learn anything on the secret s

Fiat-Shamir Transform

Prover P

Verifier V



Objective - Transform a public coin interactive proof of knowledge into a digital signature

Fiat-Shamir Transform

Signer S

CMT

$$CH = \mathcal{H}(m \parallel pk \parallel CMT)$$

RSP

CMT, RSP
→

Objective - Transform a public coin interactive proof of knowledge into a digital signature

Main Idea - If the verifier V only returns strings sampled uniformly at random, it can be replaced by a hash function (modelled as random oracle)

Figure 1.2: Fiat-Shamir Transform [FS86]

Fiat-Shamir Transform

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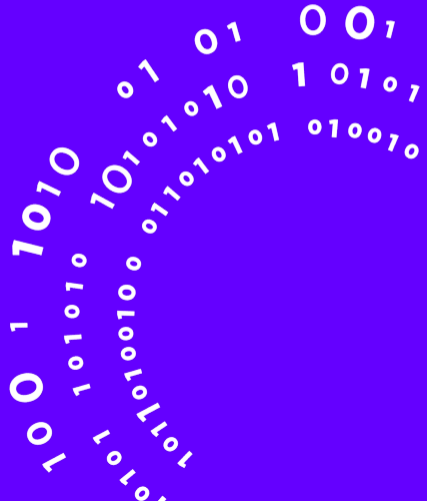
Figure 1.2: Fiat-Shamir Transform [FS86]

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Security - Proven secure in the ROM for PoK using 3-rounds [PS96] and n -rounds [DGV⁺16, AFK22] Studied in the QROM [DFMS19, DFM20]

ZK PoK from MPC



Multi-Party Computation

Let x be a secret that can be recomputed from N shares $(\llbracket x_1 \rrbracket, \dots, \llbracket x_N \rrbracket)$

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Secure MPC [GMW87] allows a set of parties (P_1, \dots, P_N) with inputs $(\llbracket x_1 \rrbracket, \dots, \llbracket x_N \rrbracket)$ to

- ◇ Compute $y = f(x)$ for some function f [correctness]
- ◇ Without leaking anything on x beyond what can be learned from $f(x)$ [privacy]

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For Fiat-Shamir based signature schemes, adversaries are modelled as *Honest-but-Curious*

MPC-in-the-Head Transform

Prover P

Verifier V

Objective - Transform a MPC protocol computing $y = f(x)$ into a ZK PoK verifying if $y = f(x)$

MPC-in-the-Head Transform

Prover P

Verifier V

Generates MPC shares
Run MPC "in-its-Head"

CMT
→

Choose a random party α

CH
←

Reveal the shares of
all parties except α
and the output of α
in the MPC protocol

Objective - Transform a MPC protocol computing $y = f(x)$ into a ZK PoK verifying if $y = f(x)$

Main Idea - Prover P generates and commits to shares of x then emulates "in its head" the MPC protocol and reveals the views of $(N - 1)$ parties

MPC-in-the-Head Transform

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Generates MPC shares
Run MPC "in-its-Head"

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Choose a random party α

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Reveal the shares of
all parties except α
and the output of α
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RSP



Check commitments

Check computation and
result of the MPC protocol

Objective - Transform a MPC protocol computing $y = f(x)$ into a ZK PoK verifying if $y = f(x)$

Main Idea - Prover P generates and commits to shares of x then emulates "in its head" the MPC protocol and reveals the views of $(N - 1)$ parties

Verifier V checks that the received views are consistent with commitments and checks the computation and result of the MPC protocol

Figure 2.1: MPC-in-the-Head [IKOS07]

MPC-in-the-Head Transform

Resulting PoK

- ◇ **Correctness** - From the correctness of the MPC protocol
- ◇ **Zero-Knowledge** - From the $(N - 1)$ -privacy of the MPC protocol
- ◇ **Soundness** - Soundness error equal to $1/N$
Can be made negligible by repeating the protocol τ times

MPC-in-the-Head Transform

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Reducing the PoK size [KKW18]

- ◇ Compress commitments by hashing them together
- ◇ Compress seeds associated to each party using a Merkle tree

ZK PoK for r-IPKP



IPKP & r-IPKP

The **P**ermuted **K**ernel **P**roblem was initially introduced in [Sha90]

IPKP & r-IPKP

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Definition (Inhomogeneous Permuted Kernel Problem)

Input - $\mathbf{H} \in \mathbb{F}_q^{m \times n}$, $(\mathbf{x}_i, \mathbf{y}_i) \in \mathbb{F}_q^n \times \mathbb{F}_q^m$ for $i \in [t]$ with $\mathbf{X} = (\mathbf{x}_1 \mid \cdots \mid \mathbf{x}_t)$ a full rank matrix
 $\pi \in \mathcal{S}_n$ such that $\mathbf{H}(\pi[\mathbf{x}_i]) = \mathbf{y}_i$ for $i \in [t]$

Goal - Find $\tilde{\pi}$ such that $\mathbf{H}(\tilde{\pi}[\mathbf{x}_i]) = \mathbf{y}_i$ for $i \in [t]$

IPKP & r-IPKP

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Goal - Find $\tilde{\pi}$ such that $\mathbf{H}(\tilde{\pi}[\mathbf{x}_i]) = \mathbf{y}_i$ for $i \in [t]$

- Mono-dimensional IPKP [$t = 1$]
- Multi-dimensional IPKP [$t > 1$]

IPKP & r-IPKP

Definition (Relaxed Inhomogeneous Permuted Kernel Problem)

Input - $\mathbf{H} \in \mathbb{F}_q^{m \times n}$, $(\mathbf{x}_i, \mathbf{y}_i) \in \mathbb{F}_q^n \times \mathbb{F}_q^m$ for $i \in [t]$ with $\mathbf{X} = (\mathbf{x}_1 \mid \cdots \mid \mathbf{x}_t)$ a full rank matrix
 $\pi \in \mathcal{S}_n$ such that $\mathbf{H}(\pi[\mathbf{x}_i]) = \mathbf{y}_i$ for $i \in [t]$

Goal - Find $\tilde{\pi}$ such that $\mathbf{H} \left(\tilde{\pi} \left[\sum_{i \in [t]} \kappa_i \cdot \mathbf{x}_i \right] \right) = \sum_{i \in [t]} \kappa_i \cdot \mathbf{y}_i$ for any $(\kappa_1, \dots, \kappa_t) \in \mathbb{F}_q^t \setminus \mathbf{0}$

Known Attacks against IPKP & r-IPKP

Attacks on IPKP

- ◇ Mono-dimensional case studied in [Geo92, BCCG93, PC94,]01, LP11, KMP19, SBC22]
- ◇ Existing attacks generalized to the multi-dimensional case in [SBC22]

Known Attacks against IPKP & r-IPKP

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Attacks on r-IPKP

- ◇ Existing attacks generalized to the relaxed case in PERK

Known Attacks against IPKP & r-IPKP

Attacks on IPKP

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- ◇ Existing attacks generalized to the multi-dimensional case in [SBC22]

Attacks on r-IPKP

- ◇ Existing attacks generalized to the relaxed case in PERK

Parameter sets considered in PERK use $t = 3$ or $t = 5$

PoK for r-IPKP

PERK is derived from a 5-rounds ZK PoK introduced in [BG23]

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Overview

- ◇ PoK uses r-IPKP for challenge space amplification [BG23]
 - First challenge space has size $|\mathcal{C}_1| = q^t - 1$

PoK for r-IPKP

PERK is derived from a 5-rounds ZK PoK introduced in [BG23]

Overview

- ◇ PoK uses **r-IPKP for challenge space amplification** [BG23]
 - First challenge space has size $|\mathcal{C}_1| = q^t - 1$
- ◇ PoK uses **shared permutation** [FJR23] to compute $\pi[\mathbf{x}]$ without leaking anything on π
 - From $(\pi_i, \mathbf{v}_i)_{i \in [N]}$, compute $\pi = \pi_1 \circ \dots \circ \pi_N$ and $\mathbf{v} = \mathbf{v}_N + \sum_{i \in [N-1]} \pi_N \circ \dots \circ \pi_{i+1}[\mathbf{v}_i]$
 - Compute $\mathbf{s}_N = \pi[\mathbf{x}] + \mathbf{v}$ by recurrence from $\mathbf{s}_0 = \mathbf{x}$ and $\mathbf{s}_i = \pi_i[\mathbf{s}_{i-1}] + \mathbf{v}_i$

PoK for r-IPKP (High Level Description)

Prover P_0

1. Sample shares (π_i, \mathbf{v}_i) with $\pi_1 = \pi_2^{-1} \circ \dots \circ \pi_N^{-1} \circ \pi$
2. Compute $\mathbf{v} = \mathbf{v}_N + \sum_{i \in [N-1]} \pi_N \circ \dots \circ \pi_{i+1}[\mathbf{v}_i]$
3. Commit to $(\pi_i, \mathbf{v}_i)_{i \in [N]}$ and $\mathbf{H}\mathbf{v}$

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Verifier V_0

4. Sample $(\kappa_i)_{i \in [t]} \xleftarrow{\$} \mathbb{F}_q^t \setminus \mathbf{0}$

PoK for r-IPKP (High Level Description)

Prover P₀

1. Sample shares (π_i, \mathbf{v}_i) with $\pi_1 = \pi_2^{-1} \circ \dots \circ \pi_N^{-1} \circ \pi$
2. Compute $\mathbf{v} = \mathbf{v}_N + \sum_{i \in [N-1]} \pi_N \circ \dots \circ \pi_{i+1}[\mathbf{v}_i]$
3. Commit to $(\pi_i, \mathbf{v}_i)_{i \in [N]}$ and $\mathbf{H}\mathbf{v}$

Verifier V₀

4. Sample $(\kappa_i)_{i \in [t]} \xleftarrow{\$} \mathbb{F}_q^t \setminus \mathbf{0}$

Prover P₁

5. Compute $\mathbf{s}_i = \pi_i[\mathbf{s}_{i-1}] + \mathbf{v}_i$ using $\mathbf{s}_0 = \sum_{i \in [t]} \kappa_i \cdot \mathbf{x}_i$
6. Commit to $(\mathbf{s}_i)_{i \in [N]}$

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Verifier V₁

7. Sample $\alpha \xleftarrow{\$} [1, N]$

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6. Commit to $(\mathbf{s}_i)_{i \in [N]}$

Verifier V₁

7. Sample $\alpha \xleftarrow{\$} [1, N]$

Prover P₂

8. Compute $\mathbf{z}_1 = \mathbf{s}_\alpha$ and $z_2 = (\pi_i, \mathbf{v}_i)_{i \in [N] \setminus \alpha}$
9. Output $\text{rsp} = (\mathbf{z}_1, z_2, \text{com}_\alpha)$

PoK for r-IPKP (High Level Description)

Prover P₀

1. Sample shares (π_i, \mathbf{v}_i) with $\pi_1 = \pi_2^{-1} \circ \dots \circ \pi_N^{-1} \circ \pi$
2. Compute $\mathbf{v} = \mathbf{v}_N + \sum_{i \in [N-1]} \pi_N \circ \dots \circ \pi_{i+1}[\mathbf{v}_i]$
3. Commit to $(\pi_i, \mathbf{v}_i)_{i \in [N]}$ and $\mathbf{H}\mathbf{v}$

Verifier V₀

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5. Compute $\mathbf{s}_i = \pi_i[\mathbf{s}_{i-1}] + \mathbf{v}_i$ using $\mathbf{s}_0 = \sum_{i \in [t]} \kappa_i \cdot \mathbf{x}_i$
6. Commit to $(\mathbf{s}_i)_{i \in [N]}$

Verifier V₁

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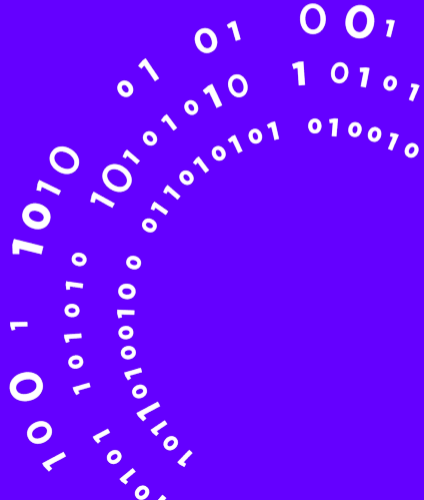
Prover P₂

8. Compute $\mathbf{z}_1 = \mathbf{s}_\alpha$ and $z_2 = (\pi_i, \mathbf{v}_i)_{i \in [N] \setminus \alpha}$
9. Output $\text{rsp} = (\mathbf{z}_1, z_2, \text{com}_\alpha)$

Verifier V₂

10. Check commitments to $(\pi_i, \mathbf{v}_i)_{i \in [N]}$ using z_2 and com_α
11. Check commitments to $(\mathbf{s}_i)_{i \in [N]}$ using \mathbf{s}_0 and \mathbf{z}_1
12. Check commitment to $\mathbf{H}\mathbf{v}$ using $\mathbf{H}\mathbf{s}_N - \sum_{i \in [t]} \kappa_i \cdot \mathbf{y}_i$

Sizes & Performances



Resulting Sizes

$$\diamond |\text{sk}| = \underbrace{\lambda}_{\text{Seed for } \pi}$$

$$|\text{pk}| = \underbrace{\lambda}_{\text{Seed for } \mathbf{H} \text{ and } (\mathbf{x}_i)_{i \in [t]}} + \underbrace{t \cdot m \lceil \log_2(q) \rceil}_{\text{Vectors } (\mathbf{y}_i)_{i \in [t]} \text{ in } \mathbb{F}_q^m}$$

Resulting Sizes

$$\diamond |\text{sk}| = \underbrace{\lambda}_{\text{Seed for } \pi} \quad |\text{pk}| = \underbrace{\lambda}_{\text{Seed for } \mathbf{H} \text{ and } (\mathbf{x}_i)_{i \in [t]}} + \underbrace{t \cdot m \lceil \log_2(q) \rceil}_{\text{Vectors } (\mathbf{y}_i)_{i \in [t]} \text{ in } \mathbb{F}_q^m}$$

$$\diamond |\sigma| \approx \tau \cdot \left(\underbrace{n \lceil \log_2(q) \rceil}_{\text{Vector } \mathbf{s}_\alpha \text{ in } \mathbb{F}_q^n} + \underbrace{n \lceil \log_2(n) \rceil}_{\text{Permutation } \pi_1} + \underbrace{\lambda \lceil \log_2(N) \rceil}_{\text{Seeds for parties } i \in [1, N] \setminus \alpha} + \underbrace{2\lambda}_{\text{Commitment for party } \alpha} \right)$$

Short Parameters

NIST level	sk	pk	σ	Keygen	Sign	Verify
PERK L1 [t = 3]	16 B	0.15 kB	6.56 kB	80 k	39 M	27 M
PERK L1 [t = 5]	16 B	0.24 kB	6.06 kB	91 k	36 M	25 M
PERK L3 [t = 3]	24 B	0.23 kB	15.0 kB	175 k	82 M	65 M
PERK L3 [t = 5]	24 B	0.37 kB	13.8 kB	194 k	77 M	60 M
PERK L5 [t = 3]	32 B	0.31 kB	26.4 kB	300 k	185 M	143 M
PERK L5 [t = 5]	32 B	0.51 kB	24.2 kB	328 k	171 M	131 M

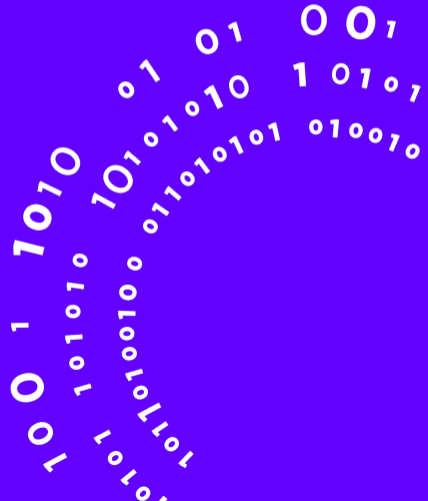
Table 1: Sizes and performances (CPU cycles)
[Constant-Time implementation using AVX2 @3GHz]

Fast Parameters

NIST level	sk	pk	σ	Keygen	Sign	Verify
PERK L1 [t = 3]	16 B	0.15 kB	8.35 kB	77 k	7.6 M	5.3 M
PERK L1 [t = 5]	16 B	0.24 kB	8.03 kB	90 k	7.2 M	5.1 M
PERK L3 [t = 3]	24 B	0.23 kB	18.8 kB	167 k	16 M	13 M
PERK L3 [t = 5]	24 B	0.37 kB	18.0 kB	185 k	15 M	12 M
PERK L5 [t = 3]	32 B	0.31 kB	33.3 kB	304 k	36 M	28 M
PERK L5 [t = 5]	32 B	0.51 kB	31.7 kB	324 k	34 M	26 M

Table 2: Sizes and performances (CPU cycles)
[Constant-Time implementation using AVX2 @3GHz]

Advantages & Limitations



Advantages & Limitations

Advantages

- ◇ **Good public key + signature size** & small public and private keys
- ◇ Underlying **hardness assumption is unstructured**
- ◇ **Resilience against IPKP and r-IPKP attacks** - Increasing the r-IPKP parameters has a limited impact on the signature size

Advantages & Limitations

Limitations

- ◇ **Relatively slow** similarly to most MPC based schemes
- ◇ **Relatively large signature size** similarly to most MPC based schemes
- ◇ Rely on a **variant of the IPKP problem**

What's Next?

Disclaimer - Work in progress, results are not guaranteed

Expected update

- ◇ Improved signature size (approx. **-5%**)
- ◇ Improved implementation

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Thank you for your attention.

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